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Examining Teacher Interventions in Teaching Mathematical Modeling: A Case of Middle School Teacher*

Seda ŞAHİN**, Muhammed Fatih DOĞAN***, Ramazan GÜRBÜZ****

***Abstract*:** The teachers' pedagogical understanding of modeling applications is one of the most critical issues in teaching mathematical modeling. The interventions used by the teacher during the implementation of the tasks are the mirror of their pedagogical knowledge. For this reason, teacher interventions, which have a decisive effect on defining the teacher's role in teaching mathematical modeling, are a crucial issue to investigate. This study examines the types of interventions used by a middle school mathematics teacher who completed professional development in mathematical modeling. The data was collected in an eighth-grade classroom consisting of twenty students. The mathematical modeling task, called Intersection Arrangement, was implemented for 2 hours, and both video and audio recordings were used to collect data. All recordings were transcribed and analyzed using the content analysis method, supported by observer notes and student worksheets. The results revealed that the teacher mostly had effective environmental and classroom interaction interventions. The teacher avoided having content-oriented or strategic interventions or did not intervene during the modeling process. These intervention types might be because of the teacher's unwillingness to affect the modeling process. On the other hand, the fact that the teacher was more active in the presentation and evaluation stage supports his hesitations and difficulties about where and how to intervene in the modeling process. The relevant literature and the results of this research show that teachers must have unique pedagogical knowledge in teaching mathematical modeling.
Keywords: Mathematical modeling, teacher intervention, classroom implementation.

Matematiksel Modelleme Öğretiminde Öğretmen Müdahalelerinin İncelenmesi: Bir Ortaokul Öğretmeni Örneği

***Öz*:** Matematiksel modelleme öğretiminde en önemli konulardan biri matematiksel modelleme uygulamalarında öğretmenin sahip olması gereken pedagojik bilgidir. Ders uygulaması sırasında öğretmenin kullandığı müdahaleler ise pedagojik bilgisinin dışavurumudur. Bu sebeple matematiksel modelleme öğretiminde öğretmen rolünün tanımlanmasında belirleyici etkiye sahip olan öğretmen müdahaleleri araştırılması gereken önemli bir faktördür. Bu çalışmada matematiksel modelleme eğitimine katılan bir ortaokul matematik öğretmeninin eğitimden sonra sınıf içi matematiksel modelleme uygulamalarının birinde kullandığı müdahale türleri incelenmiştir. Sekizinci sınıf öğrencilerinden oluşan 20 kişilik bir sınıfta "Kavşak Düzenleme" probleminin (2 ders saati) uygulamasına ait videolar transkript edilmiş, gözlemci notları ve öğrenci çalışma kağıtları ile desteklenerek içerik analizi yöntemi ile analiz edilmiştir. Elde edilen bulgular öğretmenin en çok duyuşsal müdahaleler ile ortam ve etkileşime yönelik müdahalelerde bulunduğunu göstermektedir. Öğretmenin içeriğe yönelik müdahalelerden kaçınması, stratejik müdahalelerde bulunmaması ya da müdahale etmemesi ise öğretmenin modelleme sürecini etkilemek istememesinden kaynaklandığı düşünülmektedir. Öte yandan sunum ve değerlendirme aşamasında daha etkin olması modelleme sürecinde nerede nasıl müdahale edeceği konusunda tereddütler ve zorluklar yaşadığını desteklemektedir. Yapılan çalışmalar ve bu araştırmanın sonuçları matematiksel

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** Asst. Prof. Dr., Kilis 7 Aralık University, Kilisli Muallim Rıfat Faculty of Education, ORCID: 0000-0003-3202-8852, e-mail: seda.sahin@kilis.edu.tr
*** Assoc. Prof. Dr., Adıyaman University, Faculty of Education, ORCID: 0000-0002-5301-9034, e-mail: mfatihdogan@adiyaman.edu.tr
**** Prof. Dr., Adıyaman University, Faculty of Education, ORCID: 0000-0002-2412-5882, e-mail: rgurbuz@adiyaman.edu.tr

modellemenin öğretiminde öğretmenlerin özel bir pedagojik bilgiye sahip olmaları gerektiğini ortaya koymaktadır.
Anahtar Kelimeler: Matematiksel modelleme, öğretmen müdahalesi, sınıf içi uygulamalar.

Introduction

The importance of mathematical modeling in mathematics education is increasing in Turkey, as in many countries, and modeling skills are becoming an essential part of curricula. Despite the increase in the variety and quality of studies on mathematical modeling, this needs to be adequately reflected in classroom practices. There may be many reasons for this, but one of the most important is that mathematical modeling differs from traditional problem-solving. Compared to traditional problem solving, mathematical modeling has a complex structure and a cyclical process that may cause a cognitive obstacle (Galbraith & Stillman, 2006), and this is a common problem that students have difficulties in the modeling process (Tropper et al., 2015). Difficulties experienced in teaching mathematical modeling are not limited to student difficulties; teachers also experience various problems in this process (Borromeo Ferri, 2018; Tropper et al., 2015). Considering that the most critical factor in having effective teaching is teaching practices (in-classroom practices) (Blum & Borromeo Ferri, 2009), the teachers have a crucial role in this process (Niss et al., 2007; Blum & Leiss, 2005). Research on the issues that the teacher's role in implementing mathematical modeling problems draws attention to *the balance* between students' independent work and teachers' directly guiding them. (e.g., Borromeo Ferri, 2018; Blum & Borromeo Ferri, 2009; Leiss, 2007). One of the difficulties teachers experiences while enacting the task is teachers' decisions about whether or how to intervene during an autonomous activity such as mathematical modeling (Leiss & Wiegard, 2005). Thus, it is essential to eliminate the teachers' lack of knowledge in choosing the appropriate method when following the students' solution processes and intervening in their difficulties (Blum & Leiss, 2005). The relevant research on teachers' pedagogical approaches while enacting mathematical modeling activities and methods of identifying and responding to emerging student difficulties (e.g., Blum, 2005; Blum & Leiss, 2003; Borromeo Ferri, 2018; Garfunkel et al., 2016; Leiss & Wiegand, 2005; Stender, 2018) point out recommendations that might serve as a guide for teachers on enacting modeling activities and the types of interventions that the teachers may have during those activities. The results from all these studies show that a *balance between "teacher guidance" and "independent student work"* should be achieved based on successful practices. In this context, strategic interventions that give students metacognitive hints are sufficient to establish the *balance*. The strategic interventions might be such as "Imagine the situation," "What are you aiming for?", "What is still missing?", "Does this result fit real life?" and should be interventions that do not violate the students' independent work (Blum & Borromeo Ferri, 2009). Of course, there is no set of rules to follow in teaching mathematical modeling, and it is not true that one intervention is always more effective than the other. However, strategic support based on the correct diagnosis has a decisive effect on the students' success (Stender & Kaiser, 2016). The important thing is that the teacher evaluates the situation within the context and uses the most appropriate intervention (Blum & Leiss, 2003; Blum & Borromeo Ferri, 2009; Leiss & Wiegard, 2005). Presenting an intervention framework to teachers is the first and most crucial step in understanding the situations that the teacher will face (Leiss & Wiegard, 2005). Because even the best and most experienced teachers have the space for improvement, criteria (such as intervention framework) in quality mathematics teaching should be a part of teacher education (Blum & Leiss, 2003). The purpose of this study is to examine the interventions of a mathematics teacher who attended a mathematical modeling workshop and discuss these interventions' effects. This research aims not only to reveal teacher interventions but also to show how mathematical modeling activities are enacted in the classrooms. Thus, the research question is, "What are the types of interventions a mathematics teacher uses during the classroom implementation of the mathematical modeling activity?".

