

Design of a Toolbox for Kinematic Analysis of Jansen's Linkage

Semir Sünkün^{*1} , Berke Oğulcan Parlak¹ , Alper Yıldırım² , Hüseyin Ayhan Yavaşoğlu¹ 

¹Mechatronics Engineering, Yıldız Technical University, Istanbul, Turkey

²Mechatronics Engineering, Marmara University, Istanbul, Turkey

(semir.sunkun@yildiz.edu.tr, boparlak@yildiz.edu.tr, yildirim.alper@marmara.edu.tr, hayhan@yildiz.edu.tr)

Received:Sep.08,2022

Accepted:Sep.16,2022

Published:Oct.10,2022

Abstract— Utilizing industrial robots is an efficient method for addressing the labor crisis and advancing industrial technologies. As a result, industrial robots are becoming increasingly popular. Additionally, the widespread use of industrial robots will increase the interest in robot propulsion mechanisms. Legged robots should be primarily investigated because of their potential advantages. Among leg mechanisms, Jansen's linkage (JL) has gained popularity due to its organic walking motion, scalable design, and simple drive by rotary input. However, the highly nonlinear nature of JL makes its analysis challenging. The research provides a user-friendly toolbox design that visualizes the toe trajectory and simultaneously calculates the step height by performing a kinematic analysis of the JL using the user-supplied link lengths. In this way, the study contributes significantly to the design phase of legged robots and reduces the amount of time required.

Keywords : *Toolbox, Kinematic Analysis, Jansen's Linkage, Path Simulation, Step Height*

1. Introduction

The mechanisms used in many industries such as agriculture (Guo et al., 2020), automotive (Jiang et al., 2022), healthcare (Zhang et al., 2021) and aerospace (Meng et al., 2022) are tools that convert input motion, force or torque into output motion, force or torque. As one of the building blocks of industry, these tools include linkages, gears and gear trains, cams and followers, and combinations of these. Among these, linkage mechanisms are the most widely used mechanisms in walking robots.

Leg (linkage) mechanisms are mechanisms that are driven by one or more crank links and convert rotary motion into gait motion. The advantages of legged robots over other types of robots, such as wheeled robots, can be summed up as improved mobility, the capacity to handle obstacles, active suspension, energy efficiency, and potential speed (Song & Waldron, 1989). Due to these advantages, leg mechanisms are frequently encountered in the literature. The most popular leg mechanisms are Chebyshev Lambda Linkage (CLL) (Liang et al., 2012), Rygg's Mechanism (RM) (Rygg, 1893), Klann Linkage (KL) (Lokhande & Emche, 2013) and JL (Parekh et al., 2014). Developed by Chebyshev in 1870, CLL converts rotational motion into translational motion with constant velocity based on the four-bar mechanism. Developed by Rygg in 1893, RM was used in a walking machine known as a mechanical horse, with the rider transferring power to the machine by turning pedals. Developed by Klann in 1994, KL is a planar six link mechanism modified from the Stephenson type III kinematic chain (Klann, 2001). Developed by Jansen in 1991, JL is a planar eleven link mechanism designed to simulate the walking motion of mammals (Jansen, 2007). The JL is superior to other leg mechanisms with its advantages such as scalable design, energy efficiency, load carrying capacity and natural toe trajectory (Nansai et al., 2013). Of these parameters, the toe trajectory is an adjustable output of the JL and is important for the walking characteristics of the legged robot.

The trajectory of the mechanism is determined by JL's big toe. The crucial step height for the leg mechanism is determined by this trajectory. Therefore, the analysis of the trajectory is of great importance in terms of transforming the leg into a walking structure (Mehdigholi & Akbarnejad, 2012). Numerous studies have been conducted on trajectory analysis due to its significance. Using Pro Engineer software, Moldovan and Dolga (Moldovan & Dolga, 2010) performed trajectory analysis for specific link lengths. Regulan et al. (Regulan et al., 2016) designed a robot based on the JL and performed a kinematic analysis of the mechanism using MATLAB

software. Patle et al. (Patle et al., 2017) analyzed the effect of different link lengths on the trajectory. Since many of these studies are based on CAD analysis, the mechanism must be reconstructed for different trajectory experiences. In addition, the step height, which is of great importance for leg mechanisms, is measured manually.

