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# Panel flutter numerical study of thin isotropic flat plates and curved plates with various edge boundary conditions

*Çeşitli kenar sınır koşullarına sahip ince izotropik düz plakaların ve eğri plakaların panel çarpıntı sayısal çalışması*

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# Panel Flutter Numerical Study of Thin Isotropic Flat Plates and Curved Plates with Various Edge Boundary Conditions

## Highlights

- ❖ When a thin panel (a plate or shell-like structure) is exposed to supersonic flow on one side of the panel, a type of self-exciting dynamic instability that occurs in certain ranges of critical dynamic pressure due to the interaction of panel's inertia force, elastic restoring force, and the supersonic airflow is known as panel flutter.
- ❖ Supersonic panel flutter of flat and curved plates with different edge boundary conditions is investigated in this paper using efficient, high-precision triangular shallow shell finite elements. Whenever possible, compare present numerical results with available literature data.
- ❖ It has been found that the fixed condition in the cross-flow direction of square panel have a notable effect on the critical dynamic pressure parameters and flutter frequencies.
- ❖ The outcome of present results show that the flutter limit is strongly influenced by the boundary conditions.

## Graphical Abstract

A common and practical type of structural component with important applications in aerospace vehicles, such as high-speed aircrafts, rockets, and spacecraft, are flat plate (Figure A) or curved plate or shell (Figure B) panels. So it is interesting to study the panel flutter dynamic stability of aforementioned structures subjected to supersonic flow for various edge boundary conditions.

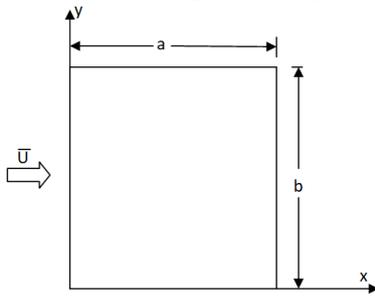


Figure. A Flat Plate

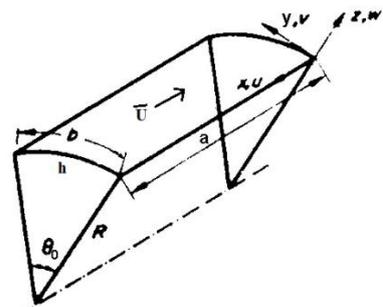


Figure.B Curved Plate or Shell

## Aim

It aims to find the panel flutter results( critical flutter frequency and critical dynamic pressure parameter) of flat and curved plates with different edge boundary conditions subjected to supersonic flow.

## Design & Methodology

Efficient, high-precision triangular shallow shell finite elements of Cowper has been used and coded to find the panel flutter results of flat and curved plates with different edge boundary conditions.

## Originality

This study is an addition to the existing literature on the panel flutter analysis of plates subjected to supersonic flow. The novelty of the work is that it considers various boundary conditions for flat and curved panels.

## Findings

The critical dynamic pressure and coalescing frequency of various edge boundary conditions are determined for flat plate (square and rectangular) and curved plate. In square flat plate, the flutter results for the C-S-C-S and S-C-S-C boundary conditions should be the same. However, the critical flutter frequency and critical pressure parameters of the S-C-S-C constraint are higher than the C-S-C-S constraint. Therefore, fixed condition in the cross-flow direction (S-C-S-C) have a significant effect on the critical dynamic pressure and flutter frequencies.

## Conclusion

The results of the current study demonstrate that the boundary conditions have a significant impact on the flutter limit. The stronger the edge boundary of plates, the higher the flutter stability of the supersonic panel.

## Declaration of Ethical Standards

The author of this article declares that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

