Universal Journal of Mathematics and Applications, 1 (4) (2018) 273-279

UJMA

Universal Journal of Mathematics and Applications

Journal Homepage: www.dergipark.gov.tr/ujma
ISSN 2619-9653
DOI: http://dx.doi.org/10.32323/ujma.399596

Multiple Soliton Solutions of Some Nonlinear Partial Differential Equations

İbrahim Enam İnan[a*]

[a]*Faculty of Education, Fırat University, Elazığ 23119, Turkey*
[*]*Corresponding author*

Article Info

Keywords: *New coupled Konno-Oono equation, extended (3+1)-dimensional KdV-type equation, Improvedtanh function method, Multiple soliton solutions.*
2010 AMS: *00000*
Received: *28 February 2018*
Accepted: *7 May 2018*
Available online: *20 December 2018*

Abstract

In this paper, we implemented an improved tanh function method for multiple soliton solutions of new coupled Konno-Oono equation and extended (3+1)-dimensional KdV-type equation.

1. Introduction

Nonlinear partial differential equations (NPDEs) have an important place in applied mathematics and physics [1], [2]. Many analytical methods have been found in literature [3]-[11]. Besides these methods, there are many methods which reach to solution by using an auxiliary equation. Using these methods, partial differential equations are transformed into ordinary differential equations. These nonlinear partial differential equations are solved with the help of ordinary differential equations. These methods are given in [12]-[39].
We used the improved tanh function method to find the multiple soliton solutionsof new coupled Konno-Oono equation and extended (3+1)-dimensional KdV-type equationin this study. This method is presented by Chen and Zhang [15].

2. Analysis of method

Let's introduce the method briefly. Consider a general partial differential equation of two variables,

$$\varphi\left(v, v_t, v_x, v_{xx}, \ldots\right) = 0. \tag{2.1}$$

and transform equation (2.1) with

$$v(x,t) = v(\varnothing),\ \varnothing = k(x - wt)$$

where k, w are constants. With this conversion, we obtain a nonlinear ordinary differential equation for $v(\varnothing)$

$$\varphi'\left(v', v'', v''', \ldots\right) = 0. \tag{2.2}$$

We can express the solution of equation (2.2) as below,

$$v(\varnothing) = \sum_{i=0}^{n} a_i F^i(\varnothing),$$

here n is a positive integer and is found as the result of balancing the highest order linear term and the highest order nonlinear term found in the equation.

Email address: **ieinan@yahoo.com, (İ. E. İnan)**

If we write these solutions in equation (2.2), we obtain a system of algebraic equations for $F(\varnothing)$, $F^2(\varnothing),\ldots,F^i(\varnothing)$, after, if the coefficients of $F(\varnothing)$, $F^2(\varnothing),\ldots,F^i(\varnothing)$ are equal to zero, we can find the $k,w,a_0,a_1,\ldots,a_n$ constants. The basic step of the method is to make full use of the Riccati equation satisfying the tanh function and to use $F(\varnothing)$, solutions. The Riccati equation required in this method is given below

$$F'(\varnothing)=A+BF(\varnothing)+CF^2(\varnothing)$$

where, $F'(\varnothing)=\frac{dF(\varnothing)}{d\varnothing}$ and A, B and C are constants. The authors expressed the solutions [15].

Example 2.1. *Example 1. Weconsider the new coupled Konno-Oono equation,*

$$v_t+2uu_x=0$$

$$u_{xt}-2uv=0. \tag{2.3}$$

Using the wave variable $v(x,t)=v(\varnothing)$ and $u(x,t)=u(\varnothing)$, $\varnothing=k(x-wt)$, the equation (2.3) turns into an ordinary differential equation,

$$-kwv'+2kuu'=0$$

$$-k^2wu''-2uv=0. \tag{2.4}$$

When balancing v' with uu' and u'' with uv then $n_1=1$ and $n_2=2$ gives. The solution is as follows:

$$u=a_0+a_1F(\varnothing)$$

$$v=b_0+b_1F(\varnothing)+b_2F^2(\varnothing) \tag{2.5}$$

(2.5) are substituted in equations (2.4), a system of algebraic equations for k,w,a_0,a_1,b_0,b_1,b_2 are obtained. The obtained systems of algebraic equations are as follows

