Journal of New Theory
ISSN: 2149-1402

43 (2023) 54-62
Journal of New Theory
https://dergipark.org.tr/en/pub/jnt
Open Access

Construction of Analytical Solutions to the Conformable New (3+1)-Dimensional Shallow Water Wave Equation

Mehmet Şenol[1], **Mehmet Gençyiğit**[2]

Article Info
Received: 15 Mar 2023
Accepted: 18 Jun 2023
Published: 30 Jun 2023
doi:10.53570/jnt.1265715
Research Article

Abstract — This study investigates the new (3+1)-dimensional shallow water wave equation. To do so, the definitions of conformable derivatives and their descriptions are given. Using the Riccati equation and modified Kudryashov methods, exact solutions to this problem are discovered. The gathered data's contour plot surfaces and related 3D and 2D surfaces emphasize the result's physical nature. To monitor the problem's physical activity, exact and complete solutions are necessary. The results demonstrate the potential applicability of additional nonlinear physical models from mathematical physics and under-investigation in real-world settings. In order to solve fractional differential equations, it may prove helpful to use these methods in various situations.

Keywords *Riccati equation method, modified Kudryashov method, new* $(3+1)$*-dimensional shallow water wave (SWW) equation, conformable derivative*

Mathematics Subject Classification (2020) 35R11, 35C07

1. Introduction

Determining a created model's analytical solution enables the physical phenomena expressed by that model to be understood and interpreted. As a result of modeling problems encountered in applied science fields, nonlinear fractional differential equations are usually found. The fractional differential equation has applications in biology, chemistry, medicine, pharmacy, psychology, economics, statistics, and natural sciences, especially engineering and physics. Examples of these application areas are the movements of fluids, earthquake vibration movements, shallow water waves, and propagation movements of acoustic sound vibrations. Research on this and other topics continues rapidly, and thus new fields of applications are also used.

Hence, it is crucial to solve nonlinear fractional differential equations analytically. Thanks to the developed analytical methods, exact solutions of fractional differential equations have been found. Thereby, it has become easier to understand and interpret physical phenomena.

Many different types of fractional derivative operators have been described in the literature. Some of them are Caputo derivative [1], Riemann-Liouville derivative [2], Caputo-Fabrizio derivative [3], modified Riemann Liouville derivative [4], Atangana-Baleanu derivative [5], and conformable derivative [6]. With the help of these derivative operators, various techniques have been developed that provide

[1]msenol@nevsehir.edu.tr (Corresponding Author); [2]mgencyigit@hotmail.com
[1,2]Department of Mathematics, Faculty of Arts and Sciences, Nevşehir Hacı Bektaş Veli University, Nevşehir, Türkiye

exact solutions of partial differential equations with nonlinear fractional derivatives. Moreover, these equations are used with the help of fractional derivatives with various analytical methods to get better results. Among these techniques are Generalized (G'/G)-expansion Method [7], Riccati Equation Method [8,9], Exp-function Method [10], Generalized Riccati Equation Mapping Method [11], Jacobi Elliptic Function Method [12], Improved F-expansion Method [13], tanh-sech Method [14], Hirota Bilinear Method [15], Inverse Scattering Method [16], and Extended tanh Method [17], etc.

This study presents analytical solutions to the conformable form of the following new (3+1)-dimensional shallow water wave equation (SWW) [18]

$$\alpha_1((u_xu_t)_x + u_{xxxt}) + \alpha_2((u_xu_y)_x + u_{xxxy}) + \alpha_3u_{yt} + \alpha_4u_{xx} + \alpha_5u_{xy} + \alpha_6u_{xt} + \alpha_7u_{yy} + \alpha_8u_{zz} = 0 \quad (1)$$

We conducted a study by applying analytical solution methods using the conformable derivative of this new equation.

The following is the layout of the paper. The preliminaries appear in Section 2. The Riccati Equation and Modified Kudryashov Methods are presented in Section 3. In Section 4, the solutions to the considered equation are provided. Finally, the paper includes a discussion in Section 5.

2. Preliminaries

This section provides a basic definition of the conformable derivative and some of its properties.

