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İlerleyen Tür Sansürlü Verilere Dayalı Aralık Tahmini

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Öz: Bu çalışmada, literatürde olan bir pivot kullanılarak Weibull, Burr XII ve Gompertz dağılımı için güven aralıkları oluşturulmuştur. İstatistiksel sonuç çıkarımı ilerleyen sansürleme altında tartışılmıştır. Pivot elemanın dağılımı literatürde verilen tablolara göre farklı sansür şemaları için daha da genişletilmiştir. Tam örneklem durumunda, regresyon modelleri pivot elemanının sınır değerlerini elde etmek için tahmin edilmiştir. Ayrıca sayısal bir örnek verilmiştir.

Anahtar kelimeler: Burr XII dağılımı, Gompertz dağılımı, güven aralığı, ilerleyen sansürleme, pivot, Weibull dağılımı,

Interval Estimation Based on Progressively Censored Data

Abstract: In this paper, a pivot in the literarture is used to construct the confidence intervals for Weibull, Burr XII and Gompertz distribution. Statistical inference are discussed under progressive censoring. The tables for the distribution of pivotal quantity are extended according to tables of existing papers with different censoring schemes. Regression models are estimated to get cut off point of pivotal quantity for complete sample. Numerical examples are also provided.

Keywords: Progressive censoring, Gompertz distribution, Burr XII distribution, Weibull distribution, pivot, confidence interval

1. Introduction

In reliability analysis, life testing with censoring scehemes are performed when the cost of the test items is high or/and test time is restricted. A type II censored sample is one for which:

1. Only the m smallest observations in a sample of size n are observed, $1 \le m \le n$.

2. m is determined before the data are collected.

The model of progressive type-II right censoring is also defined as follows: Suppose n identical units are placed on a lifetime test. At the time of ith failure, r_i surviving units are randomly withdrawn from the experiment, $1 \le m \le n$. Thus, if m failures are observed then $n-m$ units are progressively censored. $X_{1:m:n}^{\mathbf{R}} < X_{2:m:n}^{\mathbf{R}} < \cdots < X_{m:m:n}^{\mathbf{R}}$ describe the progressively censored failure times where $\mathbf{R} = (r_1, r_2, \ldots, r_m)$ denotes the censoring scheme. Extensive publications can be found in the literature which discuss the statistical inference for progressively censored data under several lifetime distributions. Some of them are (Ali and Jaheen, 2002), (Jaheen, 2003), (Soliman, 2005), (Wu et al., 2006), (Asgharzadeh, 2006), (Wu, 2008), (Kundu, 2008). A recent account on progressive censoring can be found in the book by Balakrishnan and Aggarwala (2000). As a special case if $\mathbf{R} = (0, 0, \ldots, m)$, we obtain the Type-II censored order statistics (Bairamov and Eryılmaz, 2006). For more details see Balakrishnan and Aggarwala (2000). Chen (1997) intorduced a pivotal quantity for interval estimation of Weibull parameters based on tip-II right censored sample as follows:

$$\xi(\beta; n, m) = \frac{\sum_{i=1}^{m-1}\left(X_{i:n}{}^{\beta} + (n-m+1)X_{m:n}{}^{\beta}\right)/n}{\left(\prod_{i=1}^{m} X_{i:n} X_{m:n}{}^{n-m+1}\right)^{\beta/n}}, \quad (1)$$

where $X_{i:n}$ is the ith order statistics in a random sample of size n. In the Chen's (1997) paper, distribution of ξ can not be derived and some tables are provided by using Monte Carlo simulation. Later, the pivot (1) is modifed by Wu et al. (2011) for progressive censoring pattern as

$$\Phi(\beta) = \frac{\sum_{i=1}^{m}(1+r_i)\, X_{i:m:n}^{\mathbf{R}} / n}{\prod_{i=1}^{m} X_{i:m:n}^{\mathbf{R}\ (1+r_i)/n}}. \quad (2)$$

Under progressive censoring, the pivot (2) is used to construct the confidence interval for shape parameter of Chen distribution by Wu et al. (2011) and Weibull, Burr XII and Gompertz distributions by Akdoğan et al. (2013).

In this paper, we consider the pivot (2) for Weibull, Burr XII and Gompertz distribution as in Akdoğan et al. (2013). In Section 2, The distribution of pivotal quanity are invesitagted for different censoring scheme in progressive censoring by simulations. The distribution tables are extended comapred to Akdoğan et al. (2013). The cut of are also determined by using regression analysis. Usability of the pivot (2) are discussed in Section 3. In Section 4, conclusion is provided to close the paper.

2. Distribution Tables for the Pivotal Quantity

In this section, distribution quantiles are provided for pivotal quantity (2) through a simulation under progressive censoring. Indeed, Wu et al. (2011) gave some tables like this but these are extended the tables here. Several regression equations are obtained to get the cut off points of the distribution of pivotal quantity (2) in complete sample. The Matlab code is written to get the tables in this section. The code is given in Appendix. In all tables in this section, α is thougth to be upper tail probability.

Table 1. Several quantiles of distribtuion of pivotal quantity for different censoring schemes

		$\alpha \rightarrow$	0.9950	0.9900	0.9750	0.9500	0.0500	0.0250	0.0100	0.0050
n	m	**R** $\downarrow$								
10	5	(5,0,0,0,0)	1.069	1.105	1.175	1.252	11.598	17.577	31.178	48.183
		(0,0,0,0,5)	1.008	1.013	1.023	1.034	1.681	1.831	2.068	2.243
		(0,0,5,0,0)	1.022	1.033	1.052	1.079	2.844	3.457	4.418	5.274
		(0,5,0,0,0)	1.035	1.055	1.092	1.131	4.303	5.664	8.122	10.774
		(0,0,0,5,0)	1.013	1.019	1.034	1.051	2.088	2.462	2.926	3.378
		(4,1,0,0,0)	1.073	1.095	1.151	1.228	8.657	12.861	21.499	31.468
		(4,0,1,0,0)	1.061	1.077	1.132	1.188	8.134	11.426	17.749	24.468
		(4,0,0,0,1)	1.052	1.071	1.120	1.178	7.403	10.935	16.328	20.414
		(3,0,2,0,0)	1.049	1.069	1.113	1.160	6.001	8.336	12.295	17.286
		(3,0,0,0,2)	1.045	1.058	1.093	1.140	5.259	6.915	10.575	14.078
		(2,0,0,0,3)	1.035	1.054	1.092	1.135	4.327	5.550	7.495	9.763
		(2,2,1,0,0)	1.042	1.064	1.103	1.158	5.087	6.544	8.902	11.844
		(2,0,0,2,1)	1.025	1.040	1.069	1.102	3.579	4.517	5.854	7.099
		(1,2,0,0,2)	1.028	1.039	1.063	1.093	3.273	4.027	5.329	6.189
		(1,1,1,1,1)	**1.027**	**1.038**	**1.059**	**1.090**	**3.073**	**3.647**	**4.505**	**5.508**
		(1,4,0,0,0)	1.045	1.065	1.102	1.150	4.932	6.388	9.016	11.504
		(1,0,4,0,0)	1.032	1.043	1.072	1.106	3.423	4.210	5.662	6.596
		(1,0,0,4,0)	1.017	1.024	1.042	1.061	2.399	2.797	3.424	4.052
		(1,0,0,0,4)	1.021	1.033	1.055	1.080	2.753	3.288	4.252	5.221
		(3,1,1,0,0)	1.058	1.083	1.120	1.178	6.302	8.696	12.679	16.351
		(3,0,1,1,0)	1.044	1.065	1.107	1.157	5.531	7.473	10.351	13.879
		(3,0,0,1,1)	1.037	1.057	1.091	1.132	5.064	6.744	10.073	12.836
		(1,3,1,0,0)	1.044	1.062	1.096	1.142	4.312	5.436	7.427	8.792

Table 1. Continue

		$\alpha \rightarrow$	0.9950	0.9900	0.9750	0.9500	0.0500	0.0250	0.0100	0.0050
n	m	**R** $\downarrow$								
10	8	(2,0,0,0,0,0,0,0)	1.124	1.159	1.221	1.296	4.973	6.286	7.912	9.483
		(0,2,0,0,0,0,0,0)	1.107	1.130	1.178	1.230	3.533	4.186	5.321	5.992
		(0,0,0,2,0,0,0,0)	1.074	1.089	1.129	1.170	2.667	3.017	3.495	3.846
		(0,0,0,0,0,0,2,0)	1.060	1.074	1.103	1.132	2.277	2.556	2.940	3.223
		(0,0,0,0,0,0,0,2)	1.053	1.069	1.099	1.124	2.180	2.420	2.780	3.042
		(1,1,0,0,0,0,0,0)	1.119	1.145	1.203	1.257	4.008	4.812	6.129	7.296
		(1,0,1,0,0,0,0,0)	1.107	1.134	1.183	1.241	3.731	4.428	5.378	6.300
		(1,0,0,1,0,0,0,0)	1.099	1.126	1.173	1.232	3.595	4.242	5.267	6.380
		(1,0,0,0,1,0,0,0)	1.091	1.117	1.164	1.221	3.461	4.126	5.097	5.794
		(1,0,0,0,0,1,0,0)	1.093	1.112	1.159	1.205	3.395	4.058	5.039	6.092
		(0,0,0,0,0,0,1,1)	1.060	1.071	1.100	1.131	2.244	2.524	2.911	3.240
		(0,0,0,0,0,1,0,1)	1.060	1.074	1.100	1.134	2.269	2.550	2.927	3.227
		(0,0,0,0,1,0,0,1)	1.066	1.078	1.107	1.140	2.341	2.268	3.031	3.374
		(0,0,0,1,0,0,0,1)	1.062	1.077	1.108	1.144	2.427	2.740	3.204	3.529

Table 1. Continue

		$\alpha \rightarrow$	0.9950	0.9900	0.9750	0.9500	0.0500	0.0250	0.0100	0.0050
n	m	**R** $\downarrow$								
12	5	(7,0,0,0,0)	1.084	1.123	1.201	1.298	15.305	26.669	49.111	71.402
		(0,7,0,0,0)	1.039	1.057	1.089	1.137	4.745	6.588	9.106	11.238
		(0,0,7,0,0)	1.021	1.031	1.052	1.076	2.873	3.52	4.505	5.526
		(0,0,0,7,0)	1.012	1.016	1.03	1.045	2.011	2.327	2.881	3.46
		(0,0,0,0,7)	1.008	1.012	1.018	1.027	1.535	1.662	1.857	2.028
		(5,2,0,0,0)	1.072	1.108	1.167	1.233	9.35	13.758	21.214	30.532
		(0,5,2,0,0)	1.034	1.05	1.079	1.119	3.952	5.017	6.661	8.023
		(0,0,5,2,0)	1.018	1.026	1.042	1.062	2.45	2.869	3.56	4.048
		(0,0,0,5,2)	1.01	1.015	1.025	1.037	1.758	1.964	2.293	2.602
		(5,1,1,0,0)	1.071	1.097	1.149	1.218	8.615	12.842	20.147	28.535
		(5,0,0,1,1)	1.051	1.073	1.121	1.174	7.298	10.404	17.185	24.058
		(0,0,5,1,1)	1.015	1.02	1.036	1.053	2.263	2.664	3.366	3.9
		(2,2,2,1,0)	1.034	1.049	1.085	1.129	4.106	5.352	7.174	9.03
		(0,2,2,2,1)	1.017	1.024	1.04	1.061	2.408	2.849	3.472	3.977
		(4,1,1,1,0)	1.056	1.075	1.121	1.176	6.334	8.931	13.262	17.381
		(4,0,1,1,1)	1.048	1.067	1.101	1.148	5.725	7.766	11.971	16.292
		(1,1,1,4,0)	1.022	1.032	1.056	1.083	2.778	3.296	4.121	4.96
		(1,1,1,0,4)	1.018	1.026	1.044	1.064	2.414	2.793	3.425	3.914

Table 1. Continue

		$\alpha \rightarrow$	0.9950	0.9900	0.9750	0.9500	0.0500	0.0250	0.0100	0.0050
n	m	**R** ↓								
12	7	(5,0,0,0,0,0,0)	1.155	1.189	1.271	1.36	10.128	14.194	23.417	30.853
		(0,5,0,0,0,0,0)	1.05	1.064	1.096	1.129	2.638	3.089	3.789	4.455
		(0,0,5,0,0,0,0)	1.027	1.034	1.049	1.069	1.784	1.955	2.16	2.303
		(0,0,0,5,0,0,0)	1.146	1.185	1.255	1.343	8.018	11.361	17.055	22.435
		(0,0,0,0,5,0,0)	1.134	1.18	1.252	1.339	7.48	10.124	15.023	20.419
		(0,0,0,0,0,0,5)	1.112	1.153	1.221	1.301	6.727	9.099	13.577	18.188
		(4,1,0,0,0,0,0)	1.13	1.168	1.235	1.318	6.559	8.501	12.594	15.822
		(4,0,0,1,0,0,0)	1.106	1.138	1.199	1.261	5.281	6.752	9.028	11.281
		(0,0,0,0,0,4,1)	1.065	1.081	1.12	1.16	3.364	4.234	5.489	6.511
		(0,0,0,0,0,1,4)	1.119	1.158	1.221	1.288	5.652	7.184	9.885	12.862
		(4,0,0,0,1,0,0)	1.084	1.108	1.158	1.214	3.988	4.834	6.572	7.987
		(4,0,0,0,0,1,0)	0.94	0.969	1.021	1.082	3.527	4.293	5.361	6.158
		(3,2,0,0,0,0,0)	1.101	1.132	1.193	1.263	5.042	6.29	8.269	10.564
		(3,0,0,2,0,0,0)	1.07	1.093	1.132	1.176	3.224	3.765	4.809	5.526
		(3,0,0,0,2,0,0)	1.044	1.058	1.085	1.113	2.442	2.796	3.298	3.711
		(0,0,0,0,0,3,2)	1.109	1.153	1.22	1.292	9.221	12.821	21.587	31.248
		(0,0,0,0,0,2,3)	1.043	1.059	1.088	1.121	3.046	3.622	4.766	5.824
		(2,0,0,0,0,0,3)	1.02	1.028	1.043	1.059	1.84	2.036	2.351	2.533
		(2,3,0,0,0,0,0)	1.09	1.119	1.177	1.253	7.058	9.267	14.184	17.329
		(2,0,0,3,0,0,0)	1.082	1.115	1.176	1.244	6.524	8.809	13.718	17.329
		(1,1,1,1,1,0,0)	1.073	1.1	1.147	1.206	5.834	7.889	11.516	14.729
		(1,4,0,0,0,0,0)	1.083	1.118	1.17	1.23	5.609	7.256	9.8	12.349
		(1,0,0,4,0,0,0)	1.049	1.068	1.104	1.153	3.826	4.792	6.164	7.627
		(1,0,0,0,0,0,4)	1.051	1.068	1.1	1.138	3.24	3.901	5.121	6.234
		(1,0,0,0,4,0,0)	1.034	1.047	1.069	1.098	2.641	3.084	3.76	4.352
		(1,0,0,0,0,4,0)	0.872	0.902	0.957	1.011	3.698	4.569	5.961	7.404