Literature Review and Theoretical Framework

Teacher support is indispensable in the mathematical modeling process so that students can work as autonomously as possible and be successful in mathematical modeling problems based on real situations. However, how this support can be realized is not a fully answered question (Stender, 2018). In research examining the role of the teacher in managing the whole process in mathematical modeling practices (e.g., Blum, 2005; Borromeo Ferri, 2018; Manouchehri et al., 2020; Leiss & Wiegand, 2005) and in research on teacher interventions that can be demonstrated when faced with student difficulties (e.g., Didiş et al., 2016) emphasized that teachers should have pedagogical competence in teaching mathematical modeling. Manouchehri et al. (2020) investigated the nature of teacher interventions in the modeling process and the effects of these interventions on fifth-grade students' modeling practices. They revealed that the interventions directly affected students' reasoning, structuring, mathematical work, and testing of their ideas.

One of the teacher competencies in teaching mathematical modeling, which Borromeo Ferri and Blum (2009) laid the foundations for and which Borromeo Ferri (2018) examined in detail, is the instructional dimension. The components of this competency are classified as (a) planning a lesson based on mathematical modeling activity, (b) implementing the activity, and (c) making appropriate interventions during the implementation and giving support and feedback to the students. In the instructional dimension, the competencies of making and implementing a lesson plan on mathematical modeling problems are at the forefront. This is a critical competency for maintaining the balance between theory and practice. The instructional dimension covers all practices, including how teachers enact modeling activities in the classroom, support students during the implementation, and give feedback. Borromeo Ferri (2018) defined competence directly related to this teaching competence as the diagnostic dimension. The diagnostic dimension includes the ability of teachers to see the difficulties that arise in the mathematical modeling process and to evaluate mathematical modeling activities. Diagnosing the students' difficulty is the first step in providing appropriate support and feedback interventions to students during instruction. Diagnosis provides an understanding of the students' solutions, and only after diagnosing the difficulty teachers may give individual support, feedback, or intervention to their students.

Deciding which activity to implement, recognizing different models and possible students difficulties that may be encountered during the modeling process, and deciding how to implement the activity are included in the lesson planning process. In particular, anticipating students' difficulties is essential for effectively implementing the activity. Because anticipating such difficulties enables teachers to understand student questions or their explanations during the implementation and to have effective intervention (Borromeo Ferri, 2018; Leavitt & Ahn, 2013; Stender, 2018). In addition, teachers' knowledge and interpretation of the activity directly affect students' diagnostic and intervention actions in the problem-solving process (Blum & Borromeo Ferri, 2009; Blum & Leiss, 2005; Krauss et al., 2008). The first step in implementing the mathematical modeling activity is ensuring that all students can access the problem text and understand it individually (Blum & Borromeo Ferri, 2009). While the teacher or volunteered students can read the problem situation, all students may be asked to read the problem individually (Blum, 2011; Borromeo Ferri, 2018; Leavitt & Ahn, 2013; Schukajlow et al., 2012). It is vital to ensure that students understand the problem and form their ideas before starting group work. Students may work on modeling activities individually, but it is more appropriate to have students work in groups as this process is open to interpretation, idea generation, and discussion (Borromeo Ferri, 2018; Didiş Kabar et al., 2021; Ikeda et al., 2007; Koellner-Clark & Lesh, 2003; Zawojevski et al., 2003). One of the issues that the teacher should pay attention to is that group discussions are not teacher-oriented; to be student-centered. In this way, students will freely explore the problem situation and have the opportunity to discuss their ideas. Thus, the teacher should minimize the time she is involved in the group discussion (Blum & Borromeo Ferri, 2009; Leavitt & Ahn, 2013; Stender & Kaiser, 2016) but should not completely isolate herself from the practice (Blum & Leiss, 2005). The success of students who work independently with teacher support and those who work entirely alone is not the same. Students who lack the teacher's support may have motivational, social, and cognitive problems that may cause them to be unsuccessful (Blum & Leiss, 2005). For this reason, the teacher should adjust the level of helping students well (Blum, 2011; Stender & Kaiser, 2016;). This support should be adapted based on the students' performance (Van de Pol et al., 2010). Teachers should decide which intervention to use and at what level by evaluating the existing situation (Leiss & Wiegand, 2005). For example, in the study of Leiss and Wiegand (2005), the teacher preferred not to intervene and only observed in a situation where students could find solutions independently. However, when the teacher thought that the students could not reach the desired solution by discussing, the authors showed examples that the teacher intervened in the content by explaining.

