CHAOS
Theory and Applications
in Applied Sciences and Engineering

e-ISSN: 2687-4539
RESEARCH ARTICLE
Vol.4 / No.4 / 2022 / pp.212-225
Special Issue (EDIESCA 2022)
https://doi.org/10.51537/chaos.1193078

The Effect of Agents' Psychology and Social Environment on the Opinion Formation: C/PA Relative Agreement Model in SW and SF Societies

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ABSTRACT Opinion dynamics in relative agreement models seen as an extension of bounded confidence ones, involve a new agents' variable usually called *opinion uncertainty* and have higher level of complexity than that of bounded confidence models. After revising the meaning of the opinion uncertainty variable we conclude that it has to be interpreted as the agent's opinion toleration, that changes the type of the variable from the social to the psychological one. Since the convergence rates to the stationary states in dynamics of sociological and psychological variables are in general different, we study the effect of agents' psychology and social environment interaction on the opinion dynamics, using concord and partial antagonism relative agreement model in small-world and scale-free societies. The model considers agents of two psychological types, concord and partial antagonism, that differs it from other relative agreement models. The analysis of opinion dynamics in particular scenarios was used in this work. Simulation results show the importance of this approach, in particular, the effect of small variations in initial conditions on the final state. We found significant mutual influence of opinion and toleration resulting in a variety of statistically stationary states such as quasi consensus, polarization and fragmentation of society into opinion and toleration groups of different configurations. Consensus was found to be rather rare state in a wide range of model parameters, especially in scale-free societies. The model demonstrates different opinion and toleration dynamics in small-world and scale-free societies.

KEYWORDS
Opinion dynamics
Psychological type
Psychological profile
Networked societies
Self-organization of opinion groups

INTRODUCTION

Public and personal opinions are key elements in a decision taking or making over a subject of interest. A decision is followed by actions that can be of crucial importance for the behavior or even existence of a social group or the whole society. As a consequence, both the empirical and theoretical, including mathematical, study of opinion dynamics is of great significance. Human psychology and sociology sciences consider the following factors to be important in formation of opinion: the status of a topic to be considered, a person awareness of the theme, the structure of the society, psychological type and profile of society members, pair or mixed way of interaction between persons, the influence of opinion leaders and/or mass media, a way of thinking of persons, among others. So, the process of opinion formation turns out to be a complex dynamical system when a combination of these factors is taken into account.

From nineties of the past century, the mathematical modeling of opinion dynamics turns out to be one of the important and efficient tools in studying of opinion formation, considering some of the features mentioned above. To formalize the study of the problem, different models of opinion dynamics have been proposed, which are used to explore the processes of opinion diffusion and evolution in human populations. Research of opinion dynamics covers a wide range of social phenomena: rise and popularity of subjects, spread and preservation of minority opinion, decision taking, consensus formation, emergence of political parties, spread of rumors, rise and influence of extremists, among others.

Manuscript received: 24 October 2022,
Revised: 25 November 2022,
Accepted: 5 December 2022.

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A formal theory of social power by French (French Jr 1956) can be considered as the origin of formal opinion dynamics research followed by a series of opinion dynamics models and their variations; those differ each other in representation of opinion space (discrete or continuous), opinion updating rules, dynamics (regimes) of agents' interaction and the structure of a social group, basically. The key models of opinion dynamics in discrete opinion space are the voter model (Clifford and Sudbury 1973; Galam 2008), the Sznajd model (Sznajd-Weron and Sznajd 2000), the Axelrod cultural diffusion (dissemination) model (Axelrod 1997); the bounded confidence (BC) and relative agreement (RA) models take opinion as a continuous variable (Dittmer 2001; Deffuant *et al.* 2001, 2002; Hegselmann *et al.* 2002). For more detailed and structured review of models on opinion dynamics we refer to the work of Xia et al. (Xia *et al.* 2011), S. Galam (Galam 2022) and, Dong et al. (Dong *et al.* 2018).

In the beginning of sociophysics different models and methods of mathematical physics were adapted to study various social phenomena, in particular, to explore the processes of opinion diffusion (propagation, spreading) and evolution in human populations. Later on new models, based on theories and principles of social sciences, have arisen to give more realistic features to simulation dynamics. In continuous opinion space, bounded confidence (BC) and relative agreement (RA) models, being ones of the closest to the social experience and sociological theories, were proposed for the studying of opinion dynamics in networked societies; BC models (Deffuant *et al.* 2001; Hegselmann *et al.* 2002; Wang 2022) being predecessors of the RA ones (Deffuant *et al.* 2002) are still popular in opinion evolution simulation. As in any dynamical system, the convergence and final states of opinion evolution are considered to be one of the main problems to study. Both BC and RA opinion models demonstrate convergence to the states of consensus, polarization or fragmentation of opinion at different combinations of their parameters and with different convergence rate (Dittmer 2001; Deffuant *et al.* 2001, 2002; Deffuant 2006; Hegselmann *et al.* 2002; Douven and Riegler 2010; Pineda *et al.* 2013). The state variable of an agent in BC models is the opinion only, while in RA ones the uncertainty of opinion is used as an additional variable along with the opinion; the psycho-social meaning of the latter variable was not formally defined in psycho-social sense but interpreted as a range of self-reliance of an agent on its own opinion. Nevertheless, the meaning of that variable as the opinion uncertainty is somehow confusing if not wrong, because it is not measurable (see Section *Conclussions and Discussion* at the end of article). In addition, in this case it is considered as one more sociological variable analogous to the opinion, leaving out of the consideration the influence of agents' psychology on opinion formation.

In order to overcome these constraints of RA models, the differentiation of agents in psychological types was first considered in (Kurmyshev *et al.* 2011; Abrica-Jacinto *et al.* 2017) assigning to each agent one of the two psychological types, Concord (C) or Partial Antagonism (PA); the latter was reflected in the rules of opinion updating at agents' interaction. In addition, after being analyzed the notion of the psychological profile of human individuals (open or closed mind persons), we reinterpret the meaning of the variable *opinion uncertainty* giving to it the notion of personal *toleration* to the opinions of others. New interpretation gives mayor conceptual consistency to the RA models since it allows to study effects of interrelation and mutual influence of the opinion (social variable) and the personal toleration (psychological variable) in evolution of opinion in artificial societies of different structures.

The main conceptual difference between BC and RA models is the criterion of the opinion updating. In BC models (homogeneous or heterogeneous) an agent changes its opinion if and only if the distance between the opinions of influenced (passive) agent and influential (active) ones is less than a certain threshold, that depends on closeness of opinions only. In RA models an influenced (passive) agent changes its opinion if and only if the opinion intervals of interacting agents overlap each other, that depends on both the opinion closeness and the opinion uncertainty. The rate of convergence to a stationary opinion state is regulated by a convergence parameter $\mu \in (0, 0.5]$ along with other factors. The μ parameter is the intensity of agents' opinion interaction and it shows how much other opinions influence the opinion of an agent. In homogeneous and heterogeneous BC models the influence of μ on convergence rate and opinion patterns in opinion dynamics was studied in (Urbig and Lorenz 2007; Deffuant 2006; Lorenz 2008; Huang *et al.* 2018). It was found that its value, along with other parameters of the model, influence both the convergence rate and the final opinion groups distribution; even though, most of the works on opinion dynamics in BC and RA models use the only value $\mu = 0.5$. Systematic study of this influence in RA models was not done. Moreover, when the *opinion uncertainty* is now reinterpreted as the toleration to others' opinions we have to admit the difference in convergence rates of variables, expressing them through the two convergence parameters μ_1 and μ_2 for the opinion and toleration respectively. Because the evolution of persons' psychology is used to be slower than that of their opinions, we consider opinion and toleration dynamics through the variation of μ_2 under the condition $\mu_r = \frac{\mu_2}{\mu_1} \leq 1$.

Another essential feature of models are communication regimes in opinion dynamics that in real life can be quite different, ranging from pair interactions to meetings of agents or including various combinations between them (Urbig and Lorenz 2007; Yu *et al.* 2017). Random selection of agents for the updating of opinion is usually used in simulation. The latter turns out the system into a stochastic one, and as a consequence one has to choose between the analysis of particular scenarios or the averaging of results of many similar experiments. In this respect, the influence of initial conditions on opinion dynamics in a stochastic system has to be taken into account (Yu *et al.* 2020).

This work is aimed mainly to the studying of toleration (psychological variable) and opinion (sociological variable) dynamics of agents in artificial societies of different structure (SW and SF networks) in the frame of C/PA relative agreement model. We pay special attention to particular scenarios at small variation in initial conditions in a wide range of model parameters. The rest of this document is organized as follows. In next section (*C/PA model*) we set out the problem to be studied and briefly describe the C/PA relative agreement opinion dynamics model. Later, in section *Desing of experiments* describes the design of computational experiments and parameters of the model. The results of extensive simulations are presented and analyzed in *Simulation results and analysis*. Finally, *Conclusions and discussion* are given.

C/PA MODEL

Agent based mathematical models of opinion dynamics in networked societies are characterized by four basic elements (Kurmyshev *et al.* 2011):

- *Networked society* – represents a communication system between agents of a society by means of a graph where nodes represent agents and communication channels between agents are represented by links.

- *Opinion space* – a discrete or continuous set of values that represents the opinions of agents. The *toleration* and respective space is used along with the opinion in RA models.
- *Interaction dynamics* – establish the manner and sequence of agents' interaction and conditions under which agents update their state variables.
- *Updating rule* – basically, the model equations describing opinion change as a result of agents' interaction.

Opinion dynamics models are usually distinguished by their specific updating rules and updating dynamics, while other elements are shared. In this article, we use the C/PA relative agreement opinion dynamics agent based model (Kurmyshev *et al.* 2011) that is an extension of the DW model (Deffuant *et al.* 2002). The C/PA model contemplates societies of agents of two psychological types, concord C-agents and partial antagonism agreement PA-agents; any substrate network is admitted. Given a society of $N = N_C + N_{PA}$ agents, a subset of $N_C = p \cdot N$ agents are C-type agents and the rest, $N_{PA} = (1-p) \cdot N$ are PA-agents, where $p \in [0,1]$ is the fraction (proportion) of C-agents. At each instant of time t, the state of i-agent is described by the two continuous variables, its opinion $x_i(t) \in [-1,1]$ and toleration (ex-uncertainty) $u_i(t) \in (0,1]$, where $i = 1,2,\ldots,N$.