Definition 2.1. Let $\gamma : [0,\infty) \to \mathbb{R}$ be a function, $t > 0$, and $\omega \in (0,1)$. Then, ω^{th} order conformable fractional derivative of the γ function is defined by

$$\mathscr{D}_t^\omega(\gamma)(t) = \lim_{\chi\to 0} \frac{\gamma(t+\chi t^{1-\omega}) - \gamma(t)}{\chi}$$

Lemma 2.2. [19–21] For $\omega \in (0,1)$ and $t > 0$, let γ_1 and γ_2 be ω^{th} order conformable fractional differentiable functions. Then,

i. $\mathscr{D}_t^\omega(t^{\varOmega_1}) = \varOmega_1 t^{\varOmega_1-\omega}$, $\varOmega_1 \in \mathbb{R}$

ii. $\mathscr{D}_t^\omega(\varOmega_1\gamma_1 + \varOmega_2\gamma_2) = \varOmega_1\mathscr{D}_t^\omega(\gamma_1) + \varOmega_2\mathscr{D}_t^\omega(\gamma_2)$, $\varOmega_1, \varOmega_2 \in \mathbb{R}$

iii. $\mathscr{D}_t^\omega(\frac{\gamma_1}{\gamma_2}) = \frac{\gamma_2.\mathscr{D}_t^\omega(\gamma_1) - \gamma_1\mathscr{T}_t^\omega(\gamma_2)}{\gamma_2^2}$

iv. $\mathscr{D}_t^\omega(\gamma_1.\gamma_2) = \gamma_1.\mathscr{D}_t^\omega(\gamma_2) + \gamma_2.\mathscr{D}_t^\omega(\gamma_1)$

v. If w is differentiable, then $\mathscr{D}_t^\omega(\gamma_1)(t) = \frac{t^{1-\omega}d\gamma_1(t)}{dt}$

vi. $\mathscr{D}_t^\omega(\mathscr{K}) = 0$ such that $\mathscr{K} \in \mathbb{R}$

3. The Procedures of the Analytical Methods

In this section, the procedures of the Riccati equation and modified Kudryashov methods are presented. A general form of a partial differential equation (PDE) is as follows:

$$\mathscr{B}(u, u_t, u_x, u_y, u_{xx}, u_{yy}, \cdots) = 0 \quad (2)$$

If a special wave transform is defined as, for $h \neq 0$,

$$u(x, \cdots, t) = u(\xi), \quad \xi = kx + \cdots + h\frac{t^\omega}{\omega} \quad (3)$$

then a nonlinear ordinary differential equation (ODE) of the form is obtained:

$$\mathscr{F}(u(\xi), u'(\xi), u''(\xi), \cdots) = 0 \quad (4)$$

3.1. Riccati Equation Method

The technique is based on the following equation:

$$\varphi'(\xi) = \sigma + \varphi(\xi)^2 \tag{5}$$

Suppose that the following is the general form of a nonlinear conformable PDE

$$\mathscr{Q}(f, \mathscr{D}_t^{\omega}, \mathscr{D}_x f, \mathscr{D}_y f, \mathscr{D}_x^2 f, \mathscr{D}_y^2 f, \cdots) = 0$$

In this case, the derivative operator denoted by $\mathscr{D}_t^{\omega}$ appears in any order. Equation 3 provides the definition of conformable transformations. Making use of the chain rule, $k, \cdots ., h$ constants represent arbitrary values that will be determined later. Equation 2 is transformed into a nonlinear ODE as shown below. Thus, Equation 4 should have the following solution:

$$u(\xi) = \sum_{i=0}^{N} a_i \varphi^i(\xi),\ a_N \neq 0 \tag{6}$$

Then, N is calculated by using the balancing rule in Equation 4, where $\varphi(\xi)$ is solution to the Riccati equation. A list of solutions satisfying Equation 5 is provided below.

$$\varphi(\xi) = \begin{cases} -\sqrt{-\sigma}\tanh(\sqrt{-\sigma}\xi), & \sigma < 0 \\ -\sqrt{-\sigma}\coth(\sqrt{-\sigma}\xi), & \sigma < 0 \\ \sqrt{\sigma}\tan(\sqrt{\sigma}\xi), & \sigma > 0 \\ -\sqrt{\sigma}\cot(\sqrt{\sigma}\xi), & \sigma > 0 \\ -\frac{1}{\xi+\theta},\ \theta \text{ is a constant}, & \sigma = 0 \end{cases} \tag{7}$$

By inserting all of the values found in the terms in Equation 7, we can determine the exact solutions of Equation 2. A polynomial of $\varphi(\xi)$ is produced by combining all of the results. A system of nonlinear algebraic equations for $a_i, \cdots, h, (i \in \{0, 1, \cdots, N\})$ is produced when the coefficients of $\varphi(\xi)$ disappear. These nonlinear algebraic equations solutions are calculated, and the results are used to determine the values of $a_i, \cdots, h, (i \in \{0, 1, \cdots, N\})$.