Table 1. Continue

		$\alpha \rightarrow$	0.9950	0.9900	0.9750	0.9500	0.0500	0.0250	0.0100	0.0050
n	m	**R** ↓								
18	12	$r_1 = 6\ r_i = 0$	1.321	1.372	1.495	1.611	8.554	11.663	16.85	22.034
		$r_3 = 6\ r_i = 0$	1.185	1.23	1.292	1.353	3.563	4.144	4.982	5.599
		$r_6 = 6\ r_i = 0$	1.125	1.15	1.191	1.236	2.48	2.766	3.187	3.462
		$r_9 = 6\ r_i = 0$	1.109	1.124	1.157	1.189	2.174	2.352	2.658	2.9
		$r_{12} = 6\ r_i = 0$	1.079	1.091	1.118	1.141	1.818	1.933	2.106	2.248
		$r_1 = 4\, r_2 = 2\, r_i = 0$	1.293	1.349	1.453	1.551	6.512	8.1	10.779	13.046
		$r_1 = 4\, r_5 = 2\, r_i = 0$	1.274	1.312	1.39	1.477	5.403	6.654	8.559	10.689
		$r_1 = 4\, r_{12} = 2\, r_i = 0$	1.222	1.262	1.333	1.412	4.81	5.842	7.771	9.565
		$r_1 = 4\, r_2 = 1$ $r_3 = 1\, r_i = 0$	1.274	1.335	1.419	1.52	6.163	7.602	10.443	12.895
		$r_1 = 4\, r_5 = 1$ $r_6 = 1\, r_i = 0$	1.247	1.299	1.379	1.465	5.326	6.539	8.526	10.624
		$r_1 = 4\, r_{11} = 1$ $r_{12} = 1\, r_i = 0$	1.222	1.263	1.327	1.414	4.776	5.901	7.584	8.762
		$r_1 = 3\, r_2 = 3\, r_i = 0$	1.276	1.333	1.424	1.531	5.83	6.923	9.108	11.547
		$r_1 = 4\, r_2 = 2\, r_i = 0$	1.078	1.09	1.115	1.143	1.84	1.978	2.154	2.306
		$r_1 = 3\, r_2 = 2$ $r_3 = 1\, r_i = 0$	1.258	1.314	1.41	1.508	5.418	6.525	8.417	9.864
		$r_{10} = 3\, r_{11} = 2$ $r_{12} = 1\, r_i = 0$	1.083	1.1	1.125	1.153	1.895	2.037	2.231	2.411
		$r_1 = 3\ r_2 = 1$ $r_3 = 1\, r_4 = 1\, r_i = 0$	1.253	1.314	1.399	1.483	5.047	6.082	7.808	8.989
		$r_9 = 1\ r_{10} = 1$ $r_{11} = 1\, r_{12} = 3$ $r_i = 0$	1.08	1.094	1.121	1.147	1.862	1.988	2.147	2.298
		$r_1 = 2\, r_2 = 2$ $r_3 = 2\, r_i = 0$	1.248	1.305	1.386	1.471	4.673	5.601	6.968	8.361
		$r_{10} = 2\, r_{11} = 2$ $r_{12} = 2\, r_i = 0$	1.083	1.098	1.125	1.15	1.857	1.996	2.174	2.305

Table 1. Continue

		$\alpha \rightarrow$	0.9950	0.9900	0.9750	0.9500	0.0500	0.0250	0.0100	0.0050
n	m	$\mathbf{R}\downarrow$								
20	11	$r_1 = 9\ r_i = 0$	1.349	1.414	1.554	1.723	12.873	18.428	30.649	44.77
		$r_{11} = 9\ r_i = 0$	1.054	1.064	1.083	1.105	1.641	1.756	1.89	2.033
		$r_3 = 9\ r_i = 0$	1.152	1.178	1.23	1.286	3.187	3.661	4.321	4.771
		$r_6 = 9\ r_i = 0$	1.099	1.122	1.161	1.203	2.407	2.649	3.05	3.351
		$r_9 = 9\ r_i = 0$	1.073	1.088	1.112	1.137	1.866	2.009	2.206	2.341
		$r_1 = 7\ r_2 = 2\ r_i = 0$	1.317	1.385	1.51	1.651	9.756	12.942	19.028	24.739
		$r_1 = 7\ r_5 = 2\ r_i = 0$	1.288	1.349	1.472	1.584	8.412	11.606	16.903	22.226
		$r_1 = 7\ r_{11} = 2\ r_i = 0$	1.254	1.301	1.404	1.513	7.954	10.79	15.122	19.009
		$r_1 = 7\ r_2 = 1\ r_3 = 1\ r_i = 0$	1.306	1.381	1.506	1.639	9.319	12.536	18.659	23.973
		$r_1 = 7\ r_5 = 1\ r_6 = 1\ r_i = 0$	1.304	1.361	1.46	1.571	8.305	10.959	15.402	18.657
		$r_1 = 7\ r_{10} = 1\ r_{11} = 1\ r_i = 0$	1.273	1.319	1.416	1.517	7.715	9.829	14.107	20.297
		$r_1 = 5\ r_2 = 4\ r_i = 0$	1.313	1.382	1.482	1.613	7.873	10.009	13.626	17.192
		$r_1 = 5\ r_6 = 4\ r_i = 0$	1.23	1.271	1.366	1.443	5.563	6.913	9.289	11.743
		$r_1 = 5\ r_{11} = 4\ r_i = 0$	1.177	1.216	1.283	1.362	4.862	6.245	8.194	10.52
		$r_{10} = 5\ r_{11} = 4\ r_i = 0$	1.055	1.065	1.088	1.109	1.665	1.763	1.916	2.014
		$r_1 = 5\ r_2 = 2\ r_3 = 2\ r_i = 0$	1.302	1.356	1.449	1.56	7.215	9.116	12.116	14.714
		$r_1 = 5\ r_5 = 2\ r_6 = 2\ r_i = 0$	1.233	1.28	1.365	1.458	5.641	6.934	9.548	11.455
		$r_9 = 5\ r_{10} = 2\ r_{11} = 2\ r_i = 0$	1.061	1.073	1.094	1.116	1.724	1.845	2.007	2.108
		$r_1 = 5\ r_2 = 1\ r_3 = 1\ r_4 = 1\ r_5 = 1\ r_i = 0$	1.275	1.326	1.426	1.541	6.449	7.986	10.974	13.445
		$r_1 = 5\ r_8 = 1\ r_9 = 1\ r_{10} = 1\ r_{11} = 1\ r_i = 0$	1.184	1.228	1.305	1.382	4.975	6.178	8.192	10.186
		$r_7 = 5\ r_8 = 1\ r_9 = 1\ r_{10} = 1\ r_{11} = 1\ r_i = 0$	1.07	1.081	1.11	1.138	1.903	2.055	2.257	2.444
		$r_1 = 2\ r_2 = 2\ r_3 = 2\ r_4 = 2\ r_5 = 1\ r_i = 0$	1.217	1.265	1.345	1.422	4.418	5.21	6.426	7.395
		$r_7 = 2\ r_8 = 2\ r_9 = 2\ r_{10} = 2\ r_{11} = 1\ r_i = 0$	1.059	1.074	1.099	1.123	1.763	1.882	2.062	2.205
		$r_1, r_{2,} \ldots r_9 = 1\ r_i = 0$	1.147	1.176	1.222	1.273	2.296	3.36	3.898	4.366

Table 1. Continue

		$\alpha \rightarrow$	0.9950	0.9900	0.9750	0.9500	0.0500	0.0250	0.0100	0.0050
n	m	**R**$\downarrow$								
25	6	(19,0,0,0,0,0)	1.222	1.3	1.405	1.558	30.787	53.063	109.011	184.669
		(18,1,0,0,0,0)	1.2	1.271	1.413	1.554	25.736	42.489	85.379	148.972
		(17,1,1,0,0,0)	1.191	1.23	1.358	1.495	24.154	39.585	73.937	114.672
		(16,1,1,1,0,0)	1.166	1.222	1.332	1.453	20.254	36.04	67.257	104.986
		(15,1,1,1,1,0)	1.146	1.197	1.293	1.417	15.564	25.215	45.964	747.552
		(14,1,1,1,1,1)	1.137	1.173	1.258	1.359	13.048	20.45	36.587	53.625
		(0,0,0,0,0,19)	1.007	1.01	1.015	1.022	1.267	1.32	1.381	1.433
		(0,0,0,0,1,18)	1.007	1.01	1.015	1.022	1.266	1.312	1.38	1.444
		(0,0,0,1,1,17)	1.007	1.01	1.016	1.022	1.286	1.347	1.419	1.479
		(0,0,1,1,1,16)	1.009	1.012	1.019	1.026	1.327	1.394	1.48	1.552
		(0,1,1,1,1,15)	1.01	1.014	1.022	1.031	1.419	1.501	1.631	1.714
		(1,1,1,1,1,14)	1.016	1.021	1.031	1.044	1.63	1.765	1.926	2.046
		(9,2,2,2,2,2)	1.082	1.111	1.17	1.231	6.143	8.339	12.253	16.498
		(2,9,2,2,2,2)	1.047	1.077	1.111	1.154	3.793	4.742	6.178	7.23
		(2,2,9,2,2,2)	1.04	1.057	1.086	1.119	2.891	3.42	4.099	4.795
		(2,2,2,9,2,2)	1.04	1.051	1.076	1.105	2.575	2.984	3.555	3.994
		(2,2,2,2,9,2)	1.03	1.038	1.059	1.084	2.273	2.608	3.106	3.468
		(2,2,2,2,2,9)	1.025	1.033	1.052	1.074	2.094	2.367	2.728	2.998

Table 1. Continue

		$\alpha \rightarrow$	0.9950	0.9900	0.9750	0.9500	0.0500	0.0250	0.0100	0.0050
n	m	**R** $\downarrow$								
25	20	$r_1 = 5\ r_i = 0$	1.419	1.48	1.577	1.673	5.215	6.177	7.649	9.017
		$r_3 = 5\ r_i = 0$	1.303	1.339	1.414	1.476	3.182	3.522	3.991	4.426
		$r_6 = 5\ r_i = 0$	1.244	1.28	1.333	1.39	2.562	2.81	3.136	3.43
		$r_9 = 5\ r_i = 0$	1.203	1.228	1.276	1.321	2.269	2.431	2.65	2.774
		$r_{20} = 5\ r_i = 0$	1.162	1.181	1.215	1.247	1.957	2.084	2.264	2.363
		$r_1 = 4\ r_2 = 1\ r_i = 0$	1.39	1.452	1.546	1.64	4.779	5.615	6.881	8.151
		$r_1 = 4\ r_{10} = 1\ r_i = 0$	1.362	1.409	1.506	1.596	4.382	5.083	6.095	6.875
		$r_1 = 4\ r_{20} = 1\ r_i = 0$	1.365	1.414	1.496	1.584	4.214	4.868	5.978	6.956
		$r_{19} = 4\ r_{20} = 1\ r_i = 0$	1.16	1.179	1.22	1.251	1.972	2.083	2.27	2.403
		$r_1 = 3\ r_2 = 2\ r_i = 0$	1.386	1.445	1.529	1.622	4.392	5.18	6.151	7.028
		$r_1 = 3\ r_2 = 1\ r_3 = 1$ $r_i = 0$	1.391	1.441	1.531	1.627	4.242	4.902	5.887	6.722
		$r_1 = 3\ r_{19} = 1\ r_{20} = 1$ $r_i = 0$	1.306	1.355	1.412	1.483	3.436	3.868	4.573	5.134
		$r_{18} = 3\ r_{19} = 1$ $r_{20} = 1\ \ r_i = 0$	1.161	1.182	1.221	1.255	1.952	2.071	2.234	2.358
		$r_1 = 2\ r_2 = 2\ r_3 = 1$ $r_i = 0$	1.381	1.433	1.512	1.593	4.003	4.551	5.326	5.847
		$r_{18} = 2\ r_{19} = 2$ $r_{20} = 1\ \ r_i = 0$	1.165	1.184	1.218	1.251	1.967	2.095	2.272	2.38
		$r_{17} = 2\ r_{18} = 1$ $r_{19} = 1\ r_{20} = 1\ \ r_i = 0$	1.161	1.179	1.215	1.249	1.973	2.093	2.263	2.358
		$r_1, r_{2,} \ldots . r_5 = 1\ r_i = 0$	1.319	1.357	1.437	1.51	3.333	3.672	4.153	4.549
		$r_{16}, r_{17,} \ldots . r_{20} = 1$ $r_i = 0$	1.156	1.185	1.222	1.253	1.986	2.106	2.251	2.383

