Cumhuriyet Üniversitesi Fen Fakültesi
Fen Bilimleri Dergisi (CFD), Cilt 37 (2016)
ISSN: 1300-1949

Cumhuriyet University Faculty of Science
Science Journal (CSJ), Vol. 37 (2016)
ISSN: 1300-1949

http://dx.doi.org/10.17776/csj.37746

The Total β-Half-Lives for Some Nickel Isotopes

Sevilay KAYHAN, Necla ÇAKMAK

Department of Physics, Faculty of Science, Karabuk University, Karabuk, Turkey

Received: 30.09.2016; Accepted: 15.11.2016

Abstract. The first-forbidden transitions can play an important role in decreasing the calculated half-lives specially in environments. We calculate the allowed Gamow-Teller transitions in pn-QRPA model for even-even neutron-rich isotopes of nickel. The first forbidden transitions for even-even neutron-rich isotopes of nickel are calculated assuming the nuclei to be spherical. The Woods-Saxon potential basis has been used in our calculations. The calculated half-lives have been compared with the schematic model calculations and corresponding experimental data.

Keywords: pn-QRPA approximation, Gamow-Teller, First forbidden transitions

1. INTRODUCTION

Beta (β) decay processes are very important to understand the nuclear structure. The first forbidden (FF) beta transition ensures useful information in checking the feasibility of theories related to the r-processes and 2νββ [1,2]. Recent studies have shown the importance of forbidden transitions also at orders of magnitude lower densities [3,4]. The QRPA studies based on the Fayans energy functional has been extended by Borzov recently for a consistent treatment of allowed and first-forbidden (FF) contributions to r-process half-lives [5]. The β-decay properties, under terrestrial conditions, of allowed weak interaction and U1F [6] led to a better understanding of the r-process. The pn-QRPA model was developed by Halbleib and Sorensen [7] by generalizing the usual RPA to describe charge-changing transitions. A microscopic approach based on the proton neutron-quasi-particle random phase approximation (pn-QRPA), have so far been successfully used in studies of nuclear β-decay properties of stellar weak-interaction mediated rates [8,9]. In the present study, the allowed β-decay and the first forbidden beta transitions have been investigated for some nickel isotopes. The calculations have been performed within the framework of the pn-QRPA method with the separable residual effective interaction in the particle hole (ph) channel.

2. FORMALISM

Allowed beta decay half-lives have been calculated using spherical schematic model (SSM) and spherical Pyatov's method (SPM) within the framework of pn-QRPA(WS) method. The Woods-Saxon potential with Chepurnov parameterization has been used as a mean field basis in numerical calculations [10]. The eigenvalues and eigenfunctions of the Hamiltonian with separable residual Gamow-Teller (GT) effective interactions in particle-hole (ph) channel were solved within the framework of pn-QRPA model.

Charge-exchange spin-spin correlations are added to the model Hamiltonian in the following form:

$$\hat{V}_{\beta} = 2\chi\beta \sum_{\beta} \beta_{\mu}^{+} \beta_{\mu}^{-}, \qquad \mu = 0, \pm 1 \tag{1}$$

* Corresponding author. *Email address:* kayhansevilay@gmail.com

Sivas: IX. International Workshop on Nuclear Structure Properties (NSP2016), Sivas, Turkey, 1-3 September 2016

where $\beta^{+}_{\mu}(\beta^{-}_{\mu})$ is the positron (electron) decay operatör

$$\beta^{+}_{\mu} = \sum_{np}\sum_{pp'}\langle np|\sigma_{\mu} + (-1)^{\mu}\sigma_{-\mu}|pp'\rangle a^{+}_{np}a_{pp'} \qquad \beta^{-}_{\mu} = \left(\beta^{+}_{\mu}\right)^{\dagger} \tag{2}$$

and σ_{μ} is and the spherical component of the Pauli operatör. The main formulae are given here. Details mathematical formalism are available in [11,12].

The total Hamiltonian in Pyatov's method is given by

$$\widehat{H}_{SPM} = \widehat{H}_{av} + \widehat{V}_{\beta} + \widehat{h}_{0} \tag{3}$$

where $\widehat{H}_{av}$ is the single quasi-particle Hamiltonian in a spherical symmetric average field with pairing forces. The third term comes from the restoration of broken commutation relation between the nuclear Hamiltonian and the GT operator. The schematic method Hamiltonian for GT excitations in the neighbor odd-odd nuclei is given by

$$\widehat{H}_{SSM} = \widehat{H}_{av} + \widehat{V}_{\beta} \tag{4}$$

Details of solution of allowed GT formalism can be seen in [13,14].