The teacher must be open to all students' ideas during the implementation. Mathematical modeling problems differ from traditional problems, and students should not be directed to use a particular method (Garfunkel et al., 2016; Leavitt & Ahn, 2013). Students should experience complexity and uncertainty about which variables to use and how to use them. Students should be allowed to express themselves to clarify these ambiguities while walking around the groups. When students are asked to explain, expressing their thoughts aloud enables them to review or clarify their ideas (Stender, 2018; Stender & Kaiser, 2015; Stender & Kaiser, 2016). This intervention is the most used and influential teacher intervention in Stender and Kaiser's (2016) experimental research. Manouchehri et al. (2020) also determined that the most used intervention by the teacher in the fifth-grade students' modeling process was asking questions for explanation (30.2%). At the same time, the teacher has an idea about students' thoughts and works with this intervention, and it has a diagnostic role in making a different intervention when necessary (Stender & Kaiser, 2015). If the student is taking the wrong path or making a meaningless explanation, instead of saying that the student's ideas are not correct, the teacher might say, "I do not quite understand what you mean." Such a prompt might enable students to reinterpret their ideas. This way, the student or her groupmates can realize the inconsistency or mistakes and make the necessary changes (Leavitt & Ahn, 2013). Deci et al., (1999) emphasize that the feedback given to students should be wrong or correct and informative about how to proceed. In addition, when faced with such a situation, teachers may encourage students to think about it by presenting the student idea to group discussion with questions such as "What do you think about your friend's idea? Do you agree with her?". In the modeling process, questions that support the reasoning, rather than guiding questions, should be preferred, enabling students to connect their mathematical model with the real-world context. However, such interventions are less frequently used by teachers. In the studies of Manouchehri et al. (2020), the interventions for discussion (3.5%) were the least preferred. In summary, the main criterion that the teacher will pay attention to, including giving information or explanation to the students during the implementation phase of the activity, should be to contribute without affecting the students' modeling process.

Reporting is the last step in the modeling process. Although presenting students' reports and discussing their models is a neglected stage in classroom practices, its necessity cannot be ignored (Hestenes, 2010). Because at this stage, the student groups can recheck their model's functionality while explaining what, why, and how they handled it in the modeling process. At the same time, presenting their models allows them to communicate mathematically with their friends (Garfunkel et al., 2016; Hestenes, 2010). For this reason, when students finish their solutions, it should be ensured that they share their models with their classmates. If time is sufficient, all groups may present their models (Garfunkel et al., 2016). However, if there is no such opportunity, it is sufficient for the teacher to have the groups that have different models present their group works. First, the teacher should clearly state the expectations from students in their presentations. When students complete their solutions, they should prepare their presentation by knowing what is asked from the presentation (Borromeo Ferri, 2018; Leavitt & Ahn, 2013). It is crucial to hold a question and answer session at the end of each presentation with the whole classroom participation. This will enable the students to listen to better understand the presented model and the opportunity to look at the presentation and models prepared by the students presenting from a different perspective. Thus, teachers should encourage students to ask questions and participate in discussions (Leavitt & Ahn, 2013).

Analytical Framework

The relevant research on teachers' role and interventions in teaching mathematics, specifically in teaching mathematical modeling, classified teachers' interventions under different categories. Leiss et al., (2010) classified teacher interventions under four main categories: collecting information (e.g., what data is given? What the problem ask for?), selecting information (e.g., What is the most important part here?), linking information (e.g., link information from different sources.) and knowledge of the process (e.g., put yourself in the given situation and think about it.). Leiss and Wiegand (2005) compiled research on teacher interventions in mathematics teaching. They classified all types of interventions into four categories: affective (e.g., increasing motivation), metacognitive (e.g., encouraging thinking), related to content (e.g., providing an explanation), and related to the organization (e.g., guiding discussion). The researchers defined a fifth category as a prerequisite for these four categories: diagnostic intervention. This intervention means that teachers choose the most appropriate intervention during the current situation. When the teachers cannot make the correct diagnosis, this may lead them to have an ineffective intervention; thus, an accurate diagnosis is a prerequisite for successful intervention (Stender & Kaiser, 2016). Leiss and Wiegand (2005) emphasize that the correct intervention is as necessary as the correct diagnosis. Having information about students' achievement levels or personal characteristics is as important as a diagnostic intervention in determining the proper intervention. There are also parallel factors that influence teachers' interventions. Leiss (2005) lists these factors as the timing of intervention, the type of problem students engaged in, the level of intervention, and the intervention method. Blum and Leiss (2005) examined teacher interventions during the mathematical modeling process and classified the interventions as interventions for content, environment and interaction, motivation, and metacognition. Metacognitive or meta-level interventions are similar to strategic intervention concepts used by Stender (2018), defined as support that enables students to think independently at a high level. Stender (2018) emphasizes that strategic interventions correspond to the steps of the mathematical modeling cycle. For example, "What does the mathematical result you found mean in the real world?" is one of the reference questions in strategic intervention, the steps of modeling cycles for the interpretation/evaluation or validation. Blum and Borromeo Ferri (2009), in their project (DISUM, COM [2]), in which they examined teacher interventions, found that spontaneous teacher interventions were mostly content or environment-oriented and reported that no interventions aimed to ensure students' independence. Stender (2018) stated that in cases where motivational and strategic interventions are insufficient, strategic intervention should be made regarding the content, which is the most challenging intervention for teachers. Blum and Borromeo Ferri (2009) and Leiss (2007) stated that more than the teacher's experience is needed to diversify the intervention types or use the appropriate one as the interventions were content-oriented.

This study follows Borromeo Ferri's (2018) teacher competencies framework in teaching mathematical modeling (for detailed information, see Borromeo Ferri, 2018, p.5) and utilizes the research on teacher interventions (Borromeo Ferri, 2018; Blum & Leiss, 2005; Leiss & Wiegard, 2005; Stender, 2018). Accordingly, the types of interventions for the analytical framework of this research are *diagnostic intervention* (*from* Leiss & Wiegard, 2005), *affective intervention* (*from* Blum & Leiss, 2005; Leiss & Wiegard, 2005), *environment and interaction-oriented intervention (from* Blum & Leiss, 2005), *content-oriented intervention (from* Blum & Leiss, 2005; Leiss &

Wiegard, 2005) *and strategic intervention (from* Blum & Leiss, 2005; Leiss & Wiegard, 2005; Stender, 2018).

Method

This study aims to determine the types of intervention teachers use in a lesson in which the mathematical modeling problem is enacted. This research objective focused on just one of the classroom implementations of a larger project called *Designing, Applying, and Evaluating a Learning Environment Through Mathematical Modeling: Interdisciplinary Transition*. A case study methodology as a qualitative research design was used to identify teacher interventions (Denzin & Lincoln, 2005).

Participants

The participants of this study were a mathematics teacher with six years of experience participating in mathematical modeling workshops and twenty 8th Grade students. The names of the participants in the study are pseudonyms.

Data Collection

The teacher training program, which included theoretical and practical lessons on the theoretical structure of mathematical modeling, the characteristics of modeling problems, planning a lesson to be conducted with these problems, and implementing the lesson, lasted *six* weeks (for detailed information, see Sahin, 2019). After receiving theoretical training on implementing the mathematical modeling problems, the participant teacher conducted a small teaching experiment (4 students from 8th grade) to practice how to implement modeling tasks. Then, the participant teachers and researchers involved in the project analyzed the small teaching experiment. The teaching experiment video was evaluated from different aspects, such as the teacher's classroom and time management, teacher-student interaction, and student-student interaction. The teaching experiment, in which the teacher had the opportunity to evaluate himself, contributed to the teacher's experience in enacting modeling tasks before moving on to the whole class implementation.