3.2. Modified Kudryashov Method

The general form of the solution to Equation 4 is as follows:

$$u(\xi) = \sum_{r=0}^{N} B_r \varphi^r(\xi), \quad B_N \neq 0 \tag{8}$$

$\varphi(\xi)$ meets the ODE in Equation 8.

$$\varphi'(\xi) = \log(\vartheta)\varphi(\xi)(\varphi(\xi) - 1) \tag{9}$$

The solution to this equation is given by

$$\varphi(\xi) = \frac{1}{d\vartheta^{\xi} + 1}, \quad \vartheta > 0, \quad \vartheta \neq 0, \quad d \text{ is a constant} \tag{10}$$

The homogeneous balancing concept is used to compute N at Equation 4. We may compute a polynomial of $\varphi^r(\xi)$ by putting Equation 8 into Equation 4 without ignoring Equation 9. Setting all of the coefficients of $\varphi^r(\xi)$ to zero results in a series of algebraic equations in $k, \cdots, h$ and B_r [22]. Finally, the soliton-type solutions of the given model are attained.

4. Solutions to the New (3+1)-Dimensional SWW Equation

Take into account below equation, the conformable form of Equation 1:

$$\alpha_1((u_x\mathscr{D}_t^\omega u)_x+\mathscr{D}_t^\omega u_{xxx})+\alpha_2((u_xu_y)_x+u_{xxxy})+\alpha_3\mathscr{D}_t^\omega u_y+\alpha_4u_{xx}+\alpha_5u_{xy}+\alpha_6\mathscr{D}_t^\omega u_x+\alpha_7u_{yy}+\alpha_8u_{zz}=0$$

Having the transformation $u(x,y,z,t)=u(\xi)$, for $\xi=kx+wy+sz+h\frac{t^\omega}{\omega}$, and integrating yields

$$\begin{aligned}0 =& \alpha_1hk^3u^{(3)}(\xi)+\alpha_1hk^2u'(\xi)^2+\alpha_6hku'(\xi)+\alpha_3hwu'(\xi)+\alpha_2k^3wu^{(3)}(\xi)+\alpha_4k^2u'(\xi)\\ &+\alpha_2k^2wu'(\xi)^2+\alpha_5kwu'(\xi)+\alpha_8s^2u'(\xi)+\alpha_7w^2u'(\xi)\end{aligned}$$

Balancing $u^{(3)}=N+3$, $(u')^2=2(N+1)$ gives $N=1$. If it is replaced in Equation 6 and Equation 8, the outcomes are as follows:

4.1. Riccati Equation-based Analytical Solutions

For $N=1$, the series of sums that result from substituting Equation 6 appears,

$$u=a_0+a_1\varphi,\quad a_1\neq 0$$

In combination with Equation 5, the following algebraic system is created

$$\begin{aligned}0 =& 2a_1\alpha_1hk^3\sigma^2+a_1^2\alpha_1hk^2\sigma^2+a_1\alpha_6hk\sigma+a_1\alpha_3h\sigma w+2a_1\alpha_2k^3\sigma^2w+a_1\alpha_4k^2\sigma+a_1^2\alpha_2k^2\sigma^2w\\ &+a_1\alpha_5k\sigma w+a_1\alpha_8s^2\sigma+a_1\alpha_7\sigma w^2\end{aligned}$$

$$\begin{aligned}0 =& 8a_1\alpha_1hk^3\sigma+2a_1^2\alpha_1hk^2\sigma+a_1\alpha_6hk+a_1\alpha_3hw+8a_1\alpha_2k^3\sigma w+a_1\alpha_4k^2+2a_1^2\alpha_2k^2\sigma w+a_1\alpha_5kw\\ &+a_1\alpha_8s^2+a_1\alpha_7w^2\end{aligned}$$

$$0=6a_1\alpha_1hk^3+a_1^2\alpha_1hk^2+6a_1\alpha_2k^3w+a_1^2\alpha_2k^2w$$

Here, we obtain one case and one set of solutions for a_0, a_1, and h.

Case 1.

$$a_1=-6k,\quad \text{and}\quad h=\frac{-4\alpha_2k^3\sigma w+\alpha_4k^2+\alpha_5kw+\alpha_8s^2+\alpha_7w^2}{4\alpha_1k^3\sigma-\alpha_6k-\alpha_3w}$$

Set 1.