The interaction dynamics in C/PA-model is stochastic. Agents can change their states as a result of pair interaction in a discrete time. At each instant t, M edges of a network are selected at random. Each edge connects a pair (i,j) of interacting agents and one of them is selected at random to be receptive (influenced), say j-agent, and the other, i-agent, to be influential. So, unidirectional pair interaction of agents is chosen.

For a pair of selected agents (i,j), the social condition for their interaction in RA models is defined by the overlap of the opinion segments,

$$s_i(t) = [x_i(t) - u_i(t), x_i(t) + u_i(t)]. \tag{1}$$

Toleration $u_i(t)$ defines the borderlines of acceptability of other agents' opinions. The overlap of segments is calculated as

$$\begin{aligned} h_{ij}(t) = \min\{x_i(t) + u_i(t), x_j(t) + u_j(t)\} \\ - \max\{x_i(t) - u_i(t), x_j(t) - u_j(t)\}. \end{aligned} \tag{2}$$

When $h_{ij}(t) \leq 0$, neither opinion nor toleration of the influenced agent j from (i,j) pair are modified. If $h_{ij}(t) > 0$, then the receptive j-agent of the interacting pair updates the opinion $x_j(t)$ and toleration $u_j(t)$ according to its psychological C or PA-type, following the equations:

$$x_j(t+1) = x_j(t) + \mu_1 \cdot ra_{ij}^{C,PA}(t) \cdot [x_i(t) - x_j(t)], \tag{3}$$

$$u_j(t+1) = u_j(t) + \mu_2 \cdot ra_{ij}^{C,PA}(t) \cdot [u_i(t) - u_j(t)], \tag{4}$$

where $\mu_1, \mu_2 \in (0, 1/2]$ are convergence parameters (intensity of interactions) for the opinion and toleration, respectively. Relative agreement $ra_{ij}^{C,PA}(t)$ of receptive agent with an active one depends on the psychological type of the former and is calculated as

$$ra_{ij}^{C}(t) = \frac{h_{ij}(t)}{u_i(t)}, \tag{5}$$

$$ra_{ij}^{PA}(t) = \frac{h_{ij}(t)}{2u_i(t)} \left[\frac{h_{ij}(t)}{u_i(t)} - 1 \right], \tag{6}$$

In the C/PA model (Kurmyshev *et al.* 2011), the interaction of passive C-agents is always attractive in the opinion space, its opinion always gets closer to that of the active one as in the DW model (Deffuant *et al.* 2002). Nevertheless, dynamics of passive PA-agents can be repulsive-attractive in accord to the relative agreement $ra_{ij}^{PA}(t)$, depending on the overlap of opinion intervals $h_{ij}(t)$ (see Eq. 2).

Usually, most of the BC and RA opinion dynamics models handle the convergence parameter equal to $\mu = 0.5$. The exceptions are (Lorenz 2010; Huang *et al.* 2018). In (Lorenz 2010) the convergence of opinion in function of different values of the convergence parameter was studied in the BC Deffuant model. In (Huang *et al.* 2018) the heterogeneous convergence parameter, depending on the distance between the opinions of interacting agents, was proposed for the BC Deffuant model. The use of heterogeneous parameters has converted the BC model into a kind of RA one. In general, the RA models use the same convergence parameter equal to $\mu = \mu_1 = \mu_2 = 0.5$ for both variables, x_i and u_i (Deffuant *et al.* 2002; Meadows and Cliff 2012; Kurmyshev *et al.* 2011). But x_i describes the social manifestation and u_i corresponds to the psychological profile of agent; so they can have different time scales in evolution. In addition, agents' social manifestation (opinion) and psychological profile (toleration) influence each other (Abrica-Jacinto *et al.* 2017). In our work, we study the opinion and toleration dynamics and their mutual influence varying the ratio of convergence parameters $\mu_r = \frac{\mu_2}{\mu_1}$ in the frame of C/PA relative agreement model on the SW and SF networks. With the features being integrated into the model, complex system dynamics emerge that has resemblance to the real social processes, at least qualitatively.

DESING OF EXPERIMENTS

We study opinion and toleration dynamics in artificial societies of two types, small world (SW) and scale free (SF), consisting of $N = 10^3$ agents each. It is an intermediate size society that can get the insight into particularities of evolution of both small and large societies. SW-network (undirected graph) is generated according to the Watts-Strogatz algorithm with the probability of reconnection $\beta = 0.25$ and average degree $< k >= 40$ (average number of neighbors of each node); it has $M_{SW} = 20 \times 10^3$ links (Watts and Strogatz 1998). SW-network can be considered as a prototype of democratic society without noticeable leadership. SF-network is constructed according to the Barabási-Albert model (Barabási and Albert 1999), with following parameters: $N_0 = 2$, with $m = m_0 = 1$; that has $M_{SF} = 999$ links. SF-network is a *structured* network with an intention of hubs to leadership.

Mixed societies composed of C- and PA-agents with the C-agents' fractions $p = 0.3$ and 0.7 are studied. To our opinion, the two compositions are quite representative to see the difference in dynamics of opinion and toleration in societies composed of agents of different psychological types. We understand the term society as a network (graph), with a particular psychological type (C or PA) assignment to each agent. For a given value of the p parameter, two societies represented by the same graph are considered different if they have different psychological type assigning to the agents.

In C/PA model, the initial mean value of agents' toleration is an important parameter, to which was given the following values $U = 0.3, 0.5, 0.7$. Societies with $U = 0.3$ can be considered as composed of agents with relatively low toleration (enclosed agent), while $U = 0.7$ corresponds to high tolerant agents (open agent).

In order to evaluate the effect of convergence parameters on the opinion and toleration dynamics and their mutual influence, we set the convergence parameter of opinion at the value $\mu_1 = 1/2$ and varied the convergence of toleration, $\mu_2 = 1/2, 1/6, 1/10, 1/20$, so that the ratio of convergence parameters was $\mu_r = \frac{\mu_2}{\mu_1} = 1, 1/3, 1/5, 1/10$. Finally, each of the experiments are characterized by the quaternion $(Network, p, U, \mu_r)$ in parametric space.

The updating dynamics of the model is a stochastic one, and we were faced with the decision of studying particular scenarios or the averaged results of many individual experiments. Pilot simulations showed that the averaging of experimental results (see also (Kurmyshev *et al.* 2011; Abrica-Jacinto *et al.* 2017)) capture opinion evolution tendencies, while interesting and important characteristics of each particular scenario can be lost. On the other hand, the analysis of particular scenarios shows rather general trends of opinion evolution in addition to salient particularities. For that reason, we decided to explore individual scenarios systematically in a wide range of parameters offered by the model. For a given society, a *particular scenario* of opinion and toleration dynamics is defined by the following elements: given initial opinion and toleration conditions, and the particular realization of updating dynamics (a sequence of nodes selected for the opinion and toleration updating). *Particular experiment* begins with the setting of initial conditions for the variables:

- Uniform distribution on the interval $[-1, 1]$ is used to assign the initial opinion to agents.
- Uniform distribution on the interval $[U - 0.15, U + 0.15]$ is used to assign the initial toleration to agents.

Particular realizations of the uniform distribution are not identical, being of the same type but with variations due to the generator of random numbers. To see how much the result of experiment is sensitive to initial conditions (IC) in the frame of stochastic updating dynamics, we carry out each experiment at four particular realizations (A, B, C, D) of IC.

After setting the initial conditions, the simulation is conducted according to the following algorithm:

- M edges are chosen at random.
- On each selected edge (i, j), the receptive agent is chosen at random.
- The overlap h_{ij} of opinion intervals is calculated, Eq. 2.
- If $h_{ij} > 0$, the opinion and toleration of the receptive agent are updated according to their psychological type, Eqs. 3 and 4. If $x_i(t) \leq -1$ or $x_i(t) \geq 1$, then we take $x_i(t)$ to be -1 or 1, respectively. Similarly, the toleration is retained in the interval $u_i(t) \leq [0.05, 1]$.
- In regard to $h_{ij} \leq 0$, neither opinion nor toleration of the receptive agent are modified.

Agents with $u_i(t) = 0$, if they were, are unexpressive. They are not willing to change their opinion neither toleration despite being connected in the network, because they have zero opinion interval overlap with other agents. These agents can be considered as apathetic or socially closed because they do not admit interaction with other agents. That was the reason to maintain at least a nominal interaction, so that a small margin (0.05) was left in toleration for keeping opinion exchange.

Unlike the SW, the SF networks have hubs as "distinguished" members. In order to maintain the degree, psychological type and position of hubs in the network, and, finally, to have a detailed control over the influence of parameters μ_r and U on the dynamics of system, we design experiments in SF societies in the following manner. The SF network was generated once for all experiments. The three largest degree nodes (Hubs) had 35, 25 and 23 links and we assigned them PA, C, C and psychological type, respectively. The uniform random distribution was used to assign the psychological type to the rest of the nodes of network at each of the two values $p = 0.3$ and $p = 0.7$. The structure of the society is preserved for a part of experiments; afterwards, the psychological type of Hubs is inverted, C to PA and PA to C, maintaining the psychological type of the rest of agents. So, we have conducted experiments with two societies, differed each other in opposed psychological type of Hubs.

The uniform initial conditions were generated four times (A, B, C, D) for both SW and SF networks. In order to see the effect of μ_r and IC on the dynamics, for each pair of parameters (p, U) we run $16 = 4 \times 4$ experiments simultaneously under the same (stochastic) updating dynamics for all combinations of initial conditions ($\times 4$) and values of parameter μ_r ($\times 4$). In experiments we used the following values for $p = 0, 3, 0.7$ and $U = 0.3, 0.5, 0.7$.