Therefore, the development of a toolbox to overcome these deficiencies in the existing literature is described in detail. The designed toolbox's user-friendly interface enables simple modification of input link lengths. As a result of the kinematic analysis with the given inputs, the final trajectory is visualized and the step height is calculated. Besides, the simulation is continuously repeated in an animate manner for condition $0^\circ \leq \Theta_2 \leq 360^\circ$. During the animation, the link positions can be displayed momentarily, while the animation can be stopped at certain positions to examine the posture of the mechanism. If the mechanism drifts into singularity as a result of the given link lengths, the toolbox warns that the link lengths are not correct to generate the JL. With all these features, the toolbox provides a rapid method for determining the leg mechanism's desired trajectory and step height. In this way, it makes a great contribution to the design phase of legged robots and minimizes time consumption.

2. Kinematic Analysis of Jansen's Linkage

The kinematic analysis of a mechanism consists of calculating the position, velocity or acceleration of its links. Kinematic analysis methods can be summarized as graphical (Arrouk et al., 2018) or analytical (Gaikwad et al., 2004) methods. Graphical methods are easy to implement compared to analytical methods. However, it produces instant results and requires recalculation according to the movement of the mechanism. Analytical methods produce parametric results, giving accurate results for every movement of the mechanism. In the literature, there are hybrid methods such as graphical-analytical (Alaci et al., 2018) that combine the benefits of both approaches.

In the study, an analytical method was used for kinematic analysis, since JL includes repetitive loops. Closed-loop mechanisms such as JL can be mathematically modeled by expressing mechanism links with vectors. These mathematical equations are referred to as Loop Closure Equations (LCE). For a position analysis, the LCE includes the known lengths and unknown positions of the links. Thus, the LCE can be solved for two unknown position variables.

2.1. Position Analysis

The study focuses only on the position analysis of the JL. The analysis can be simplified by using trigonometric identities. These identities are given in Equation 1 and Equation 2 where $t = \tan\left(\frac{\theta}{2}\right)$. After these transformations, Equation 3 can be used to easily determine the unknown link position.

$$\sin(\theta) = \frac{2t}{1+t^2} \quad (1)$$

$$\cos(\theta) = \frac{1-t^2}{1+t^2} \quad (2)$$

$$\theta = 2\arctan(t) \quad (3)$$

The first stage of analytical position analysis is to write the LCE of a particular closed loop. Figure 1 gives a representation of the linkages of the JL. The LCE of loop 1 formed by the vectors L_2 , L_{12} , L_{11} and m can be written as

$$\vec{L}_2 + \vec{L}_{12} + \vec{m} = \vec{L}_{11}. \quad (4)$$

Equation 4 in vector form can be divided into x

$$L_2 \cos\theta_2 + L_{12} \cos\theta_{12} + m \cos\alpha = L_{11} \cos\theta_{11}, \quad (5)$$

and y axis

$$L_2 \sin\theta_2 + L_{12} \sin\theta_{12} + m \sin\alpha = L_{11} \sin\theta_{11}, \quad (6)$$

components. The positions of θ_{11} and θ_{12} are unknown while θ_2 , m , and α are known values given by the user. The way to find one of the unknowns is to take the squares of Equation 5 and Equation 6 and add them.

If the constants are defined as

$$\begin{cases} C_1 = m^2 + L_2^2 + L_{12}^2 - L_{11}^2 + 2mL_2(\cos\alpha\cos\theta_2 + \sin\alpha\sin\theta_2), \\ C_2 = 2mL_{12}\cos\alpha + 2L_2L_{12}\cos\theta_2, \\ C_3 = 2mL_{12}\sin\alpha + 2L_2L_{12}\sin\theta_2, \end{cases}$$