The whole class implementation continued for four weeks. One of the researchers participated in all classroom sessions as an observer. There were, in total, four mathematical modeling problems enacted each week for two hours. This study focuses on one of the problems called the "Intersection Problem[1]" which was implemented in the second week for two hours (see Appendix). The students worked on the problems in groups of four formed by the teacher. All sessions were video and audio recorded, and the recordings were for the whole classroom and the group work so that each group's work could be examined in detail.

Data Analysis

All the recordings of the implementation of the problem were transcribed, and the sections where teacher-student interaction was specified. These sections were analyzed using descriptive analysis (Mayring, 2015). The coding scheme, taking into account the relevant literature used in the analysis for the categories, explanations, and sample intervention types, is given in Table 1.

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[1] This problem was desingned by participants of *Designing, Applying, and Evaluating a Learning Environment Through Mathematical Modeling: Interdisciplinary Transition Project* (Gürbüz et al., 2018, pp. 101-102).

Table 1
Coding Scheme for Teacher Interventions

Type of intervention	Explanation	Example Intervention Type
Diagnostic intervention	Preliminary intervention to determine the most appropriate response to the situation	Can you explain what you are doing here?
Affective intervention	Trigger intervention that will keep students in their work	Read more carefully what is asked in the problem.
Environment and interaction-oriented intervention	Intervention that supports students' communication with each other	Do you agree with your friend's opinion? Why?
Content-oriented intervention	Situational intervention that will affect the modeling process	It might be easier if you make a circle instead of a rectangle.
Strategic intervention	Intervention in students' metacognitive thinking	Do you think this idea is suitable for real life?
No-intervention	Being unresponsive to the situation or making evasive explanations	You should have known the answer to that.

A consensus was reached by discussing the different codings. The coding was done separately by the researchers, and the level of consistency was checked by the cross-comparison method. In addition, the observer researcher's field notes and the students' solution papers supported the analysis to increase the comments' consistency level by approaching the data in depth and holistically.

Results

This section presents the findings of teacher interventions with examples from the sections where teacher-student interaction happened. First, the following descriptive information is necessary to help readers imagine the classroom atmosphere: Before starting the lesson, the teacher determined the groups and arranged the classroom setting. In the first part of the lesson, before handing out the problem, the teacher started with a short reminder. Since the students had experience engaging in the mathematical modeling problems from the previous class, the teacher should have talked about how to engage in group work and lesson structure.

1 **Teacher**: All, you are in a group of four. You may have different ideas,
2 but in the end, we want your group’s conclusion that you all agree on. Okay?
3 You can discuss the question among yourselves, argue, and scribble, but
4 first of all, you need to read the question I handed out. Everyone should read the question individually.
5 If there is a point that you do not understand, ask for clarification on the question in advance.
6 ***After a few minutes…***
7 **Teacher:** Yes, did you read the question?
8 (some of the class said yes, and some said no)
9 **Teacher:** Is there anything not understood here?
10 **Melih:** Sir, will you give the area of this here? (showing the intersection in the picture)
11 **Teacher:** No. You will try to find the area. So you see the picture there. Based on that,
12 you can make a logical conclusion.
13 **Ali:** (showing the intersection picture to the Teacher) Teacher, are we going to deal with this?
14 **Teacher:** Yes, we will plant flowers there. You will plant the inside of the intersection you see.
15 A student: Sir, there is no information, no information. No meter information, nothing, Sir.
16 **Teacher:** Maybe not. What I want from you is to be able to guess it.

The teacher, who wanted each student to read the problem individually, distributing the problem to all students. From the teacher's first explanations (1st-3rd lines), he emphasized that he encouraged students to work in groups (*environment and interaction-oriented intervention*) but first wanted that they should understand the problem individually before moving on to group work (lines 3rd-5th) (*affective intervention*). After giving his students a few minutes to read and understand the problem, he asked the class if they understood the problem (*diagnostic intervention*). When one of the students asked about the intersection area, the teacher effectively intervened with the student (lines 11-12). The lack of numerical data in the problem generally needed to be clarified for the students. When a student expressed this confusion, the teacher avoided giving a guiding explanation and again made an affective intervention (line 16). After this dialogue, the teacher started the group work by stating that the flower types and unit prices are given in the Appendix and that the unit represents a flower seedling.

During the implementation, the teacher followed each group's work by observing both from a distance and closely. The teacher, who preferred not to be involved in the discussions between the students, mainly intervened when they asked questions. For example, the third group discussed how to calculate the area covered by the flowers, but when they could not agree, they wanted to ask the teacher:

1 **Ahmet:** Sir, can you look? What would the area covered by the flowers be?
2 **Melih:** Sir, the area covered by the flowers?
3 **Teacher:** (To the whole class) All, we have all passed this intersection dozens of times. We have seen it in bloom, too. I have an approximation of the area covered by the flowers there.
4 **Ali:** I do not solve this question (in a way that the teacher will not hear).
5 **Melih: T**here is no such thing; we will solve it.

As seen in this dialogue, the teacher did not explain to the students in the group but the whole class (3rd line). This explanation, considered an affective intervention, was not sufficient or meaningful for the students (4th-5th lines).

While the teacher followed the groups' solutions, he watched the students in the fifth group without interfering in the discussion about whether they should use the flowers with the lowest cost or make a design they wanted and determine the cost accordingly, but after a while, he was included in the discussion at the request of the students.

1 **Adil:** Which is the cheapest?
2 **Ferit:** Violet.
3 **Adil:** We can have violets.
4 **Barış:** He did not say anything like that.
5 **Adil:** How do we do it conveniently, then?
6 **Barış:** For one thing, he never said anything about the most convenient or the cheapest. Why are you saying such a thing?
7 **Adil**: Look, it says to determine the cost.
8 **Barış:** Okay, it says, determines the cost. It does not mean it is cheap. He asks how we
9 determine the cost.
10 **Ferit:** We will choose the cheap one. Cheaper is better.
11 **Barış:** Okay, it says to determine the cost; it does not say to do what is cheapest.
After a few minutes…
12 **Barış:** (He turns to the teacher when he cannot convince his friends) It did not give any information.
13 **Adil:** Yes, Sir. There needs to be more information.
14 **Teacher:** Guys, there is enough information. You have to conduct some of this information yourself. Isn't it?
15 **Barış:** Teacher, how do I know how much space this tassel flower occupies?
16 **Teacher:** From your picture. You are smart kids. You have a photograph.
17 **Barış:** Sir, it did not say the cheapest either.