The advantage of this scheme is that the influence of parameters can be analyzed separately, under the same stochastic updating dynamics. To see the influence of μ_r, one has to analyze the results of experiments along the lines at fixed IC, but the influence of variation in IC is analyzed along the columns. Through the preliminary experimentation we noticed that the evolution time in SF networks is greater than that for the SW. So, for the SF experiments we extended the evolution time from 2000 to 6000 steps.

In order to compare the convergence of opinion and toleration in networks of different type, SW and SF, one has to choose equal number of edges to ensure near the same number of agents update their opinion and tolerance. The empiric rule in most of the publications is to choose at random in each time step the number of edges equal to the number of agents in the network. So, near a half of agents has an opportunity to update their opinion and tolerance.

We use a SF network of 1000 agents that has 999 edges in accord to the Barabasi-Albert algorithm (Barabási and Albert 1999); the number of edges in a SF network is much smaller than that of SW with equal numbers of agents. In opinion dynamics, random selection (with regression) of network edges for a unit time step can result in a multiple selection of a link between one agent and the others. So, one agent can have a number of interactions with others during unit time step; in particular, it is quite possible between a hub and common agents, and a hub can participate as a passive or active agent several times for a time step. Nevertheless, in accord to the Eqs. 3 and 4, only the ultimate interaction has an effect on changing the opinion and tolerance; one can see all previous interactions of the stage as an exploration of the issue (opinion and toleration situation).

SIMULATION RESULTS AND ANALYSIS

The number of simulated particular scenarios is 192: 2 (networks, SW and SF) $\times 2$ (composition of society, $p = 0.3$ and 0.7) $\times 3$ (U's values, $U = 0.3, 0.5, 0.7$) $\times 4$ (μ_r's values) $\times 4$ (initial conditions, $A, B, C, D) = 2 \cdot 2 \cdot 3 \cdot 4 \cdot 4 = 192$. In order to exclude the influence of variations in generation of networks on results of simulation, we use the same SW or SF graph in all experiments with SW and SF societies; each was generated only once. In this work, no specific quantitative criteria are used to analyze multiple aspects of the opinion and uncertainty dynamics; analysis and conclusions are qualitative more than quantitative. Each experiment was carrying out up to 2000 generations for SW and 6000 generations for SF networks. These numbers were chosen on base of preliminary simulations, because the tracking of evolution trajectories has shown convergence to a steady state.

Along with the data files, each experiment provides two plots of opinion and toleration trajectories of each and all agents, two color palette histograms of opinion and toleration evolution and two histograms of final distributions of opinion and toleration – those are for the visual qualitative analysis. We see that the plots of agents' opinion and toleration trajectories can be often confusing and difficult for individual tracking because of their multiple overlapping. In that cases the evolutionary histograms are of great benefit. Since the representation of simulation results in a graphic manner is extensive, we resume them by some instructive examples and qualitative description. Histograms of opinion are given in the interval $[-1, 1]$ and for the toleration in the interval $[0.05, 1]$, both with 21 bins. In order to facilitate comparison of simulation results, we comment them by similar phrases in a repetitive manner.

Experiments in SW-networks

Each experiment is characterized by the quaternion (SW, p, U, μ_r) and by the same set of four realizations of initial conditions (A, B, C, D); we chose at random $M = 1000$ links of total number $M_{SW} = 20 \times 10^3$ links in each experiment. We think that the selection of links in each experiment does not influence much on the updating dynamics, since each node of the network has in average the same degree.

SW society at $p = 0.3$. The cross-analysis of plots and histograms of the opinion and toleration evolution in SW society with C-agents' fraction equal to $p = 0.3$, for the set of parameters $\mu_r = \{1, 1/3, 1/5, 1/10\}$ ($\mu_r = \mu_2/\mu_1$) reveals the following features in opinion and toleration dynamics (see Figure 1).

At $U = 0.3$, for the decreasing ratio of convergence coefficients μ_r's: 1. We observe the opinion fragmentation; the number of opinion groups is decreasing from 7 to 4 with the decrease of μ_r. 2. No regular change in the rate of toleration convergence is detected with the decrease of the convergence parameter μ_r; toleration converges to the values smaller than U and agents show more toleration ($u_i \approx U$) for the societies with less convergence parameter μ_r, meanwhile most of the agents become low tolerant ($u_i \approx 0.06$) at $\mu_r = 1$. Sometimes toleration tends to split into few close groups. 3. The four generations of initial conditions, even being each of the same type uniform distribution, are not identical, and the effects of relatively small variations in IC and stochastic updating dynamics on the final opinion and toleration distributions were observed through the variation in size and position of principal peaks.

At $U = 0.5$, for the decreasing ratio μ_r (Figure 2 as an example): 1. We observe polarization and, sometimes, fragmentation of opinion into three groups. The slowing down of toleration convergence is observed as μ_r decreases and, as consequence, the opinion evolution is elongated also (Figure 2.b). 2. Toleration shows a slowing down of convergence with the decrease of μ_r; toleration converges to values smaller than U, so agents become less tolerant in average; the final values of toleration frequently split into two groups, one of them is a group of low toleration agents. 3. Between groups dynamics is observed in evolution of opinion and tolerance, that is seen in plots of trajectories but almost not seen in histograms; trajectories of agents migrating from one to another opinion group are observed – those are bridges connecting groups of different opinions. 4. The effects of IC on the final opinion and toleration distributions are observed as the variation in size and position of principal peaks, those are more noticeable than that in case of $U = 0.3$. That is an indication of instability in toleration and opinion dynamics, $U = 0.5$ looks to be near the bifurcation point.

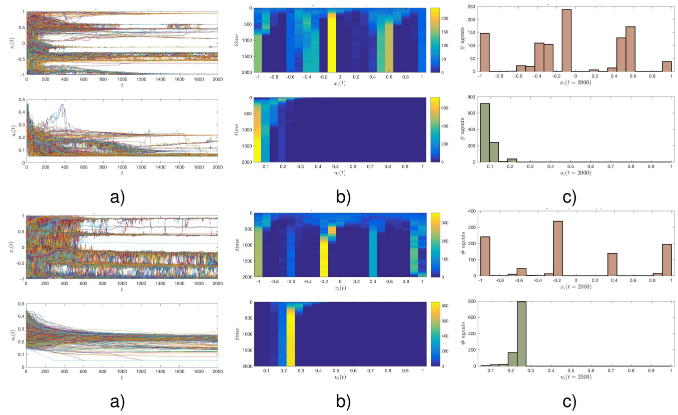

Figure 1 Opinion and toleration evolution in SW network at $(p, U) = (0.3, 0.3)$ IC$-A$. Columns: a) trajectories and b) color palette histograms of agents' opinion and toleration evolution, c) final distribution of opinion and toleration. First double line for $\mu_r = 1$, second double line for $\mu_r = 1/10$.

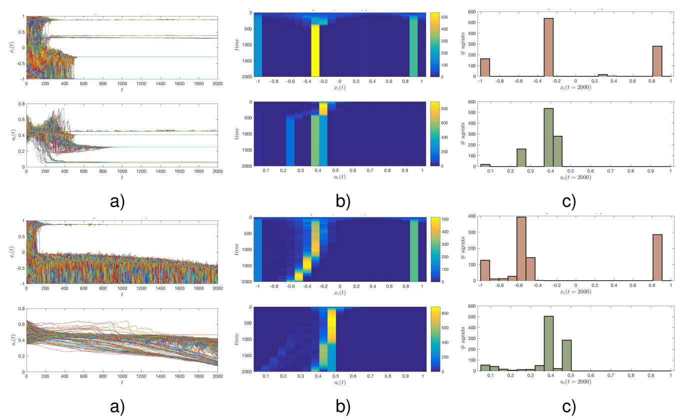

Figure 2 Opinion and toleration evolution in SW network at $(p, U) = (0.3, 0.5)$, IC$-A$. Other conditions are the same as in Figure 1.

At $U = 0.7$, for convergence coefficients μ_r (see Figure 3) we observe: 1. Opinion polarization into two asymmetric groups with final positions generally fluctuating in the opinion space at small variations of IC (sometimes, fragmentation into three groups at $\mu_r = 1$); decreasing of μ_r (slowing down of toleration convergence) causes increasing of opinion convergence. 2. Toleration shows the slowing down of convergence with the decrease of μ_r, converging to values smaller but close to U. At $\mu_r = 1$ toleration sometimes converges to two values, $u_i \approx 0.65$ and $u_i \approx 0.05$, the latter corresponds to low toleration agents, enclosed or unwilling to collaborate. 3. Between groups dynamics is observed in evolution of opinion and tolerance, that is seen in plots of trajectories but almost not seen in histograms; trajectories of agents migrating from one to another opinion group are observed – those are bridges connecting groups of different opinions.

It is important to note that in SW societies at $p = 0.3$ (societies with a predominant number of PA agents) an opinion consensus was not observed at any value of U and μ_r.

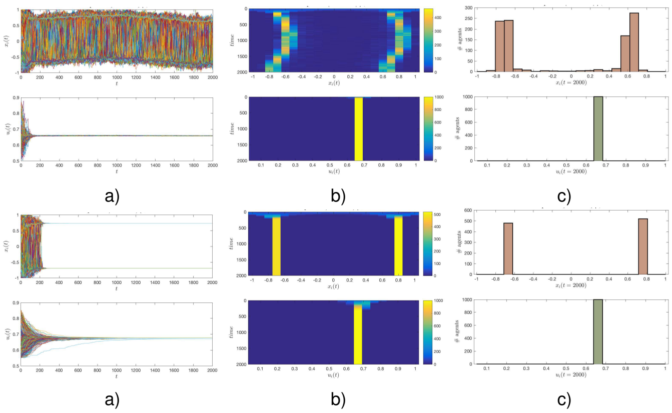

Figure 3 Opinion and toleration evolution in SW network at $(p, U) = (0.3, 0.7)$, IC$-A$. Other conditions are the same as in Figure 1.