the result of the sum of squares will be

$$(7) \quad C_2\cos\theta_{12} + C_3\sin\theta_{12} + C_1 = 0.$$

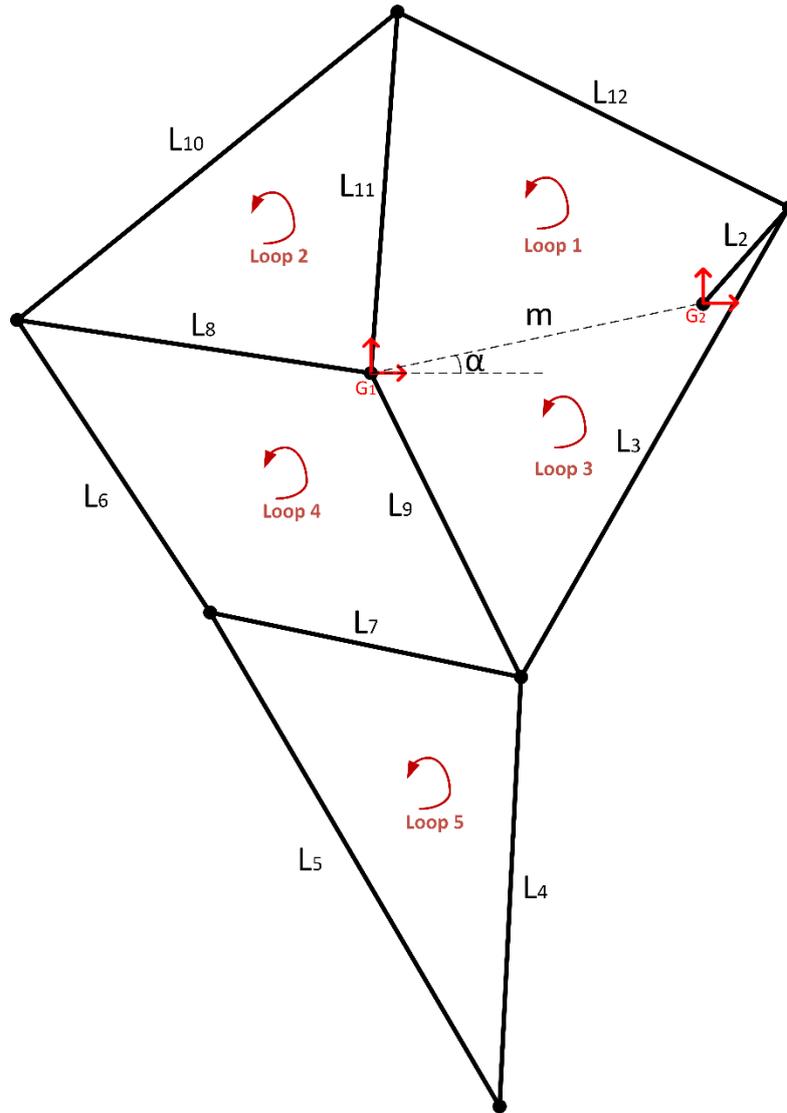


Figure 1. Loops and linkages of the JL

If Equation 1 and Equation 2 transformations are applied in Equation 7,

$$(8) \quad (t_{12})^2(C_1 - C_2) + t_{12}(2C_3) + (C_1 + C_2) = 0$$

is obtained. Equation 8 will have two roots and therefore two solutions, t_{12}^1 and t_{12}^2 . This results in two solutions for θ_{12} ,

$$(9) \quad \theta_{12}^1 = 2\arctan(t_{12}^1)$$

and

$$\Theta_{12}^2 = 2\arctan(t_{12}^2), \quad (10)$$

with the help of Equation 3. The two solutions of Θ_{12} refer to two distinct constructions of the loop. Since the physical posture of the JL is obvious, one of these two solutions can be easily chosen and the Θ_{11} can be determined by substituting the selected Θ_{12} angle in Equation 5 or Equation 6.

The LCE given in Equation 4 can be written for Loop 2, Loop 3, Loop 4 and Loop 5. By following the steps for Loop 1, Θ_8 and Θ_{10} can be found with the help of Loop 2, Θ_3 and Θ_9 with the help of Loop 3, Θ_6 and Θ_7 with the help of Loop 4, Θ_4 and Θ_5 with the help of Loop 5. For the two sets of inputs given in Table 1, a complete position analysis was performed with the help of Equation 4–10 and instant outputs were calculated for $\Theta_2^1 = 269.29^\circ$ and $\Theta_2^2 = 272.16^\circ$.