For $\sigma<0$,

$$u_1=a_0+6k\sqrt{-\sigma}\tanh\left(\sqrt{-\sigma}\left(\frac{t^\omega\left(-4\alpha_2k^3\sigma w+\alpha_4k^2+\alpha_5kw+\alpha_8s^2+\alpha_7w^2\right)}{\omega\left(4\alpha_1k^3\sigma-\alpha_6k-\alpha_3w\right)}+kx+sz+wy\right)\right) \tag{11}$$

$$u_2=a_0+6k\sqrt{-\sigma}\coth\left(\sqrt{-\sigma}\left(\frac{t^\omega\left(-4\alpha_2k^3\sigma w+\alpha_4k^2+\alpha_5kw+\alpha_8s^2+\alpha_7w^2\right)}{\omega\left(4\alpha_1k^3\sigma-\alpha_6k-\alpha_3w\right)}+kx+sz+wy\right)\right)$$

for $\sigma>0$,

$$u_3=a_0-6k\sqrt{\sigma}\tan\left(\sqrt{\sigma}\left(\frac{t^\omega\left(-4\alpha_2k^3\sigma w+\alpha_4k^2+\alpha_5kw+\alpha_8s^2+\alpha_7w^2\right)}{\omega\left(4\alpha_1k^3\sigma-\alpha_6k-\alpha_3w\right)}+kx+sz+wy\right)\right)$$

$$u_4=a_0+6k\sqrt{\sigma}\cot\left(\sqrt{\sigma}\left(\frac{t^\omega\left(-4\alpha_2k^3\sigma w+\alpha_4k^2+\alpha_5kw+\alpha_8s^2+\alpha_7w^2\right)}{\omega\left(4\alpha_1k^3\sigma-\alpha_6k-\alpha_3w\right)}+kx+sz+wy\right)\right) \tag{12}$$

and for $\sigma=0$,

$$u_5=a_0+\frac{6k}{\theta+\frac{t^\omega(\alpha_4k^2+\alpha_5kw+\alpha_8s^2+\alpha_7w^2)}{\omega(\alpha_3(-w)-\alpha_6k)}+kx+sz+wy}$$

4.2. The Modified Kudryashov Method-based Analytical Solutions

For $N = 1$, the series of sums that result from substituting Equation 8 appears,

$$u = B_0 + B_1\varphi(\xi), \quad B_1 \neq 0 \tag{13}$$

The below system is attained with combining Equation 9,

$$\begin{aligned}
0 =\ & 7\alpha_1 B_1 hk^3 \log^3(a) + \alpha_1 B_1^2 hk^2 \log^2(a) + \alpha_6 B_1 hk \log(a) + \alpha_3 B_1 hw \log(a) + 7\alpha_2 B_1 k^3 w \log^3(a) \\
& + \alpha_4 B_1 k^2 \log(a) + \alpha_2 B_1^2 k^2 w \log^2(a) + \alpha_5 B_1 kw \log(a) + \alpha_8 B_1 s^2 \log(a) + \alpha_7 B_1 w^2 \log(a) \\
0 =\ & \alpha_1 B_1(-h)k^3 \log^3(a) - \alpha_6 B_1 hk \log(a) - \alpha_3 B_1 hw \log(a) - \alpha_2 B_1 k^3 w \log^3(a) - \alpha_4 B_1 k^2 \log(a) \\
& - \alpha_5 B_1 kw \log(a) - \alpha_8 B_1 s^2 \log(a) - \alpha_7 B_1 w^2 \log(a) \\
0 =\ & -12\alpha_1 B_1 hk^3 \log^3(a) - 2\alpha_1 B_1^2 hk^2 \log^2(a) - 12\alpha_2 B_1 k^3 w \log^3(a) - 2\alpha_2 B_1^2 k^2 w \log^2(a) \\
0 =\ & 6\alpha_1 B_1 hk^3 \log^3(a) + \alpha_1 B_1^2 hk^2 \log^2(a) + 6\alpha_2 B_1 k^3 w \log^3(a) + \alpha_2 B_1^2 k^2 w \log^2(a)
\end{aligned}$$

Here, we obtain one case and one set of solutions for B_0, B_1 and h.

Case 2.

$$B_1 = -6k\log(a) \quad \text{and} \quad h = -\frac{\alpha_2 k^3 w \log^2(a) + \alpha_4 k^2 + \alpha_5 kw + \alpha_8 s^2 + \alpha_7 w^2}{\alpha_1 k^3 \log^2(a) + \alpha_6 k + \alpha_3 w}$$

Using Equation 10 and these values with Equation 13, the solutions are obtained as follows:

Set 2.

$$u_5 = B_0 - \frac{6k\log(a)}{d a^{-\frac{t^{\omega}\left(\alpha_2 k^3 w \log^2(a) + \alpha_4 k^2 + \alpha_5 kw + \alpha_8 s^2 + \alpha_7 w^2\right)}{\omega\left(\alpha_1 k^3 \log^2(a) + \alpha_6 k + \alpha_3 w\right)} + kx + sz + wy} + 1} \tag{14}$$