$$\begin{array}{l}
2Aka_0a_1-Akwb_1=0,\\
2Bka_0a_1+2Aka_1^2-Bkwb_1-2Akwb_2=0,\\
2Cka_0a_1+2Bka_1^2-Ckwb_1-2Bkwb_2=0,\\
2Cka_1^2-2Ckwb_2=0-ABk^2wa_1-2a_0b_0=0,\\
-B^2k^2wa_1-2ACk^2wa_1-2a_1b_0-2a_0b_1=0,\\
-3BCk^2wa_1-2a_1b_1-2a_0b_2=0,
\end{array}$$

$$-2C^2k^2wa_1-2a_1b_2=0.$$

Solving the above system with the help of Mathematica, the coefficients are found as two cases:
Case 1:

$$\mathrm{a}_0=0,\mathrm{B}=0,\mathrm{b}_1=0,A\neq 0,b_2=\frac{Cb_0}{A},k=\frac{ib_0}{Aa_1},w=\frac{a_1^2}{b_2},a_1\neq 0,b_2\neq 0.$$

Case 2:

$$a_0=0,\ B=0,\ b_1=0,\ A=0,\ b_2\neq 0,\ k=\frac{ib_2}{Ca_1},\quad w=\frac{a_1^2}{b_2},\ b_0=0,\ Ca_1\neq 0.$$

After these procedures, the solutions:
Solution 1:

$$u_1(x,t)=a_1\left[Coth\left(\frac{2ib_0}{a_1}x+2ia_1t\right)\pm Cosech\left(\frac{2ib_0}{a_1}x+2ia_1t\right)\right]$$

$$v_1(x,t)=b_0-b_0\left[Coth\left(\frac{2ib_0}{a_1}x+2ia_1t\right)\pm Cosech\left(\frac{2ib_0}{a_1}x+2ia_1t\right)\right]^2$$

$$u_2(x,t)=a_1\left[Tanh\left(\frac{2ib_0}{a_1}x+2ia_1t\right)\pm iSech\left(\frac{2ib_0}{a_1}x+2ia_1t\right)\right]$$

$$v_2(x,t) = b_0 - b_0\left[Tanh\left(\frac{2ib_0}{a_1}x + 2ia_1t\right) \pm iSech\left(\frac{2ib_0}{a_1}x + 2ia_1t\right)\right]^2$$

Solution 2:

$$u_3(x,t) = a_1\left[Sec\left(\frac{2ib_0}{a_1}x - 2ia_1t\right) \pm Tan\left(\frac{2ib_0}{a_1}x - 2ia_1t\right)\right]$$

$$v_3(x,t) = b_0 + b_0\left[Sec\left(\frac{2ib_0}{a_1}x - 2ia_1t\right) \pm Tan\left(\frac{2ib_0}{a_1}x - 2ia_1t\right)\right]^2$$

$$u_4(x,t) = a_1\left[Cosec\left(\frac{2ib_0}{a_1}x - 2ia_1t\right) \pm Cot\left(\frac{2ib_0}{a_1}x - 2ia_1t\right)\right]$$

$$v_4(x,t) = b_0 + b_0\left[Cosec\left(\frac{2ib_0}{a_1}x - 2ia_1t\right) \pm Cot\left(\frac{2ib_0}{a_1}x - 2ia_1t\right)\right]^2$$

$$u_5(x,t) = a_1\left[Sec\left(-\frac{2ib_0}{a_1}x + 2ia_1t\right) \pm Tan\left(-\frac{2ib_0}{a_1}x + 2ia_1t\right)\right]$$

$$v_5(x,t) = b_0 + b_0\left[Sec\left(-\frac{2ib_0}{a_1}x + 2ia_1t\right) \pm Tan\left(-\frac{2ib_0}{a_1}x + 2ia_1t\right)\right]^2$$

$$u_6(x,t) = a_1\left[Cosec\left(-\frac{2ib_0}{a_1}x + 2ia_1t\right) \pm Cot\left(-\frac{2ib_0}{a_1}x + 2ia_1t\right)\right]$$

$$v_6(x,t) = b_0 + b_0\left[Cosec\left(-\frac{2ib_0}{a_1}x + 2ia_1t\right) \pm Cot\left(-\frac{2ib_0}{a_1}x + 2ia_1t\right)\right]^2$$