Table 1. Calculated instant outputs for two different input sets

1 st Input Set	Instant 1 st Output Set for $\Theta_2^1 = 269.29^\circ$	2 nd Input Set	Instant 2 st Output Set for $\Theta_2^2 = 272.16^\circ$
$m = 10.0$ cm	$\Theta_3 = 19.35^\circ$	$m = 38.0$ cm	$\Theta_3 = 26.25^\circ$
$L_2 = 2.6$ cm	$\Theta_4 = 85.84^\circ$	$L_2 = 15.0$ cm	$\Theta_4 = 70.71^\circ$
$L_3 = 15.2$ cm	$\Theta_5 = 113.54^\circ$	$L_3 = 61.9$ cm	$\Theta_5 = 104.18^\circ$
$L_4 = 11.2$ cm	$\Theta_6 = 65.04^\circ$	$L_4 = 49.0$ cm	$\Theta_6 = 68.26^\circ$
$L_5 = 17.5$ cm	$\Theta_7 = 148.02^\circ$	$L_5 = 65.7$ cm	$\Theta_7 = 151.61^\circ$
$L_6 = 10.2$ cm	$\Theta_8 = 140.53^\circ$	$L_6 = 39.4$ cm	$\Theta_8 = 151.82^\circ$
$L_7 = 9.2$ cm	$\Theta_9 = 60.20^\circ$	$L_7 = 36.7$ cm	$\Theta_9 = 63.31^\circ$
$L_8 = 10.2$ cm	$\Theta_{10} = 41.50^\circ$	$L_8 = 40.1$ cm	$\Theta_{10} = 19.74^\circ$
$L_9 = 8.8$ cm	$\Theta_{11} = 95.20^\circ$	$L_9 = 39.3$ cm	$\Theta_{11} = 65.55^\circ$
$L_{10} = 9.0$ cm	$\Theta_{12} = 126.42^\circ$	$L_{10} = 55.8$ cm	$\Theta_{12} = 114.45^\circ$
$L_{11} = 12.5$ cm	Step height = 7.56 cm	$L_{11} = 41.5$ cm	Step height = 39.41 cm
$L_{12} = 18.7$ cm		$L_{12} = 50.0$ cm	
$\alpha = 0^\circ$		$\alpha = 11^\circ$	

2.2. Design of the Toolbox

Python, which is an object-oriented and interpreted programming language, was preferred for the design of toolbox. Python-based and open-source software packages have been chosen as NumPy for scientific computing, Matplotlib for graphical representations, and Tkinter for building graphical user interface. Figure 2 shows the toolbox's general interface and running state.