The students tried to determine the variables in the modeling process. However, they could not succeed because of the difficulties they experienced in understanding the problem. Especially in the first part of the dialogue (Lines 1-11), when Barış could not convince his friends after the differences of opinion among the students, he turned to the teacher. Barış and his other friend asked the teacher for help because they believed there was insufficient information on the problem (Lines 12-13). The teacher had an affective intervention here. However, it is clear from the next student question that this intervention was insufficient for the students (Line 16). Upon this, the teacher again made a motivating affective intervention (You are smart kids) and a content-oriented intervention to give the students a clue that they could benefit from the flower pictures in the problem. The problem that the students experienced in the understanding step, which is the first step of the modeling process, was not resolved, which led to in-group conflict for a while. Eventually, they decided that the least cost would be the most appropriate solution and created a suitable model.

The students tended to continue with the solution by getting approval from the teacher about the correctness of their ideas throughout the modeling process. In such cases, the teacher made affective interventions that encouraged students to do what they thought without directing them. For example, while calculating the radius of the intersection, a student from the first group asked the teacher whether it would be correct to determine the diameter after counting the paving stones surrounding the intersection and calculating the circumference of the intersection. Before this question, the student shared his opinion with his group of friends but wanted to ask the teacher before having a final decision.

1 **Gazi:** Sir, can't we first count these (paving stones), calculate the circumference, and then find the radius?
2 **Teacher:** Count. Why not count?

As in this short dialogue, even if the students discussed their ideas in the group when they could not convince each other, they tended to ask the teacher for their ideas, which they believed to be correct, with the desire to always get approval from the teacher. In such cases, the intervention shown by the teacher was generally affective, short-answer interventions (yes, you are right, you are right, etc.).

While walking around the class, the teacher observed that the students were trying to remember the area formula of the circle and arguing among themselves. However, he did not directly interfere with any group. After the first group decided that the radius of the intersection in the picture was six meters on average based on the dimensions of the car and the paving stones in the picture, they asked the teacher for help because they could not remember the area formula of the circle. The following dialog was recorded at this time:

1 **Metin:** Sir, is not the area (area of a circle) diameter x pi?
2 **Mustafa:** Area is radius x radius x pi
3 **Teacher:** We learned this topic last year.
4 **Metin:** Radius squared x pi squared.
5 **Mustafa:** (turning to Ali) No. Isn't it radius equal to 6? 6 x 6 = 36. It will be 36 x 3.
6 **Teacher:** (He asks the class to listen to him) If you remember last year, which most of you do not remember… The area of a circle with a radius of r is πr^2 (written on the board). You can approximate the value of Pi.

The teacher did not first intervene in the students' questions regarding the area formula of the circle (Line 3) and then make a statement to the whole class and remind them about what they learned in the previous year. While following the groups, the teacher noticed that most could not remember this formula and argued among themselves, but he did not intervene. The teacher intervened in this difficulty by explaining to the whole class together after the first student question (Line 6). This intervention, which is necessary for students to continue their work, was considered a content-oriented intervention.

The teacher mainly had affective interventions during the implementation but also used different interventions, albeit in small numbers. While visiting the groups, he encouraged students to work together. For example, when the fifth group said that they completed the solution, the teacher visited the group, and the following dialogue took place:

1 **Ferit:** Sir, we are done.
2 **Teacher:** Very nice. Check your solution. Maybe you can come up with a better idea.
3 **Adil:** Sir, we thought a lot. As a result, we have an approximate result.
4 **Teacher:** Did Hüseyin have any contribution? (Hüseyin was a student who did not care much about the problem during the process) Hüseyin seemed to be dealing with the phone all the time.
5 **Group:** He helped a lot (with a sarcastic tone).
6 **Ferit:** Honestly, I did it alone, Sir.
7 **Teacher:** If you think you did most of it, discuss it with your group and get their opinion.

The following dialogue occurred between the researcher and the fifth group: in this dialogue, the teacher had one affective intervention to the group who said that they finished their solutions in 20 minutes (Line 2) and had one environment and interaction-oriented intervention (Line 7). The indecision experienced by this group at the understanding stage of the problem was reflected in the whole process, and two students wanted to avoid contributing to the solution because of the disagreement between the group members. This can be seen from the above dialogue; the teacher was also aware of this situation. Since the teacher did not interact with their work after completing their group solution, the students wanted to interact with the researcher in the classroom.

1 **Group:** Sir, can you look? This is how we did it (they show their solution).
2 **Researcher:** Did you plant one flower per square meter?
3 **Group:** Yes.
4 **Researcher:** What do you think it will look like? Just imagine. There is one flower in 1 square meter.
5 One flower per 1 square meter…
6 **Group:** (no answer)
7 **Researcher:** So how much is 1 square meter?
8 **Ferit:** Something like this (drawing an imaginary square on the table with an average size of 50 cm)
9 **Adil:** Something like this (drawing an imaginary square with dimensions similar to Ferit's)
10 **Researcher:** Ferit, How long can 1 meter be?
11 **Ferit:** 1 meter… something like this (it shows 50 cm on average)
12 **Researcher:** What do you think?
13 **Mert:** I think that is it.
14 **Adil:** Sir, something like this (he shows an average length of 50 cm with his hand)
15 **Researcher:** How did you calculate?
16 **Adil:** Spanning.
17 **Researcher:** How many cm is your (hand) span?
18 **Adil:** I do not know, but it is about 25 cm.
19 **Ferit:** If it is 25 cm, it will be 50 cm by two spans. We planted a flower in this area. It already takes up space.
20 **Researcher:** Wait a minute. I could not understand this. You took one side 50 cm. Then did you take the area 50 cm by 50 cm?
21 **Ferit:** Yes, sir, something like that (other students also approve). This is how I planted it. (showing the picture).
22 **Researcher:** Well, let us review it. Discuss again. Does it make sense to you? Write your report later.