Table 1. Continue

		$\alpha \rightarrow$	0.9950	0.9900	0.9750	0.9500	0.0500	0.0250	0.0100	0.0050
n	m	**R** ↓								
30	8	(22,0,0,0,0,0,0,0)	1.37	1.452	1.629	1.846	35.948	62.342	119.874	181.106
		(0,22,0,0,0,0,0,0)	1.199	1.254	1.346	1.442	8.605	11.433	16.584	21.37
		(0,0,22,0,0,0,0,0)	1.129	1.156	1.216	1.279	4.688	5.877	7.944	9.43
		(0,0,0,0,22,0,0,0)	1.076	1.092	1.133	1.175	3.201	3.825	4.68	5.289
		(0,0,0,0,0,0,22,0)	1.023	1.028	1.039	1.051	1.589	1.727	1.943	2.114
		(0,0,0,0,0,0,0,22)	1.014	1.018	1.025	1.033	1.285	1.331	1.39	1.411
		(20,1,1,0,0,0,0,0)	1.308	1.411	1.562	1.775	25.657	42.799	75.251	111.098
		(20,0,0,0,0,0,1,1)	1.328	1.392	1.536	1.691	24.852	43.462	8.185	144.978
		(0,0,0,0,0,20,1,1)	1.028	1.035	1.049	1.064	1.774	1.965	2.253	2.488
		(18,1,1,1,1,0,0,0)	1.2998	1.38	1.534	1.683	19.546	31.328	59.915	93.34
		(18,0,0,0,1,1,1,1)	1.253	1.319	1.444	1.583	18.384	28.894	52.914	79.535
		(0,0,0,1,1,1,1,18)	1.056	1.071	1.099	1.132	2.577	3.003	3.587	3.977
		(0,0,1,1,1,1,18,0)	1.084	1.11	1.155	1.201	3.452	4.113	5.098	5.814
		(18,0,0,0,1,1,1,1)	1.255	1.323	1.448	1.577	18.313	30.973	54.309	84.157
		(17,0,0,1,1,1,1,1)	1.2375	1.295	1.412	1.533	14.871	22.912	43.163	66.158
		(17,1,1,1,1,1,0,0)	1.274	1.341	1.472	1.614	16.463	26.115	47.314	74.036
		(0,0,1,1,1,1,1,17)	1.0185	1.024	1.033	1.044	1.384	1.459	1.558	1.63
		(15,1,1,1,1,1,1,1)	1.202	1.257	1.353	1.488	12.17	18.203	29.075	41.233
		(1,1,1,1,1,1,1,15)	1.032	1.042	1.057	1.075	1.671	1.789	1.967	2.086

Table 1. Continue

		$\alpha \rightarrow$	0.9950	0.9900	0.9750	0.9500	0.0500	0.0250	0.0100	0.0050
n	m	**R** ↓								
30	18	$r_1 = 9\ r_2 = 3\ r_i = 0$	1.536	1.621	1.79	1.948	9.809	12.69	17.656	22.502
		$r_1 = 9\ r_{10} = 3\ r_i = 0$	1.462	1.556	1.658	1.78	7.556	9.285	12.832	15.961
		$r_1 = 9\ r_{18} = 3\ r_i = 0$	1.414	1.484	1.593	1.712	7.15	9.001	12.033	15.279
		$r_{17} = 9\ r_{18} = 3\ r_i = 0$	1.101	1.114	1.14	1.163	1.666	1.739	1.848	1.939
		$r_1 = 9\ r_2 = 2\ r_3 = 1$ $r_i = 0$	1.533	1.613	1.775	1.952	9.047	11.699	16.099	19.795
		$r_{16} = 9\ r_{17} = 2\ r_{18} = 1$ $r_i = 0$	1.109	1.122	1.148	1.173	1.689	1.781	1.917	2.005
		$r_1 = 9\ r_2 = 1$ $r_3 = 1\ r_4 = 1\ \ r_i = 0$	1.543	1.631	1.761	1.923	9.098	11.36	15.674	18.844
		$r_1 = 9\ r_{16} = 1$ $r_{17} = 1\ r_{18} = 1\ \ r_i = 0$	1.423	1.487	1.598	1.724	7.369	9.487	12.428	17.29
		$r_{15} = 9\ r_{16} = 1$ $r_{17} = 1\ r_{18} = 1\ \ r_i = 0$	1.12	1.131	1.155	1.179	1.733	1.814	1.931	2.047
		$r_1 = 7\ r_2 = 4\ r_3 = 1$ $r_i = 0$	1.512	1.592	1.741	1.884	8.079	10.098	13.176	15.946
		$r_1 = 7\ r_2 = 3$ $r_3 = 1\ r_4 = 1\ \ r_i = 0$	1.51	1.578	1.719	1.857	7.454	9.251	12.407	14.951
		$r_1 = 7\ r_2 = 1\ r_3 = 1$ $r_4 = 1\ r_5 = 1 r_6 = 1$ $r_i = 0$	1.472	1.536	1.658	1.797	6.838	8.522	11.315	14.276
		$r_{13} = 7\ r_{14} = 1\ r_{15} = 1$ $r_{16} = 1\ r_{17} = 1 r_{18} = 1$ $r_i = 0$	1.125	1.139	1.168	1.196	1.78	1.884	2.03	2.137
		$r_1 = 6\ r_2 = 6\ r_i = 0$	1.528	1.599	1.726	1.868	7.74	9.429	11.906	14.19
		$r_1 = 6\ r_{10} = 6\ r_i = 0$	1.37	1.438	1.528	1.61	4.839	5.716	7.088	8.268
		$r_1 = 6\ r_{18} = 6\ r_i = 0$	1.306	1.351	1.427	1.499	4.307	5.075	6.283	7.16
		$r_{17} = 6\ r_{18} = 6\ r_i = 0$	1.095	1.112	1.139	1.162	1.646	1.722	1.814	1.889
		$r_1 = 4\ r_2 = 4\ r_3 = 4$ $r_i = 0$	1.467	1.542	1.665	1.798	6.095	7.293	8.827	10.006
		$r_1 = 3\ r_2 = 3$ $r_3 = 3\ r_4 = 3\ \ r_i = 0$	1.437	1.496	1.609	1.714	5.117	6.044	7.339	8.472

Table 1. Continue

		$\alpha \rightarrow$	0.9950	0.9900	0.9750	0.9500	0.0500	0.0250	0.0100	0.0050
n	m	**R** $\downarrow$								
50	30		1.978	2.094	2.305	2.506	14.406	19.589	30.445	40.178
		$r_1 = 20\ r_i = 0$	1.143	1.159	1.181	1.202	1.581	1.631	1.701	1.769
		$r_{30} = 20\ r_i = 0$	1.895	1.987	2.166	2.346	8.89	10.803	13.973	16.516
		$r_1 = 10\ r_2 = 10$ $r_i = 0$	1.145	1.158	1.179	1.202	1.585	1.64	1.713	1.768
		$r_{29} = 10\ r_{30} = 10$ $r_i = 0$	1.843	1.931	2.087	2.265	8.167	9.798	12.274	14.771
		$r_1 = 10\ r_2 = 5$ $r_3 = 5\ r_i = 0$	1.791	1.875	2.047	2.2	7.796	9.349	11.564	13.517
		$r_1 = 10\ r_{29} = 5$ $r_{30} = 5\ r_i = 0$	1.144	1.159	1.183	1.203	1.598	1.644	1.726	1.765
		$r_{28} = 10\ r_{29} = 5$ $r_{30} = 5\ r_i = 0$	1.721	1.787	1.932	2.062	6.231	7.393	8.787	9.931
		$r_1 = 10$ $r_2, r_{3,} \dots r_{11} = 1$ $r_i = 0$	1.371	1.413	1.483	1.539	2.707	2.895	3.119	3.292
		$r_{30} = 10$ $r_{20}, r_{21,} \dots r_{29} = 1$ $r_i = 0$	1.152	1.167	1.191	1.213	1.612	1.664	1.752	1.805
		$r_1, r_{2,} \dots r_{20} = 1$ $r_i = 0$	1.197	1.215	1.245	1.274	1.805	1.895	2.016	2.09
		$r_{11}, r_{21,} \dots r_{30} = 1$ $r_i = 0$	1.697	1.76	1.886	2.011	5.301	6.086	7.174	8.04
		$r_1, r_{2,} \dots r_5 = 4$ $r_i = 0$	1.148	1.162	1.186	1.208	1.569	1.657	1.729	1.782
		$r_{26}, r_{27,} \dots r_{30} = 1$ $r_i = 0$	1.527	1.58	1.673	1.766	3.711	4.092	4.547	4.941
		$r_1, r_{2,} \dots r_{10} = 2$ $r_i = 0$	1.159	1.175	1.199	1.223	1.644	1.712	1.79	1.846
		$r_{21}, r_{22,} \dots r_{30} = 2$ $r_i = 0$								

In Table 2, cut off points for the distribution of pivotal quantity (2) are simulated under complete sample. Weibull growth model is used to modelling cut off point (dependent variable) via sample size of n (independent variable), where model is given by

$$Y = A - (A - B) * EXP\left(-\left(C\left|X\right|\right)^D\right)$$

Since there a lot of schemes in progressive censoring, the idea of regression analysis is only applied on complete sample that is $n = m$. Results of all anaysis are given below (See Analysis 1-8). Since the cut off points are decreasing for $\alpha \leq 0.05$, negative cutt of points are used in the regression analysis. Because Weibull growth model is used in the analysis and the model known as increasing. The predicted cut off points from regression analysis are given in Table 3. From the regression analysis results and from Table 2 and Table 3, estimated models can be used to get cut of point for distribution of pivotal quantity (2).

Table 2. Estimated **c**ut off points for pivotal quantity (2) by simulation

	$\alpha \rightarrow$	0.9950	0.9900	0.9750	0.9500	0.0500	0.0250	0.0100	0.0050
n	m								
5	5	1.02192	1.033878	1.060421	1.089206	3.044611	3.717473	4.664872	5.706714
6	6	1.044445	1.060344	1.090297	1.122046	2.911891	3.404233	4.085361	4.811534
7	7	1.060181	1.077478	1.115032	1.154945	2.837262	3.28627	3.931282	4.568732
8	8	1.072864	1.095962	1.132289	1.173293	2.816747	3.311122	4.046945	4.637621
9	9	1.087829	1.112013	1.15298	1.194758	2.751475	3.123506	3.722727	4.110333
10	10	1.111126	1.132327	1.173743	1.218514	2.658829	2.996896	3.534319	3.873166
11	11	1.123193	1.14801	1.194641	1.239866	2.651593	2.94393	3.362648	3.687189
12	12	1.139119	1.163613	1.20738	1.255991	2.576132	2.864645	3.176465	3.449742
13	13	1.156323	1.177634	1.223258	1.270395	2.546718	2.797979	3.184933	3.499178
14	14	1.169664	1.195998	1.237282	1.27904	2.528879	2.791535	3.137718	3.35942
15	15	1.175378	1.199656	1.244282	1.292598	2.502323	2.726135	3.011406	3.259363
16	16	1.193654	1.218845	1.268764	1.313612	2.476593	2.710495	3.010064	3.307846
17	17	1.197534	1.224502	1.270337	1.315338	2.441075	2.655958	2.961509	3.188395
18	18	1.2112	1.236419	1.284827	1.329489	2.413168	2.621614	2.920868	3.178713
19	19	1.215508	1.244054	1.289106	1.333301	2.380159	2.558698	2.830232	3.023663
20	20	1.229164	1.254563	1.302852	1.347068	2.367923	2.550781	2.83237	3.052136
21	21	1.223859	1.255289	1.304171	1.348883	2.366104	2.544655	2.782405	2.983382
22	22	1.235733	1.262728	1.313928	1.360653	2.368437	2.562789	2.817247	3.049198
23	23	1.244607	1.278833	1.325521	1.370086	2.35011	2.520895	2.753719	2.961425
24	24	1.255644	1.27704	1.328501	1.374891	2.322542	2.492926	2.707409	2.902272
25	25	1.257829	1.286877	1.339246	1.382568	2.321017	2.482005	2.683551	2.827139

Table 2. Continue

	$\alpha \rightarrow$	0.9950	0.9900	0.9750	0.9500	0.0500	0.0250	0.0100	0.0050
n	m								
26	26	1.260294	1.296192	1.340919	1.386363	2.298795	2.459184	2.667453	2.805275
27	27	1.27144	1.298308	1.351922	1.395432	2.280106	2.419757	2.582323	2.723977
28	28	1.27144	1.298308	1.351922	1.395432	2.280106	2.419757	2.582323	2.723977
29	29	1.28743	1.315145	1.362633	1.404833	2.269211	2.394025	2.568498	2.771884
30	30	1.284609	1.31915	1.362015	1.408667	2.266517	2.413265	2.606444	2.721019
31	31	1.299134	1.329677	1.376109	1.421359	2.259329	2.400269	2.586252	2.738272
32	32	1.305955	1.329672	1.378151	1.421277	2.2374	2.360466	2.545716	2.650244
33	33	1.302553	1.331216	1.375976	1.423953	2.233494	2.361429	2.500998	2.631448
34	34	1.308452	1.341578	1.386071	1.430439	2.243563	2.381306	2.549678	2.670031
35	35	1.321912	1.350967	1.395042	1.436219	2.218183	2.347207	2.495278	2.61917
36	36	1.323	1.35053	1.396842	1.438781	2.208048	2.323734	2.466329	2.555403
37	37	1.319176	1.350367	1.394956	1.438745	2.211503	2.324163	2.469683	2.584375
38	38	1.330021	1.362042	1.40167	1.445367	2.208211	2.311964	2.442007	2.548877
39	39	1.328581	1.360835	1.411756	1.451635	2.18755	2.300918	2.456565	2.607586
40	40	1.331483	1.359288	1.409274	1.453983	2.193648	2.3162	2.450178	2.55559
41	41	1.339752	1.37077	1.419473	1.462465	2.208307	2.318495	2.470181	2.565774
42	42	1.345142	1.376126	1.417897	1.457133	2.186948	2.281621	2.412441	2.510228
43	43	1.35379	1.374159	1.419347	1.459506	2.172289	2.288884	2.414002	2.511798
44	44	1.345358	1.379889	1.424423	1.469275	2.171431	2.273734	2.416481	2.513286
45	45	1.36096	1.38562	1.428995	1.469935	2.174786	2.285181	2.398211	2.515057
46	46	1.367966	1.397075	1.43912	1.480355	2.165012	2.263174	2.383957	2.491976
47	47	1.361418	1.388501	1.431467	1.47031	2.171964	2.274184	2.392896	2.499675
48	48	1.372246	1.395912	1.434644	1.476254	2.148442	2.265594	2.393813	2.51562
49	49	1.380556	1.400504	1.447467	1.484852	2.151389	2.248703	2.379245	2.456682
50	50	1.387272	1.409206	1.447672	1.488963	2.133481	2.229226	2.350251	2.430943
51	51	1.371469	1.402913	1.446362	1.483736	2.1467	2.233562	2.361495	2.450075
52	52	1.372114	1.397266	1.447761	1.491017	2.14215	2.232855	2.338711	2.43272
53	53	1.376118	1.403543	1.448109	1.489631	2.142205	2.229083	2.33742	2.449114
54	54	1.387959	1.413436	1.453039	1.49289	2.142019	2.222397	2.339462	2.436514
55	55	1.392857	1.417168	1.461628	1.497945	2.129204	2.215255	2.327024	2.40143
56	56	1.396395	1.421909	1.459609	1.494937	2.121486	2.195359	2.310444	2.388192
57	57	1.397148	1.42014	1.46011	1.501211	2.122236	2.205801	2.294386	2.355452
58	58	1.399493	1.426299	1.463544	1.505227	2.118086	2.20631	2.313324	2.402504
59	59	1.395727	1.421948	1.464151	1.503407	2.11188	2.201575	2.322144	2.4201
60	60	1.394605	1.427493	1.469287	1.50743	2.111686	2.194725	2.30957	2.386481