The *ft* values for the allowed GT β transitions are finally calculated using

$$ft = \frac{D}{\left(\frac{g_A}{g_V}\right)^{2} 4\pi B^{GT}(I_i \to I_f, \beta^{-})} \tag{5}$$

where the reduced matrix elements of GT transitions are given by

$$B^{GT}\left(I_i \to I_f, \beta^{-}\right) = \sum_{\mu}\left|\langle 1^{+}_{i}, \mu|G^{-}_{\mu}|0^{+}\rangle\right|^{2} \tag{6}$$

The model Hamiltonian which generates the spin-isospin-dependent vibration modes with $\lambda^{\pi} = 0^{-}, 1^{-}, 2^{-}$ in odd-odd nuclei in quasi boon approximation is given as

$$\widehat{H} = \widehat{H}_{sqp} + \widehat{h}_{ph} \tag{7}$$

The single quasi-particle Hamiltonian of the system is given by

$$\widehat{H}_{sqp} = \sum_{j_\tau} \varepsilon_{j_\tau} \alpha^{\dagger}_{j_\tau m_\tau} \alpha_{j_\tau m_\tau} \qquad (\tau = p, n) \tag{8}$$

where ε_{j_τ} and $\alpha^{\dagger}_{j_\tau m_\tau}(\alpha_{j_\tau m_\tau})$ are the single quasi-particle energy of the nucleons with angular momentum j_τ and the quasi-particle cretaion (annihilation) operators, respectively.

The $\widehat{h}_{ph}$ is the spin-isospin effective interaction Hamiltonian which generates $0^{-}, 1^{-}, 2^{-}$ vibration modes in particle-hole channel and given as

$$\widehat{h}_{ph} = \frac{2\chi_{ph}}{g_A^2} \sum_{j_p j_n j_{p'} j_{n'} \mu} \left[b_{j_p j_n} A^{+}_{j_p j_n}(\lambda\mu) + (-1)^{\lambda-\mu}\bar{b}_{j_p j_n} A_{j_p j_n}(\lambda-\mu)\right] \times \left[b_{j_{p'} j_{n'}} A_{j_{p'} j_{n'}}(\lambda\mu) + (-1)^{\lambda-\mu}\bar{b}_{j_{p'} j_{n'}} A^{+}_{j_{p'} j_{n'}}(\lambda-\mu)\right]$$

where χ_{ph} is particle-hole effective interaction constant.

The quasi-boson creation $A^{+}_{j_p j_n}(\lambda\mu)$ and annihibilation $A_{j_p j_n}(\lambda\mu)$ operators are given as

$$A^{+}_{j_p j_n}(\lambda\mu) = \sqrt{\frac{2\lambda+1}{2j_p+1}} \sum_{m_n m_p} (-1)^{j_n-m_n} \langle j_n m_n \lambda\mu | j_p m_p \rangle \alpha^{+}_{j_p m_p} \alpha^{+}_{j_n - m_n}$$

$$A_{j_p j_n}(\lambda\mu) = \left\{ A^{+}_{j_p j_n}(\lambda\mu) \right\}^{\dagger}$$

The $b_{j_p j_n}, \bar{b}_{j_p j_n}$ are the reduced matrix elements f the non-relativistic multipole operators for rank 0, 1 and 2 [15] and given by

$$b_{j_p j_n} = \langle j_p(l_p s_p) \| r_k [Y_1 \sigma_k]_0 \| j_n(l_n s_n) \rangle V_{j_n} U_{j_p}$$

$$\bar{b}_{j_p j_n} = \langle j_p(l_p s_p) \| r_k [Y_1 \sigma_k]_0 \| j_n(l_n s_n) \rangle U_{j_n} V_{j_p}$$

$$b_{j_p j_n} = \langle j_p(l_p s_p) \| r_k [Y_1 \sigma_k]_1 \| j_n(l_n s_n) \rangle V_{j_n} U_{j_p}$$

$$\bar{b}_{j_p j_n} = \langle j_p(l_p s_p) \| r_k [Y_1 \sigma_k]_1 \| j_n(l_n s_n) \rangle U_{j_n} V_{j_p}$$

$$b_{j_p j_n} = \langle j_p(l_p s_p) \| r_k [Y_1 \sigma_k]_2 \| j_n(l_n s_n) \rangle V_{j_n} U_{j_p}$$

$$\bar{b}_{j_p j_n} = \langle j_p(l_p s_p) \| r_k [Y_1 \sigma_k]_2 \| j_n(l_n s_n) \rangle U_{j_n} V_{j_p}$$

where U_{j_τ} and V_{j_τ} are the standart BCS occupation amplitudes. The calculation of the transition probabilities for rank 0 and rank 1 have been performed using ξ approximation (see [15] for a detailed information about the ξ approximation).

Calculation of rank 0 FF transitions was done within the pn-QRPA (WS-SSM) formalism. Details of this calculations are dictated by the matrix elements of moments [15,16].