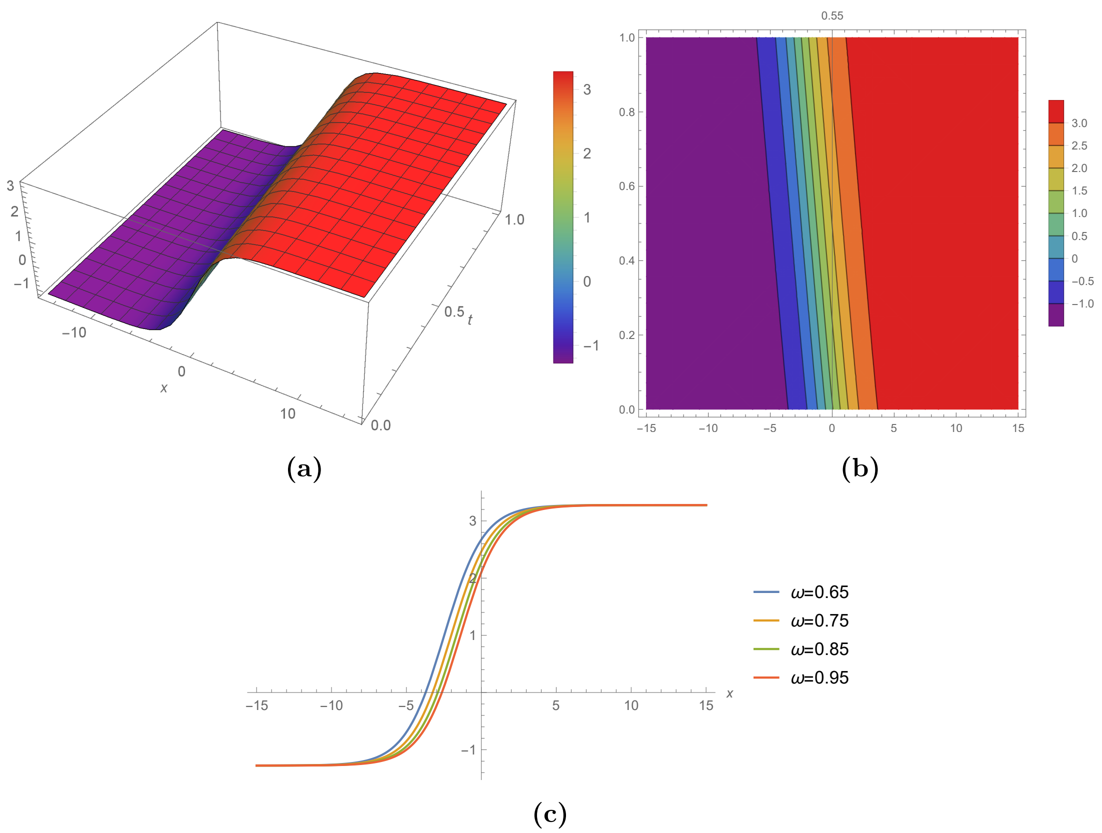

Fig. 1. The plot of Equation 11 for (a) 3D, (b) contour, and (c) 2D plot

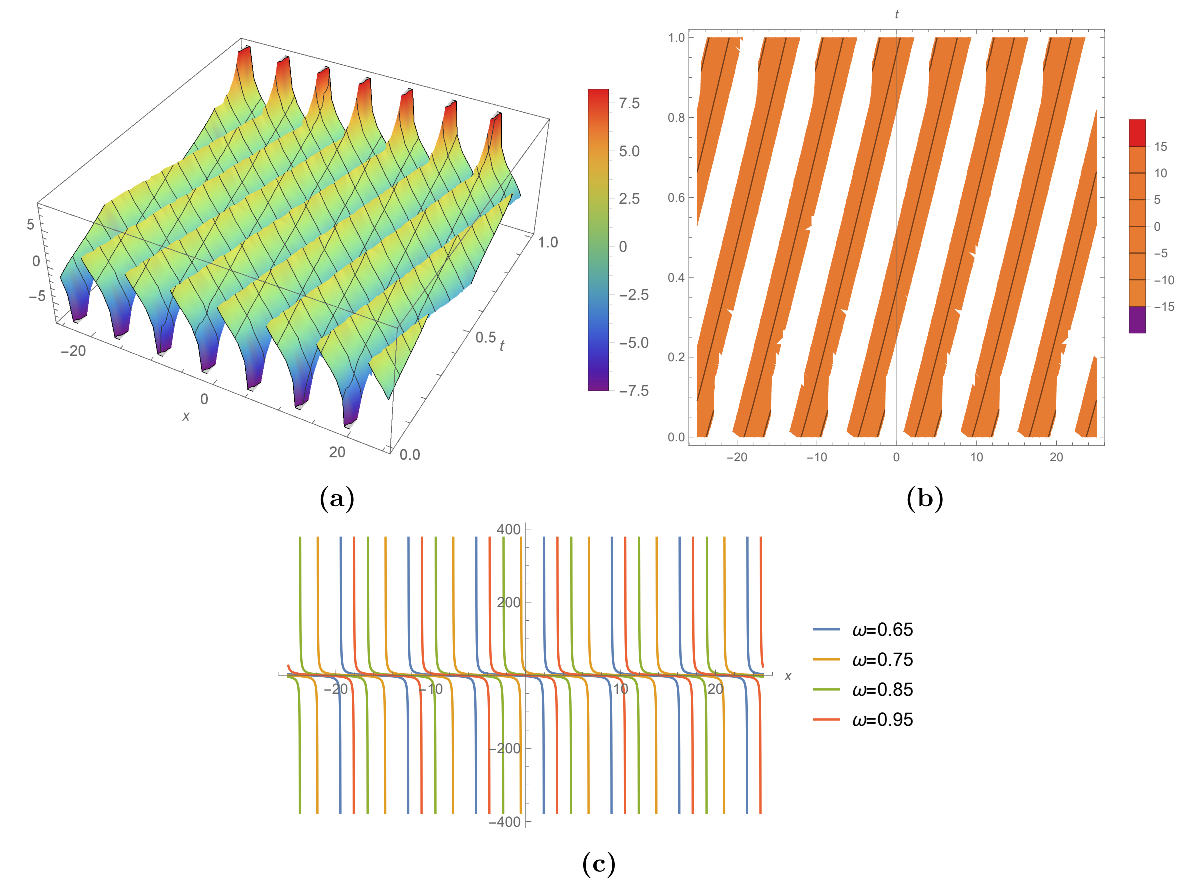

Fig. 2. The plot of Equation 12 for (a) 3D, (b) contour, and (c) 2D plot

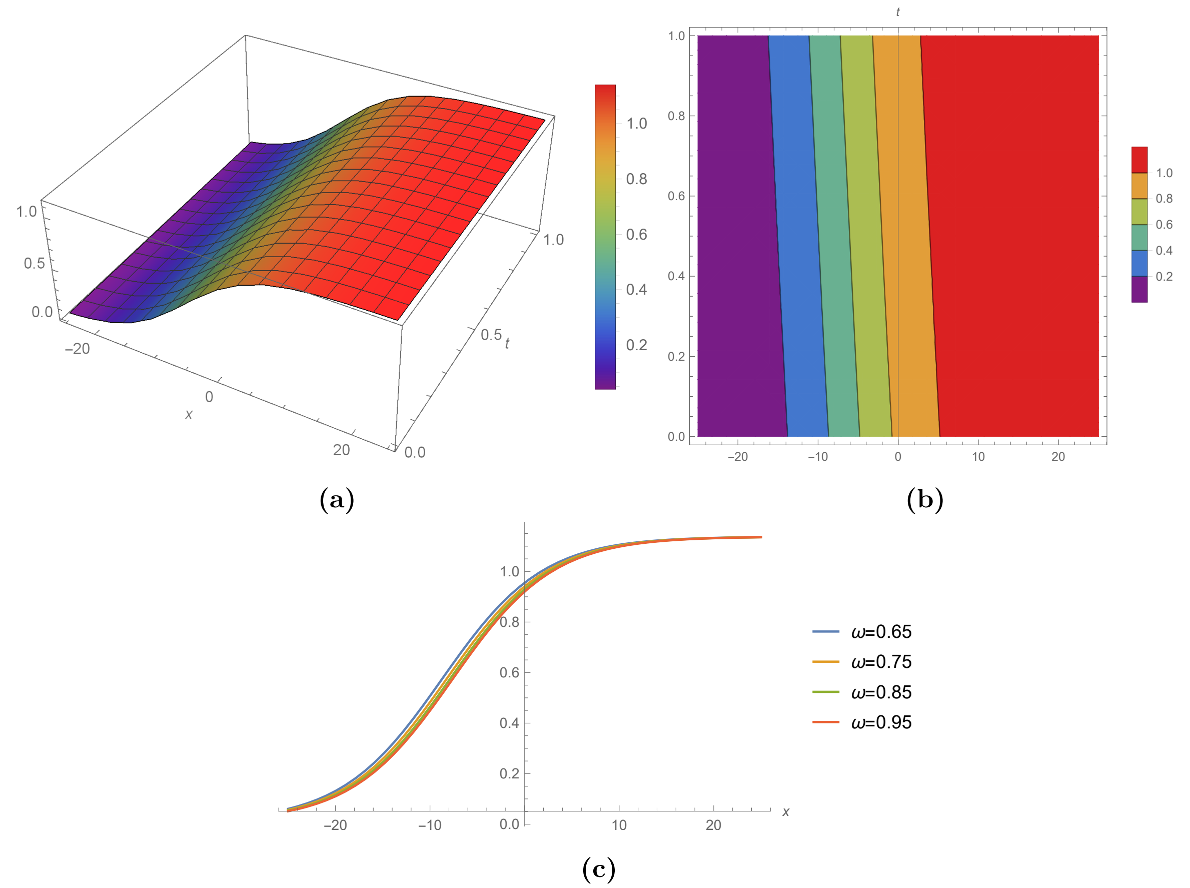

Fig. 3. The plot of Equation 14 for (a) 3D, (b) contour, and (c) 2D plot

For Figures 1-3, the following numerical values are employed:

i. In Figure 1, for (a) and (b), $a_0 = 1, k = -0.6, w = 0.1, s = 0.05, y = 0.1, z = 0.5, \alpha_1 = 0.4, \alpha_2 = -0.2, \alpha_3 = 0.6, \alpha_4 = -0.4, \alpha_5 = 0.99, \alpha_6 = 0.1, \alpha_7 = -0.7, \alpha_8 = 0.8, \sigma = -0.4$, and $\omega = 0.95$ and for (c), $t = 0.55$.

ii. In Figure 2, for (a) and (b), $a_0 = 0.1, k = 0.22, w = 0.5, s = 0.5, y = 0.1, z = 0.5, \alpha_1 = 0.95, \alpha_2 = 0.85, \alpha_3 = 0.65, \alpha_4 = 0.4, \alpha_5 = 0.99, \alpha_6 = 0.1, \alpha_7 = 0.9, \alpha_8 = 0.8, \sigma = 4$, and $\omega = 0.95$ and for (c), $t = 0.55$.