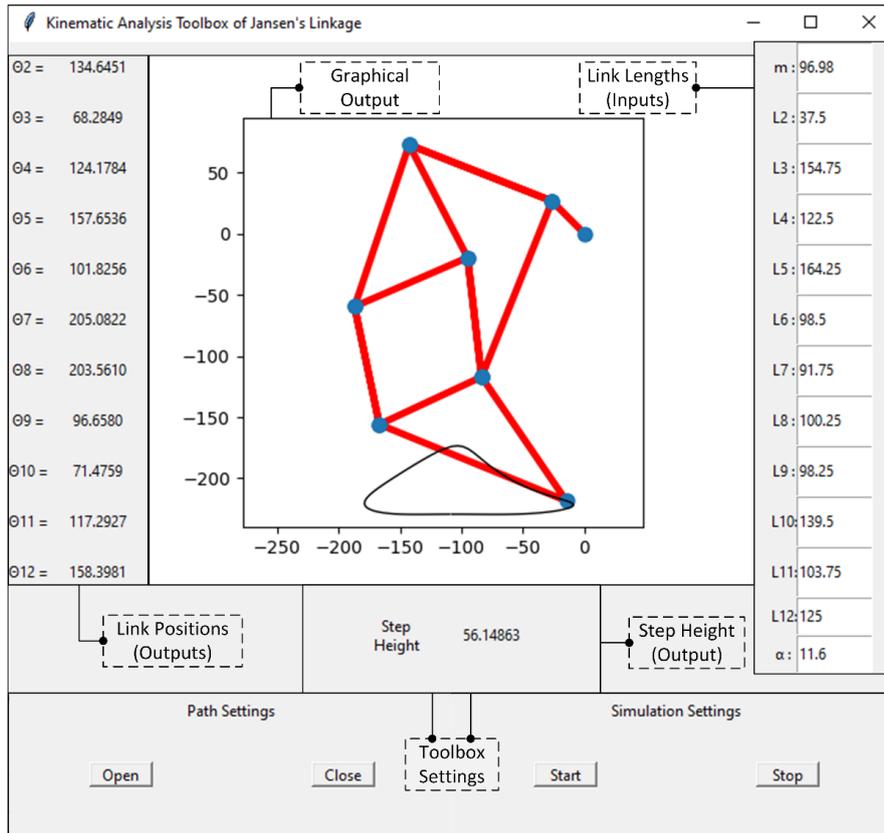


Figure 2. General interface of toolbox

The Toolbox simply takes the user-supplied link lengths. With these inputs, unknown link positions are calculated for the condition $0^\circ \leq \Theta_2 \leq 360^\circ$, using Equation 4–10 and with the help of the NumPy package. A graphical representation of the mechanism with known link lengths and calculated link positions is visualized for condition $0^\circ \leq \Theta_2 \leq 360^\circ$ with the help of the Matplotlib package. The graphical representation also includes the trajectory produced by the specified link lengths. Using the NumPy package, the step height is determined by analyzing the trajectory information. All inputs and outputs are placed in the GUI with the help of the Tkinter package. The interface also includes buttons to stop the simulation for any Θ_2 value during the $0^\circ \leq \Theta_2 \leq 360^\circ$ condition. In this way, the position of other links can be examined for a particular position of the drive link. The program has been debugged and if the mechanism drifts into singularity as a result of the given link lengths, the toolbox alerts the user that the link lengths are not correct for creating the JL. With all of these features, the toolbox provides a fast and effective solution to determine the desired trajectory and desired step height for the leg mechanism of any walking robot.

2.3. Tests of the Toolbox

The test of the toolbox requires validation data. For this purpose, the calculated data given in Table 1 can be used. Figure 3 presents a graphical representation of the mechanism at time $\Theta_2^1 = 269.29^\circ$ and $\Theta_2^2 = 272.16^\circ$ generated by the toolbox in both datasets.