The researcher's interventions in this dialogue were not analyzed as teacher interventions. However, they were shared because it reveals how effective teacher intervention is in mathematical modeling practices. The researcher first made a diagnostic intervention (Line 1) and then strategic interventions (Lines 3-4 and 6) to determine why the students thought this way. The researcher also observed the students' disagreement and lack of communication in the group. Therefore, the researcher asked the other group members to express their thoughts and asked them to justify them. Then, the researcher asked students to explain their models. The students' idea of planting one primrose per square meter did not fulfill the reality creteria for their model. Although the students made acceptable estimates while calculating the length of the vehicles, they needed to transition from the length to the concept of area. The researcher made strategic interventions that would enable the students to notice the inconsistency in their thoughts after they showed the average length of one side to be 50 cm (Lines 7-8) and experimentally tested this with the spanners (Lines 15-19) in their figurative representation of a unit square with a side of 1 m (Lines 20-22) but the intervention was unsuccessful (Lines 20-22). The researcher tried not to interfere with the students' models directly and to preserve her status as an observer and asked the students to review their models by enabling them to think metacognitively with strategic interventions. If the teacher, instead of the researcher, asked those questions, he could be aware of these difficulties and have had affective interventions to eliminate them.

The teacher periodically reminded the students about the time to complete the problem at the beginning of the class (40–45 minutes) and gave the remaining time to finish their solutions. The teacher ended the implementation part of the problem after assuring all groups had finished their solutions to the problem.

The presentation and evaluation phase was launched after completing the problem-solving and reporting processes. At this point, the teacher requested each group to choose a student or students to present, and the presentation was ordered per the students' preferences. When necessary, other students in the group helped their friends. As a result, everyone in the group participated in the presentations. The teacher made the whole class listen to the students during the presentations and encouraged them to give feedback or ask questions. Following the presentations by the two groups, the instructor led a general class discussion and requested feedback from the students on the groups' models.

The teacher frequently used affective interventions during the implementation phase, but diagnostic and strategic interventions were used in place of those interventions during the presentation and evaluation phase. He stressed the steps of choosing the variables and making assumptions while asking the students to describe how they came up with a solution. He then demonstrated diagnostic interventions with questions to challenge the students' reasoning. For example, the dialogue with the third group about how the intersection area is calculated, which he asked all groups, is as follows:

1 **Melih:** (…) We found the intersection area to be 27 square meters. We got radius three.
2 **Teacher:** How did you find it? What was your starting point? What did you base your ideas on?
3 **Melih:** Sir, there was a car.
4 **Teacher:** Then you took the car as half a meter.
5 **A student from the class:** No, Sir, it cannot be that small.
6 **Teacher:** Did you take the car as your reference point?
7 **Melih:** Yes.
8 **Teacher:** How many meters did you get the length of the car?
9 **Melih:** We got as 2 meters.
10 **Teacher:** Hmm (obviously not convinced). So, taking the length of the car by 2 m, did you consider it side by side, or did you stack the car on top of each other?

11 **Melih:** Teacher, if we got the length of the car as two meters, there is already one meter part left.
12 **Teacher:** Hmm, got it. Yes, after that.

The teacher's inquiry on how they initially determined the area of the intersection (Line 2) was considered a diagnostic intervention. When the student said that they took the length of the car as a criterion, the teacher did not find the group's assumption appropriate for real life and asked the group, "I suppose you took the car for half a meter?". This intervention was coded as a content-oriented intervention. In the continuation of the speech, they said that they took the length of the car as 2 meters, which is inappropriate for real-life situations. The teacher's speaking style indicates that, although being aware of this, he continues without intervening. When the teacher inquired about the other groups' calculations for the intersection area, the first group again engaged in the following dialogue with the teacher:

1 **Teacher:** I was wondering how the other groups found the area. Any other ideas? Let us give Gazi a voice.
2 **Gazi:** Sir, we calculated by counting the pavements.
3 **Teacher:** By pavement, do you mean stones?
4 **Gazi:** Paving stone.
5 **Teacher:** Yes, how did you calculate from there?
6 **Gazi:** We calculated each pavement as half a meter.
7 **Teacher:** Okay, you have calculated half a meter each.
8 **Gazi: As** you can see from here, it is the radius. We counted this place and found its area accordingly.
9 **Teacher:** Did you count and find the radius?
10 **Gazi:** Teacher, we counted and found the circumference. Then from there, we find the radius and find the area.
11 **Teacher:** So, what did you find in the area?
12 **Gazi:** 108.
13 **Teacher:** Did you find 108? They found almost the same result with a different method (as Group 5).
14 **Gazi:** we counted 72 of them that we can see. It is 36 meters. It has a circumference of 36 meters. When we divide it by three, we get pi as 3, so the diameter is 12, and the radius is 6 meters.

The teacher asked first to describe how they solved the problem to help the other students understand the solution; rather than as a diagnostic intervention, the teacher asked first to describe how they solved the problem. The teacher determined what this group did while following their modeling process during the implementation phase. However, the teacher guided them to explain in detail to the whole class to ensure everyone understood their solution clearly. Therefore, depending on the teacher's intention, this intervention is called environment- and interaction-oriented. While explaining the models of all groups and encouraging others to understand and ask questions, the teacher asked students to compare their solutions and models with other groups. These environment and interaction-oriented interventions were not used to find the best or the most accurate model by comparing the students' models but were used so that they could see the deficiencies and mistakes in their solutions. For example, the teacher, who realized during the implementation phase that the fourth group calculated the intersection area as 50 square meters, wanted to let them see their mistakes by asking them to present their solution. The conversation that follows is captured at this moment.

1 **Teacher:** Now, what caught my attention is this. Some of you thought about the area and said, for example, there should be one flower in 1 square meter; some of them found it by thinking about the distance, not the area. Let us also ask how other groups found the radius. How did you find the radius?
2 **4th group members:** Sir, we took a wild guess.

3 **Teacher:** Did you guess without knowledge, or do you say it is (this) because of (this)?
4 **4th group members:** Sir, we guessed the radius but chose the most logical one.
5 **Teacher:** You say you had a wild guess but also chose the logical one… a wild guess
6 means: "I have no idea; I could not make any sense." What is the logic in this?
7 **4th group members:** Sir, now it cannot be 108 square meters, so this is the intersection. It would be ridiculous.
8 **Teacher:** For example, would it be absurd if I said it is 100 meters?
9 **4th group members:** Yes.
10 **Teacher:** Why? Maybe that car is 20 meters long? So why would it be silly? Because I thought about my car, and 100 meters would be ridiculous. What number of square meters did you accept, then?
11 **4th group members:** around 50 square meters.
12 **Teacher:** Did you start from the radius, or did you guess the area directly?
13 **4th group members:** We tried to guess by looking at the area.