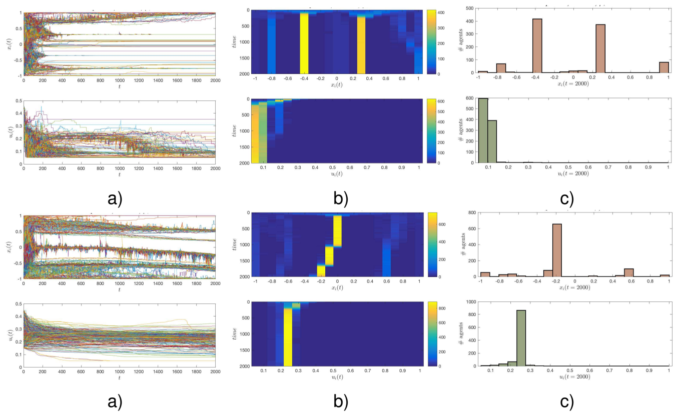

Figure 4 Opinion and toleration evolution in SW network at $(p, U) = (0.7, 0.3)$, IC$-A$. Other conditions are the same as in Figure 1.

SW society at $\boldsymbol{p = 0.7}$**.** When the psychological composition of the society is changed from $p = 0.3$ to $p = 0.7$ (C-agents are predominant), the analysis of plots and histograms of the opinion and toleration evolution, for the set of parameters $U = 0.3, 0.5, 0.7$ and $\mu_r = \{1, 1/3, 1/5, 1/10\}$, shows the following characteristics of the opinion and toleration dynamics.

At $U = 0.3$, for μ_r's (Figure 4): 1. In the range of $\mu_r = 1/3, 1/5, 1/10$ we observe the convergence of opinion into one dominating group (quasi consensus). Centrist dominating group *drifts* in opinion space to one of the extremes (see, for example Figure 4.b); the effect is most noticeable at $\mu_r = 1/5$ while it is less visible at $\mu_r = 1/10$ and absent at $\mu_r = 1, 1/3$. Final value of opinion depends on the parameter μ_r and IC, even though no regular pattern of this dependence was found. In addition, several minority groups are formed. In case of $\mu_r = 1$, polarization or fragmentation into three dominant groups is observed; several minority groups are also observed. At all values of μ_r small groups of opposed extremists emerge. 2. Decreasing in the rate of toleration convergence is observed with the decrease of the convergence parameter μ_2, that is not trivial. Predominant compact group of agents with toleration less than U is formed and, moreover, agents become less tolerant when the convergence parameter μ_r increases. In case of $\mu_r = 1$, toleration converges to small values, in general, less than 0.1, so that the society evolves into a state with low toleration agents. 3. The effects of relatively small variations in IC and stochastic updating dynamics on the final opinion and toleration distributions were observed through the variation in size and position of principal peaks.

At $U = 0.5$, for μ_r's (Figure 5 as an example): 1. We observe the convergence of opinion of agents into one dominant group that *flips* its position due to small variations in uniform IC. The final value of the dominant opinion group seems to depend on the IC mainly and on the parameter μ_r partially, even though no regular pattern of this dependence was found. In addition, one or two small extremist groups are formed. 2. Composition of SW society of agents of different psychological type has significant influence on the formation of opinion. At $p = 0.3$ (minority of C-agents), the society tends to separate into two or three opinion and tolerance groups. Nevertheless, at $p = 0.7$ (minority of PA-agents), the trend to formation of a single dominant group is observed. In some scenarios, at both $p = 0.3$ and $p = 0.7$, the formation of a compact single one or various groups are observed soon after the beginning of interaction between agents, then the group evolves (drifts) as a whole. 3. An interesting and important effect of convergence parameters in opinion evolution is observed at $(p, U) = (0.7, 0.5)$. When $\mu_r = 1, 1/10$ the dominant both opinion and toleration groups after being formed remain stable in opinion and toleration spaces (Figure 5.b), but at the intermediate values of $\mu_r = 1/3, 1/5$ a notable drift is observed, especially at $\mu_r = 1/5$. With the decrease of the convergence parameter μ_2, the opinion converges faster than the toleration due to $\mu_1 > \mu_2$. 4. Toleration of agents converges to a value smaller than U (predominant compact group of agents with close tolerances less than U is formed) and, in addition, agents show more toleration ($u_i \approx U$) in the societies with a less convergence parameter μ_2. In some cases, toleration converges to rather small values (in general, less than 0.2), so that the majority of the society advances into the group of agents with low tolerance. Drift of the dominant toleration group toward to $U \approx 0.3$ is also observed at $\mu_r = 1/5$. 5. Effects of relatively small variations in IC and stochastic updating dynamics on the final opinion and toleration distributions are observed through the variation in position of dominant groups.

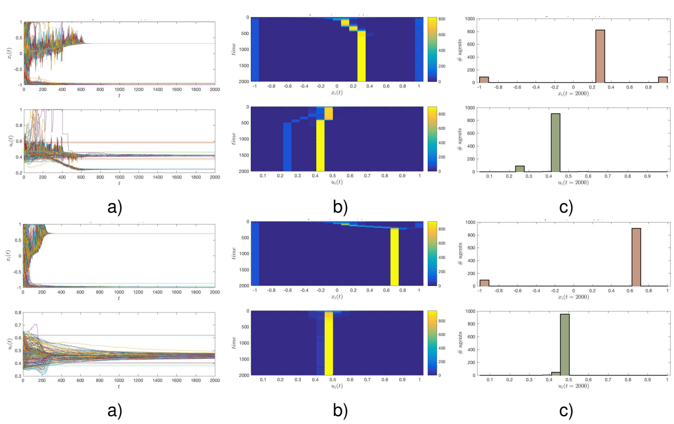

Figure 5 Opinion and toleration evolution in SW network at $(p, U) = (0.7, 0.5)$, IC$-A$. Other conditions are the same as in Figure 1.

At $U = 0.7$, for μ_r's (see Figure 6): 1. Similar to the case $U = 0.5$, we clearly observe two stages both in the opinion and toleration dynamics: first a dominant compact group of opinion and toleration is formed and, then this group evolves in opinion or toleration space as a whole; drift is obvious, especially at $\mu_r = 1/5, 1/10$ (Figure 6.b). 2. In the range of $\mu_r = 1/3, 1/5, 1/10$, the toleration convergence time grows up with the decreasing of convergence parameter μ_2; toleration of agents converges to the values smaller than U and, in addition, toleration normally converges to rather small values $u_i \approx 0.1$, so that the society evolves into a state with low toleration. 3. In the range of $\mu_r = 1, 1/3, 1/5, 1/10$ we observe the convergence of opinion into a single consensus group. Final value of the opinion depends on the parameter μ_r and IC, even though no regular pattern of this dependence was found. At $\mu_r = 1$ the position of dominant group tends to the center ($x_i \approx 0$), nevertheless, for $\mu_r = 1/3, 1/5, 1/10$ the group takes one of the extremist positions, $x_i \approx -1$ or $x_i \approx +1$, depending on IC and updating dynamics. 4. Effects of relatively small variations in IC and stochastic updating dynamics on the final opinion and toleration distributions is observed through the variation in position of dominant groups.

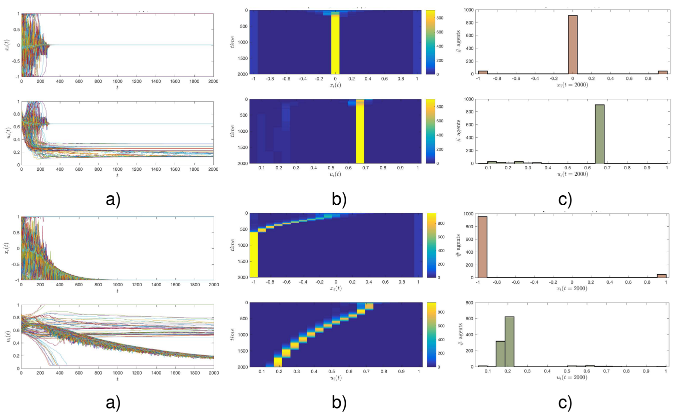

Figure 6 Opinion and toleration evolution in SW network at $(p, U) = (0.7, 0.7)$, IC$-A$. Other conditions are the same as in Figure 1.

It is instructive to revise the histograms of final distributions of opinion and toleration for the several IC and values of $U = 0.3, 0.5, 0.7$, at $p = 0.3, 0.7$ (Figures 7 and 8). Polarization of opinion is predominant at $p = 0.3$ and $U = 0.5, 0.7$ (Figure 7), while at $p = 0.7$ and $U = 0.5, 0.7$ an asymmetric quasi-consensus accompanied by small extremist groups is observed (Figure 8). Small variations in IC, A and B, cause noticeable change in position of opinion groups, at $U = 0.5, 0.7$ especially (see Figure 8, second and third columns).

Experiments on SF-networks

Trajectories and histograms of agents' opinion and toleration evolution and the histograms of final distributions of opinion and toleration were obtained in experiments. To observe the evolution of the hubs' opinion and toleration, the trajectories of the three hubs in figures of temporal evolution are presented in black, cyan and magenta, respectively. In histograms of final distributions of opinion and tolerance, the bins containing the hubs are shown in yellow.

SF society and $p = 0.3$. When the psychological composition of the society is $p = 0.3$, the cross-analysis of plots and histograms of the opinion and toleration evolution, for the set of parameters $U = 0.3, 0.5, 0, .7$ and $\mu_r = 1, 1/3, 1/5, 1/10$, shows the following tendencies and particularities of opinion and toleration dynamics.

