

## Strong Domination Number of Some Graphs

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### Abstract

In this paper, we consider strong domination number of  $P_n^k$  and  $C_n^k$ . Let  $G=(V,E)$  be a graph,  $V$  is a vertex set and  $E$  is an edge set of graph  $G$  and  $u,v \in V$ .  $u$  strongly dominates  $v$  and  $v$  weakly dominates  $u$  if (i)  $uv \in E$  and (ii)  $d(u, G) \geq d(v, G)$ . A set  $D \subset V$  is a strong-dominating set (*sd-set*) of  $G$  if every vertex in  $V-D$  is strongly dominated by at least one vertex in  $D$ . The strong domination number  $\gamma_s$  of  $G$  is the minimum cardinality of an *sd-set*.

**Keywords:** Domination Number, Strong Domination Number, Power of a Graph, Networks, *sd-set*

### 1 Introduction

Vulnerability is the most important notion in any communication network. A communication network can be modeled as a graph whose vertices represent the stations and edges represent the lines of communication. A graph  $G$  is denoted by  $G=(V,E)$ , where  $V$  and  $E$  are vertices and edges sets of  $G$ , respectively. In this paper, we only consider finite, undirected graph  $G$  without multiple edges or loops.

In graph theory, many graph parameters have been used widely in the past to describe the stability of a graph. Connectivity, integrity, domination number, etc [1,2]. Domination number is one of the important measures of vulnerability. Domination number and its various types have been studied widely.

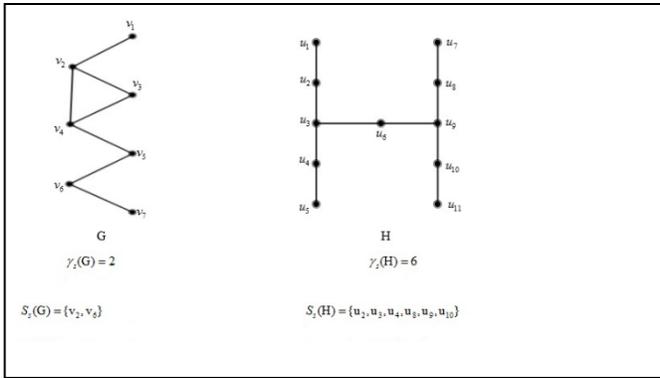
For any vertex  $v \in V$ , the open neighborhood of  $v$  is the set  $N(v) = \{u \in V \mid uv \in E\}$  and the closed neighborhood is the set  $N[v] = N(v) \cup \{v\}$ . For a set  $S \subseteq V$ , the open neighborhood is  $N(S) = \bigcup_{v \in S} N(v)$  and the closed neighborhood is  $N[S] = N(S) \cup S$ . A set  $S \subseteq V$  is a dominating set if  $N[S] = V$ , or equivalently, every vertex in

$V-S$  is adjacent to at least one vertex in  $S$ . The domination number  $\gamma(G)$  is the minimum cardinality of a dominating set in  $G$ , and a dominating set  $S$  of minimum cardinality is called a  $\gamma$ -set of  $G$ .

In this paper, we consider strong domination number of a graph which is defined by Sampathkumar and Latha in 1996 [3]. Let  $u,v \in V$ . Then,  $u$  strongly dominates  $v$  and  $v$  weakly dominates  $u$  if (i)  $uv \in E$  and (ii)  $d(u, G) \geq d(v, G)$ . A set  $D \subset V$  is a strong-dominating set (*sd-set*) of  $G$  if every vertex in  $V-D$  is strongly dominated by at least one vertex in  $D$ . The strong domination number  $\gamma_s$  of  $G$  is the minimum cardinality of an *sd-set*. Throughout this paper, we use  $S(G)$  for *sd-set* of graph  $G$ .

The maximum degree of a graph defined as,  $\Delta(G) = \max\{d(x, G), x \in V\}$ , where  $d(x, G)$  is the degree of a vertex  $x \in V$  in the graph  $G$ . Degree of a vertex is the cardinality of its neighborhood.

Below there are two examples illustrating how to calculate strong domination number of graphs.



**Figure 1.**  $S(G)$ ,  $S(H)$  sets and Strong Domination Numbers

### 2 Strong Domination Number of $P_n^k$ and $C_n^k$

The  $k$ -th power  $G^k$  of an undirected graph  $G$  is another graph that has the same set of vertices, but in which two vertices are adjacent when their distance in  $G$  is at most  $k$ . If a graph has diameter  $d$ , then its  $d$ -th power is the complete graph. In this paper, we consider  $P_n^k$  and  $C_n^k$ . Before consider strong domination numbers of these graphs we give some basic results.

**Proposition 2.1.**  $\gamma_s(P_n) = \left\lceil \frac{n}{3} \right\rceil$ , where  $P_n$  be the path graph of order  $n$ .