Solution 3:

$$u_7(x,t) = a_1\left[Tanh\left(\frac{ib_0}{a_1}x + ia_1t\right)\right]$$

$$v_7(x,t) = b_0 - b_0\left[Tanh\left(\frac{ib_0}{a_1}x + ia_1t\right)\right]^2$$

$$u_8(x,t) = a_1\left[Coth\left(\frac{ib_0}{a_1}x + ia_1t\right)\right]$$

$$v_8(x,t) = b_0 - b_0\left[Coth\left(\frac{ib_0}{a_1}x + ia_1t\right)\right]^2$$

Solution 4:

$$u_9(x,t) = a_1\left[Tan\left(\frac{ib_0}{a_1}x - ia_1t\right)\right]$$

$$v_9(x,t) = b_0 + b_0\left[Tan\left(\frac{ib_0}{a_1}x - ia_1t\right)\right]^2$$

Solution 5:

$$u_{10}(x,t) = a_1\left[Cot\left(-\frac{ib_0}{a_1}x + ia_1t\right)\right]$$

$$v_{10}(x,t)=b_0+b_0\left[Cot\left(-\frac{ib_0}{a_1}x+ia_1t\right)\right]^2$$

Solution 6:

$$u_{11}(x,t)=-\frac{a_1}{\left(\frac{ib_2}{a_1}x-ia_1t\right)+c_0}$$

$$v_{11}(x,t)=b_2\left(-\frac{1}{\left(\frac{ib_2}{a_1}x-ia_1t\right)+c_0}\right)^2$$

Example 2.2. *Now let's get the extended (3+1)-dimensional KdV-type equation,*

$$u_t+6u_xu_y+u_{xxy}+u_{xxxxz}+60u_x^2u_z+10u_{xxx}u_z+20u_xu_{xxz}+6u_xu_z+u_{xxz}=0, \tag{2.6}$$

Using the wave variable $u(x,y,z,t)=u(\varnothing)$ and, $\varnothing=k(x+\alpha y+\beta z-wt)$, the equation (2.6) turns into an ordinary differential equation,

$$-wu'+6k\alpha\left(u'\right)^2+k^2\alpha u'''+k^4\beta u^{(5)}+60k^2\beta\left(u'\right)^3+30k^3\beta u'u'''+6k\beta\left(u'\right)^2+k^2\beta u'''=0, \tag{2.7}$$

When balancing $u^{(5)}$ with $u'u'''$ then $n=1$ gives. The solution is as follows:

$$u=a_0+a_1F(\varnothing) \tag{2.8}$$

If (2.8) is substituted in equation (2.7), a system of algebraic equations for k,w,α,β,a_0,a_1 can be obtained. The obtained systems of algebraic equations are as follows

$$\begin{aligned}&-Awa_1+AB^2k^2\alpha a_1+2A^2Ck^2\alpha a_1+AB^2k^2\beta a_1+2A^2Ck^2\beta a_1+AB^4k^4\beta a_1+22A^2B^2Ck^4\beta a_1\\&+16A^3C^2k^4\beta a_1+6A^2k\alpha a_1^2+6A^2k\beta a_1^2+30A^2B^2k^3\beta a_1^2+60A^3Ck^3\beta a_1^2+60A^3k^2\beta a_1^3=0,\end{aligned}$$

$$\begin{aligned}&-Bwa_1+B^3k^2\alpha a_1+8ABCk^2\alpha a_1+B^3k^2\beta a_1+8ABCk^2\beta a_1+B^5k^4\beta a_1+52AB^3Ck^4\beta a_1+136A^2BC^2k^4\beta a_1\\&+12ABk\alpha a_1^2+12ABk\beta a_1^2+60AB^3k^3\beta a_1^2+300A^2BCk^3\beta a_1^2+180A^2Bk^2\beta a_1^3=0,\end{aligned}$$