For the decreasing ratio of convergence coefficients $\mu_r = 1, 1/3, 1/5, 1/10$ and $U = 0.3$: 1. The great majority of (if not all) individual trajectories in opinion and toleration space are straight lines of steady state after some evolution time, that is rather different of SW networks where only a stochastic steady states are ob-

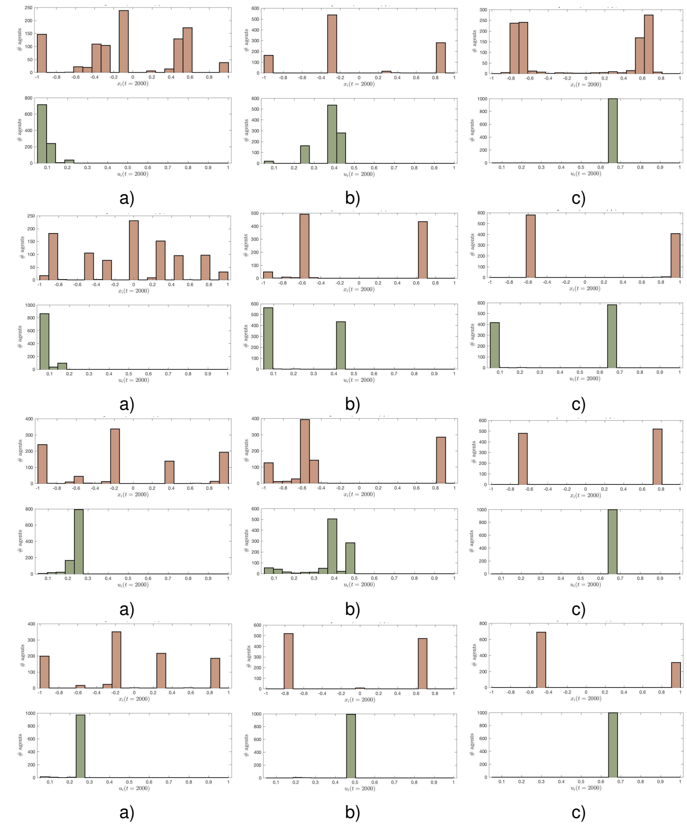

Figure 7 Final distributions of opinion and toleration on SW network at $p = 0.3$. Columns: a) $U = 0.3$, b) $U = 0.5$ and c) $U = 0.7$. First double line for IC$-A$, $\mu_r = 1$; second double line for IC$-B$, $\mu_r = 1$; third double line for IC$-A$, $\mu_r = 1/10$ and fourth double line for IC$-B$, $\mu_r = 1/10$. Brown histograms – opinion, green histograms – toleration.

served (see Figures 9 and 10). This interesting effect is the result of combination of the three causes: the structure of SF network, high proportion of PA-agents and relatively low toleration of agents. Agents with close opinion are located far each other (they are not nearest neighbors) or close to agents with rather distinct opinion in the SF network, and for that reason can't interact each other (their opinion segments have no overlap in the opinion space). 2. No substantial changes in the opinion compared to its initial distribution are observed, neither fragmentation no polarization of the opinion (Figures 9.c and 10.c). The final distributions of opinion at $\mu_r = 1, 1/3, 1/5, 1/10$ differs each other not much but in the position of hubs mainly. The opinion convergence time increases with the decreasing of μ_r. 3. The trajectory of principal PA-hub opinion is much stable and regular than that of the two smaller C-hubs, each hub behaves similar to Brownian particle (irregular interaction with neighboring agents causes chaotic motion in opinion space). We observe an irregular change of the final position of hubs in opinion space with the change of μ_r. 4. No regular change in the rate of toleration convergence is detected with the decrease of the convergence parameter μ_r; toleration converges to the values smaller than U, forming a kind of bell distribution.5. The four generations of IC are not identical (the hubs are included), even being each of the same type uniform distribution, and the effects of variations in IC on the final opinion and toleration distributions were observed, in particular, through the variation of hubs position and size of bins that include hubs (Figures 9 and 10).

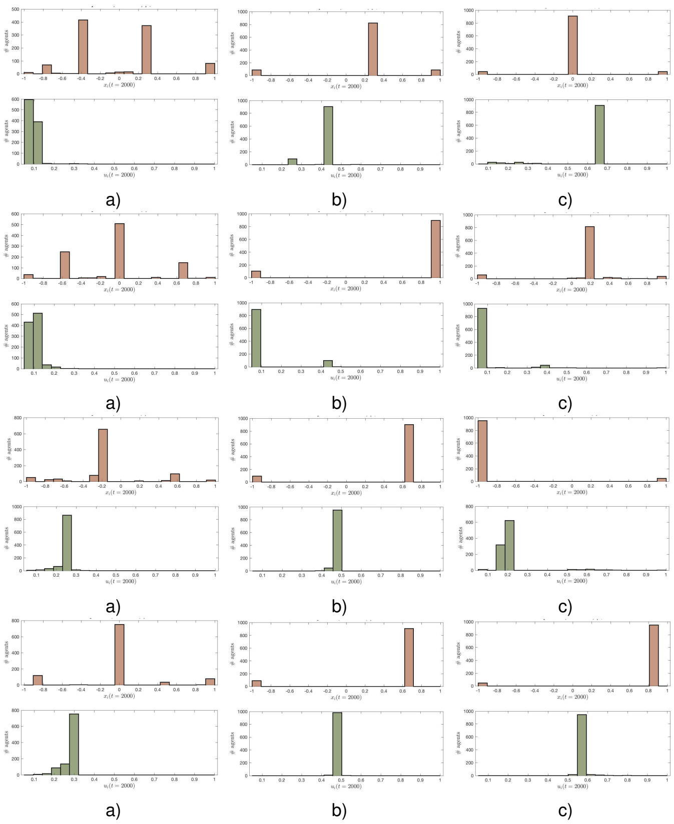

Figure 8 Final distributions of opinion and toleration on SW network at $p = 0.7$. Other conditions are the same as in Figure 7.

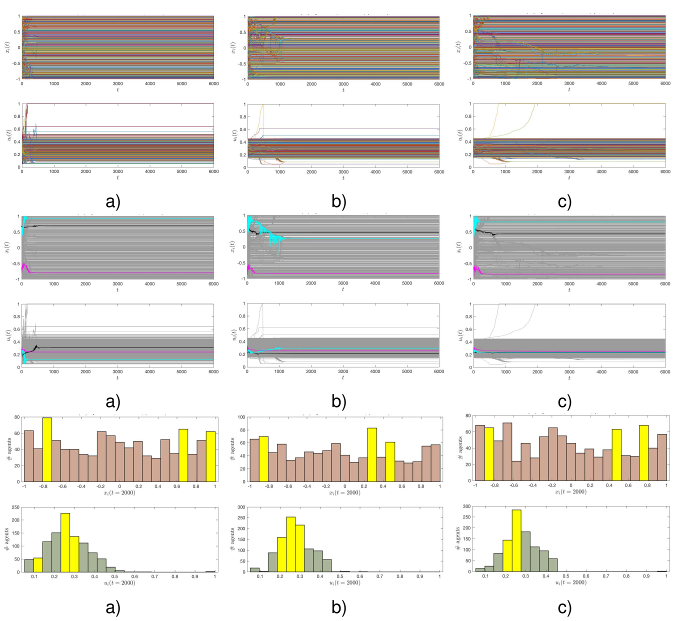

Figure 9 Opinion and toleration evolution on SF network at $(p, U) = (0.3, 0.3)$, IC$-A$. First double line – trajectories of agents' opinion and toleration evolution, second double line – trajectories of hubs' opinion and toleration evolution, third double line – final distributions of opinion (brown histograms) and toleration (green histograms), yellow bins include hubs. Columns: a) $\mu_r = 1$, b) $\mu_r = 1/5$ and c) $\mu_r = 1/10$.

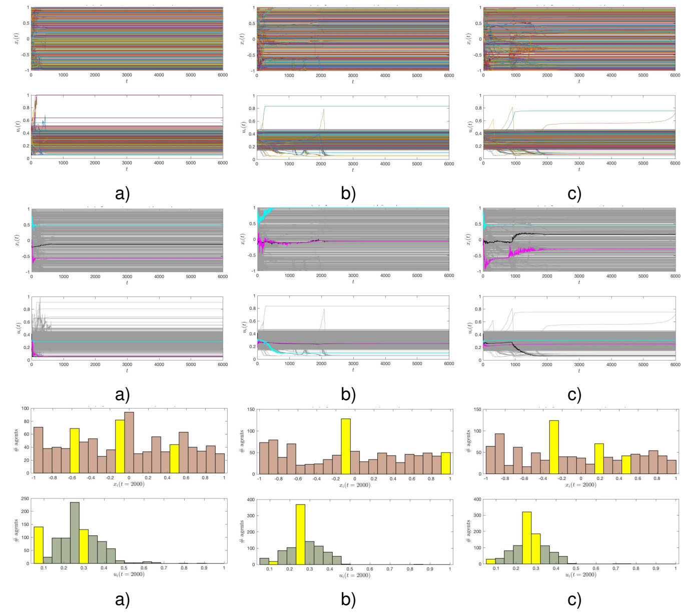

Figure 10 The same conditions as in Figure 9 but for IC$-B$.

For $U = 0.5$ and the decreasing ratio of convergence coefficients, μ_r's: 1. The time to reach an opinion stationary state increases when μ_r decreases and that is much greater than the time for SW networks. At $\mu_r = 1$ neither happens after that time, all trajectories of opinion evolution are parallel lines. Nevertheless, the evolution to stationary state is much longer and increasing for decreasing $\mu_r = 1/3, 1/5, 1/10$ (see Figure 11, first double line). 2. At $\mu_r = 1, 1/3, 1/5$, final distributions of opinion are similar each other and show a tendency to fragmentation in three opinion groups, but at $\mu_r = 1/10$ we observe a tendency to polarization (Figure 11, third double line). 3. The trajectory of principal PA-hub opinion is much stable and regular than that of the two smaller C-hubs, hubs behave similar to Brownian particles. We detect an irregular change of the final position of hubs in opinion space with the change of μ_r (Figure 11, second double line). 4. No regular change in the rate of toleration convergence is detected with the decrease of the convergence parameter μ_2. Toleration converges to the values smaller than U, forming two groups: one of them is a kind of bell distribution and the other consists of low tolerant agents. 5. The four generations of IC are not identical (the hubs are included), even being each of the same type uniform distribution, and the effects of their variations on the final opinion and toleration distributions were observed, in particular, through the variation of hubs' position and the size of bins that include hubs.