As seen in the conversations above, the fourth group was asked to find their mistake based on the criteria and solution strategies that the other groups had. The teacher, who made a strategic intervention by asking them to justify their solution, thought 108 square meters was a large area for an intersection. The teacher emphasized the importance of making logical assumptions without questioning the reason behind that idea. The teacher turned to the second group's solution without intervening when the students said they only made a wild guess. In the second group, the students calculated the intersection area as 27 m ^{2,} considering that the radius would be 3 meters. After this dialogue, the other students had to reconsider the relevance of their models for real-life:

1 **Teacher:** How did you find the girls? Did you base the length of this pylon on the car?
2 **Group 2:** It seems so.
3 **Teacher:** It seems so. Understood. Like the other group, you made a decision rather than having data.
4 **Group 4:** Sir, if they found 108 square meters, this intersection would be half as much as our house.
5 **Teacher:** Hmm… That also makes sense to me now.
6 **Group 3:** Sir, they are telling the truth.

The second group was also unsuccessful in justifying their models, using estimated values without any criteria when determining the intersection area. On the other hand, the fourth group students criticized that the first and fifth groups' models were inappropriate for real life (Line 5). The teacher gave a reaction in which he agreed with this criticism. However, students in other groups also supported this idea. The teacher did not have a classroom discussion as the time was over for the lesson.

During the presentation and evaluation stage, where all groups were allowed to explain their models, the evaluation of the models was made relatively through the discussions during the presentation. However, the lesson was finished without a precise evaluation of the models and implementation. The types of intervention used by the teacher throughout the implementation are predominantly affective interventions. The environment and interaction-oriented interventions were the second most common ones, but content-oriented and diagnostic interventions were less used. The type of strategic intervention for students' metacognitive thinking was used only once during the implementation. Furthermore, there were situations where the teacher did not intervene during the implementation. All intervention types and frequencies used by the teacher throughout the implementation are given in Table 2.

Table 2
Types of Interventions Used and Frequency of Use

Type of intervention	Explanation	Frequency (n)
Diagnostic intervention	Preliminary intervention to determine the most appropriate response to the situation	3
Affective intervention	Trigger intervention that will keep students in their studies	9
Environment and interaction-oriented intervention	Intervention that supports students' communication with each other	5
Content-oriented intervention	Situational intervention that will affect the modeling process	3
Strategic intervention	Intervention in students' metacognitive thinking	1
No-intervention	Being unresponsive to the situation or making evasive explanations	5

The total number of interventions used during the implementation of the Intersection Arrangement problem was determined as 26. These intervention types are predominantly affective (n=9); strategic intervention (n=1) is the least used.

Discussion and Conclusion

One factor that directly affects the teaching of mathematical modeling is in-class practices. Thus, the teacher must be competent in implementing mathematical modeling problems (Borromeo Ferri, 2018). Teacher interventions evaluated within the scope of this competency should support students' modeling skills and mathematical learning. Considering the open-ended nature of mathematical modeling problems, it is difficult for teachers to intervene immediately and accurately to every different idea and solution approach. The teacher's intervention repertoire and practice experience will increase the rate of making the right interventions (Krauss et al., 2008). This study aims to determine the types of interventions a teacher employs when implementing the mathematical modeling problem in the preparation, application, presentation, and evaluation stages.

As mentioned earlier, the Intersection Arrangement Problem is the second mathematical modeling problem implemented in this class. Therefore, the students partially know that the mathematical modeling problem differs from other problems. In addition, the teacher made brief reminders before the lesson about the implementation time for the mathematical modeling problem and how to work together (group work). The teacher divides the students into groups of four (Borromeo Ferri, 2018; Ikeda et al., 2007) and considers the students' characteristics and performances while forming the groups (Berry, 2013; Garfunkel et al., 2016; Zawojevski et al., 2003). However, despite the teacher's attention, one group (Group 2) remained relatively passive when solving the mathematical modeling problem compared to the other groups. In another group (Group 5), communication problems were observed among students. There may be various reasons for experiencing such difficulties, such as the problem's difficulty or the students' problem. On the other hand, the teachers' consideriations of forming the groups may need to be more suitable for a lesson on the mathematical modeling problem. As in Leavitt and Ahn's (2013) studies, students may have behaviors such as a different way of participating, level of contribution to the group, and leadership role during mathematical modeling activities than in a regular classroom.

The teacher made deliberate and effective interventions when presenting the problem during the implementation of the mathematical modeling problem. Understanding the problem situation by all students is the fundamental element of the model-building process. After reading the problem, the students stated that although they understood the problem, they were unable to solve it because they believed the information provided needed to be more and more. This situation, which can be defined as "missing data" for students, is one of the challenges encountered in many studies (e.g., Borromeo Ferri, 2020; Sahin et al., 2019). Borromeo Ferri (2020) also obtained similar results in examining a ninth-grade teacher's implementation of a mathematical modeling problem and revealed that the teacher made small non-directive interventions. This study determined that the teacher had similar interventions, which were called affective interventions. The teacher's pre-lesson preparations for presenting the problem and his foresight into potential student questions may have contributed to his affective intervention during the preparation stage.

The teacher walked around the classroom, examined every group, and generally preferred to monitor the work from a distance, as advised in the second stage of the modeling cycle, which aims to solve the problem (e.g., Blum & Borromeo Ferri, 2009; Leavitt & Ahn, 2013; Stender & Kaiser, 2016). He only listened to the students' work in the parts where he asked them to describe what they had done. However, this diagnostic intervention is one of the central interventions that will allow students to review their thoughts and assist the teacher in determining how and at what level to intervene (Stender & Kaiser, 2016); the primary motivation for the teacher to act in this way is to maintain the students' highest level of freedoms. Because following this interaction, the students were frequently blocked from their circumstances by powerful interventions that motivated them. This could result from the teacher education program's emphasis on fostering an environment in the classroom that supports students' independent work by avoiding directive interventions. There are, however, more effective ways for the instructor to entirely stepback during the implementation (Blum & Leiss, 2005). However, the relevant literature showed that the teacher should be a guide in creating such an environment in their teaching, and it is shown theoretically and practically that it is critical to establish a *balance with strategic interventions* (Blum & Borromeo Ferri, 2009; Borromeo Ferri, 2018; Leiss, 2007; Stender, 2018, 2019). As a natural consequence, it is crucial that the teacher not only intervene when students ask for help in the modeling process but also create a learning environment where they can put forward their ideas and question and discuss other ideas. However, many of the teacher's interventions during the implementation stage were reactions to the students' request for assistance and did not reach the level of strategic intervention. Leiss (2010) also stated that teachers do not prefer to use strategic interventions and found that such strategies remained in the background for teachers.

