# AN OPTIMAL APPROACH TO TARGET TRACKING PROBLEM

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**Abstract:** Target tracking is an important research area widely studied in many engineering disciplines. In this paper, we present an optimal approach to solve the target tracking problem in three-dimesional space. We initially define the problem as an optimization problem and then we solve it as a minimization problem. The performance of the approach is analyzed by simulation experiments.

**Key Words:** Target tracking, target model, optimization.

### Hedef İzleme Problemi İçin Bir En İyi Yaklaşım

**Özet:** Hedef izleme pek çok mühendislik disiplininde geniş bir biçimde çalışılan önemli bir araştırma alanıdır. Bu çalışmada, üç-boyutlu uzayda hedef izleme probleminin çözümü için bir en iyi yaklaşım sunulmaktadır. Öncelikle, problem bir eniyileme problemi olarak tanımlanmakta ve daha sonra enküçültme problemi biçiminde çözülmektedir. Önerilen yaklaşımın başarımı benzetim deneyleri ile analiz edilmiştir.

**Anahtar Kelimeler:** Hedef izleme, hedef modeli, eniyileme.

## 1. INTRODUCTION

The problem of the target tracking has received significant attention in the literature. It is an important problem for a variety of applications from military to automotive. There have been many works devoted to the solution of the target tracking problem; see, for example, (Kosko, 1992). A fuzzy rule-based, a neural-network-based and a neuro-fuzzy-based solutions are suggested in (Kóczya and Zorat, 1997), (Ma and Teng, 2000) and (Babaev and Yılmaz, 2004), respectively. A new target model in two-dimensional space and an algorithm for tracking two-dimensional rigid non-point targets are presented in (Cong and Hong 2001) and (Gu and Hong, 2001), respectively.

The whole tracking procedure is operated in two modes: seeking and tracking. In the seeking mode, the object is detected by processing the sensor's data. In the tracking mode, the detected object is followed by a tracking system. In this paper we focus our attention on the tracking mode. Initially, the target tracking problem in three-dimensional space is defined as an optimization problem and then it is solved as a minimization problem.

The rest of the paper is organized as follows. Section 2 describes the target tracking problem in three-dimensional space. Section 3 gives the optimal solution to the problem. Section 4 and 5 illustrate the performance of the approach. Finally, some concluding remarks and future works are offered in Section 6.

## 2. PROBLEM DESCRIPTION

Let us build up the model for the problem in a three-dimensional coordinate system, see Figure 1. The positions of the target and the tracking system are defined by the coordinates $(x_0, y_0, z_0)$ and $(x_t, y_t, z_t)$ respectively. The aim of the tracking system is to follow the target trajectory as accurately as possible. The parameters ($\psi$, $\varphi$) and ($\theta$, $\vartheta$) are the position and the rotation angels of the tracking system, respectively. The parameter $d$ represents the distance which is taken at each new

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step and it is defined as $d = vt$, the well-known velocity formula. The intervals for the coordinates and the position angles of the tracking system are chosen as $x_t \in (-\infty, \infty)$, $y_t \in (-\infty, \infty)$, $z_t \geq 0$, $0 \leq \varphi \leq \pi$ and $0 \leq \psi \leq 2\pi$

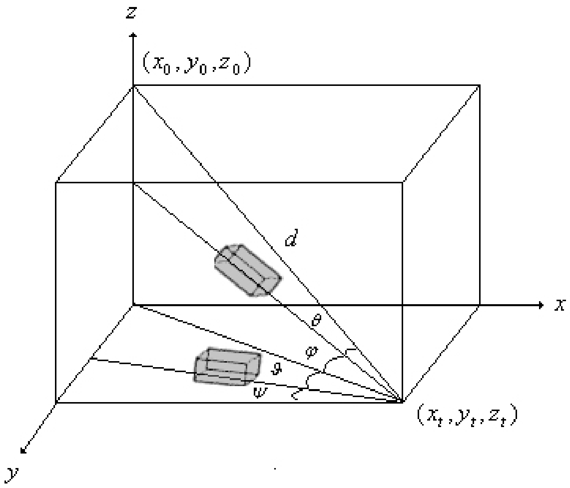


*Figure 1:*
*Model for target tracking problem in three-dimensional space*

## 3. THE SOLUTION OF THE PROBLEM

Our aim is to move the tracking system in the direction of the target trajectory. The movement of the target is controlled by the rotation angles and the velocity of the tracking system. There are the following constraints on these parameters. Because of the maneuvering capability of the tracking system, the rotation angles can be in the range of $-\pi/2 \leq \theta, \vartheta \leq \pi/2$. Due to an upper limit for the velocity $v \leq v_{\max} = d_{\max}/t$, the distance can be in the range of $0 \leq d \leq d_{\max}$. The values of the parameters $\theta$, $\vartheta$ and $d$ are computed for the existing position of the tracking system in order to move the tracking system along the trajectory of the target by solving an optimization problem.