# Panel Flutter Numerical Study of Thin Isotropic Flat Plates and Curved Plates with Various Edge Boundary Conditions

*Araştırma Makalesi / Research Article*

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## ABSTRACT

In this article, supersonic panel flutter analysis of flat plates and curved plates with different edge boundary conditions are studied, using efficient, high precision triangular shallow shell finite elements. The fluid on the underside of the plate was assumed to be stationary. The linear piston theory can be applied to the top surface of the plate. The linear piston theory was used to evaluate the aerodynamic loads. The solution of a complex eigenvalue problem was formulated according to Hamilton's principle. Lagrange's equation of motion was obtained using standard methods for finding eigenvalues. Current finite element analysis ignores aerodynamic damping. For panels, the theory of thin and small deformed shells was taken into account. To validate the developed finite element code, the results of a square and rectangular flat-panels with simply supported edges (S-S-S-S), a square plate with four fixed edges (C-C-C-C), and a square plate with the length side clamps (C-S-C-S) were compared with the published data. The flutter results of other edge boundary conditions (S-C-S-C, C-S-C-S, and C-C-C-C) for square and rectangular flat panels are evaluated for which literature data is limited. It has been found that the fixed condition in the cross-flow direction (S-C-S-C) has a significant effect on the critical flutter pressure parameters and flutter frequencies. Further, to study the aforementioned effect, the current finite element (FE) has been extended to curved plates with S-C-S-C (constrained in the cross-flow direction and exposed to supersonic flow), S-S-S-S boundary conditions to find flutter results.

**Keywords:** Panel flutter, finite element, edge boundary conditions, supersonic flow, piston theory:

## Çeşitli Kenar Sınır Koşullarına Sahip İnce İzotropik Düz Plakaların ve Eğri Plakaların Panel Çarpıntı Sayısal Çalışması

### ÖZ

Bu makale de, etkili, yüksek hassasiyetli üçgen sığ kabuk sonlu elemanlar kullanılarak, farklı kenar sınır koşullarına sahip düz plakaların ve eğri plakaların süpersonik panel çarpıntısı sayısal olarak incelenmiştir. Plakanın alt tarafındaki akışkanın durağan olduğu varsayılmıştır. Lineer piston teorisi, plakanın üst yüzeyine uygulanabilir. Aerodinamik yükleri değerlendirmek için doğrusallaştırılmış piston teorisi kullanılmıştır. Karmaşık bir özdeğer probleminin çözümü Hamilton ilkesine göre formüle edilir. Lagrange'in hareket denklemi, özdeğerleri bulmak için standart yöntemler kullanılarak elde edilir. Mevcut sonlu eleman analizi, aerodinamik sönümlenmeyi yok sayar. Geliştirilen sonlu elemanlar paneller için ince ve küçük deforme olmuş kabuklar teorisi dikkate alınır. Geliştirilen sonlu elemanlar kodunu doğrulamak için basitçe desteklenen kenarlı (S-S-S-S) kare ve dikdörtgen düz panellerin, dört sabit kenarlı kare plakanın (C-C-C-C) ve uzunluk yan kelepçeli kare plaka (C-S-C-S) sonuçları yayımlanmış veilerle karşılaştırılmıştır. Literatür verilerinin sınırlı olduğu kare ve dikdörtgen düz panel için diğer kenar sınır koşullarının (S-C-S-C, C-S-C-S ve C-C-C-C) flutter sonuçları değerlendirilmiştir. Çapraz akış yönündeki (S-C-S-C) sabit koşulun, kritik flutter basınç parametreleri ve flutter frekansları üzerinde önemli bir etkiye sahip olduğu bulunmuştur. Ayrıca, yukarıda bahsedilen etkiyi incelemek için, mevcut sonlu eleman (FE), çarpıntı sonuçlarını bulmak için S-C-S-C (çapraz akış yönünde kısıtlı ve süpersonik akışa maruz kalan), S-S-S-S sınır koşullarına sahip kavisli plakalara genişletildi.

**Anahtar Kelimeler:** Panel çarpıntısı, sonlu eleman, kenar sınır koşulları, süpersonik akış, piston teorisi

### 1. INTRODUCTION

Determining the flutter characteristics of a shallow shell is of paramount importance in the design of supersonic

aircraft and missiles. When a thin panel (a plate or shell-like structure) is exposed to supersonic flow on one side of the panel, a type of self-exciting dynamic instability due to the interaction of panel's inertia force, elastic restoring force, and the supersonic airflow that occurs in certain ranges of critical dynamic pressure is known as panel flutter. The dynamic pressure at which this type of instability occurs depends on the initial curvature and

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stiffness of the panel; the ratio of air density to panel density; the dimensions of the panel; and the thrust applied by the support at the panel's side.