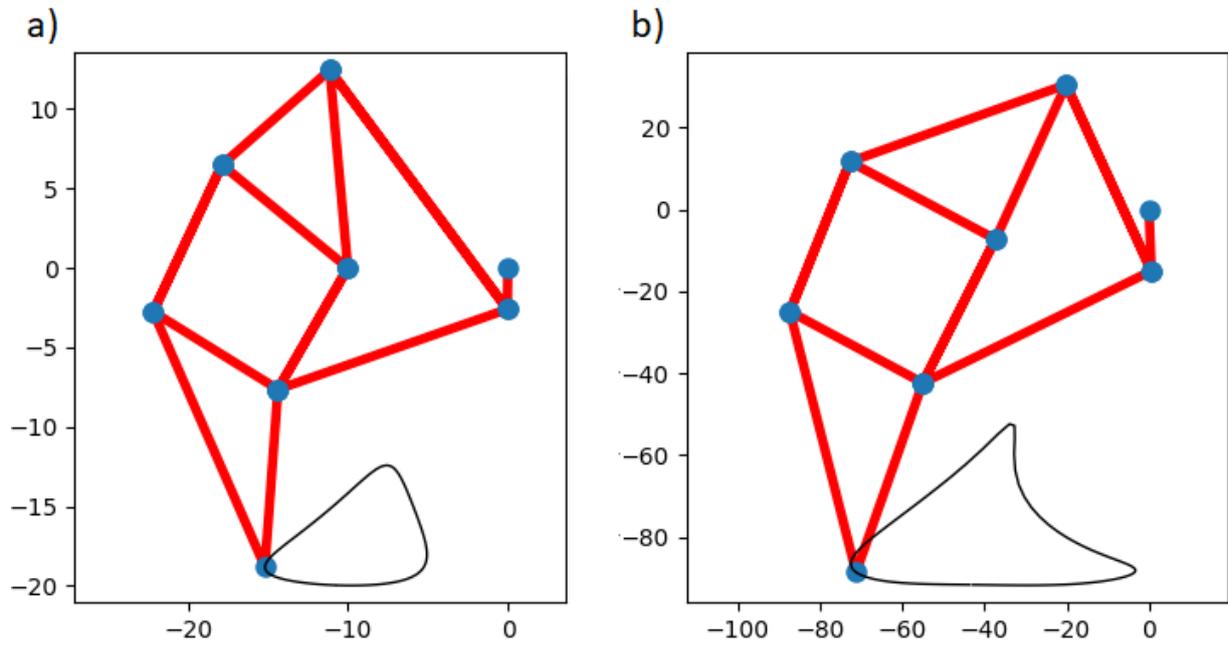


Figure 3. Graphical representation of JL for two different datasets

When outputs such as graphical representation, link positions and step height are compared with the actual results in Table 1, it was determined that the outcomes were identical. Finally, for inputs $m=95$ cm, $L_2=30$ cm, $L_3=140$ cm, $L_4=150$ cm, $L_5=120$ cm, $L_6=100$ cm, $L_7=90$ cm, $L_8=100$ cm, $L_9=98$ cm, $L_{10}=175$ cm, $L_{11}=70$ cm, $L_{12}=100$ cm and $\alpha=0^\circ$, which are known to drive the mechanism to the singularity, Toolbox warns that link lengths are not suitable for generating the JL.

3. Conclusions

Utilizing industrial robots is an efficient strategy for addressing the labor shortage and advancing industrial technology. Oxford Economics predicts that by 2030, industrial robots will replace 8.5% of the current global manufacturing workforce. Additionally, the prevalence of industrial robots will increase interest in robot propulsion mechanisms. Compared to other propulsion mechanisms, legged robots are undoubtedly the most suitable robots for rough terrain exploration, military research and other challenging conditions. Among leg mechanisms, JL has gained popularity due to its organic walking motion, scalable design, and simple drive by rotary input. Due to these factors, more research should be conducted on JL to meet the growing industrialization's demand.

The study successfully presents a toolbox design that visualizes the toe trajectory and simultaneously calculates the step height by performing a kinematic analysis of the JL with the link lengths provided from the user. The designed Toolbox's user-friendly interface makes it simple to modify input link lengths. As a result of the kinematic analysis with the given inputs, the final trajectory is visualized and the step height is calculated. Additionally, the simulation is repeated continuously for condition $0^\circ \leq \Theta_2 \leq 360^\circ$. The animation can be stopped at specific points to look at the mechanism's posture, while the link positions can be displayed momentarily during the animation. Toolbox issues a warning if the mechanism drifts into singularity as a result of the provided link lengths, indicating that link lengths are insufficient to produce the JL. With all these features, toolbox offers a quick solution to obtain the desired trajectory and step height for the leg mechanism. In this way, it makes a great contribution to the design phase of legged robots and reduces the amount of time required.

4. Nomenclature

4.1. Abbreviations

JL	Jansen's Linkage
LCE	Loop Closure Equation
CLL	Chebyshev Lambda Linkage
RM	Rygg's Mechanism

KL Klann Mechanism

4.2. Symbols

G_1	Fix Point 1
G_2	Fix Point 2
m	Distance Between Fix Point 1 and Fix Point 2
α	Position of the m link
Θ_2	Position of 2 nd Link
Θ_3	Position of 3 rd Link
Θ_4	Position of 4 th Link
Θ_5	Position of 5 th Link
Θ_6	Position of 6 th Link
Θ_7	Position of 7 th Link
Θ_8	Position of 8 th Link
Θ_9	Position of 9 th Link
Θ_{10}	Position of 10 th Link
Θ_{11}	Position of 11 th Link
Θ_{12}	Position of 12 th Link
Θ_{12}^1	1 st Solution of The Position for 12 th Link
Θ_{12}^2	2 nd Solution of The Position for 12 th Link
L_2	Length of 2 nd Link
L_3	Length of 3 rd Link
L_4	Length of 4 th Link
L_5	Length of 5 th Link
L_6	Length of 6 th Link
L_7	Length of 7 th Link
L_8	Length of 8 th Link
L_9	Length of 9 th Link
L_{10}	Length of 10 th Link
L_{11}	Length of 11 th Link
L_{12}	Length of 12 th Link
C_1	Constant 1
C_2	Constant 2
C_3	Constant 3
t_{12}	Transformation Variable to Solve Position of 12 th Link
t_{12}^1	1 st Transformation Variable to Solve Position of 12 th Link
t_{12}^2	2 nd Transformation Variable to Solve Position of 12 th Link