Table 2. Continue

	$\alpha \rightarrow$	0.9950	0.9900	0.9750	0.9500	0.0500	0.0250	0.0100	0.0050
n	m								
61	61	1.398236	1.429073	1.469786	1.508014	2.110983	2.195593	2.31097	2.383228
62	62	1.410175	1.435716	1.474449	1.510844	2.103567	2.188499	2.288641	2.372085
63	63	1.41704	1.441425	1.48148	1.514729	2.096362	2.184607	2.282162	2.346516
64	64	1.406134	1.438399	1.479808	1.516544	2.101962	2.183598	2.287461	2.360693
65	65	1.401384	1.434115	1.475615	1.515286	2.104198	2.177141	2.263078	2.351182
66	66	1.412792	1.439309	1.478132	1.51684	2.100842	2.181296	2.299489	2.357803
67	67	1.42185	1.441702	1.481445	1.520073	2.087576	2.169638	2.250458	2.317462
68	68	1.423214	1.446797	1.490279	1.523581	2.089223	2.165986	2.262323	2.302127
69	69	1.429442	1.458079	1.491763	1.526096	2.089552	2.168625	2.268789	2.326952
70	70	1.429456	1.455414	1.493009	1.527084	2.087925	2.161012	2.258894	2.335275
71	71	1.423038	1.45288	1.491213	1.529021	2.093484	2.164549	2.252925	2.325195
72	72	1.42572	1.44946	1.491324	1.527075	2.086213	2.157704	2.249085	2.295939
73	73	1.426834	1.452316	1.49401	1.530095	2.077473	2.149361	2.240318	2.31484
74	74	1.435453	1.460574	1.499626	1.534251	2.076117	2.148778	2.22298	2.294834
75	75	1.433003	1.459544	1.497703	1.534954	2.080279	2.151055	2.236471	2.304996
76	76	1.4292	1.4588	1.4956	1.5312	2.0731	2.1416	2.2396	2.3071
77	77	1.439664	1.465165	1.503548	1.539722	2.071598	2.147701	2.242156	2.303232
78	78	1.433901	1.466795	1.503444	1.53819	2.073923	2.145071	2.232468	2.299163
79	79	1.436844	1.463574	1.503543	1.539327	2.067377	2.140655	2.23727	2.31074
80	80	1.444068	1.464769	1.508377	1.540895	2.062756	2.13683	2.21131	2.270095
81	81	1.433847	1.460197	1.501589	1.541435	2.070633	2.139331	2.217828	2.273947
82	82	1.442464	1.466318	1.508148	1.543083	2.054322	2.119275	2.203941	2.275865
83	83	1.442451	1.463152	1.507626	1.54207	2.061929	2.127394	2.210412	2.262472
84	84	1.458251	1.476163	1.511322	1.545956	2.051601	2.1092	2.193873	2.258537
85	85	1.458688	1.481346	1.51606	1.54667	2.060418	2.12631	2.212648	2.263541
86	86	1.446723	1.468319	1.511038	1.548264	2.06272	2.135849	2.227847	2.281167
87	87	1.453327	1.474434	1.514947	1.545968	2.05162	2.107238	2.192846	2.259249
88	88	1.450176	1.47974	1.517669	1.554077	2.05728	2.122652	2.214716	2.260289
89	89	1.455825	1.480176	1.515187	1.551395	2.050185	2.117866	2.197247	2.249144
90	90	1.455634	1.475919	1.514304	1.553079	2.042438	2.114433	2.196932	2.24993
91	91	1.453364	1.475967	1.516932	1.550096	2.041663	2.107722	2.188264	2.242855
92	92	1.465587	1.48494	1.521538	1.554718	2.039568	2.095843	2.17162	2.23892
93	93	1.457013	1.480448	1.519246	1.553827	2.039026	2.109288	2.187618	2.25272
94	94	1.461773	1.484935	1.523765	1.560246	2.043155	2.102259	2.186969	2.241609
95	95	1.463078	1.489978	1.529259	1.561538	2.039663	2.096447	2.175294	2.222716
96	96	1.467567	1.490933	1.528861	1.557905	2.034803	2.092863	2.169897	2.213948
97	97	1.469706	1.492761	1.532295	1.563851	2.036021	2.101071	2.173052	2.220081
98	98	1.461158	1.489923	1.524355	1.55932	2.035924	2.093527	2.168113	2.217991
99	99	1.477965	1.496951	1.532544	1.565948	2.038728	2.101296	2.180465	2.232661
100	100	1.473715	1.496754	1.528777	1.562641	2.031456	2.094268	2.170524	2.222065

Table 3. Estimated cut off points by regression analysis

$\alpha \rightarrow$	0.9950	0.9900	0.9750	0.9500	0.0500	0.0250	0.0100	0.0050
m								
5	1,02192	1,033878	1,060421	1,089206	3,04461	3,71747	4,66487	-5,70671
6	1,044445	1,060344	1,090297	1,122046	2,91189	3,40423	4,08536	4,81153
7	1,060181	1,077478	1,115032	1,154945	2,83726	3,28627	3,93128	4,56873
8	1,072864	1,095962	1,132289	1,173293	2,81675	3,31112	4,04694	4,63762
9	1,087829	1,112013	1,15298	1,194758	2,75147	3,12351	3,72273	4,11033
10	1,111126	1,132327	1,173743	1,218514	2,65883	2,9969	3,53432	3,87317
11	1,123193	1,14801	1,194641	1,239866	2,65159	2,94393	3,36265	3,68719
12	1,139119	1,163613	1,20738	1,255991	2,57613	2,86465	3,17647	3,44974
13	1,156323	1,177634	1,223258	1,270395	2,54672	2,79798	3,18493	3,49918
14	1,169664	1,195998	1,237282	1,27904	2,52888	2,79154	3,13772	3,35942
15	1,175378	1,199656	1,244282	1,292598	2,50232	2,72614	3,01141	3,25936
16	1,193654	1,218845	1,268764	1,313612	2,47659	2,7105	3,01006	3,30785
17	1,197534	1,224502	1,270337	1,315338	2,44107	2,65596	2,96151	3,1884
18	1,2112	1,236419	1,284827	1,329489	2,41317	2,62161	2,92087	3,17871
19	1,215508	1,244054	1,289106	1,333301	2,38016	2,5587	2,83023	3,02366
20	1,229164	1,254563	1,302852	1,347068	2,36792	2,55078	2,83237	3,05214
21	1,223859	1,255289	1,304171	1,348883	2,3661	2,54465	2,78241	2,98338
22	1,235733	1,262728	1,313928	1,360653	2,36844	2,56279	2,81725	3,0492
23	1,244607	1,278833	1,325521	1,370086	2,35011	2,5209	2,75372	2,96142
24	1,255644	1,27704	1,328501	1,374891	2,32254	2,49293	2,70741	2,90227
25	1,257829	1,286877	1,339246	1,382568	2,32102	2,482	2,68355	2,82714
26	1,260294	1,296192	1,340919	1,386363	2,29879	2,45918	2,66745	2,80528
27	1,27144	1,298308	1,351922	1,395432	2,28011	2,41976	2,58232	2,72398
28	1,27144	1,298308	1,351922	1,395432	2,28011	2,41976	2,58232	2,72398
29	1,28743	1,315145	1,362633	1,404833	2,26921	2,39403	2,5685	2,77188
30	1,284609	1,31915	1,362015	1,408667	2,26652	2,41327	2,60644	2,72102

Table 3. Continue

$\alpha \rightarrow$	0.9950	0.9900	0.9750	0.9500	0.0500	0.0250	0.0100	0.0050
m								
31	1,299134	1,329677	1,376109	1,421359	2,25933	2,40027	2,58625	2,73827
32	1,305955	1,329672	1,378151	1,421277	2,2374	2,36047	2,54572	2,65024
33	1,302553	1,331216	1,375976	1,423953	2,23349	2,36143	2,501	2,63145
34	1,308452	1,341578	1,386071	1,430439	2,24356	2,38131	2,54968	2,67003
35	1,321912	1,350967	1,395042	1,436219	2,21818	2,34721	2,49528	2,61917
36	1,323	1,35053	1,396842	1,438781	2,20805	2,32373	2,46633	2,5554
37	1,319176	1,350367	1,394956	1,438745	2,2115	2,32416	2,46968	2,58438
38	1,330021	1,362042	1,40167	1,445367	2,20821	2,31196	2,44201	2,54888
39	1,328581	1,360835	1,411756	1,451635	2,18755	2,30092	2,45657	2,60759
40	1,331483	1,359288	1,409274	1,453983	2,19365	2,3162	2,45018	2,55559
41	1,339752	1,37077	1,419473	1,462465	2,20831	2,31849	2,47018	2,56577
42	1,345142	1,376126	1,417897	1,457133	2,18695	2,28162	2,41244	2,51023
43	1,35379	1,374159	1,419347	1,459506	2,17229	2,28888	2,414	2,5118
44	1,345358	1,379889	1,424423	1,469275	2,17143	2,27373	2,41648	2,51329
45	1,36096	1,38562	1,428995	1,469935	2,17479	2,28518	2,39821	2,51506
46	1,367966	1,397075	1,43912	1,480355	2,16501	2,26317	2,38396	2,49198
47	1,361418	1,388501	1,431467	1,47031	2,17196	2,27418	2,3929	2,49967
48	1,372246	1,395912	1,434644	1,476254	2,14844	2,26559	2,39381	2,51562
49	1,380556	1,400504	1,447467	1,484852	2,15139	2,2487	2,37924	2,45668
50	1,387272	1,409206	1,447672	1,488963	2,13348	2,22923	2,35025	2,43094
51	1,371469	1,402913	1,446362	1,483736	2,1467	2,23356	2,3615	2,45007
52	1,372114	1,397266	1,447761	1,491017	2,14215	2,23285	2,33871	2,43272
53	1,376118	1,403543	1,448109	1,489631	2,1422	2,22908	2,33742	2,44911
54	1,387959	1,413436	1,453039	1,49289	2,14202	2,2224	2,33946	2,43651
55	1,392857	1,417168	1,461628	1,497945	2,1292	2,21525	2,32702	2,40143
56	1,396395	1,421909	1,459609	1,494937	2,12149	2,19536	2,31044	2,38819
57	1,397148	1,42014	1,46011	1,501211	2,12224	2,2058	2,29439	2,35545
58	1,399493	1,426299	1,463544	1,505227	2,11809	2,20631	2,31332	2,4025
59	1,395727	1,421948	1,464151	1,503407	2,11188	2,20158	2,32214	2,4201
60	1,394605	1,427493	1,469287	1,50743	2,11169	2,19472	2,30957	2,38648

Table 3. Continue

$\alpha \rightarrow$	0.9950	0.9900	0.9750	0.9500	0.0500	0.0250	0.0100	0.0050
m								
61	1,398236	1,429073	1,469786	1,508014	2,11098	2,19559	2,31097	2,38323
62	1,410175	1,435716	1,474449	1,510844	2,10357	2,1885	2,28864	2,37208
63	1,41704	1,441425	1,48148	1,514729	2,09636	2,18461	2,28216	2,34652
64	1,406134	1,438399	1,479808	1,516544	2,10196	2,1836	2,28746	2,36069
65	1,401384	1,434115	1,475615	1,515286	2,1042	2,17714	2,26308	2,35118
66	1,412792	1,439309	1,478132	1,51684	2,10084	2,1813	2,29949	2,3578
67	1,42185	1,441702	1,481445	1,520073	2,08758	2,16964	2,25046	2,31746
68	1,423214	1,446797	1,490279	1,523581	2,08922	2,16599	2,26232	2,30213
69	1,429442	1,458079	1,491763	1,526096	2,08955	2,16863	2,26879	2,32695
70	1,429456	1,455414	1,493009	1,527084	2,08792	2,16101	2,25889	2,33528
71	1,423038	1,45288	1,491213	1,529021	2,09348	2,16455	2,25293	2,3252
72	1,42572	1,44946	1,491324	1,527075	2,08621	2,1577	2,24908	2,29594
73	1,426834	1,452316	1,49401	1,530095	2,07747	2,14936	2,24032	2,31484
74	1,435453	1,460574	1,499626	1,534251	2,07612	2,14878	2,22298	2,29483
75	1,433003	1,459544	1,497703	1,534954	2,08028	2,15106	2,23647	2,305
76	1,4292	1,4588	1,4956	1,5312	2,0731	2,1416	2,2396	2,3071
77	1,439664	1,465165	1,503548	1,539722	2,0716	2,1477	2,24216	2,30323
78	1,433901	1,466795	1,503444	1,53819	2,07392	2,14507	2,23247	2,29916
79	1,436844	1,463574	1,503543	1,539327	2,06738	2,14066	2,23727	2,31074
80	1,444068	1,464769	1,508377	1,540895	2,06276	2,13683	2,21131	2,27009
81	1,433847	1,460197	1,501589	1,541435	2,07063	2,13933	2,21783	2,27395
82	1,442464	1,466318	1,508148	1,543083	2,05432	2,11927	2,20394	2,27586
83	1,442451	1,463152	1,507626	1,54207	2,06193	2,12739	2,21041	2,26247
84	1,458251	1,476163	1,511322	1,545956	2,0516	2,1092	2,19387	2,25854
85	1,458688	1,481346	1,51606	1,54667	2,06042	2,12631	2,21265	2,26354
86	1,446723	1,468319	1,511038	1,548264	2,06272	2,13585	2,22785	2,28117
87	1,453327	1,474434	1,514947	1,545968	2,05162	2,10724	2,19285	2,25925
88	1,450176	1,47974	1,517669	1,554077	2,05728	2,12265	2,21472	2,26029
89	1,455825	1,480176	1,515187	1,551395	2,05018	2,11787	2,19725	2,24914
90	1,455634	1,475919	1,514304	1,553079	2,04244	2,11443	2,19693	2,24993
91	1,453364	1,475967	1,516932	1,550096	2,04166	2,10772	2,18826	2,24285
92	1,465587	1,48494	1,521538	1,554718	2,03957	2,09584	2,17162	2,23892
93	1,457013	1,480448	1,519246	1,553827	2,03903	2,10929	2,18762	2,25272
94	1,461773	1,484935	1,523765	1,560246	2,04316	2,10226	2,18697	2,24161
95	1,463078	1,489978	1,529259	1,561538	2,03966	2,09645	2,17529	2,22272
96	1,467567	1,490933	1,528861	1,557905	2,0348	2,09286	2,1699	2,21395
97	1,469706	1,492761	1,532295	1,563851	2,03602	2,10107	2,17305	2,22008
98	1,461158	1,489923	1,524355	1,55932	2,03592	2,09353	2,16811	2,21799
99	1,477965	1,496951	1,532544	1,565948	2,03873	2,1013	2,18047	2,23266
100	1,473715	1,496754	1,528777	1,562641	2,03146	2,09427	2,17052	2,22207

Analyis 1. The model estimated for $\alpha = 0.99$ is given below. In the estimated model, C1 is sample size of n, C2 is the desired cut off value.