In the aerospace industry, missiles and launch vehicles are used for a variety of purposes. It can carry payloads into orbit and space, as well as be used for military purposes. Aerodynamic forces act on an object engrossed in a fluid medium as a result of the relative motion of an object and the fluid. The ANSYS CFX solver was used to design and analyse the subsonic missile [1]. The finite element method (FEM) has been used to analyse aeroelastic flutter in shells and plates exposed to supersonic flow [2]. The main issue with cylindrical shells (and curved plates) is that the minimum circumferential (or cross-flow) mode is not always the most important, as it is with flat plates. It was observed that the in-plane boundary conditions and panel shape have a substantial impact on the panel's flutter properties. Non-linear flutter analysis of 2D and 3D curved plates was performed using quasi-static aerodynamic theory [3]. A study by Dowell [3] showed that the in-plane edges restraint has a substantial impact on the flutter limit, a phenomenon resulting from analyzed shell frequency spectrum. The concept of aerodynamic matrix was introduced. The aerodynamic matrix concept was introduced. In the field of supersonic panel flutter, Olson [4] applied the FEM to square flat plates and Pany and Parthan [5] to periodically supported curved plates. FEM analysis of a conical sandwich composite shell with flexible honeycombs is performed for fixed and cantilever boundary conditions subjected to supersonic flow [6]. Sabri and Lakis [7] investigated the supersonic flutter aspects of curtailed conical shape shells that were partially filled with fluid and exposed to an external airflow using FEM with linear piston theory. They noticed that increasing the interior pressure improves the aero-elastic controllability of conical shells. The panel flutter study of layered composite panels and shells have been carried out utilising 8-node isoparametric finite elements (FE) [8].

Hassan et al. [9] studied the aeroelastic behavior (flutter analysis) of carbon nanotube (CNTs) reinforced polymeric cone shells under supersonic fluid flow using FSDT and the differential quadrature method with linear piston theory. Long, thick-walled composite cylinders with internally or externally pressurized fixed ends have been investigated [10]. The free vibrational behaviour of axially compressed cross-laminated composite cylinders has been studied using FSDT-based shell FEs with various boundary conditions [11]. Vibration and flutter in composite cylindrical and flat plates are presented using FEM for various edge

boundary conditions [12]. The aeroelastic characteristics of 2D laminated composite panels in supersonic flow have been investigated making use of the piston theory [13]. It has been reported that fiber orientation has a significant impact on the dynamic behavior of panels. Vibration of a cylindrical multi-supported shell has been investigated [14]

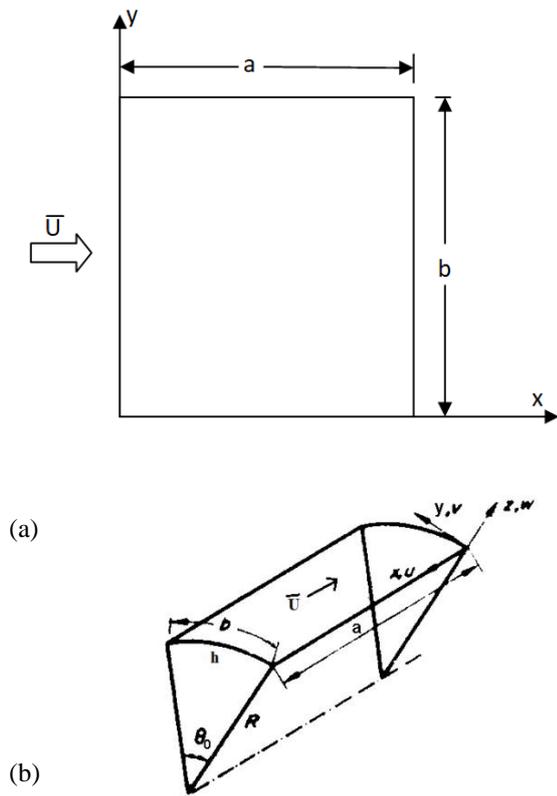
A high-precision triangular arbitrary shell FE of Cowper, Lindberg and Olson [15,16] is successfully applied to line-supported periodic plate [17], orthogonally line-supported curved panels [18, 19], and curved panels with axially periodic support [20] for free vibration analysis. In the present work, the same high precision triangular shell FE [15,16] is extended to include the supersonic flow based on linear piston theory for the flutter analysis of isotropic flat (square and rectangular) and cylindrically curved panels for different edge boundary conditions. The linear Piston theory is used to calculate the aerodynamic force. The QR method is employed for the solution of complex eigenvalue problems. Square flat plate flutter solutions are proffered for limited elastic edge boundary conditions [4,8,21] using the FE approach.