References

- Alaci, S., Ciornei, F.-C., Siretean, S.-T., & Ciornei, M.-C. (2018). Graphical-analytical analysis of the mechanism with rotating cam and flat-face follower. *MATEC Web of Conferences*, 184, 1010.
- Arrouk, K. A., Bouzgarrou, B. C., & Gogu, G. (2018). Workspace characterization and kinematic analysis of

- general spherical parallel manipulators revisited via graphical based approaches. *Mechanism and Machine Theory*, 122, 404–431.
- Gaikwad, J. L., Dasgupta, B., & Joshi, U. (2004). Static equilibrium analysis of compliant mechanical systems using relative coordinates and loop closure equations. *Mechanism and Machine Theory*, 39(5), 501–517.
- Guo, S., Wang, G., Qi, Z., Zhuo, Y., & Wu, Z. (2020). A quasi-static model for kinematic analysis of a feed driving mechanism. *Mechanism and Machine Theory*, 148, 103780.
- Jansen, T. (2007). *The Great Pretender*. 010 Uitgeverij. Rotterdam, Holland.
- Jiang, J., Wu, D., He, T., Zhang, Y., Li, C., & Sun, H. (2022). Kinematic analysis and energy saving optimization design of parallel lifting mechanism for stereoscopic parking robot. *Energy Reports*, 8, 2163–2178.
- Klann, J. C. (2001). *Patent No. 6.260. 862*. USA.
- Liang, C., Ceccarelli, M., & Takeda, Y. (2012). Operation analysis of a Chebyshev-Pantograph leg mechanism for a single DOF biped robot. *Frontiers of Mechanical Engineering*, 7(4), 357–370.
- Lokhande, N. G., & Emche, V. B. (2013). Mechanical spider by using Klann mechanism. *International Journal of Mechanical Engineering and Computer Applications*, 1(5), 13–16.
- Mehdigholi, H., & Akbarnejad, S. (2012). Optimization of watt's six-bar linkage to generate straight and parallel leg motion. *International Journal of Advanced Robotic Systems*, 9(1), 22.
- Meng, Q., Liu, X.-J., & Xie, F. (2022). Structure design and kinematic analysis of a class of ring truss deployable mechanisms for satellite antennas based on novel basic units. *Mechanism and Machine Theory*, 174, 104881.
- Moldovan, F., & Dolga, V. (2010). Analysis of Jansen walking mechanism using CAD. *Solid State Phenomena*, 166, 297–302.
- Nansai, S., Elara, M. R., & Iwase, M. (2013). Dynamic analysis and modeling of Jansen mechanism. *Procedia Engineering*, 64, 1562–1571.
- Parekh, B. J., Thakkar, P. N., & Tambe, M. N. (2014). Design and analysis of the Jansen's mechanism based sports ground (pitch) marking robot. *2014 Annual IEEE India Conference (INDICON)*, 1–5.
- Patle, M. R., Mehekare, M. S., Bisen, R. Y., & Shette, U. N. (2017). Analysis of stride length and step height of Theo Jansen leg motion. *International Journal of Innovative Research in Science, Engineering and Technology*, 4184–4189.
- Regulan, G. K., Kaliappan, G., & Santhakumar, M. (2016). Development of an Amphibian Legged Robot Based on Jansen Mechanism for Exploration Tasks. *International Conference on Advancements in Automation, Robotics & Sensing*, 74–91.
- Rygg, L. A. (1893). *Mechanical Horse*, United States Patent 491927. February.
- Song, S.-M., & Waldron, K. J. (1989). *Machines that walk: the adaptive suspension vehicle*. MIT press.
- Zhang, J., Liu, C., Liu, T., Qi, K., Niu, J., & Guo, S. (2021). Module combination based configuration synthesis and kinematic analysis of generalized spherical parallel mechanism for ankle rehabilitation. *Mechanism and Machine Theory*, 166, 104436.