Curve Fit Report

Dependent alpha=0,99

Model Estimation Section

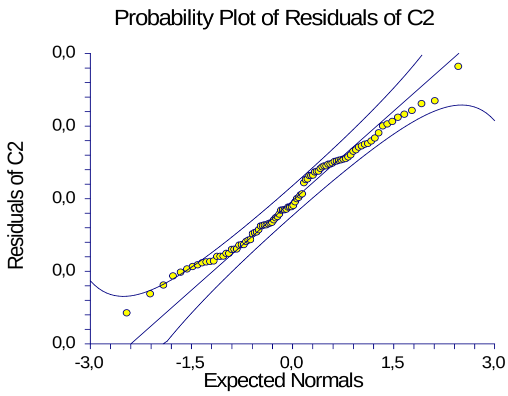
Parameter Name	**Parameter Estimate**	**Asymptotic Standard Error**	**Lower 95% C.L.**	**Upper 95% C.L.**
A	1,572425	1,918742E-02	1,534317	1,610533
B	0,7438802	0,043417	0,6576502	0,8301101
C	3,815007E-02	1,810748E-03	3,455377E-02	4,174637E-02
D	0,5508873	4,583861E-02	0,4598479	0,6419268

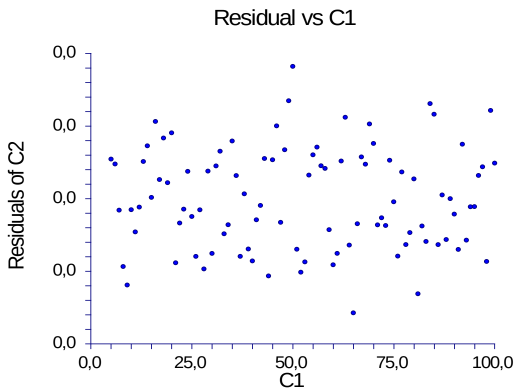
Dependent	C2
Independent	C1
Model	C2=A-(A-B)*EXP(-(C*|C1|)^D)
R-Squared	0,997904
Iterations	5

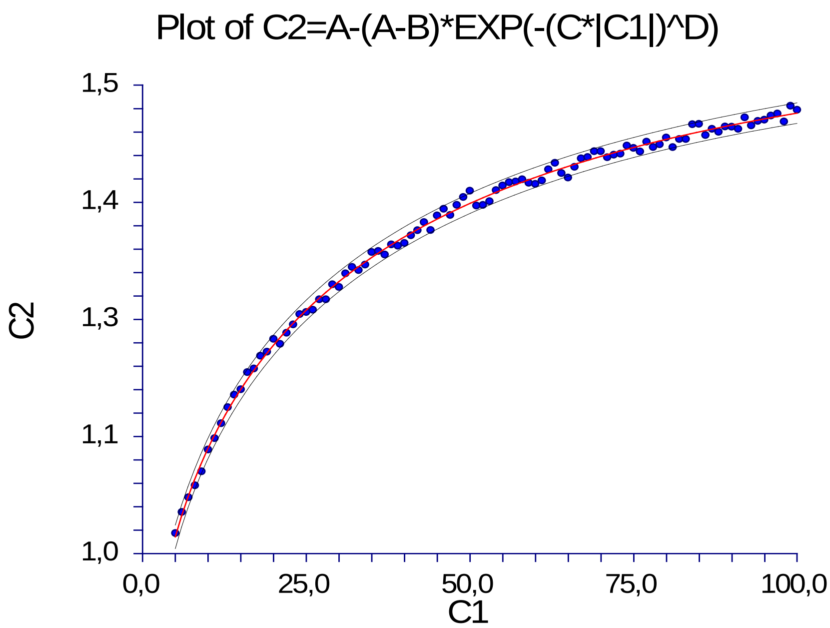
Estimated Model

(1.572425)-((1.572425)-(.7438802))*EXP(-((3.815007E-02)*(ABS(C1)))^(.5508873))

Plot Section







Analyis 2. The model estimated for $\alpha = 0.975$ is given below. In the estimated model, C1 is sample size of n, C3 is the desired cut off value.

Curve Fit Report

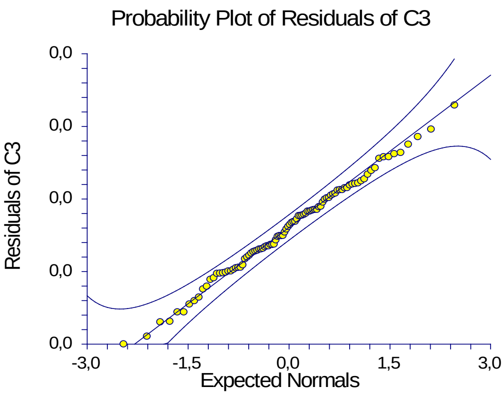
Dependent alpha=0,975

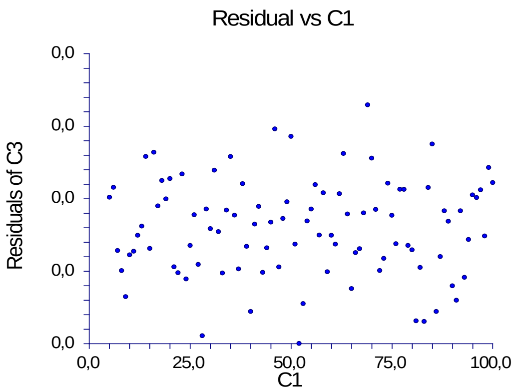
Model Estimation Section

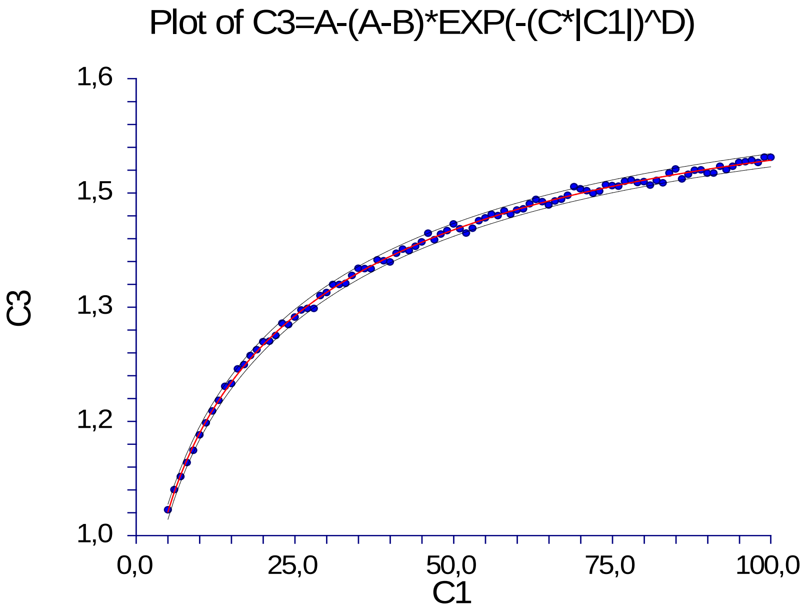
Parameter Name	Parameter Estimate	Asymptotic Standard Error	Lower 95% C.L.	Upper 95% C.L.
A	1,585828	1,351398E-02	1,558988	1,612668
B	0,6947767	0,043154	0,6090692	0,7804843
C	4,779498E-02	2,677602E-03	4,247703E-02	5,311292E-02
D	0,5212668	3,535855E-02	0,4510416	0,5914919

Dependent	C3
Independent	C1
Model	C3=A-(A-B)*EXP(-(C*|C1|)^D)
R-Squared	0,998702
Iterations	5

Estimated Model
(1.585828)-((1.585828)-(.6947767))*EXP(-((4.779498E-02)*(ABS(C1)))^(.5212668))







Analyis 3. The model estimated for $\alpha = 0.95$ is given below. In the estimated model, C1 is sample size of n, C4 is the desired cut off value.

Curve Fit Report

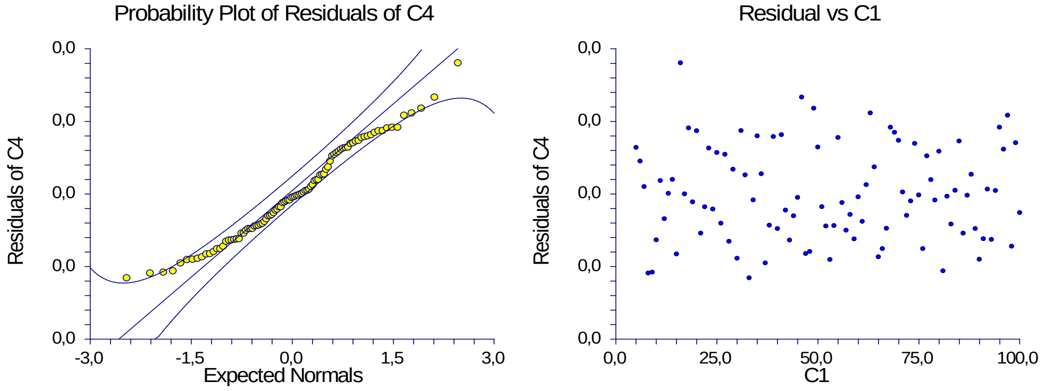
Dependent alpha=0,95

Model Estimation Section

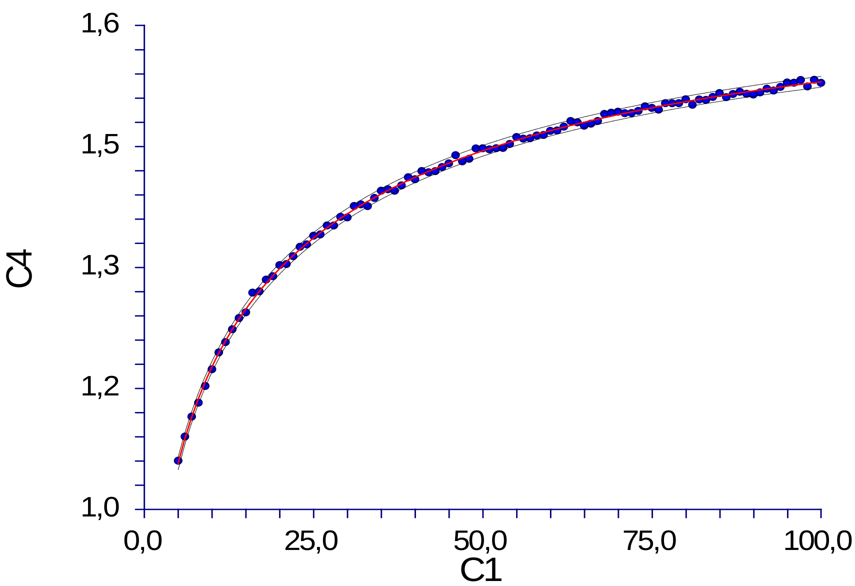
Parameter Name	**Parameter Estimate**	**Asymptotic Standard Error**	**Lower 95% C.L.**	**Upper 95% C.L.**
A	1,634777	1,299179E-02	1,608974	1,66058
B	0,4651898	7,827758E-02	0,3097237	0,6206558
C	8,551861E-02	1,058859E-02	6,448875E-02	0,1065485
D	0,4104668	3,023028E-02	0,3504269	0,4705068

Dependent	C4
Independent	C1
Model	C4=A-(A-B)*EXP(-(C*|C1|)^D)
R-Squared	0,999186
Iterations	9

Estimated Model
(1.634777)-((1.634777)-(.4651898))*EXP(-((8.551861E-02)*(ABS(C1)))^(.4104668))



Plot of C4=A-(A-B)*EXP(-(C*|C1|)^D)



Analyis 4. The model estimated for $\alpha = 0.90$ is given below. In the estimated model, C1 is sample size of n, C5 is the desired cut off value.