Flutter results for plates with different edge boundary conditions are limited in the literature. Therefore, in this study, four different edge boundary conditions for the flat plate were attempted. A flat plate with the following edge boundary conditions: That is, (i) four sides are simply supported (S-S-S-S), (ii) the width side is fixed and the length side is simply supported (S-C-S-C), (iii) the simply supported in width sides and fixed in the length sides (C-S-C-S) and (iv) the four fixed sides (C-C-C-C) were studied using the high precision triangular FE formulation. The panel flutter results using the present FE with all edges simply-supported and S-C-S-C edge boundary conditions are also presented for curved plates with supersonic flow along the generator.

Whenever possible, the present numerical results were compared with the results published in the literature. The material is steel of  $E=210 \text{ N/m}^2$ ,  $\rho=7800 \text{ Kg/m}^3$  and  $\nu=0.3$ .

## 2. EQUATION OF MOTION

Consider thin isotropic flat plates (length  $a$  and width  $b$ ) and cylindrically curved plates (length  $a$  and width  $\theta_0$ ), as shown in Figure 1. Supersonic airflow flows over the top surface at zero angle of attack and is parallel to the edge.



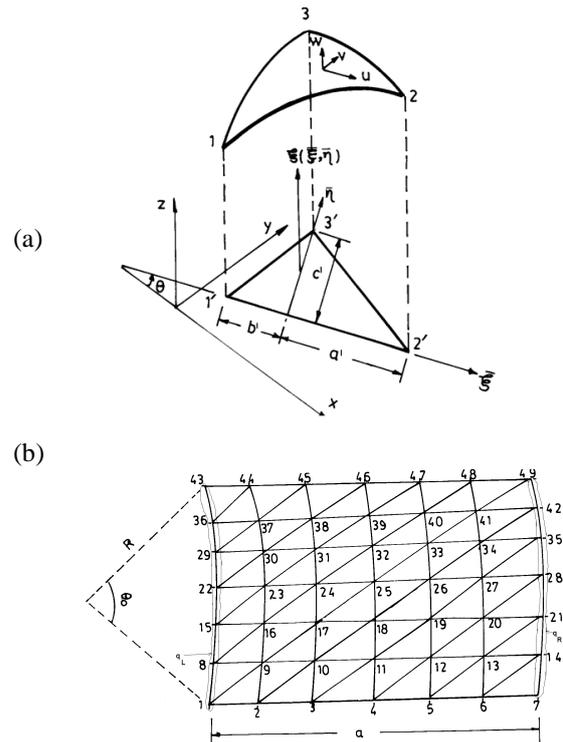
**Figure 1.** (a) Flat panel and (b) Curved panel; with supersonic flow

The fluid underside of the plate is assumed to be stationary. If a perturbation is present, the plate begins the perturbation along the lateral  $w(x, y, t)$ . It is interesting to study the stability of flat plates and curved plates under such motion. The aeroelastic equation of motion was derived according to Hamilton's principle. For non-conservative systems, this leads to the following variational forms, which determine the dynamics of the system.

$$\delta \int_{t_1}^{t_2} L dt + \int_{t_1}^{t_2} \delta W dt = 0 \tag{1}$$

Where  $L$  is a Lagrangian function defined as  $L=T-U$ .  $T$  and  $U$  are the total kinetic energy and strain energy of the system, respectively.  $t_1$  and  $t_2$  are the start and end times of motion.  $W$  is work done by the non-conservative aerodynamic force fields.

The basic shell element used here is a conforming higher-order arbitrary triangular-shaped shallow shell FE (Figure 2a) with 12 degrees of freedom per node [15]. The stiffness and mass matrix of an element result from strain energy ( $U$ ) and kinetic energy ( $T$ ), respectively. Details of the stiffness and mass matrix have been previously reported [15,16,17].