For $U = 0.7$ and decreasing ratio of convergence coefficients, μ_r's: 1. The time to reach an opinion stationary state (the hubs included) increases when μ_r decreases and that is much greater than the time for SW networks. Neither happens after that time, all trajectories of opinion evolution are parallel lines. The evolution to stationary state takes more time and is increasing for decreasing $\mu_r = 1, 1/3, 1/5, 1/10$ (see Figure 12). 2. Final distributions of opinion, being dependent of initial conditions noticeably, show a tendency to fragmentation or polarization at $\mu_r = 1, 1/3, 1/5, 1/10$ (Figure 12, third double line). 3. The trajectory of principal PA-hub opinion is more stable and regular than that of the two smaller C-hubs, hubs behave similar to Brownian particles. We detect an irregular change of the final position of hubs in opinion space with the change of μ_2, that depends of initials conditions also (igure

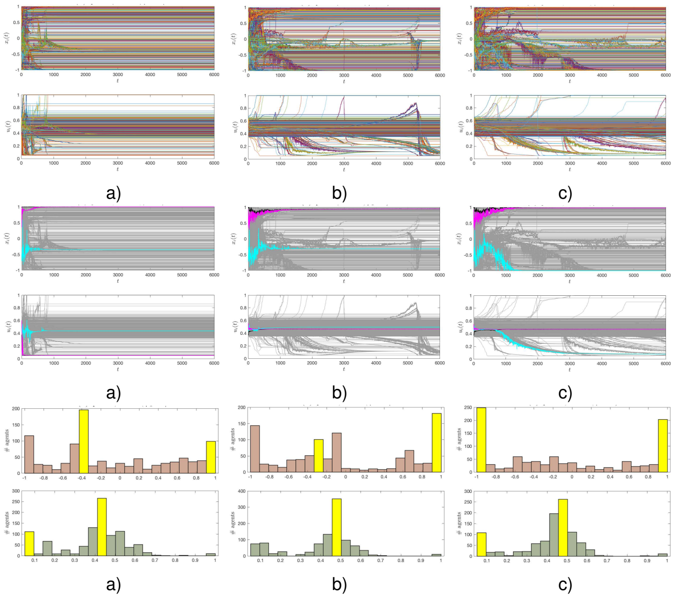

Figure 11 Opinion and toleration evolution in SF network at $(p, U) = (0.3, 0.5)$, IC$-A$. Other conditions are the same as in Figure 9.

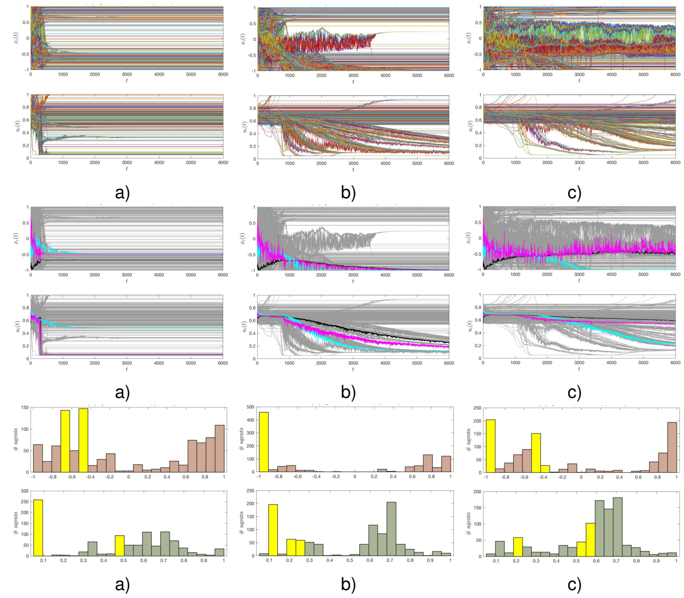

Figure 12 Opinion and toleration evolution in SF network at $(p, U) = (0.3, 0.7)$, IC$-A$. Other conditions are the same as in Figure 9.

12, second double line). A hub is frequently located in one of the important bins. Nevertheless, a hub does not represent an attractor of opinions because we observe final states when it belongs to a minority group. 4. Convergence of toleration is decreasing with decrease of the convergence parameter μ_2. Toleration converges to the values smaller than U, forming two groups: one of them is a kind of bell distribution and the other consists of a compact group of low tolerant agents, except for the case of low rate evolution of tolerance, $\mu_r = 1/10$ .5. The four generations of IC are not identical (the hubs are included), even being each of the same type uniform distribution, and the effects of initial conditions' variation on the final opinion and toleration distributions were observed, in particular, through the variation of hubs position and the size of bins that include hubs.

Finally, in SF societies at $p = 0.3$ no compact centrist opinion groups (a kind of local consensus) were detected and, in general, final opinion distributions look wide-ranging at almost all U and μ_r.

SF society and $\boldsymbol{p = 0.7}$**.** When the psychological composition of the society is $p = 0.7$ (C-agents are predominant), the cross-analysis of plots and histograms of the opinion and toleration evolution, for the set of parameters $U = 0.3, 0.5, 0.7$ and $\mu_r = 1, 1/3, 1/5, 1/10$, shows the following tendencies and peculiarities of opinion and toleration dynamics.

For $U = 0.3$ and the ratio of convergence coefficients, μ_r's: 1. The time to reach an opinion stationary state increases when decreases and that is greater than the time for SW networks. Nothing happens after that time, all trajectories of opinion and toleration are parallel lines (see Figure 13). The reason is that agents with close opinion have no common links or they have no overlap of opinion segments due to the distant opinions, so they are not pairs of interacting agents. 2. The trajectory of principal PA-hub opinion is much stable and regular than that of the two smaller C-hubs, all hubs behave similar to Brownian particles. We detect an irregular change of the final position of hubs in opinion space with the change of μ_2 (Figure 13, second double line). 3. We observe a tendency to fragmentation in the final opinion distribution, with noticeable opinion groups associated to the hubs. The final distributions of opinion for different $\mu_r = 1, 1/3, 1/5, 1/10$ differ each other not much, being different in position and size of hubs mainly (Figure 14). 4. No regular change in the rate of toleration convergence is detected with the decrease of the convergence parameter μ_2; toleration converges to the values smaller than U, with a tendency to form a kind of bell distribution.5. The four generations (A, B, C, D) of IC are not identical (the hubs are included), even being each of the same type uniform distribution, and the effects of variations in IC on the final opinion and toleration distributions were observed, in particular, through the variation in the position of hubs and size of bins that include hubs. In addition, groups of centrists are observed in contrast to that for the case of $(p, U) = (0.3, 0.3)$ (Figure 14).

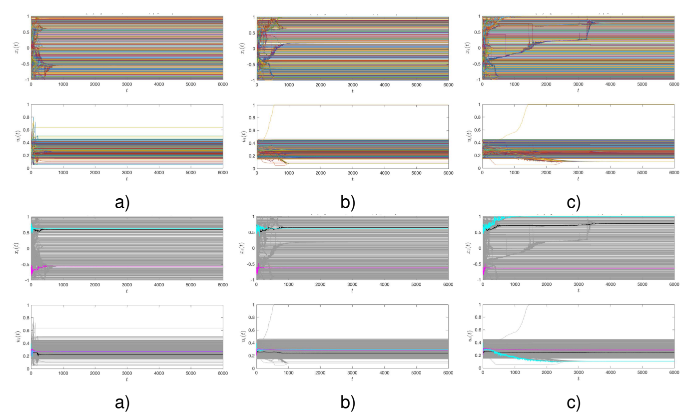

Figure 13 Opinion and toleration evolution in SF network at $(p, U) = (0.7, 0.3)$, IC$-A$. First double line – trajectories of agents' opinion and toleration evolution, second double line – trajectories of hubs' opinion and toleration evolution. Columns: a) $\mu_r = 1$, b) $\mu_r = 1/5$ and c) $\mu_r = 1/10$.

Figure 14 Final distributions of opinion and toleration (yellow bins include hubs) in network at $(p, U) = (0.7, 0.3)$: first double line is for the initial conditions A, second double line for IC$-B$, third double line for IC$-C$ and the fourth double line is for IC$-D$. Columns: a) $\mu_r = 1$, b) $\mu_r = 1/5$ and c) $\mu_r = 1/10$.

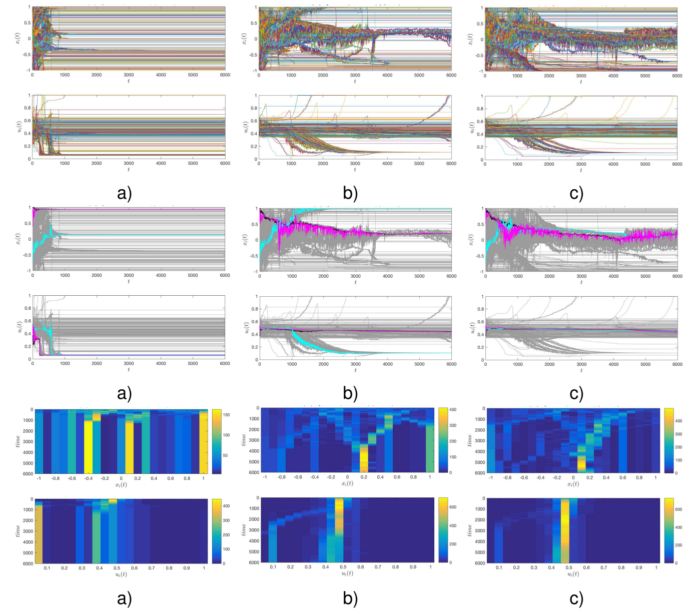

Figure 15 Opinion and toleration evolution in SF network at $(p, U) = (0.7, 0.5)$, IC$-A$. First double line – trajectories of agents' opinion and toleration evolution, second double line – trajectories of hubs' opinion and toleration evolution, third double line – color palette histogram of opinion and toleration evolution. Columns: a) $\mu_r = 1$, b) $\mu_r = 1/5$ and c) $\mu_r = 1/10$.