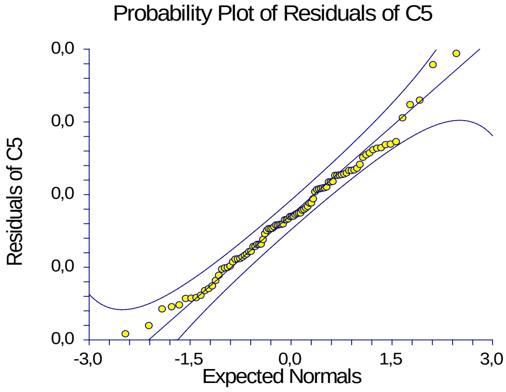
Curve Fit Report

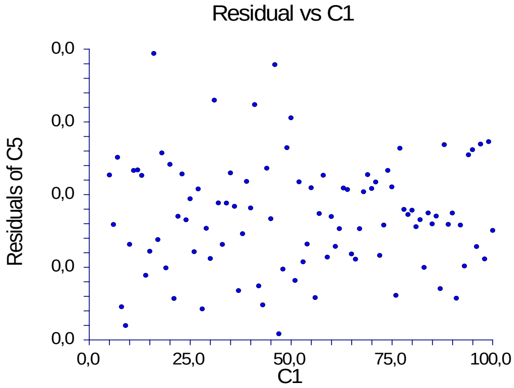
Dependent alpha=0,90

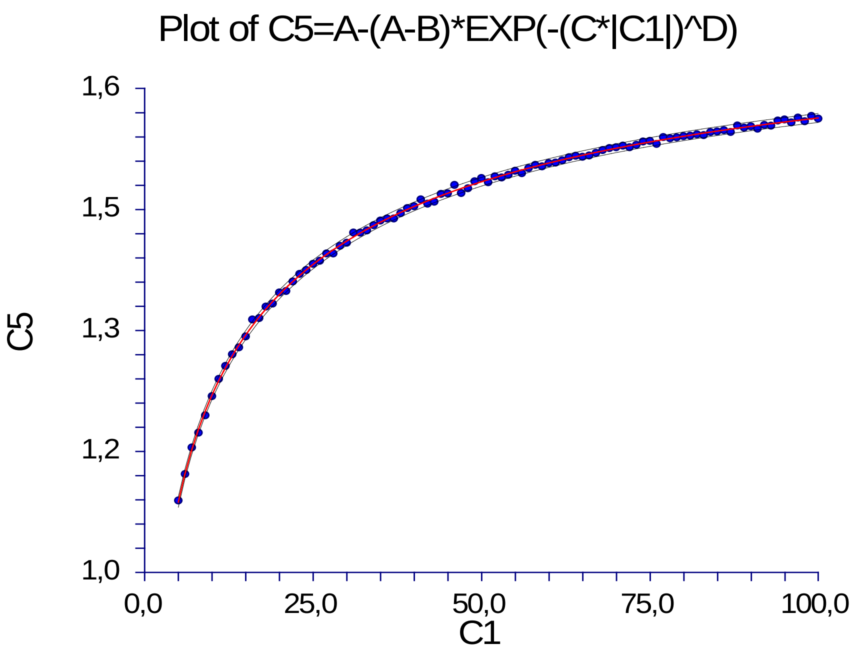
Model Estimation Section

Parameter Name	Parameter Estimate	Asymptotic Standard Error	Lower 95% C.L.	Upper 95% C.L.
A	1,672461	1,203583E-02	1,648556	1,696365
B	6,269884E-02	0,1567126	-0,2485458	0,3739435
C	0,2073718	5,260377E-02	0,1028962	0,3118474
D	0,3265727	2,701914E-02	0,2729104	0,380235

Dependent C5
Independent C1
Model C5=A-(A-B)*EXP(-(C*|C1|)^D)
R-Squared 0,999430
Iterations 44
Estimated Model
(1.672461)-((1.672461)-(6.269884E-02))*EXP(-((.2073718)*(ABS(C1)))^(.3265727))







Analyis 5. The model estimated for $\alpha = 0.10$ is given below. In the estimated model, C1 is sample size of n, C6 is the desired cut off value.

Curve Fit Report

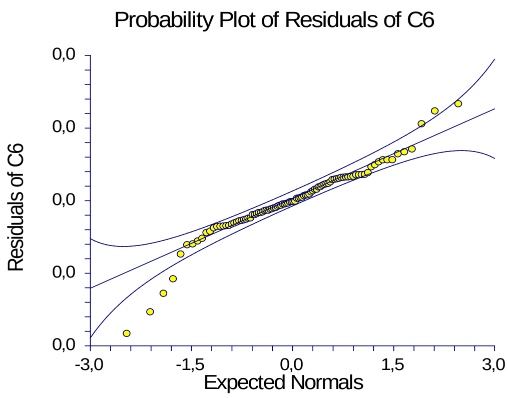
Dependent alpha=0,10

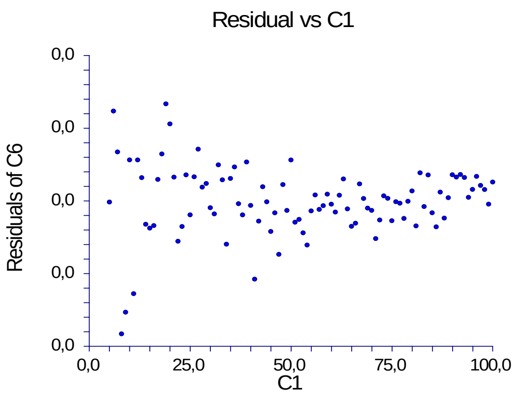
Model Estimation Section

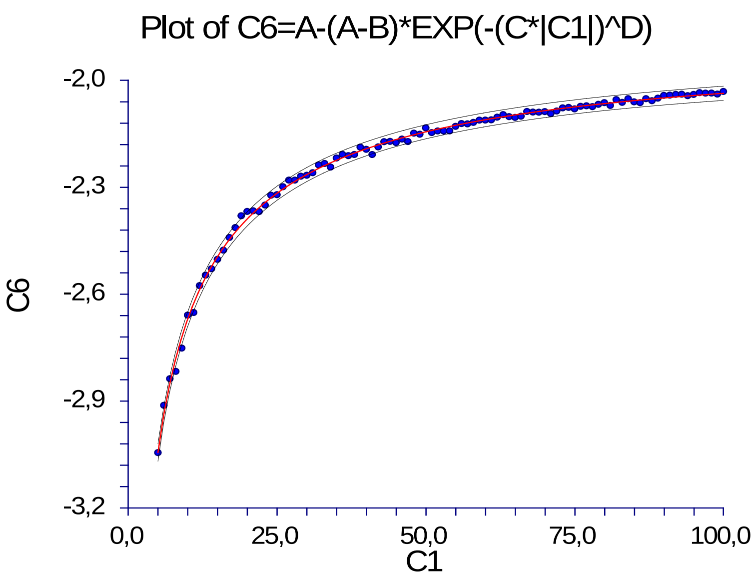
Parameter Name	Parameter Estimate	Asymptotic Standard Error	Lower 95% C.L.	Upper 95% C.L.
A	-1,923085	2,135232E-02	-1,965493	-1,880678
B	-15,66164	8,009557	-31,56931	0,2460351
C	13,87601	27,07766	-39,90255	67,65457
D	0,2166733	4,676172E-02	0,1238005	0,3095461

Dependent	C6
Independent	C1
Model	C6=A-(A-B)*EXP(-(C*|C1|)^D)
R-Squared	0,997967
Iterations	239

Estimated Model
(-1.923085)-((-1.923085)-(-15.66164))*EXP(-((13.87601)*(ABS(C1)))^(.2166733))







Analyis 6. The model estimated for $\alpha = 0.05$ is given below. In the estimated model, C1 is sample size of n, C7 is the desired cut off value.

Curve Fit Report

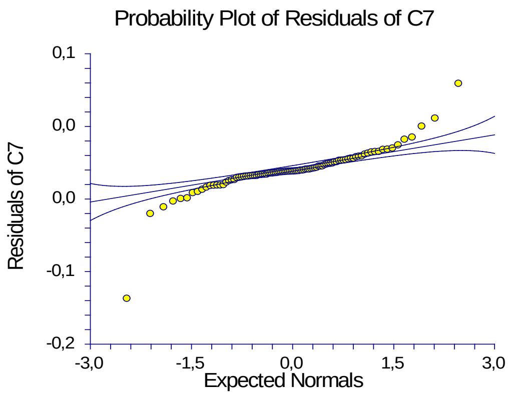
Dependent alpha=0,05

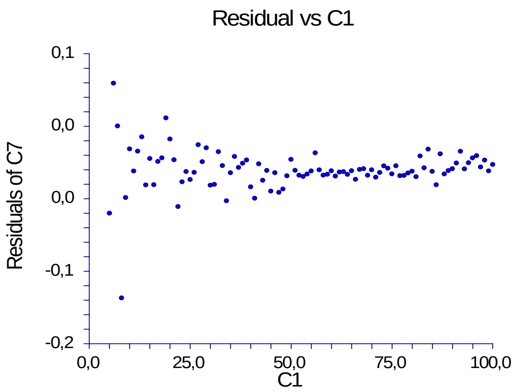
Model Estimation Section

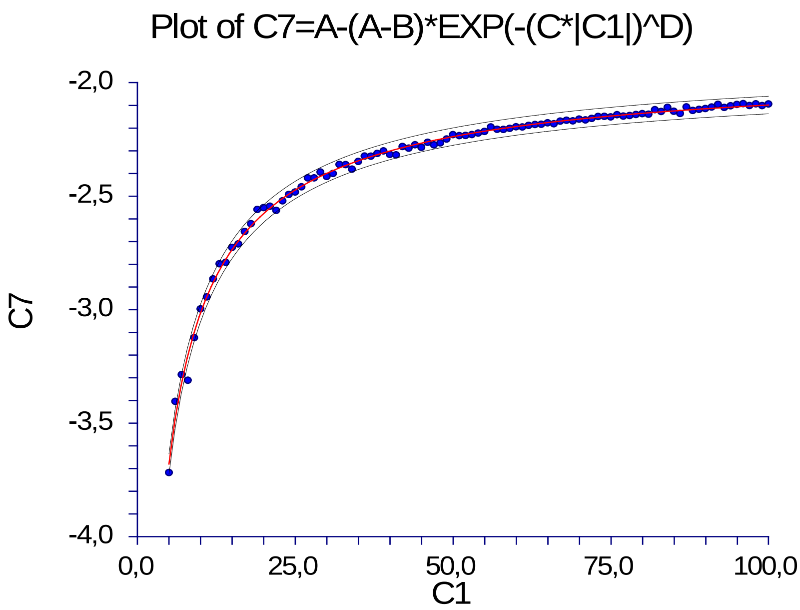
Parameter Name	Parameter Estimate	Asymptotic Standard Error	Lower 95% C.L.	Upper 95% C.L.
A	-1,941461	3,948557E-02	2,019883	-1,863039
B	-474,0543	1615,416	-3682,41	2734,301
C	385831	5412299	-1,036346E+07	1,113512E+07
D	0,1190874	7,480658E-02	2,948492E-02	0,2676597

Dependent	C7
Independent	C1
Model	C7=A-(A-B)*EXP(-(C*|C1|)^D)
R-Squared	0,996498
Iterations	1000

Estimated Model
(-1.941461)-((-1.941461)-(-474.0543))*EXP(-((385831)*(ABS(C1)))^(.1190874))







Analyis 7. The model estimated for $\alpha = 0.025$ is given below. In the estimated model, C1 is sample size of n, C8 is the desired cut off value.

Curve Fit Report

Dependent alpha=0,025

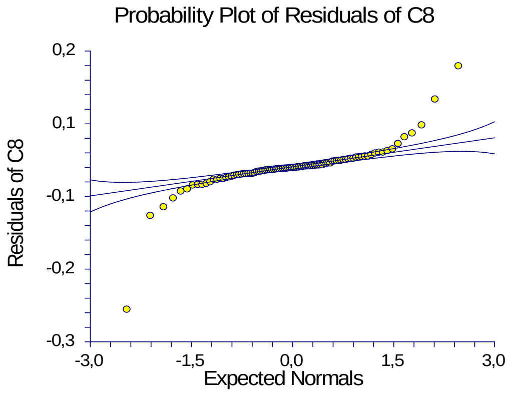
Model Estimation Section

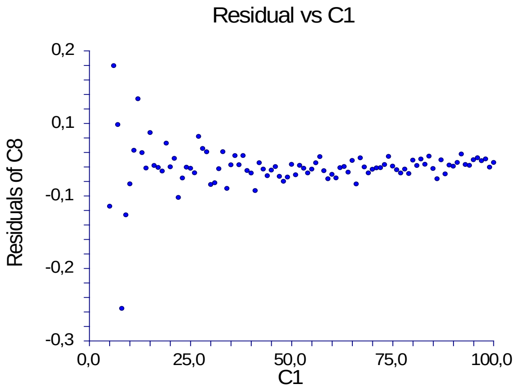
Parameter Name	Parameter Estimate	Asymptotic Standard Error	Lower 95% C.L.	Upper 95% C.L.
A	2,01684	5,569311E-02	2,127452	-1,906229
B	-974,6003	4298,349	-9511,492	7562,292
C	220153,8	3519451	-6769780	7210088
D	0,1279754	9,822115E-02	-6,710024E-02	0,3230511

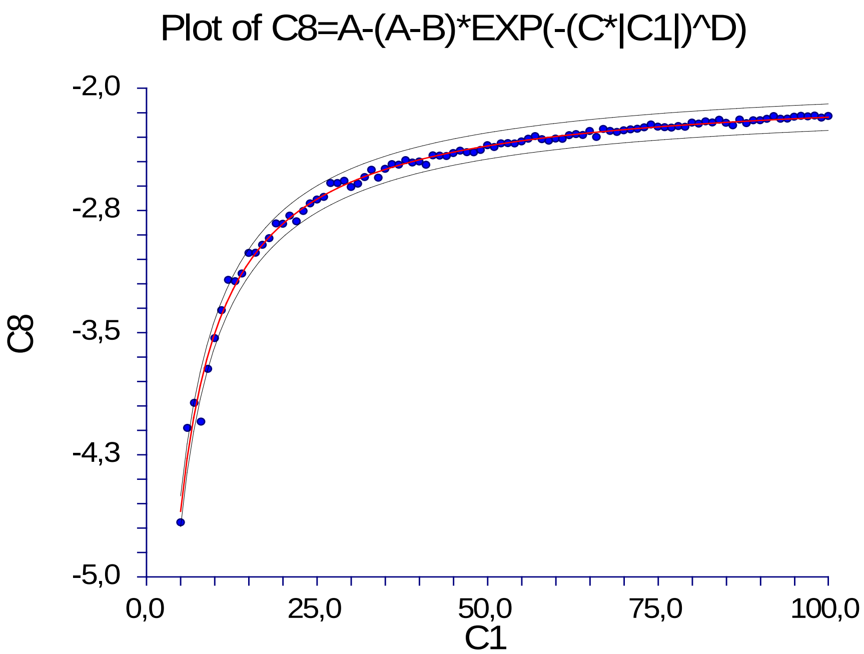
Dependent	C8
Independent	C1
Model	C8=A-(A-B)*EXP(-(C*|C1|)^D)
R-Squared	0,992935
Iterations	1000

Estimated Model

(2.01684)-((2.01684)-(-974.6003))*EXP(-((220153.8)*(ABS(C1)))^(.1279754))







Analyis 8. The model estimated for $\alpha = 0.01$ is given below. In the estimated model, C1 is sample size of n, C9 is the desired cut off value.