**Figure 2.** (a) Triangular high-precision shallow shell element of Cowper[15] considered in the analysis (b) FE model using (6x6) mesh for curved plate

Assuming the linear piston theory, the work done  $W$  by aerodynamic load is given as follows:

$$W = \int p(x, y, t) w dx dy \tag{2}$$

Using a two-dimensional quasi-steady-state theory [22] with a first-order approximation by Ashley and Zartarian, the aerodynamic pressure intensity acting on the infinitesimal element  $dA = dx dy$  is expressed as:

$$p(x, y, t) = - \frac{2q}{(\bar{M}_\infty^2 - 1)^{\frac{1}{2}}} \left[ \frac{\partial w}{\partial x} + \frac{1}{\bar{U}} \frac{\bar{M}_\infty^2 - 2}{\bar{M}_\infty^2 - 1} \frac{\partial w}{\partial t} \right] \tag{3}$$

Where  $q$ ,  $\bar{M}_\infty$  and  $\bar{U}$  are dynamic pressure, Mach number, and flow velocity, respectively. Note that  $p(x,y,t)$  is the positive aerodynamic pressure in the  $w$  (lateral deflection) direction of the plate,  $q = (1/2) \rho_a \bar{U}^2$  is the dynamic pressure of free-flowing air,  $\rho_a$  free stream density and  $\bar{U}$  is the free stream velocity. In the above equation, the first term is the contribution of steady-state flow due to the local flow gradient, and the second transient term is associated with aerodynamic damping. The free-flow dynamic pressure parameter and the aerodynamic damping parameters are expressed as:

$$\bar{\Lambda} = \frac{2q}{(\bar{M}_{\infty}^2 - 1)^{1/2}}; \text{ dynamic pressure parameter}$$

$$g_d = \frac{\bar{\Lambda} \bar{M}_{\infty}^2 - 2}{\bar{U} \bar{M}_{\infty}^2 - 1}; \text{ aerodynamic damping parameter}$$

(4 a,b)

The aerodynamic pressure intensity  $p(x,y,t)$  in local coordinates ignoring the aerodynamic damping ( $g_d$ ) will be [4]

$$p(x, y, t) = -\Lambda \left[ \frac{\partial w}{\partial \xi} \cos(\theta) + \frac{\partial w}{\partial \eta} \sin(\theta) \right] \quad (5)$$

Based on the virtual work principle, the aerodynamic virtual work (W) done by aerodynamic force for the elements nodal vector, can be written as:

$$\delta W = \{\delta q_1\}^T [a] \{q_1\} \quad (6)$$

The generalized element displacement  $\{q_1\}$  is: [4,15]

$$\{q_1\}^T = (u_1, u_{\xi_1}, u_{\eta_1}, v_1, v_{\xi_1}, v_{\eta_1}, w_1, w_{\xi_1}, w_{\eta_1}, w_{\xi\xi_1}, w_{\xi\eta_1}, w_{\eta\eta_1}, u_2, \dots, u_3, \dots, u_c, v_c). \quad (7)$$

Where,

$$u_{\xi} = \frac{\partial u}{\partial \xi}, \quad u_{\eta} = \frac{\partial u}{\partial \eta}, \quad w_{\xi\xi} = \frac{\partial^2 w}{\partial \xi^2}, \text{ etc.}$$

Using the notation from references [15, 4, 5], the elements of the plate's aerodynamic matrix are represented by the polynomial displacement functions  $u$ ,  $v$ , and  $w$ . Expected expressions for  $u$ ,  $v$ , and  $w$  are available in references [15,16,4].

$$[a]_{i,j} = 0; \text{ for } i, j = 1, \dots, 20$$

$$[a]_{i,j} = -\bar{\Lambda} \left[ r_j F(\bar{r}_i + \bar{r}_j - 1, \bar{s}_i + \bar{s}_j) \cos(\theta) - \bar{s}_j F(\bar{r}_i + \bar{r}_j, \bar{s}_i + \bar{s}_j - 1) \sin(\theta) \right] \text{ for } i, j = 21, \dots, 40. \quad (8)$$

Where  $\bar{r}_i$  to  $\bar{s}_j$  have been described in [4].