Let us define the function to be minimized as

$$F(\theta, \vartheta, d) = (z_0 - z_t - d\sin(\varphi + \theta))^2 + ..$$
$$.. + (x_0 - x_t + d\cos(\varphi + \theta)\cos(\psi + \vartheta))^2 + (y_0 - y_t + d\cos(\varphi + \theta)\sin(\psi + \vartheta))^2 \tag{1}$$

Our goal is to find the values of the parameters $\theta$, $\vartheta$ and $d$ which minimize the function $F(\theta, \vartheta, d)$. We use the particle swarm optimization (PSO) to solve this minimization problem. The updated position of the tracking system is computed by using the following formulas:

$$\varphi_{n+1} = \varphi_n + \theta_{n+1} \tag{2}$$

$$\psi_{n+1} = \psi_n + \vartheta_{n+1} \tag{3}$$

$$z_{t(n+1)} = z_{t(n)} + d_{n+1}\sin(\varphi_{n+1}) \tag{4}$$

$$x_{t(n+1)} = x_{t(n)} - d_{n+1}\cos(\varphi_{n+1})\cos(\psi_{n+1}) \tag{5}$$

$$y_{t(n+1)} = y_{t(n)} - d_{n+1}\cos(\varphi_{n+1})\sin(\psi_{n+1}) \tag{6}$$

where $\theta_{n+1}$, $\vartheta_{n+1}$ and $d_{n+1}$ are the optimum values which are obtained by the PSO at each iteration step.

## 4. PARTICLE SWARM OPTIMIZATION (PSO)

PSO is a population based stochastic optimization algorithm proposed in (Kennedy and Eberhart, 1995), motivated by social behavior of bird flocking and fish schooling. PSO is initialized with a population (swarm) of random solutions (particles) and searches for optimum by updating generations.

The current position and the velocity of the $ith$ particle in $N$ dimensional space are represented as $\lambda_i = [\lambda_{i1} \quad ,..., \quad \lambda_{in}]$ and $V_i = [V_{i1} \quad ,..., \quad V_{iN}]$, respectively. The best particle of the swarm and the best position of the $ith$ particle are represented as $G = [G_1 \quad ,..., \quad G_N]$ and $P_i = [P_{i1} \quad ,..., \quad P_{iN}]$, respectively.

In each iteration, the swarm is updated using the following equations:

$$V_i^{k+1} = \omega V_i^k + c_1 r_1^k (P_i^k - \lambda_i^k) + c_2 r_2^k (G^k - \lambda_i^k) \tag{7}$$

$$\lambda_i^{k+1} = \lambda_i^k + V_i^{k+1} \tag{8}$$

The inertia weight $\omega$ provides a balance between global exploration and local exploitation. A larger value of inertia weights facilitate global search while a smaller inertia weight facilitate local search.

The positive constants $c_1$ and $c_2$ are cognitive and social learning (accelerating) factors which controls the maximum step size. $r_1$ and $r_2$ are random numbers generated from uniform distribution on the interval $[0,1]$.

The optimal solution of the target tracking problem is realized by the following sequential steps:

- The values $\min_{\theta,\vartheta,d} F(\theta,\vartheta,d)$ are solved by using the PSO for the existing position of the tracking system and the optimum values of the parameters $\theta, \vartheta$ and $d$ are obtained as $\theta_{n+1}, \vartheta_{n+1}$ and $d_{n+1}$.
- The updated position of the tracking system, which is given by the equations (2)-(6), is computed.
- The updated position is chosen as the existing position of the tracking system and go to the first step for the new position of the target.

## 5. SIMULATION EXPERIMENTS

The performance of the approach is analyzed via computer simulations. The simulation program has been developed in Matlab. For each PSO, 125 particles are used and the parameters are chosen as $\omega = 0.75$, $c_1 = c_2 = 2$.

*Example:* Consider the case of initial velocity of the tracking system $v = 0$ with two different choices of the start-up positions for two different trajectories:

(a) $(x_t, y_t, z_t) = (100,50,0)$, $\psi = 0$, $\varphi = pi/2$

(b) $(x_t, y_t, z_t) = (-100,-50,0)$, $\psi = 0$, $\varphi = pi/2$

The simulation results are given in Figure 2 and Figure 3.

## 6. CONCLUSIONS

We provide an optimal approach to the target tracking problem in three-dimensional space. The problem is initially defined as an optimization problem and then it is solved as a minimization problem. The effectiveness of the approach is illustrated by the simulation experiments.

The approach can also be used to generate the optimum training data set for neural network solutions for this tracking problem. In a future work, we will design an optimal neural network controller for the problem.