Curve Fit Report

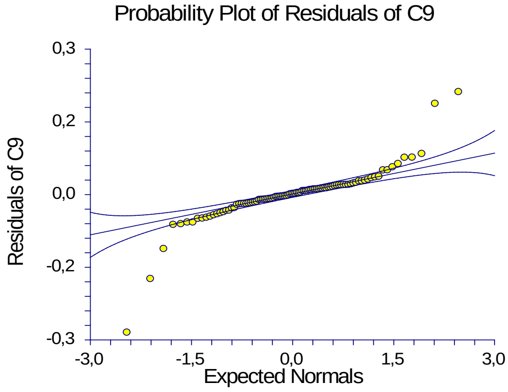
Dependent alpha=0,010

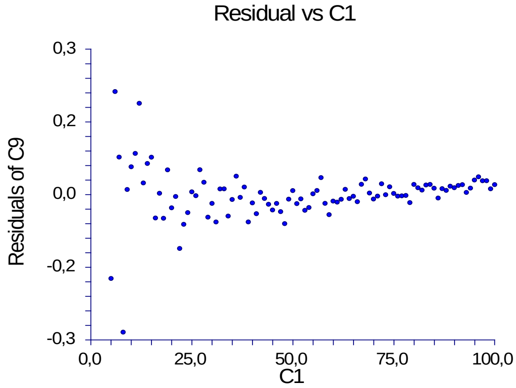
Model Estimation Section

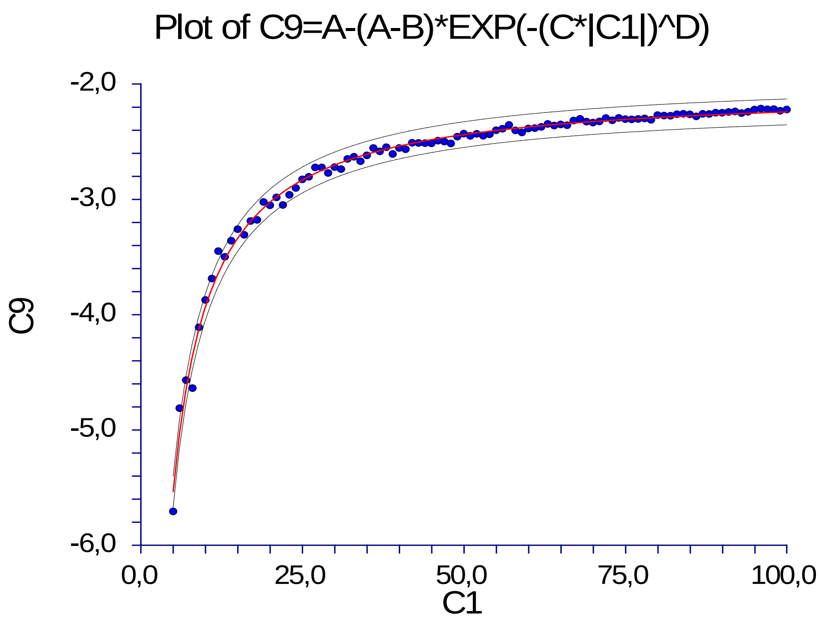
Parameter Name	**Parameter Estimate**	**Asymptotic Standard Error**	**Lower 95% C.L.**	**Upper 95% C.L.**
A	2,083265	5,790014E-02	2,19826	-1,968271
B	-9672,191	74881,91	-158394,1	139049,8
C	3,410865E+07	9,654103E+08	-1,88328E+09	1,951497E+09
D	0,1092983	0,1154059	-0,1199078	0,3385045

Dependent	C9
Independent	C1
Model	C9=A-(A-B)*EXP(-(C*|C1|)^D)
R-Squared	0,992194
Iterations	1000

Estimated Model
(2.083265)-((2.083265)-(-9672.191))*EXP(-((3.410865E+07)*(ABS(C1)))^(.1092983))







3. Interval Estimation Under Progressive Censoring

In this section, interval estimation is discussed for Weibull, Burr XII and Gompertz distribution based on progressive censored sample.

3.1. Weibull Case

The pdf and cdf of Weibull dsitribution are given, respectively, by

$$f(x) = \beta\lambda^{-\beta} x^{\beta-1} \exp\left\{-\left(\lambda^{-1}x\right)^{\beta}\right\} \quad , \quad x > 0 \ , \ \lambda > 0 \ , \ \beta > 0 \tag{3}$$

$$F(x) = 1 - \exp\left\{-\left(\lambda^{-1}x\right)^{\beta}\right\}. \tag{4}$$

Let $X^{\mathbf{R}}_{1:m:n} < X^{\mathbf{R}}_{2:m:n} < \cdots < X^{\mathbf{R}}_{m:m:n}$ be the progressive censored order statistics from Weibull distribution. Let us define following transformation.

$$Y^{\mathbf{R}}_{i:m:n} = \left(\frac{X^{\mathbf{R}}_{i:m:n}}{\lambda}\right)^{\beta} , \qquad i = 1, 2, \ldots, m. \tag{5}$$

It is easily seen that $Y^{\mathbf{R}}_{i:m:n}, i = 1, 2, \ldots, m$ are standard exponential progressive censored order statistics. The Eq. (5) is substitute in (2) then one can obtain following pivot:

$$\Phi(\beta) = \frac{\sum_{i=1}^{m}(1+r_i)Y_i / n}{\prod_{i=1}^{m} Y_i^{(1+r_i)/n}} = \frac{\sum_{i=1}^{m}\left\{(1+r_i)\left(\frac{X_i}{\lambda}\right)^{\beta}\right\} / n}{\prod_{i=1}^{m}\left\{\left(\frac{X_i}{\lambda}\right)^{\beta}\right\}^{(1+r_i)/n}}$$

$$= \frac{\sum_{i=1}^{m}(1+r_i)\left(X_i^{\beta}\right) / n}{\prod_{i=1}^{m}\left(X_i^{\beta}\right)^{(1+r_i)/n}} \tag{6}$$

where X_i and Y_i are used instead of $X^{\mathbf{R}}_{i:m:n}$ and $Y^{\mathbf{R}}_{i:m:n}$ for abbreviation respectively and this notation will be used in later sections.

Confidence interval for parameter β can be obtained using pivot (6). Confidence interval for parameter β with confidence level $(1-\alpha)\%$ is

$$P\left(\Phi^{*}_{1-\alpha/2} < \frac{\sum_{i=1}^{m}(1+r_i)\left(X_i^{\beta}\right) / n}{\prod_{i=1}^{m}\left(X_i^{\beta}\right)^{(1+r_i)/n}} < \Phi^{*}_{\alpha/2}\right) = 1-\alpha \tag{7}$$

where Φ_a^* is quantile which satisfies $P\left(\Phi > \Phi_a^*\right) = a$ Akdoğan et al. (2013).

If the pivot (6) is strictly monotone in β then the upper bound and lower bound of the confidence interval (7) can be obtined uniqely. In this paper, it can not proved the strictly monotonicity of (6) in β. But some figures of (6) are provided for different patern of data and censoring schemes. From Figs.1-3. it can be conclude that pivot (6) seems to be strictly increasing but this does not mean that ways exactly.

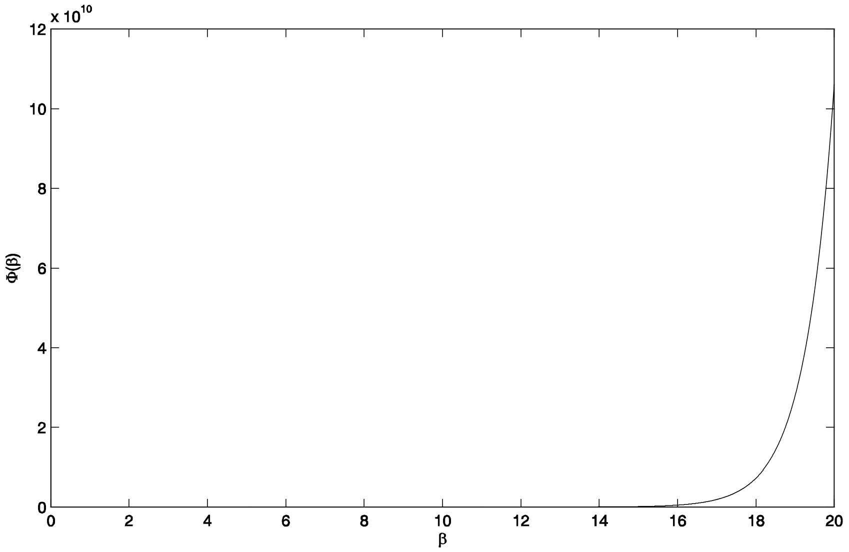


Fig 1. Plot of (6) for $x_i = 0.1, 0.3, 2.4, 3.2$, $n = 8$ $m = 4$ and $\mathbf{R} = (1, 0, 2, 1)$

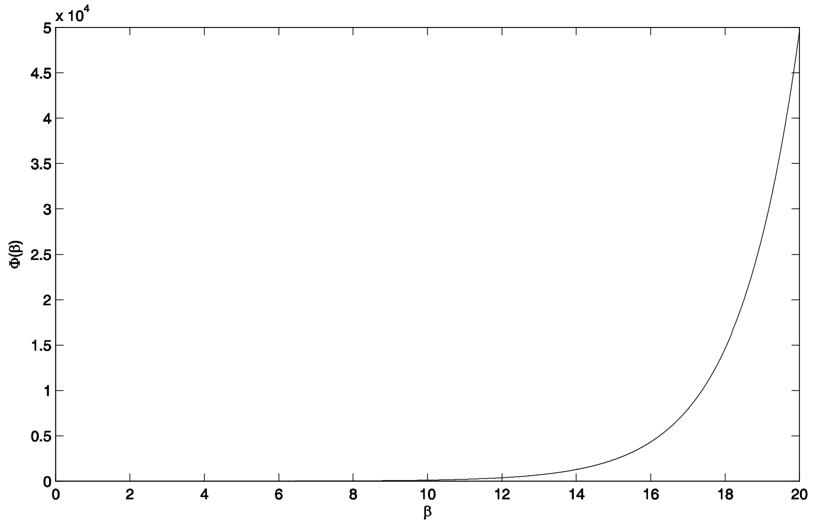


Fig 2. Plot of (6) for $x_i = 1.2, 3.2, 6.5, 7.8$, $n = 8$, $m = 4$ and $\mathbf{R} = (1, 0, 2, 1)$

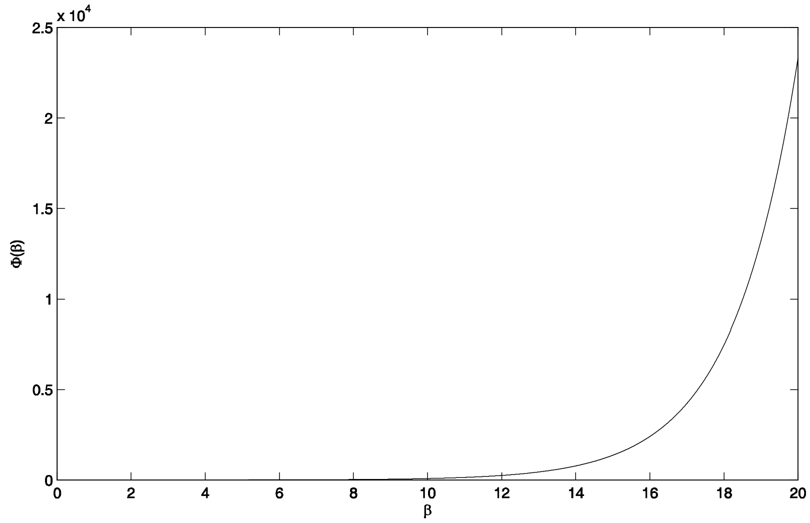


Fig 3. Plot of (6) for $x_i = 0.1, 0.3, 0.6, 0.9$, $n = 8$ $m = 4$ and $\mathbf{R} = (1, 0, 2, 1)$

An Application

For $n = 10$ and $\mathbf{R} = (r_1, r_2, r_3, r_4, r_5) = (1,1,1,1,1)$, generated progressively censored data from Weibull distribution with parameters $\lambda = 1$ and $\beta = 4$ are given in Table 4.

Table 4. Progresively censored data from Weibull distribution

i	1	2	3	4	5
x_i	0.3662	0.6783	0.6807	0.8338	1.0870
r_i	1	1	1	1	1

From Table 1, $\Phi^*_{(0.05)} = 3.073$ and $\Phi^*_{(0.95)} = 1.090$. Using interval (7) it can be obtained 90% confidence interval for parameter β is $(1.2165, 5.1727)$.

3.2. Burr XII Case

The pdf and cdf of Weibull distribution are given, respectively, by

$$f(x) = \beta\lambda x^{\beta-1}\left(1 + x^{\beta}\right)^{-(\lambda+1)}, \quad x > 0,\ \beta > 0,\ \lambda > 0 \tag{8}$$

$$F(x) = 1 - \left(1 + x^{\beta}\right)^{-\lambda} \tag{9}$$

Let $X^{\mathbf{R}}_{1:m:n} < X^{\mathbf{R}}_{2:m:n} < \cdots < X^{\mathbf{R}}_{m:m:n}$ be the progressive censored order statistics from Burr XII distribution. Let us define following transformation.

$$Y^{\mathrm{R}}_{i:m:n} = \lambda \log\left\{1 + \left(X^{\mathrm{R}}_{i:m:n}\right)^{\beta}\right\}, \qquad i = 1, 2, \ldots, m$$

(10)

It is easily seen that $Y^{\mathbf{R}}_{i:m:n}, i = 1, 2, \ldots, m$ are standard exponential progressive censored order statistics. The Eq. (10) is substitute in (2) then one can obtain following pivot:

$$\Phi(\beta)=\frac{\sum_{i=1}^{m}(1+r_i)Y_i/n}{\prod_{i=1}^{m}Y_i^{(1+r_i)/n}}=\frac{\sum_{i=1}^{m}\left\{(1+r_i)\lambda\log\left(1+X_i^{\beta}\right)\right\}/n}{\prod_{i=1}^{m}\left\{\lambda\log\left(1+X_i^{\beta}\right)\right\}^{(1+r_i)/n}}$$

$$=\frac{\sum_{i=1}^{m}(1+r_i)\log\left(1+X_i^{\beta}\right)/n}{\prod_{i=1}^{m}\log\left(1+X_i^{\beta}\right)^{(1+r_i)/n}} \tag{11}$$

Confidence interval for parameter β can be obtained by using pivot (10). Confidence interval for parameter β with confidence level $(1-\alpha)\%$ is

$$P\left(\Phi^{*}_{1-\alpha/2}<\frac{\sum_{i=1}^{m}(1+r_i)\log\left(1+X_i^{\beta}\right)/n}{\prod_{i=1}^{m}\log\left(1+X_i^{\beta}\right)^{(1+r_i)/n}}<\Phi^{*}_{\alpha/2}\right)=1-\alpha \tag{12}$$

where Φ_{α}^{*} is quantile which satisfies $P\left(\Phi>\Phi_{\alpha}^{*}\right)=\alpha$ Akdoğan et al. (2013).