The equation(8) can be written as

$$[a]_{i,j} = -\bar{\Lambda} [A]_{i,j} \quad (9)$$

The transformation matrix [T] and rotation matrix [R] of [15] are used to transform local corner displacements of the to the global coordinate system of the finite element. The aerodynamic matrix( equation(9)) can be described as a generalized displacement with respect to the global coordinates as follows[4,15]:

$$[AA] = -\bar{\Lambda} [R]^T [T]^T [A]_{i,j} [T][R] = -\bar{\Lambda} [K_a] \quad (10)$$

### 2.1. Flutter Equation of Panel:

The dynamic instability ( interaction of panel with supersonic airflow) equation of motion for a non-damping elastic structural system with small displacements and linear assumptions can be expressed as:

$$[M] \{\ddot{q}\} + [K] \{q\} = -\bar{\Lambda} [K_a] \{q\} \quad (11)$$

Where [M], [K], and [K<sub>a</sub>] are the global mass, stiffness, and aerodynamics matrices respectively. These matrices are obtained by assembling an element matrix for the entire structural system. The global acceleration and displacement vectors are  $\{\ddot{q}\}$  and  $\{q\}$ , respectively. Suppose the solution has the following form:

$$\{q\} = \{\bar{q}\} e^{\omega t} \quad (12)$$

Putting equation (12) in equation(11) becomes

$$[\omega^2 [M] + ([K] + \bar{\Lambda} [K_a])] \{\bar{q}\} = \{0\} \quad (13)$$

The aerodynamic stiffness matrices [K<sub>a</sub>] is asymmetric.

### 2.2. Application of Boundary Condition:

The meaning of this section is the presence or absence of displacement  $(u, u_{\xi}, u_{\eta}, v, v_{\xi}, v_{\eta}$

,  $w, w_{\xi}, w_{\eta}, w_{\xi\xi}, w_{\xi\eta}, w_{\eta\eta}$ ) in all elements of different nodes. All displacements above are assumed to exist at all nodes except those along the supports. Given support constraints are mathematically imposed by assuming that the corresponding displacement is zero.

### 2.3. Solution Procedure to Search for Critical Dynamic Pressure:

The problem defined by the equations of motion is an eigenvalue problem i.e. equation(13), the solution of which provides the eigenvalue for a particular value of dynamic pressure. When the dynamic pressure parameter  $\Lambda = 0$ , the problem of evaluating the free vibration frequency of a plate or panel is degenerate. If  $\Lambda$  is greater than 0, an asymmetric aerodynamic matrix is displayed, and some of the eigenvalues will become complex in a certain range of  $\Lambda$ . Structural stability is known to be determined by the real part of the complex frequency. As the real part of the frequency becomes positive, the amplitude of the structural motion increases with time. The minimum value of  $\Lambda$  that produces a pair of complex conjugate eigenvalues is called the critical dynamic pressure  $\Lambda_{cr}$ . In the absence of aerodynamic damping, the flutter limit corresponds to the minimum value of the dynamic pressure parameter  $\Lambda_{cr}$  where the first coalescence occurs.

### 3. RESULTS AND DISCUSSION

#### 3.1. Finite Element Method:

Using the FEM, numerical results are generated. The effect of various parameters such as curvature, aspect ratio, edge boundary conditions, etc. on the flutter characteristics of flat and curved isotropic panels (Figure 1a,b) is studied. A 6x6 triangular mesh (Figure 2b) is used to model all the panels. Four types of edge boundary conditions are considered for flat panels: S-S-S-S, S-C-S-C, C-S-C-S and C-C-C-C. For curved panels, S-S-S-S and S-C-S-C boundary conditions are investigated. The direction of the airflow is assumed to be parallel to the x-axis (Figure 1). The dimensionless flutter parameters  $\Omega$  and  $\Lambda$  are described below.

For isotropic plates and curved plates, the dimensionless frequency is expressed as:

$$\Omega_{plate} = \Omega_f = \omega a^2 \sqrt{\frac{\rho h}{D}} \tag{14}$$

$$\Omega_{shell} = \Omega_f = \omega R \sqrt{\frac{\rho(1-\nu^2)}{E}} \tag{15}$$

Where,  $\omega$  is the eigenfrequency in Hertz;  $\rho$  is the density of material;  $h$  is the thickness of plate;  $R$  is the radius of curvature;  $a$  is the length of plate or shell in x-direction;  $\nu$  is the Poisson's ratio;  $E$  is the Young's modulus of elasticity;  $D(= E h^3 / 12(1-\nu^2))$  is the bending stiffness of the plate.

The dynamic pressure in nondimensional form is defined as  $\Lambda = \frac{\bar{\Lambda} a^3}{D}$ .