iii. In Figure 3, for (a) and (b), $B_0 = 0.01, a = 0.625, k = 0.4, w = -0.02, s = 0.55, y = 0.5, z = -0.09, d = 0.36, \alpha_1 = 0.25, \alpha_2 = 0.55, \alpha_3 = 0.45, \alpha_4 = -0.65, \alpha_5 = 0.35, \alpha_6 = 0.75, \alpha_7 = 0.15, \alpha_8 = -0.55$, and $\omega = 0.95$ and for (c), $t = 0.95$.

The presented methods have several novel solutions revealed by the graphical representations and may be applied to other kinds of equations.

5. Conclusion

This work investigated the new $(3 + 1)$-dimensional shallow water wave equation with conformable derivative's soliton characteristics using the Riccati equation and modified Kudryashov methods. Then, to visualize some of the solutions with the proper values, 3D, contour, and 2D graphics are presented. Graphical representations and analytical solutions have been provided to show these techniques' accuracy. Furthermore, the physical characteristics of these solutions are distinctive and significant, and they have been researched in the literature. Consequently, future studies may use the proposed approaches to handle and solve various additional fractional differential equations.

Author Contributions

All authors contributed equally to this work. They all read and approved the final version of the paper.

Conflicts of Interest

All authors declare no conflict of interest.

References

[1] M. Şenol, O. Taşbozan, A. Kurt, *Comparison of Two Reliable Methods to Solve Fractional Rosenau-Hyman Equation*, Mathematical Methods in the Applied Sciences 44 (10) (2021) 7904–7914.

[2] G. Jumarie, *Fractional Partial Differential Equations and Modified Riemann-Liouville Derivative New Methods for Solution*, Journal of Applied Mathematics and Computing 24 (1-2) (2007) 31–48.

[3] A. Atangana, D. Baleanu, *Caputo-Fabrizio Derivative Applied to Groundwater Flow within Confined Aquifer*, Journal of Engineering Mechanics 143 (5) (2017) 1–5.

[4] Y. Khan, Q. Wu, N. Faraz, A. Yıldırım, M. Madani, *A New Fractional Analytical Approach via a Modified Riemann–Liouville Derivative*, Applied Mathematics Letters 25 (10) (2012) 1340–1346.

[5] A. Atangana, I. Koca, *Chaos in a Simple Nonlinear System with Atangana–Baleanu Derivatives with Fractional Order*, Chaos, Solitons & Fractals 89 (2016) 447–454.