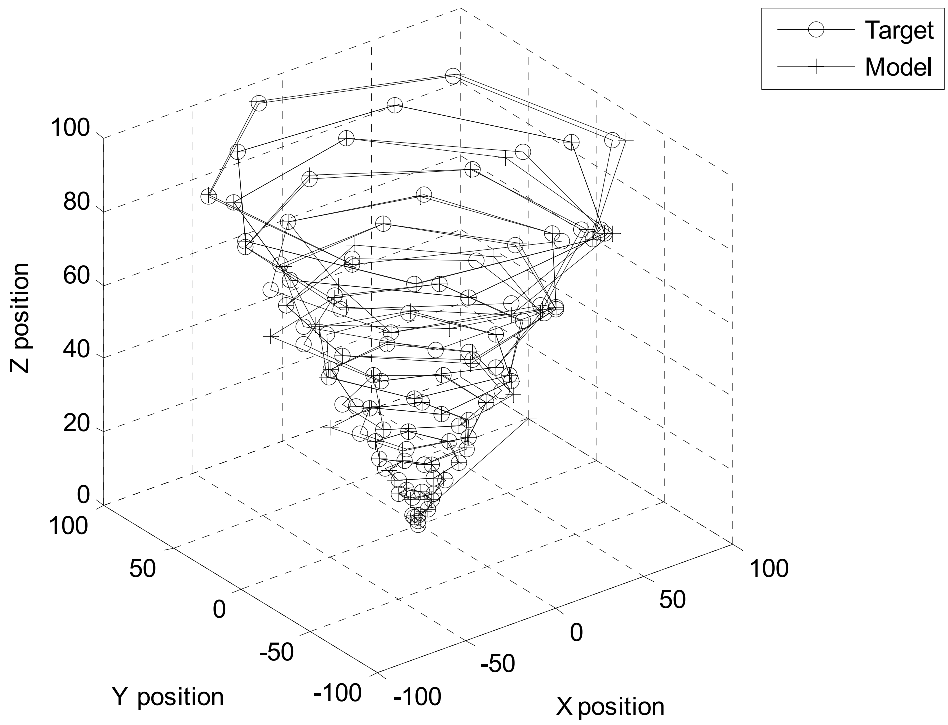


*Figure 2:*
*Optimal solution for* $(x_t, y_t, z_t) = (100,50,0)$, $\psi = 0$, $\varphi = pi/2$

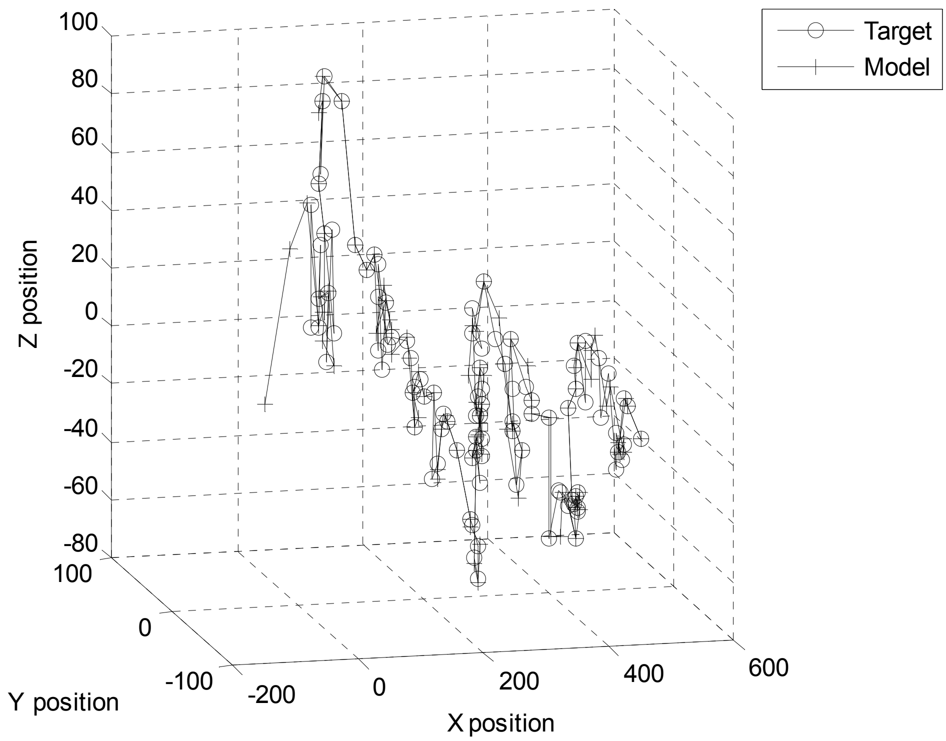


*Figure 3:*
*Optimal solution for* $(x_t, y_t, z_t) = (-100,-50,0)$, $\psi = 0$, $\varphi = pi/2$

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