For the decreasing ratio of convergence coefficients, $\mu_r = 1, 1/3, 1/5, 1/10$ and $U = 0.5$: 1. The time to reach a quasi-stationary opinion state increases when μ_r decreases and that is much greater than the time for SW networks; an increasing number of bridges between opinion groups is also observed (Figure 15). 2. The trajectory of principal PA-hub opinion is much stable and regular than that of the two smaller C-hubs, all hubs behave similar to Brownian particles. We detect an irregular change of the final position of hubs in opinion space with change of μ_r (Figures 15 and 16, second double line). 3. A decrease in the rate of toleration convergence is detected with the decrease of the convergence parameter μ_2; toleration converges to the values smaller than U, with a tendency of forming a group of low tolerant agents, while the rest of agents are aggregated in a group with the toleration near U; in the case of $\mu_r = 1/10$ the toleration converges to a compact distribution centered almost at U (Figures 15, 16 and 17). 4. Significant difference in final distributions of opinion is observed as a result of change of μ_2 and relatively small variations in initial conditions (A, B, C, D) (Figure 17). In general, hubs are located in majority opinion groups. The states of polarization and consensus at different values of opinion and, sometimes, a tendency to fragmentation are observed. The latter indicates a kind of instability of the opinion in the C-PA society at $(p, U) = (0.7, 0.5)$.

For the decreasing ratio of convergence coefficients, $\mu_r = 1, 1/3, 1/5, 1/10$ and $U = 0.7$: 1. The time to reach a quasi-stationary opinion state increases when μ_r decreases and that is much greater than the time for SW networks; a number of bridges between opinion groups is large and increasing, so that the opinion variations are continuing for a long time (Figures 18 and 19). 2.

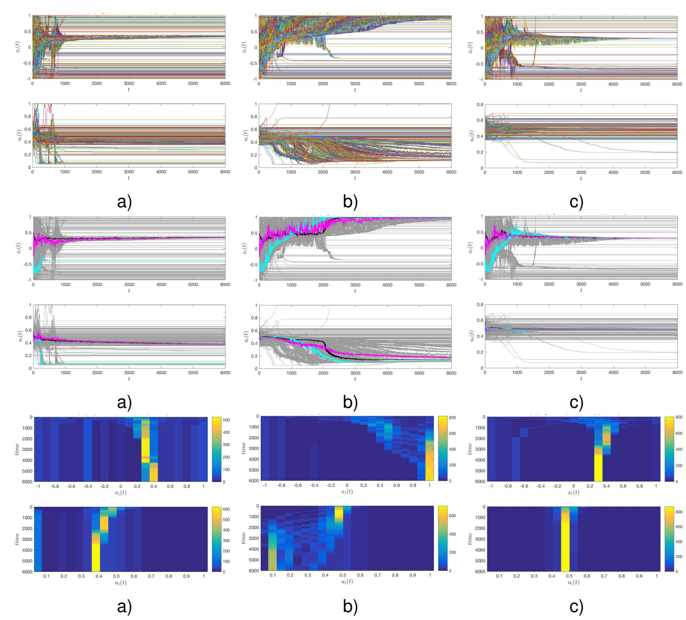

Figure 16 The same conditions as in Figure 15 but for IC$-B$.

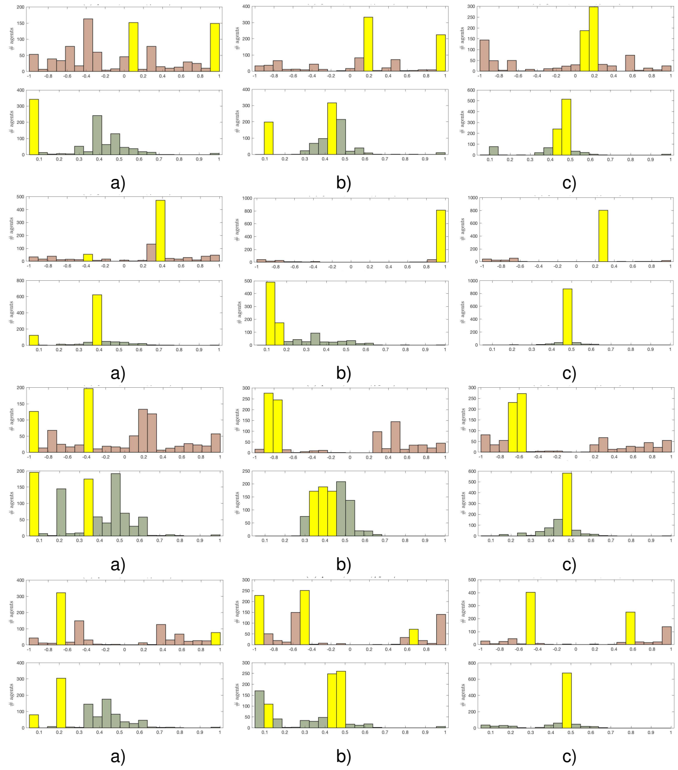

Figure 17 Final distributions of opinion and toleration (yellow bins include hubs) in network at $(p, U) = (0.7, 0.5)$: other conditions are the same as in Figure 14.

The trajectory of principal PA-hub opinion is much stable and regular than that of the two smaller C-hubs, all hubs behave similar to Brownian particles. We detect an irregular change of the final position of hubs in opinion space with change of μ_r (Figures 18 and 19, second double line; Figure 20). 3. A decrease in the rate of toleration convergence is detected with the decrease of the convergence parameter μ_2; toleration converges to the values smaller than U, with a tendency of forming a group of low tolerant agents at $\mu_r = 1, 1/3, 1/5$ especially, while the rest of agents are aggregated in a group with the toleration near U; large variations in final distributions of tolerance, associated to small variations in IC, are observed (Figures 18, 19 and 20). 4. Significant difference in final distributions of opinion is observed as a result of change of μ_r and of small variations in IC (Figure 20-(A, B, C, D)). In general, hubs are located in opinion majority groups. The states of polarization and consensus at different values of opinion and, sometimes a tendency to fragmentation are observed. The latter indicates a kind of instability of the opinion in the C-PA society at $(p, U) = (0.7, 0.7)$. Groups of centrists are not observed.

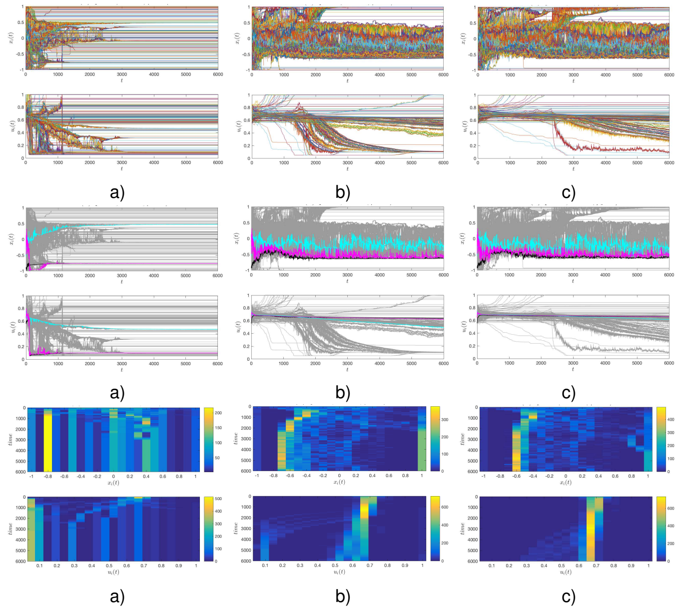

Figure 18 Opinion and toleration evolution in SF network at $(p, U) = (0.7, 0.7)$. Other conditions are the same as in Figure 15.

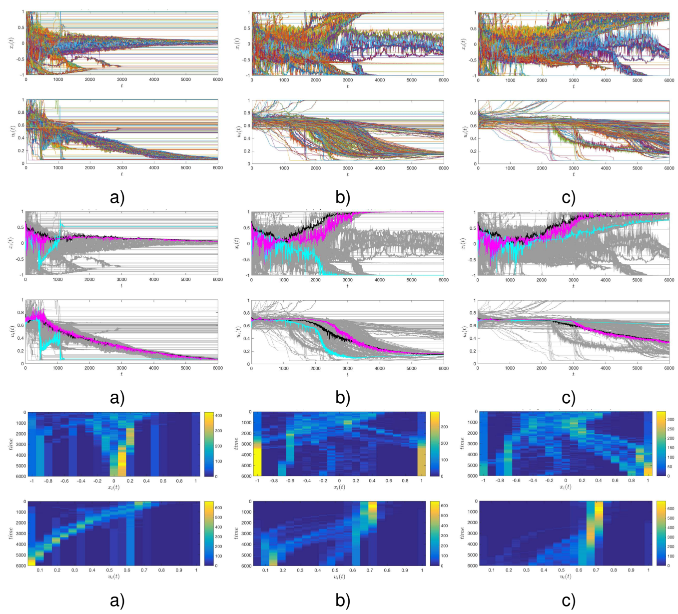

Figure 19 The same conditions as in Figure 18 but for IC$-B$.

Turning over the psychological type of hubs in society

Hubs of a social network, having a large number of links to other agents, interact more frequently with other agents than that do ordinary agents. We have turned over the psychological type of three hubs from PA, C and C to C, PA and PA, in order to see if it is important in opinion dynamics and run a series of experiments identical to that of the previous section. Comparative cross analysis of trajectories and final distributions of opinion and toleration of previous section show that evolution of opinion and toleration of hubs are notably affected by their psychological type and the ratio of timescales μ_r (Figure 21 as an example).