**Proposition 2.2.**  $\gamma_s(C_n) = \left\lceil \frac{n}{3} \right\rceil$  where  $C_n$  be the  $n$ -cycle.

**Proposition 2.3.**  $\gamma_s(K_n) = 1$ , where  $K_n$  be the complete graph of order  $n$ .

**Proposition 2.4.**  $\gamma_s(K_{m,n}) = 2$ , where  $K_{m,n}$  be the complete bipartite graph.

**Theorem 1.** Let  $P_n^k$  be the  $k$ -th power of the path graph of order  $n$ . Then,

$$\gamma_s(P_n^k) = \left\lceil \frac{n}{\Delta(P_n^k)+1} \right\rceil = \left\lceil \frac{n}{2k+1} \right\rceil.$$

**Proof.** Maximum degree of the graph  $P_n^k$  is  $\Delta(P_n^k) = 2k$ . A vertex which has maximum degree must be chosen to dominate maximum possible vertices and this vertex dominates  $2k+1$  vertices including itself. There is two cases.

Case 1. If we consider vertices of  $P_n^k$  as a group of  $2k+1$  number of vertices that are neighbours, then we obtain  $n/(2k+1)$  group of vertices. Middle vertex of each vertices group strongly dominates its own vertices group, so middle vertex must be element of  $S(P_n^k)$ . Therefore,  $n/(\Delta(P_n^k)+1)$  number of vertices must be element of  $S(P_n^k)$ , where every vertex of  $P_n^k$  belongs to only one of these groups.

Case 2. If we use previous process, then middle vertex of each vertices group strongly dominates its own vertices group, so middle vertex must be element of  $S(P_n^k)$ , but at most  $\Delta(P_n^k)-1$  number of vertices can not be dominated by these middle vertices. Then, one more vertex must be added to strong domination set to dominate them. Hence,  $\gamma_s(P_n^k) = n/(\Delta(P_n^k)+1)$ . Then, we obtain

$$\gamma_s(P_n^k) = \left\lceil \frac{n}{\Delta(P_n^k)+1} \right\rceil = \left\lceil \frac{n}{2k+1} \right\rceil$$

from these two cases. ■

**Theorem 2.** Let  $C_n^k$  be the  $k$ -th power of the  $n$ -cycle.

Then,  $\gamma_s(C_n^k)$  is equal to

$$\begin{cases} \left\lceil \frac{n}{k+3} \right\rceil, & \frac{n-1}{2} > k \\ 1, & \frac{n-1}{2} \leq k \end{cases}, \text{ when } n \text{ is odd and } n > 3$$

$$\begin{cases} \left\lceil \frac{n}{k+3} \right\rceil, & \frac{n}{2} > k \\ 1, & \frac{n}{2} \leq k \end{cases}, \text{ when } n \text{ is even.}$$

**Proof.** Let  $C_n^k$  be the  $k$ -th power of the  $n$ -cycle. The  $C_n^k$  is the complete graph when  $\Delta(C_n^k) = n-1$  and

$n > 3$ . Obviously, strong domination number of complete graph is 1.

$\Delta(C_n^k) = k+2$ , where  $\frac{n-1}{2} > k$ . A vertex which has maximum degree must be chosen to dominate maximum possible vertices and this vertex dominate  $k+3$  vertices including itself. If we consider vertices of  $C_n^k$  as a group of  $k+3$  number of vertices that are neighbours, then  $\gamma_s(C_n^k)$  is  $\frac{n}{k+3}$  by previous idea. If  $\frac{n-1}{2} \leq k$ , then degree of all vertices of  $C_n^k$  are maximum and  $C_n^k$  is complete graph. Similarly, if  $n$  is even and  $\frac{n}{2} > k$ , then

$\gamma_s(C_n^k) = \left\lceil \frac{n}{k+3} \right\rceil$  and when  $\frac{n}{2} \leq k$ , then  $C_n^k$  is complete graph, so  $\gamma_s(C_n^k) = 1$ .

Consequently,  $\gamma_s(C_n^k)$  is equal to

$$\begin{cases} \left\lceil \frac{n}{k+3} \right\rceil, & \frac{n-1}{2} > k \\ 1, & \frac{n-1}{2} \leq k \end{cases}, \text{ where } n \text{ is odd and } n > 3$$

$$\begin{cases} \left\lceil \frac{n}{k+3} \right\rceil, & \frac{n}{2} > k \\ 1, & \frac{n}{2} \leq k \end{cases}, \text{ where } n \text{ is even.}$$

### 3 Conclusion

In this paper, we give strong domination number of well-known graphs which are given by propositions. We consider strong domination number of  $P_n^k$  and  $C_n^k$ . For further study, we it is planned to check strong and weak dominations number of other graph classes.

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