$$\begin{aligned}&-Cwa_1+7B^2Ck^2\alpha a_1+8AC^2k^2\alpha a_1+7B^2Ck^2\beta a_1+8AC^2k^2\beta a_1+31B^4Ck^4\beta a_1+\\&292AB^2C^2k^4\beta a_1+136A^2C^3k^4\beta a_1+6B^2k\alpha a_1^2+12ACk\alpha a_1^2+6B^2k\beta a_1^2+12ACk\beta a_1^2+\\&30B^4k^3\beta a_1^2+480AB^2Ck^3\beta a_1^2+300A^2C^2k^3\beta a_1^2+180AB^2k^2\beta a_1^3+180A^2Ck^2\beta a_1^3=0,\end{aligned}$$

If the system is solved, the coefficients are found as

$$B=0,\ \ a_1=\frac{1}{2}\sqrt{\frac{C}{A}},\ \ a_1\neq 0,\ \ A\neq 0,\ \ k=-\frac{1}{4Aa_1},\ \ \alpha=-w,\ \ \beta\neq 0.$$

with the help of the Mathematica program. After these operations, the solutions of equation (2.6) as follow:
Solution 1:

$$u_1(x,t)=\frac{i}{2}\left[Coth(ix-iwy+i\beta z-iwt)\pm Cosech(ix-iwy+i\beta z-iwt)\right]$$

$$u_2(x,t)=\frac{i}{2}\left[Tanh(ix-iwy+i\beta z-iwt)\pm iSech(ix-iwy+i\beta z-iwt)\right]$$

Solution 2:

$$u_3(x,t)=\frac{1}{2}\left[Sec(-x+wy-\beta z+wt)+Tan(-x+wy-\beta z+wt)\right] \tag{2.9}$$

$$u_4(x,t)=\frac{1}{2}\left[Cosec(-x+wy-\beta z+wt)-Cot(-x+wy-\beta z+wt)\right]$$

$$u_5(x,t)=\frac{1}{2}\left[Sec(x-wy+\beta z-wt)-Tan(x-wy+\beta z-wt)\right] \tag{2.10}$$

$$u_6(x,t) = \frac{1}{2}\left[Cosec\left(x - wy + \beta z - wt\right) + Cot\left(x - wy + \beta z - wt\right)\right] \tag{2.11}$$

Solution 3:

$$u_7(x,t) = \frac{i}{2}\left[Tanh\left(\frac{i}{2}x - \frac{i}{2}wy + \frac{i}{2}\beta z - \frac{i}{2}wt\right)\right] \tag{2.12}$$

$$u_8(x,t) = \frac{i}{2}\left[Coth\left(\frac{i}{2}x - \frac{i}{2}wy + \frac{i}{2}\beta z - \frac{i}{2}wt\right)\right] \tag{2.13}$$

Solution 4:

$$u_9(x,t) = \frac{1}{2}\left[Tan\left(-\frac{1}{2}x + \frac{1}{2}wy - \frac{1}{2}\beta z + \frac{1}{2}wt\right)\right] \tag{2.14}$$

Solution 5:

$$u_{10}(x,t) = \frac{1}{2}\left[Cot\left(\frac{1}{2}x - \frac{1}{2}wy + \frac{1}{2}\beta z - \frac{1}{2}wt\right)\right] \tag{2.15}$$

3. Explanations and graphical presentments of some of the solutions obtained

The graphical demonstrations of some obtained solutions are shown in Figures 1-3.