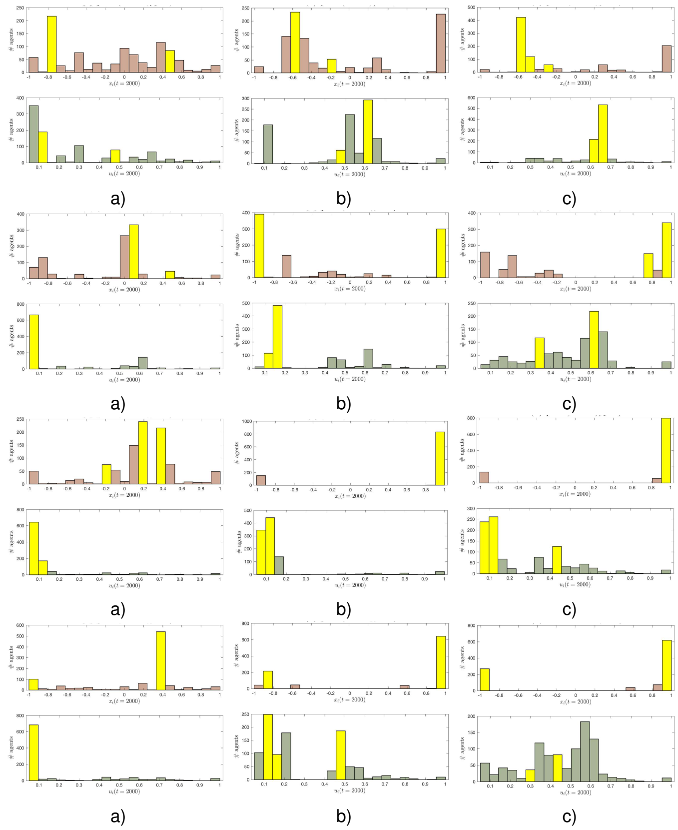

Figure 20 Final distributions of opinion and toleration (yellow bins include hubs) in network at $(p, U) = (0.7, 0.7)$: other conditions are the same as in Figures 14 and 17.

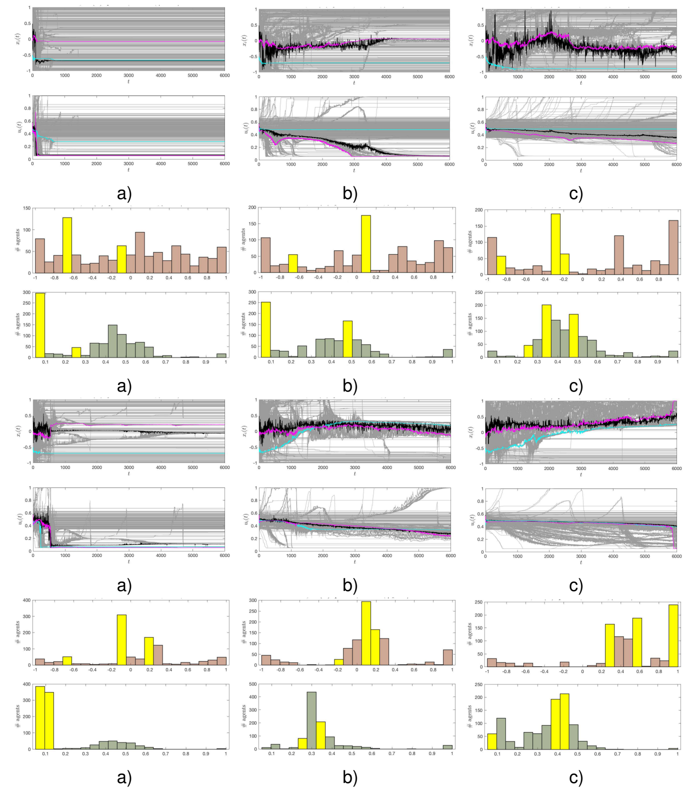

Figure 21 Opinion and toleration evolution in SF network for IC$-B$ at $(p, U) = (0.3, 0.5)$ and $(p, U) = (0.7, 0.5)$. Psychological type of hubs is C, PA and PA, that is inverted compared to Figures 11 and 15, respectively. Double lines one and three show trajectories of hubs' opinion and toleration evolution, lines two and four show final distributions of opinion and toleration. Columns: a) $\mu_r = 1$, b) $\mu_r = 1/3$ and c) $\mu_r = 1/10$.

CONCLUSION AND DISCUSSION

In the frame of C/PA relative agreement model of opinion dynamics we have analyzed mutual influence of social environment (SW and SF societies) and psychological aspects (psychological C/PA type and psychological profile) of agents on the opinion and toleration evolution in SW and SF artificial societies.

Mutual influence of social and psychological aspects of agents in mathematical models of opinion dynamics was treated systematically for the first time. Agents of C and PA psychological type were organized in SW and SF artificial societies. Psychological profile of each agent was represented by the *toleration* variable $u_i(t)$, that was interpreted vague as the opinion uncertainty in all previous works; the initial average toleration of society was regulated by the parameter U. C or PA psychological type responds for the agents' reaction on the opinion of others during agents' interaction, while the toleration shows the range of acceptability of others' opinions (wideness of agent's opinion interval). To study the mutual influence of opinion x_i and toleration u_i we took into account different time scale of opinion and toleration evolution by varying the relative parameter $\mu_r = \mu_2/\mu_1$, keeping in mind that opinion is the social characteristic and toleration expresses the psychological profile of agent.

1. Results of simulation demonstrate notable mutual influence of opinion and toleration on the dynamics of both, in particular, showing a split of toleration in two or three groups that was not revealed in other models. In general, the final toleration of agents shows a tendency to values lower than the initial U. In SW and SF societies the effects appear in different manner, being influenced by C/PA composition also. In other words, we observe how social environment influences psychology of agents, and vice versa. The peculiarity of opinion and toleration dynamics on SF networks comes from its tree-type topology, with communication channels that can be obstructed by the lack of relative agreement between the adjacent neighbors of network; the effect is observed through the straight line parallel individual trajectories in opinion and toleration evolution.

2. Consensus, being an opinion state desirable in some real life situations, unfortunately is not a typical one in a real society. In this concern, we found the consensus in mixed C/PA societies is the state rare to reach, in contrast to the results reported by other models (Yu *et al.* 2017). Opinion polarization and fragmentation accompanied by the formation of extremist groups resulted to be more recurrent states. In addition, the model shows the formation of groups of agents with a low toleration (agents closed for the interaction with others). In recent work (Huang *et al.* 2018) focused on the study of probability of opinion consensus emergence in SW societies, authors reported the consensus as a dominant state in a wide range of parameters of a modified DW model. It should be noted that modifications done to the original bounded confidence DW model have transformed it to a kind of relative agreement one, but without explicit use of toleration or uncer-

tainty as an agent's state variable. To some extent that model is similar to our one when the latter being applied to pure concord societies. When C-agents are predominant in C/PA societies, in our model the consensus is also observed more frequently.

3. The C/PA model used in this work is a dynamical system with stochastic elements; those are initial conditions for the variables and a stochastic updating dynamics through the random selection of interacting pairs of agents, even when the distribution of C and PA agents is fixed on a given network. When many simulations are done and the results of all scenarios are averaged, the latter will represent statistical tendencies in opinion and toleration evolution. The averaging of final opinion and toleration distributions, indicates general tendencies of opinion dynamics and can help to detect the bifurcation points of distributions (fragmentation, polarization or consensus) versus the parameter U. However, the averaged results can lead to confusing interpretation and explanation of opinion dynamics with respect to an individual scenario. For example, when single simulations show a dominant opinion bin in two alternating locations, then the averaging of these results will give exactly two groups of opinion and that is interpreted as splitting or polarization; that is not true for a single scenario, see the flips of position of predominant opinion groups $(SW, 0.7, 0.5, 1/10)$. This indicates the importance of particular scenarios analysis, one by one.

4. Dynamics of opinion and toleration in the C/PA societies on the SF networks is more complex and diverse than that on the SW networks. Both the process and final results of opinion and toleration evolution in C/PA model show significant difference for the SW and SF societies. That is due to the structural differences in organization of these societies; SF network has tree-like structure and agents-hubs. Having high degree of connection, hubs could be expected to be natural leaders of opinion, but our simulation experiments have shown it is not true. Even though hubs used to belong to majority but not outstanding groups, they behave like Brownian particles in opinion space more than leaders, their final states are not predictable. The size of the group of opinion followers depends on the degree of a hub and its psychological type. Sometimes, two hubs meet each other in the same group despite of different initial opinions. The final opinion and toleration states of SF society depend notably on small variations in initial conditions for the hubs (quasi leaders) and their psychological type, C or PA agent; the influence of timescale ratio μ_r is clearly important.

Few words in favor of toleration. In dynamical systems, any variable has to be measurable and that is evaluated by external measuring tool, in case of opinion models it is a kind of social enquiry. When the variable $u_i(t)$ is considered as an individual characteristic of i-agent at instant t and interpreted as the opinion uncertainty we meet a methodological difficulty if not a contradiction. In order to evaluate the state of a person, in sociology and psychology a specialist uses a kind of enquiry or a set of enquiries. To measure *opinion uncertainty* u_i at instant t one has to measure the opinion of an individual many times at an instant (in order to have statistical validation of the result), but that is not possible. Suppose we apply the individual enquiry for a short period of time. In this case the interval of time has to be so short that the opinion of the individual remains unvaried, and that has to be valid for each and every person of the society. That is also impossible because we don't know how fast or slow an individual opinion is changing. On the other hand, if we apply the enquiry to a set of persons, the result can't be considered as an individual characteristic. So, the "uncertainty" can't be measured instantaneously or it can't be considered as an individual variable.

The way to reconcile these contradictions is to interpret $u_i(t)$ as the toleration of i-agent to the opinions of others, that can be measured at each instant applying the same enquiry for a reasonably short interval of time (individual opinion remains unvaried) to each and every agent. *Opinion uncertainty*, as it was defined, can be self-evaluated only and, so there is no an objective criterion to validate it. Whereas the toleration (acceptability) can be measured, simply evaluating the range of opinions a person can accept. Acceptability is the base of agents' interaction and opinion exchange, but the opinion uncertainty of agent does not.

Opinion leadership is an important, if not crucial element in public opinion formation. So, for the future work we shall extent the model for the studying of the opinion leadership, using the self-organization of opinion and toleration in SW and SF societies of this work as the background. Pure mathematical study of the model is working on also.

Acknowledgments

Special thanks to the reviewers.

Availability of data and material

Not applicable.

Conflicts of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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How to cite this article: Kourmychev, E. and Abrica-Jacinto, N. L. A. The Effect of Agents' Psychology and Social Environment on the Opinion Formation: C/PA Relative Agreement Model in SW and SF Societies. *Chaos Theory and Applications*, 4(4), 212-225, 2022.