If the pivot (11) is strictly monotone in β then we can use the confidence interval (12) reliable. In this paper, it can not proved the strictly monotonicity of (6) in β. But some figures of (11) are provided for different patern of data and censoring schemes. From Figs.4-6. it can be conclude that pivot (11) is strictly increasing but this does not mean that ways exactly.

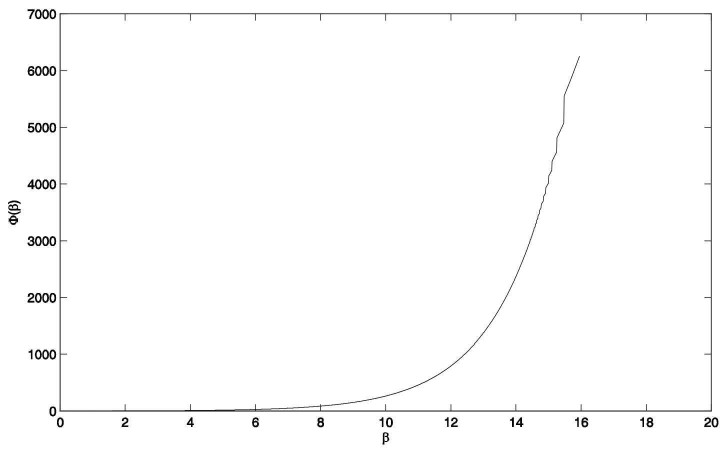


Fig 4. Plot of (11) for $x_i=0.1,0.3,2.4,3.2$, $n=8$ $m=4$ and $\mathbf{R}=(1,0,2,1)$

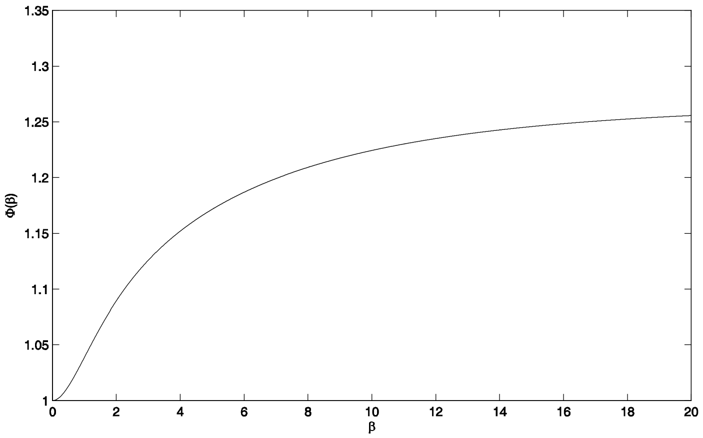


Fig 5. Plot of (11) for $x_i = 1.2, 3.2, 6.5, 7.8$, $n = 8$, $m = 4$ and $\mathbf{R} = (1, 0, 2, 1)$

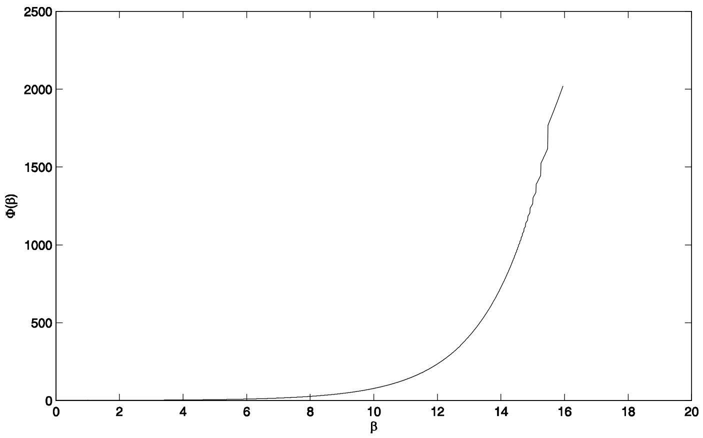


Fig 6. Plot of (11) for $x_i = 0.1, 0.3, 0.6, 0.9$, $n = 8$ $m = 4$ and $\mathbf{R} = (1, 0, 2, 1)$

An Application

For $n = 10$ and $\mathbf{R} = (r_1, r_2, r_3, r_4, r_5) = (1,1,1,1,1)$, generated progressively censored data from Burr XII distribution with parameters $\lambda = 2$ and $\beta = 2$ are given in Table 5.

Table 5. Progresively censored data from Burr XII distribution

i	1	2	3	4	5
x_i	0.2816	0.4235	0.5899	0.6755	0.8387
r_i	1	1	1	1	1

From Table 1, $\Phi^*_{(0.05)} = 3.073$ and $\Phi^*_{(0.95)} = 1.090$. Using interval (7) it can be obtained 90% confidence interval for parameter β is $(1.1324, 5.0725)$.

3.3. Gompertz Case

The pdf and cdf of Weibull dsitribution are given, respectively, by

$$f(x) = \lambda \exp(\beta x)\exp\{-\lambda\beta^{-1}[\exp(\beta x)-1]\}, \quad x>0, \beta>0, \lambda>0 \tag{13}$$

$$F(x) = 1-\exp\{-\lambda\beta^{-1}[\exp(\beta x)-1]\} \tag{14}$$

Let $X_{1:m:n}^{\mathbf{R}} < X_{2:m:n}^{\mathbf{R}} < \cdots < X_{m:m:n}^{\mathbf{R}}$ be the progressive censored order statistics from Gompertz distribution. Let us define following transformation.

$$Y_{i:m:n}^{\mathbf{R}} = \frac{\lambda}{\beta}\left\{\exp\left(\beta X_{i:m:n}^{\mathrm{R}}\right)-1\right\}, \quad i=1,2,\ldots,m \tag{15}$$

It is easily seen that $Y_{i:m:n}^{\mathbf{R}}, i=1,2,\ldots,m$ are standard exponential progressive censored order statistics. The Eq. (15) is substitute in (2) then one can be obtained by following pivot:

$$\Phi(\beta) = \frac{\sum_{i=1}^{m}(1+r_i)Y_i/n}{\prod_{i=1}^{m}Y_i^{(1+r_i)/n}} = \frac{\sum_{i=1}^{m}(1+r_i)\frac{\lambda}{\beta}\left(e^{\beta X_i}-1\right)/n}{\prod_{i=1}^{m}\frac{\lambda}{\beta}\left(e^{\beta X_i}-1\right)^{(1+r_i)/n}}$$

$$= \frac{\sum_{i=1}^{m}(1+r_i)\left(e^{\beta X_i}-1\right)/n}{\prod_{i=1}^{m}\left(e^{\beta X_i}-1\right)^{(1+r_i)/n}} \tag{16}$$

Confidence interval for parameter β can be obtained by using pivot (15). Confidence interval for parameter β with confidence level $(1-\alpha)\%$ is

$$P\left(\Phi^*_{1-\alpha/2} < \frac{\sum_{i=1}^{m}(1+r_i)\left(e^{\beta X_i}-1\right)/n}{\prod_{i=1}^{m}\left(e^{\beta X_i}-1\right)^{(1+r_i)/n}} < \Phi^*_{\alpha/2}\right) = 1-\alpha \tag{17}$$

where Φ_α^* is quantile which satisfies $P\left(\Phi > \Phi_\alpha^*\right) = \alpha$ Akdoğan et al. (2013).

If the pivot (16) is strictly monotone in β then we can use the confidence interval (17) reliable. In this paper, it can not proved the strictly monotonicity of (16) in β. But some figures of function (16) are provided for different patern of data and censoring schemes. From Figs. 7-9 it can be conclude that pivot (16) is strictly increasing but this does not mean that ways exactly.

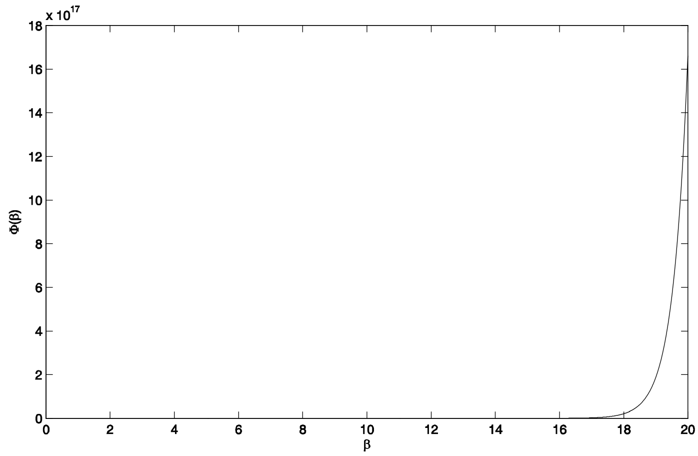


Fig 7. Plot of (16) for $x_i = 0.1, 0.3, 2.4, 3.2$, $n = 8$ $m = 4$ and $\mathbf{R} = (1, 0, 2, 1)$

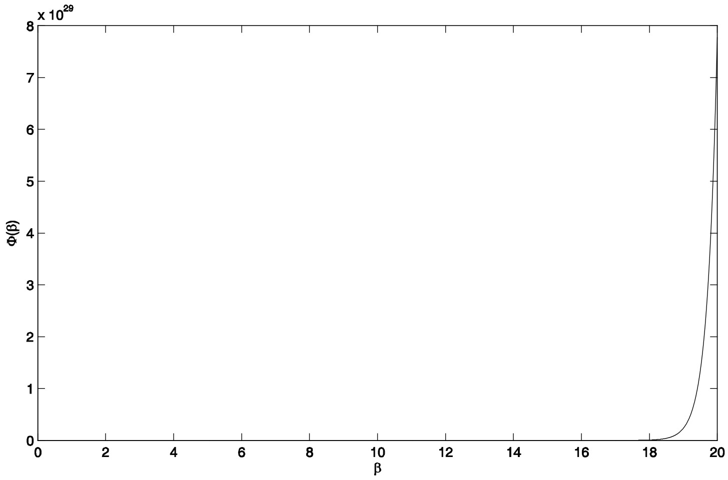


Fig 8. Plot of (16) for $x_i = 1.2, 3.2, 6.5, 7.8$, $n = 8$, $m = 4$ and $\mathbf{R} = (1, 0, 2, 1)$

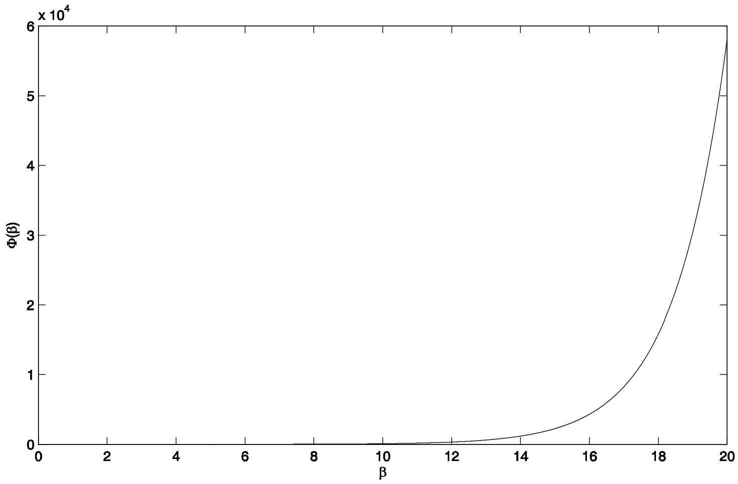


Fig 9. Plot of (16) for $x_i = 0.1, 0.3, 0.6, 0.9$, $n = 8$ $m = 4$ and $\mathbf{R} = (1, 0, 2, 1)$

An Aplication

For $n=10$ and $\mathbf{R}=(r_1,r_2,r_3,r_4,r_5)=(1,1,1,1,1)$, generated progressively censored data from Burr XII distribution with parameters $\lambda=2$ and $\beta=3$ are given in Table 6.

Table 6. Progresively censored data from Gompertz distribution

i	1	2	3	4	5
x_i	0.1029	0.1191	0.1739	0.2478	0.2996
r_i	1	1	1	1	1

From Table 1, $\Phi^*_{(0.05)}=3.073$ and $\Phi^*_{(0.95)}=1.090$. Using interval (7) it can be obtained 90% confidence interval for parameter β is $(1.0228, 9.5704)$.

4. Conclusion

In this paper, some tables containing cut of points of a pivotal quantity which are useful for constructing confidence intervals are given based on progressively censored data. For complete sample case, several regression models are estimated to predict cut off points. In the future study, the strictly monotonicity of pivots can be discussed for Weibull, Burr XII and Gompertz model. Different regression models can be estimated for Type-II and progressive censoring.

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Appendix

The Matlab code to get the Tables 1-2

```
clear all;
clc;
n=10;m=5;
r=[0 3 2 0 0];
r=r';
lambda=3;
beta=2;
ds=10000;
tc=0;
for kk=1:ds
  kk
z=random('wbl',lambda,
        beta,n,1);
y=sort(z);
yy=y;
nn=n;
t=0;
for i=1:m
 x(i)=y(1);
 nn=nn-1;
 sr=randperm(nn);
 for j=1:nn
  y(j)=y(j+1);
 end
 for j=1:nn-1
    for s=j+1:nn
     if (sr(j)>sr(s))
       i1=sr(j);
       sr(j)=sr(s);
       sr(s)=i1;
       i2=y(j);
       y(j)=y(s);
       y(s)=i2;
     end;
   end
end
for j=1:r(i)
   h(t+j)=y(j);
end
t=t+r(i);
for j=r(i)+1:nn
   y(j-r(i))=y(j);
end
nn=nn-r(i);
for j=1:nn-1
   for s=j+1:nn
      if (y(j)>y(s))
         i1=y(j);
         y(j)=y(s);
         y(s)=i1;
      end
   end
  end

end
x=x';
 t1=0;t2=1;
   for j=1:m

         t1=t1+(x(j)^beta
         ).*(r(j)+1);

         t2=t2*(x(j)^beta)
         ^(r(j)+1);
   end
   t=1/n*(t1);
   c=(t2)^(1/n);
   pivot(kk)=t/c;

end
pivot;
wdeger=sort(pivot);
lb1=wdeger(kk*0.025)
lb2=wdeger(kk*0.05)
ub1=wdeger(kk*0.95)
ub2=wdeger(kk*0.975)
```