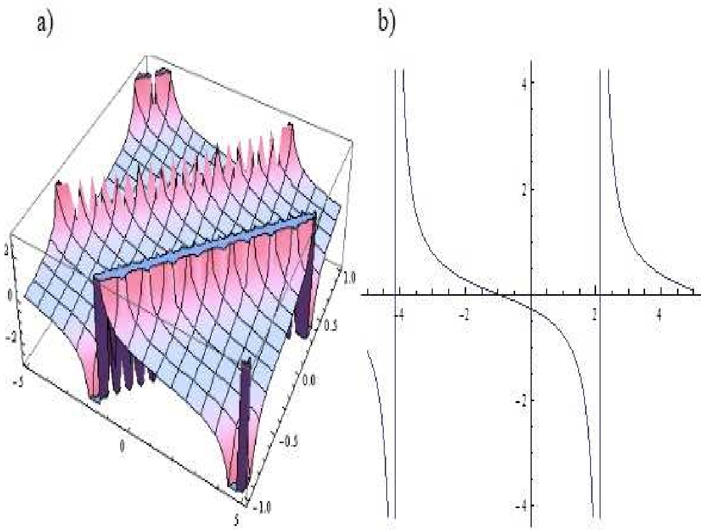

Figure 3.1: a)The 3D surfaces of Eq.(2.9)for the values $y = 1$, $z = 0$ and $w = 5$ within the interval$-5 \leq x \leq 5$, $-1 \leq t \leq 1$. b) The 2D surfaces of Eq.(2.9)for thevalues $y = 1$, $z = 0$, $w = 5$ and $t = 1$ within the interval$-5 \leq x \leq 5$.

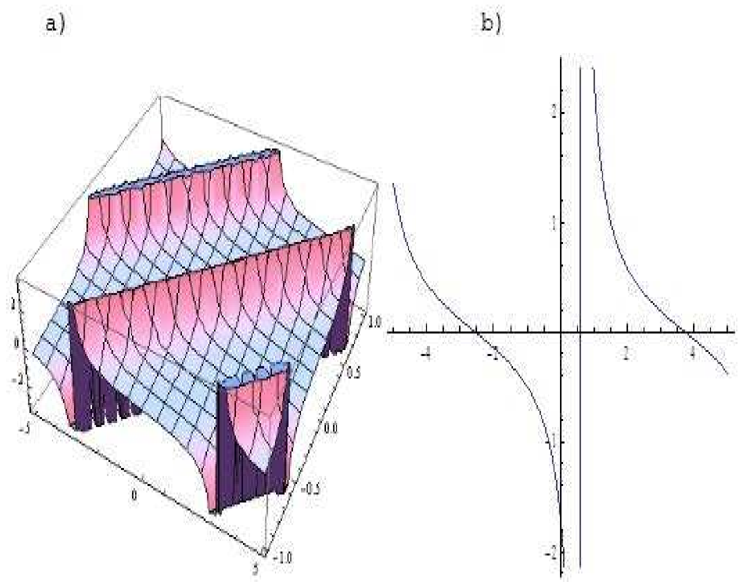

Figure 3.2: a) The 3D surfaces of Eq.(2.14)for the values $y = 1,\ z = 0$ and $w = 5$ within the interval $-5 \leq x \leq 5,\ -1 \leq t \leq 1$. b) The 2D surfaces of Eq.(2.14)for the values $y = 1,\ z = 0,\ w = 5$ and $t = 1$ within the interval$-5 \leq x \leq 5$.

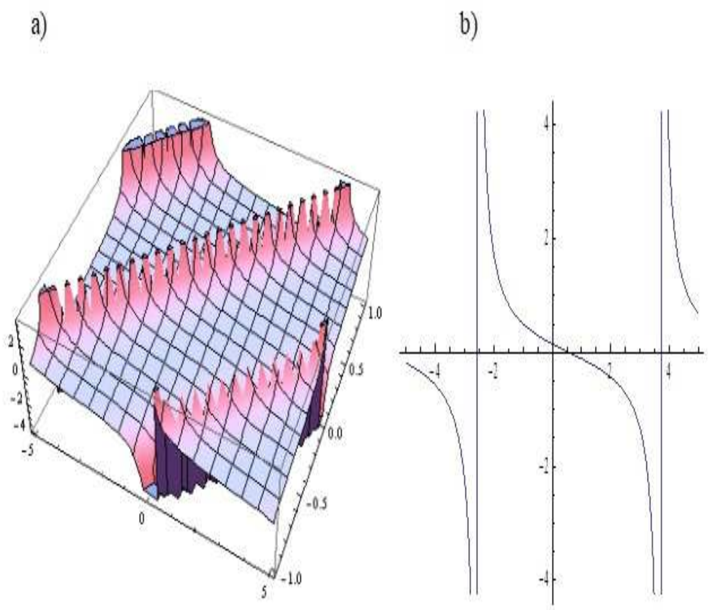

Figure 3.3: a)The 3D surfaces of Eq.(2.15)for the values and within the interval b) The 2D surfaces of Eq.(2.15) for the values and within the interval