#### 3.2. Comparison of FE Results with Flat Panel (R Tends To $\infty$ ):

The flutter limits for isotropic flat panel (Figure 1a) are determined for four boundary conditions (S-S-S-S, S-C-S-C, C-S-C-S and C-C-C-C) and two aspect ratios ( $a/b = 1/2$  and 1). Table 1 shows the flutter limits, that are, dimensionless critical dynamic pressure parameters ( $\Lambda_{cr}$ ) and dimensionless critical flutter frequencies ( $\Omega_f$ ) for square ( $a/b=1$ ) and rectangular plates ( $a/b = 1/2$ ) with various edge boundary conditions. The results of the square flat panel ( $a/b = 1$ ) were compared with the boundary conditions S-S-S-S and C-C-C-C with reference [4,8]. In the case of simply supported (S-S-S-S) and fixed (C-C-C-C) panels, the flutter frequency ( $\Omega_f$ ) is found to exist between the first and second natural frequencies and is nearer to the second natural frequency. Results of C-S-C-S were compared to reference [21]. The comparison shows that the present results are in good agreement and validate the current FE code. For square flat panels, the flutter frequency should be the same at the

C-S-C-S and S-C-S-C boundary conditions. However, the critical coalescing flutter frequency and critical pressure parameters of the S-C-S-C constraint are higher than the C-S-C-S constraint. From this, it can be concluded that the fixed state in the cross-flow direction has a great influence on the critical dynamic pressure parameter and flutter frequency.

The results ( $a/b=1/2$ ) of a flat rectangular plate with flutter boundaries satisfying the C-S-C-S, S-C-S-C, and C-C-C-C boundary conditions are shown in Table 1. Literature data is sparse in this regard.

Table 1 shows a comparison of the flutter frequency ( $\Omega_f$ ) and the critical dynamic pressure parameter ( $\Lambda_{cr}$ ) for square( $a/b=1$ ) and rectangular ( $a/b=0.5$ ) plates. From Table 1, one can see that the above parameters are higher for square plates. Therefore, increasing the aspect ratio ( $a/b$ ) increases the flutter boundary (dynamic instability).

The lowest flutter frequency ( $\Omega_f$ ) and critical dynamic pressure parameter ( $\Lambda_{cr}$ ) are observed for the S-S-S-S boundary condition and the highest for the C-C-C-C boundary condition.

**Table 1.** Comparison of flutter solution for flat isotropic panels for different edge boundary conditions.

Edge boundary conditions	Square panels		Rectangular panels	
	Flutter frequency $\Omega_f$ (real part)	Critical dynamic pressure ( $\Lambda_{cr}$ )	Flutter frequency $\Omega_f$ (real part)	Critical dynamic pressure ( $\Lambda_{cr}$ )
S-S-S-S	42.346 (42.98) 42.99*	512.651 (512.60) 512.651* 512.**	34.571 (35.26)	391.66 (387.2) 389.6*
S-C-S-C	60.045	812.52	54.587	679.16
C-S-C-S	49.128	544.16 546**	36.391	400.98
C-C-C-C	65.504 (66.38) 65.43*	853.80 (879.42) 850.418*	55.497	683.33

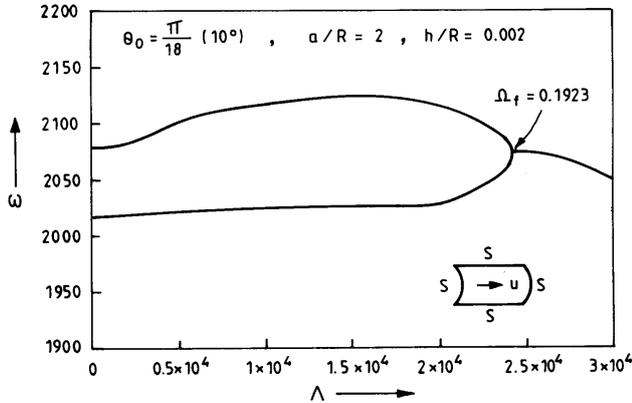
Brackets values are obtained from reference [8], \* values taken from [4] and \*\* values are from [21]

#### 3.3. Shallow Curved Panels:

A FE model of a cylindrically curved plate was created using a 6x6 mesh of high precision triangular elements for current flutter calculations. The dimensions of the curved plate are length ( $a$ ) = 1.016 m, radius ( $R$ ) = 0.508 m, thickness ( $h$ ) = 1.016e-3 m, and the inscribed angle at the center of the curved plate ( $\theta_0$ )=10°[5].