One of the difficulties encountered during the implementation phase was that the students had difficulties transferring the formulas/operations they knew mathematically to solve real-life problems. Such problems when bridging the real-world and the mathematical world are typical difficulties in other studies (e.g., Borromeo Ferri, 2020). An example of these difficulties was their inability to adapt the area formula of the circle, which should be used in solving the problem, despite having seen it in previous lessons. Similarly, the fact that students (Group 5), who did not have any problems in mathematical operations related to calculating the area of the square, took the side length of a square area of 1 m^2 as 50 cm during the modeling process was a difficulty they experience while passing from the mathematical world to the real world. The teacher did not notice this difficulty experienced by the students or did not intervene. While dealing with this challenge, the students wanted to share their models with the observer researcher. The researcher, with strategic interventions, tried to have the students question their ideas. However, cognitive inquiries were insufficient for students to understand their mistakes, as the researcher refrained from intervening due to his role as an observer. This situation, which the teacher ignored or failed to notice during the implementation phase, emerged once again during the presentation and evaluation phase. Here, the teacher only asked why they made such a design. However, he needed more information about its applicability in real life because he did not question it mathematically. When the other students were asked about planting a flower in their models in an area measuring 1 m^2, they were informed that this distance was improper. However, the presentation was finished without adequate discussion. This is a remarkable example of how a mathematical difficulty can be revealed through mathematical modeling and how important the teachers' interventions are in this process.

During the presentation and evaluation phase, the teacher asked the two groups to present their works and ensured that the other groups shared their ideas about the presented models. The teacher used more diagnostic and strategic interventions at this stage than the implementation process. While questions such as "What did you do here?" were only asked to follow how the students carried out the modeling process, questions such as "Why do you think so?" were used within the scope of the diagnostic intervention during this phase and continued with strategic interventions. The teacher specifically stressed and intervened when the students' theories regarding how the radius of the intersection was determined during the presentation and evaluation stage. This is crucial to the problem's solution since identifying the intersection area with reasonable assumptions allows models to form that apply to real-world situations. However, the model's appropriateness for the real world is affected by variables including planting distance, flower orders, and flower cost. The teacher's concentration may improve the students' ability to express their unique thoughts on factors and assumptions that are significant to them. As a result, the teacher's knowledge and expectations about the activity directly affect the diagnosis and intervention practices in the problem-solving process (Blum & Borromeo Ferri, 2009; Blum & Leiss, 2005; Bozkurt & Özbey, 2019; Krauss et al., 2008). For example, when Blum and Leiss (2005) asked the students what they wanted to say with a word in the problem text during the classroom implementation of a modeling problem, the teacher explained the problem in his way, and this situation led to the students to a standard model.

During the presentation and evaluation stage, the teacher felt more freedom to make changes. This is because the teacher withdraws from the problem-solving process to not interfere with the students' modeling process. Nevertheless, once the students had created their models, the possibility of interference with their models may no longer exist. Teachers frequently struggle with knowing when and how to intervene in implementations of mathematical modeling (Leiss & Wiegard, 2005). However, because mathematical modeling problems are a unique form of the problem and require distinct competencies, learning them takes time for both students and teachers (Borromeo Ferri, 2020; Sahin, 2019). However, considering the learning outcomes, many studies, such as this study, showed that mathematical modeling problems are worth implementing in the classroom. Successful implementation of modeling problems would require teachers to have theoretical understanding and practical ability in mathematical modeling, particularly to maintain a *balance* between students' independent work and guidance (Blum & Borromeo Ferri, 2009; Borromeo Ferri, 2018; Leiss, 2007). When assessing the teacher's interventions overall, the teacher took a more active role in the planning, implementation, and evaluation phases and presented a planned approach while selecting the different types of interventions. However, it is noteworthy that he primarily played the role of an observer during the implementation phase, followed the students' suggestions without interfering, only intervened when the students asked him to, and employed affective interventions. Considering that the intersection problem was the second problem the teacher presented to the class, the teacher attempted to maintain a balance despite his hesitation. This assertion is supported by the fact that he focused more on affective interventions and avoided or did not have content-oriented interventions. While the teachers focused on what the students did in the implementation phase, it was focused on how they did in the presentation and evaluation phases. On the other hand, his inexperience may contribute to his limited use of strategic interventions. The relevant research showed (e.g., Blum & Borromeo Ferri, 2009; Leiss, 2007; Sahin et al., 2020) that although teachers are experienced, they tend to make content and environment-focused interventions rather than strategic ones and need support in deciding on the best type of intervention. This could result from the teacher's intervention spectrum having expanded in line with their experiences and shifting from affective to strategic interventions. In the project of which this study is a part, the teacher implemented many modeling problems in the classroom. For this reason, it is crucial for teachers to practice mathematical modeling, assess themselves, and improve in order to provide high-quality mathematics instruction.

The results of this study, as discussed in detail above, are consistent with the results of other studies in the literature. The results show that the teacher's ability to enact mathematical modeling problems directly affects students' acquisition of mathematical modeling competencies. It is typical for the teacher to experience some difficulties in this process. For example, the teacher's refraining from intervening, not directing students, and not participating in group discussions without students' demand was among the teacher's difficulties in this study. Establishing the *balance* between "not violating the independence of students" and "guiding" them depends primarily on the teacher' theoretical competence. In this regard, the teacher needs to have knowledge about the problems to be enacted in the class and the types of interventions that can be used. Using appropriate intervention types at the right time is within the teacher's pedagogical knowledge. In the studies conducted (e.g., Blum & Borromeo Ferri, 2009; Leiss, 2007; Temurtas et al., 2021), the fact that even experienced teachers have difficulties in determining the appropriate intervention type in mathematical modeling practices, as in this study, reveals that special pedagogical knowledge is required in the teaching of mathematical modeling. However, before having the knowledge to implement, teachers need to have more detailed content knowledge about mathematical modeling. Thus, studies that examine the modeling process more comprehensively are needed.

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Appendix

Intersection Problem

Osman Bey, who works in Adıyaman Municipality, uses various flowers to decorate the intersections with different motifs so that the city has a beautiful appearance. The intersection seen in the photo above will be decorated with different kinds of flowers and motifs by Adıyaman municipality. The municipality asks for your help as a mathematician in this regard. Your task is to determine the number and cost of flowers needed to be able to line the intersection with flowers. Prepare a report detailing how you determined the number and cost of flowers.

NOTE: Do not take into account the bloom times when creating your model. Plants will be planted by the municipality to bloom in the same period. The unit price of the flowers to be used is given in the table.

Name of flower	Image of flower	Unit price	Name of flower	Image of flower	Unit price
Violet		**40 kr**	**Coxcomb**		**50 kr**
Tulip		**60 kr**	**Primrose**		**75 kr**
Hyacinth		**60 kr**	**Chrysanthemum**		**50 kr**