4. Conclusion

We used the improved tanh function method to find the multiple soliton solutions of new coupled Konno-Oono equation and extended (3+1)-dimensional KdV-type equation. This method has been successfully applied to solve some nonlinear wave equations and can be used to many other nonlinear equations or coupled ones.

References

[1] L. Debtnath, *Nonlinear partial differential equations for scientist and engineers*, Birkhauser, Boston, MA, 1997.
[2] A. M. Wazwaz, *Partial differential equations: methods and applications*, Balkema, Rotterdam, 2002.
[3] Y. Shang, *Backlund transformation,Lax pairs and explicit exact solutions for the shallow water wave sequation*, Appl. Math. Comput., **187** (2007), 1286-1297.
[4] T. L. Bock, M. D. Kruskal, *A two-parameter Miura transformation of the Benjamin-Onoequation*, Phys. Lett. A, **74** (1979), 173-176.
[5] V. B. Matveev, M. A. Salle, *Darboux transformations and solitons*, Springer, Berlin, 1991.

[6] A.M. Abourabia, M. M. El Horbaty, *On solitary wave solutions for the two-dimensional nonlinear modified Kortweg-de Vries-Burger equation*, Chaos Solitons Fractals, **29** (2006), 354-364.
[7] W. Malfliet, *Solitary wave solutions of nonlinear wave equations*, Am. J. Phys., **60** (1992), 650-654.
[8] Y. Chuntao, *A simple transformation for nonlinear waves*, Phys. Lett. A, **224** (1996), 77-84.
[9] F. Cariello, M. Tabor, *Painlev eexpansions for nonintegrable evolution equations*, Phys. D, **39**(1989), 77-94.
[10] E. Fan, *Two new application of the homogeneous balance method*, Phys. Lett. A, **265** (2000), 353-357.
[11] P. A. Clarkson, *New similarity solutions for the modified boussinesq equation*, J. Phys. A: Math. Gen., **22** (1989), 2355-2367.
[12] W. Malfliet, *Solitary wave solutions of nonlinear wave equations*, Am. J. Phys., **60** (1992), 650-654.
[13] E. Fan, *Extended tanh-function method and its applications to nonlinear equations*, Phys. Lett. A, **277** (2000), 212-218.
[14] S. A. Elwakil, S. K. El-labany, M. A. Zahran, R. Sabry, *Modified extended tanh- function method for solving nonlinear partial differential equations*, Phys. Lett. A, **299** (2002), 179-188.
[15] H. Chen, H. Zhang, *New multiple soliton solutions to the general Burgers-Fisher equation and the Kuramoto-Sivashinsky equation*, Chaos Solitons Fractals, **19** (2004), 71-76.
[16] Z. Fu, S. Liu, Q. Zhao, *New Jacobi elliptic function expansion and new periodic solutions of nonlinear wave equations*, Phys. Lett. A, **290** (2001), 72-76.
[17] S. Shen, Z. Pan, *A note on the Jacobi elliptic function expansion method*, Phys. Let. A, **308** (2003), 143-148.
[18] H. T. Chen, Z. Hong-Qing, *New double periodic and multiple soliton solutions of the generalized (2+1)-dimensional Boussinesq equation*, Chaos Solitons Fractals, **20** (2004), 765-769.
[19] Y. Chen, Q. Wang, B. Li, *Jacobi elliptic function rational expansion method with symbolic computation to construct new doubly periodic solutions of nonlinear evolution equations*, Z. Naturforsch. A, **59** (2004), 529-536.
[20] Y. Chen, Z. Yan, *The Weierstrass elliptic function expansion method and its applications in nonlinear wave equations*, Chaos Solitons Fractals, **29** (2006), 948-964.
[21] M. Wang, X. Li, J. Zhang, *The* $\left(\frac{G'}{G}\right)$*-expansion method and travelling wave solutions of nonlinear evolutions equations in mathematical physics*, Phys. Lett. A, **372** (2008), 417-423.