The *ft* values are given by the following expression:

$$(ft)_{\beta^{\mp}} = \frac{D}{(g_A/g_V)^2 4\pi B(I_{f,}\beta^{\mp})}$$

where

$$D = \frac{2\Pi^3 h^2 ln^2}{g_v^2 m_e^5 c^4} = 6250sec, \qquad \frac{g_A}{g_v} = -1.254.$$

Transitions with λ= n+1 are referred to as unique first forbidden transitions [15], and the *ft* values are expressed as

$$(ft)_{\beta^{\mp}} = \frac{D}{\left({}^{g_A}/{}_{g_V}\right)^2 4\Pi B(I_f \to I_f, \beta^{\mp})} \frac{(2n+1)!!}{[(n+1)!]^2 n!}$$

3. RESULTS AND COMPARIONS

The calculated allowed β-decay half-lives in pn-QRPA (WS-SSM) and pn-QRPA (WS-SPM) models are show in Table 1 for chosen nickel isotopes. A quenching factor of 0.6 was applied for all pn-QRPA (WS) calculations. The pairing correlation constants were taken as $C_n = C_p = 12/\sqrt{A}$. The strength parameters of the effective interaction are $\chi_\beta = 5.2A^{0.7} MeV$ [6]. Only particle-hole interaction strength was considered for both allowed GT and FF calculations within the pn-QRPA (WS) formalism. The strength parameters of the effective interaction are $\chi_\beta = 30A^{-5/3} MeV\, fm^{-2}$, $\chi_\beta = 55A^{-5/3} MeV\, fm^{-2}$, $\chi_\beta = 99A^{-5/3} MeV\, fm^{-2}$ for rank0, rank1 and rank2, respectively. The FF contributions to the total calculated half-lives are shown in Table 2 and 3. Here we present the GT+U1F calculation of half-lives in the GT+rank0+rank1+rank2 half-lives calculation using the pn-QRPA (WS-SSM) and the pn-QRPA (WS-SPM) models. The calculated half-lives are also compared with experimental data. The pn-QRPA (WS-SPM) model was later used to calculate the FF contribution which led to a better agreement of the calculated half-lives with the measured data.

Table 1. Allowed GT β-decay half-lives for Ni isotopes calculated using the pn-QRPA (WS-SSM) and pn-QRPA (WS-SPM) models, in comparison with experimental data [17].

A	Exp	pn-QRPA (WS-SSM) (GT)	pn-QRPA (WS-SPM) (GT)
56	6.075 d	5.32 d	11.5 d
66	54.6 h	50.1 h	61.3 h
68	29 s	25.3 s	37.8 s

Table 2. Total β-decay half-lives for Ni isotopes calculated using pn-QRPA (WS-SSM) models for allowed plus first-forbidden transitions, in comparison with experimental data [17].

A	Exp	pn-QRPA (WS-SSM) (GT)	pn-QRPA (WS-SSM) (GT+rank$_{0,1,2}$)
56	6.075 d	10.32 d	9.14 d
66	54.6 h	50.1 h	47.8 h
68	29 s	25.3 s	23.1 s

Table 3. Total β-decay half-lives for Ni isotopes calculated using pn-QRPA (WS-SPM) models for allowed plus first-forbidden transitions, in comparison with experimental data [17].

A	Exp	pn-QRPA (WS-SPM) (GT)	pn-QRPA (WS-SPM) (GT+rank$_{0,1,2}$)
56	6.075 d	11.5 d	10.09 d
66	54.6 h	61.3 h	56.7 h
68	29 s	37.8 s	31.08 s

CONCLUSIONS

The contribution of FF transitions to total β-decay becomes significant for neutron-rich isotopes. We used the Woods-Saxon potential to calculate allowed GT transitions for isotopes of nickel using the pn-QRPA (WS-SSM), pn-QRPA (WS-SPM) models. Results of pn-QRPA (WS-SPM) model was later used to calculate the FF contribution which led to a better agreement of the calculated half-lives with the measured data.

Acknowledgements

The authors would like to thank Jameel-Un Nabi and Cevad Selam for his contributions to this study.

REFERENCES

1. Dobaczewski, J. et al., 1994, Phys. Rev. Lett. 72,981
2. Kratz, K.-L., et al., 1993, Astrophys. J. 403,216
3. Nabi, J.-Un and Stoica, S., 2013, Astrophys. Space Sci. 349, 843
4. Zhi, Q., et al., 2013, Phys. Rev. C 87, 025803
5. Borzov, I., 2006, Nucl. Phys. A 777, 645
6. Homma, H., et al., 1996, Phys. Rev. C 54, 2972
7. Halbleib, J.A. and Sorensen, R.A., 1967, Nucl. Phys. A 98, 542
8. Nabi, J.-Un and Klapdor-Kleingrothaus, H.V., 1999, Eur. Phys. J.A 5, 337
9. Nabi, J.-Un and Klapdor-Kleingrothaus, H.V., 2004, At. Data Nucl. Data Tables 88, 237
10. Soloviev, V.G., 1976, Theory of Complex Nuclei, (Pergamon, New York)
11. Gabrakov, S.I. and Kuliev, A.A., Dubna, 1970, JINR Report PA-5003.
12. Selam, C., et al., 2004, Math. Comp. App. 9, 79
13. Cakmak, N., et al., 2010, Pramana J. Phys. 75, 649
14. Cakmak, N., et al., 2012, Phys. At. Nucl. 75, 8
15. Bohr, A., Mottelson, B.R., 1969, *Nuclear Structure* (Benjamin, New York)
16. Nabi, J.-Un, et al., 2016, Eur. Phys. J.A 52, 5
17. Audi, G., et al., 2012, Chin. Phys. C 36, 1287