[6] F. M. Alharbi, D. Baleanu, A. Ebaid, *Physical Properties of the Projectile Motion Using the Conformable Derivative*, Chinese Journal of Physics 58 (2019) 18–28.

[7] M. Gençyiğit, M. Şenol, M. E. Köksal, *Analytical Solutions of the Fractional (3+1)-Dimensional Boiti-Leon-Manna-Pempinelli Equation*, Computational Methods for Differential Equations 11 (3) (2023) 564–575.

[8] L. Akinyemi, M. Şenol, E. Az-Zo'bi, P. Veeresha, U. Akpan, *Novel Soliton Solutions of Four Sets of Generalized (2+1)-Dimensional Boussinesq–Kadomtsev–Petviashvili-like Equations*, Modern Physics Letters B 36 (01) (2022) 1–30.

[9] H. Durur, O. Taşbozan, A. Kurt, M. Şenol, *New Wave Solutions of Time Fractional Kadomtsev-Petviashvili Equation Arising in the Evolution of Nonlinear Long Waves of Small Amplitude*, Erzincan University Journal of Science and Technology 12 (2) (2019) 807–815.

[10] A. Kurt, M. Şenol, O. Taşbozan, M. Chand, *Two Reliable Methods for the Solution of Fractional Coupled Burgers' Equation Arising as a Model of Polydispersive Sedimentation*, Applied Mathematics and Nonlinear Sciences 4 (2) (2019) 523–534.

[11] L. Akinyemi, M. Şenol, M. S. Osman, *Analytical and Approximate Solutions of Nonlinear Schrödinger Equation with Higher Dimension in the Anomalous Dispersion Regime*, Journal of Ocean Engineering and Science 7 (2) (2022) 143–154.

[12] E. Az-Zo'bi, W. A. Alzoubi, L. Akinyemi, M. Şenol, B. S. Masaedeh, *A Variety of Wave Amplitudes for the Conformable Fractional (2+1)-Dimensional Ito Equation*, Modern Physics Letters B 35 (15) (2021) 1–13.

[13] M. Mirzazadeh, A. Akbulut, F. Taşcan, L. Akinyemi, *A Novel Integration Approach to Study the Perturbed Biswas-Milovic Equation with Kudryashov's Law of Refractive Index*, Optik 252 (2022) 1–9.

[14] S. S. Ray, S. Sahoo, *A Novel Analytical Method with Fractional Complex Transform for New Exact Solutions of Time-Fractional Fifth-Order Sawada-Kotera Equation*, Reports on Mathematical Physics 75 (1) (2015) 63–72.

[15] D. Ntiamoah, W. Ofori-Atta, L. Akinyemi, *The Higher-Order Modified Korteweg-de Vries Equation: Its Soliton, Breather and Approximate Solutions*, Journal of Ocean Engineering and Science (in press).

[16] H. Grosse, G. Opelt, *Fractional Charges in External Field Problems and the Inverse Scattering Method*, Nuclear Physics B 285 (1987) 143–161.

[17] K. R. Raslan, K. K. Ali, M. A. Shallal, *The Modified Extended tanh Method with the Riccati Equation for Solving the Space-Time Fractional EW and MEW Equations*, Chaos, Solitons & Fractals 103 (2017) 404–409.

[18] Y. Gu, S. M. Zia, M. Isam, J. Manafian, A. Hajar, M. Abotaleb, *Bilinear Method and Semi-Inverse Variational Principle Approach to the Generalized (2+1)-Dimensional Shallow Water Wave Equation*, Results in Physics 45 (2023) 1–12.

[19] M. Mirzazadeh, L. Akinyemi, M. Şenol, K. Hosseini, *A Variety of Solitons to the Sixth-Order Dispersive (3+1)-Dimensional Nonlinear Time-Fractional Schrödinger Equation with Cubic-Quintic-Septic Nonlinearities*, Optik 241 (2021) 1–15.

[20] R. Khalil, M. Al Horani, A. Yousef, M. Sababheh, *A New Definition of Fractional Derivative*, Journal of Computational and Applied Mathematics 264 (2014) 65–70.

[21] Y. Cenesiz, A. Kurt, *New Fractional Complex Transform for Conformable Fractional Partial Differential Equations*, Journal of Applied Mathematics, Statistics and Informatics 12 (2) (2016) 41–47.

[22] K. S. Nisar, L. Akinyemi, M. Inc, M. Şenol, M. Mirzazadeh, A. Houwe, S. Abbagari, H. Rezazadeh, *New Perturbed Conformable Boussinesq-like Equation: Soliton and Other Solutions*, Results in Physics 33 (2022) 1–10.