[22] S. Guo, Y. Zhou, *The extended* $\left(\frac{G'}{G}\right)$*-expansion method and its applications to the Whitham-Broer-Kaup-like equations and coupled Hirota-Satsuma KdV equations*, Appl. Math. Comput., **215** (2010), 3214-3221.
[23] H. L. Lu, X. Q. Liu, L. Niu, *A generalized* $\left(\frac{G'}{G}\right)$*-expansion method and its applications to nonlinear evolution equations*, Appl. Math. Comput., **215** (2010), 3811-3816.
[24] L. Li, E. Li, M. Wang, *The* $\left(\frac{G'}{G}, \frac{1}{G}\right)$*-expansion method and its application to travelling wave solutions of the Zakharov equations*, Appl. Math-A J. Chin. U., **25** (2010), 454-462.
[25] J. Manafian, *Optical soliton solutions for Schrodinger type nonlinear evolution equations by the tan*$\left(\frac{\phi(\varphi)}{2}\right)$*- expansion Method*, Optik, **127** (2016), 4222-4245.
[26] E. Don, *Schaum's outline of Theoryand problems of mathematica*, McGraw-Hill, 2001.
[27] E. Yasar, *New travelling wave solutions to the Ostrovsky equation*, Appl. Math. Comput., **216**(11) (2010), 3191-3194.
[28] Y. Yildirim, E. Yasar, *An extended Korteweg-de Vries equation: multi-soliton solutions and conservation laws*, Nonlinear Dynam., **90**(3) (2017), 1571-1579.
[29] E. Yasar, S. San, Y. S. Ozkan, *Nonlinear self adjointness, conservation laws and exact solutions of ill-posed Boussinesq equation*, Open Phys., **14**(1) (2016), 37-43.
[30] W. X. Ma, Y. Zhou, *Lump solutions to nonlinear partial differential equations via Hirota bilinear forms*, J. Differential Equations, **264**(4) (2018), 2633-2659.
[31] W. X. Ma, T. Huang, Y. Zhang, *A multiple exp-function method for nonlinear differential equations and its application*, Physica Scr., **82**(6) (2010), 065003.
[32] M. S. Osman, H. I. Abdel-Gawad, M. A. El Mahdy, *Two-layer-atmospheric blocking in a medium with high nonlinearity and lateral dispersion*, Results Phys., **8** (2018), 1054-1060.
[33] Hamdy I. Abdel-Gawad, M. S. Osman, *On the variational approach for analyzing the stability of solutions of evolution equations*, Kyungpook Math., **53** (2013), 661-680.
[34] M. S. Osman, *On complex wave solutions governed by the 2D Ginzburg-Landau equation with variable coefficients*, Optik, **156** (2018), 169-174.
[35] M. S. Osman, J. A. T. Machado, D. Baleanu, *On nonautonomous complex wave solutions described by the coupled Schrodinger-Boussinesq equation with variable-coefficients*, Opt. Quant. Electron, **50** (2018), 73.
[36] M. Nasir Ali, M. S. Osman, S. Muhammad Husnine, O*n the analytical solutions of conformable time-fractional extended Zakharov-Kuznetsov equation through* $\left(\frac{G'}{G2}\right)$*-expansion method and the modified Kudryashov method*, SeMA Journal, (in Press).
[37] M. S. Osman, *Multiwave solutions of time-fractional (2 + 1)-dimensional Nizhnik-Novikov-Veselov equations*, Pramana J. Phys., **88** (2017), 67.
[38] M. S. Osman, A. Majid Wazwaz, A*n efficient algorithm to construct multi-soliton rational solutions of the (2+ 1)-dimensional KdV equation with variable coefficients*, Appl. Math. Comput., **321** (2018), 282-289.
[39] M. S. Osman, *On multi-soliton solutions for the (2 + 1)-dimensional breaking soliton equation with variable coefficients in a graded-index waveguide*, Comput. Math. Appl., **75** (2018), 1-6.