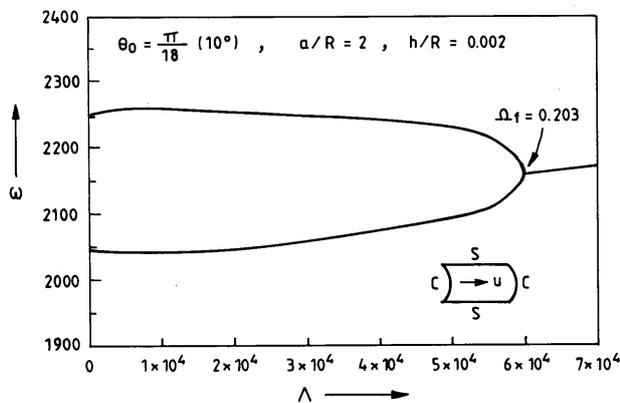
The calculation was performed with the boundary conditions (i) four simply supported edges, and (ii) fixed at the curved edges with simply supported on straight edges. The variation of the natural frequency ( $\omega$ ) versus

the dimensionless dynamic pressure parameter ( $\Lambda$ ) is plotted in Figures 3 and 4 for the above boundary conditions. The piston theory has been employed to understand coupled mode flutter. When flutter occurs, the frequencies of various modes converge and coalesce.



**Figure 3.** FE flutter frequency ( $\omega$ ) and dynamic pressure parameter ( $\Lambda$ ) of a curved plate ( $\theta_0=10^\circ$ ) with four edges simply supported.

Figure 3 shows the results of the current FE code panel flutter analysis for a simply supported four-edge curved plate. The dimensionless flutter frequency  $\Omega_f$  of a simply supported curved plate is 0.1923 and the corresponding dimensionless critical dynamic pressure parameter ( $\Lambda_{cr}$ ) is  $2.36 \times 10^4$ .



**Figure 4.** FE flutter frequency ( $\omega$ ) and dynamic pressure parameter ( $\Lambda$ ) for a curved plate ( $\theta_0=10^\circ$ ) with straight edges simply supported and curved edges fixed.

A curved plate with S-C-S-C boundary conditions (the curved side is fixed and the straight side is simply supported) that receive supersonic flow along generator was determined using the current FE code to obtain the panel flutter result. The dimensionless flutter frequency ( $\Omega_f$ ) for plate is 0.203 and the corresponding dimensionless dynamic pressure parameter ( $\Lambda_{cr}$ ) is  $6 \times 10^4$  (Figure 4). The general behavior is similar to that of simply supported panels. But the flutter boundary of S-C-S-C is observed to be higher as the clamped panel is

more stiff, in the structural sense, than the simply supported panel.

**4. CONCLUSION**

The panel flutter behavior of isotropic flat plates (square and rectangular) and curved plates subjected to supersonic flow is studied using a high-precision triangular shell FE. The critical dynamic pressure and coalescing frequency of various edge boundary conditions were determined. For a square flat plate, the flutter results for the C-S-C-S and S-C-S-C boundary conditions should be the same. However, the critical coalescing flutter frequency and critical pressure parameters of the S-C-S-C constraint are higher than the C-S-C-S constraint. From this, it can be concluded that the fixed state in the cross-flow direction has a great influence on the critical dynamic pressure parameter and flutter frequency. Also note that increasing the aspect ratio ( $a/b$ ) increases the flutter limit.

Based on the above findings, the current FE code is used to find the panel flutter results for S-C-S-C boundary conditions (fixed condition in the cross-flow direction) of a curved plate. In addition, the result of supersonic panel flutter analysis of curved plates corresponding to the S-S-S-S was also calculated. The critical dynamic pressure parameter increased under the S-C-S-C edge boundary conditions. These results show that the flutter limit is strongly influenced by the boundary conditions. Therefore, the stronger the edge boundary of plates, the higher the flutter stability of the supersonic panel.

Some recommendations for future research include the extension of the analysis to flutter of flat and curved panels and studying the influence of temperature distributions on flutter characteristics.

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**DECLARATION OF ETHICAL STANDARDS**

The author of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

**AUTHORS' CONTRIBUTIONS**

**Chitaranjan PANY:** Author has formulated the problem, performed analysis, written the paper and revised based on reviewers / editorial board comments.

**CONFLICT OF INTEREST**

There is no conflict of interest in this study.

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