GU J Sci 30(2): 247-259 (2017)

Gazi University

Journal of Science

http://dergipark.gov.tr/gujs

Investigation for the Robustness of Significance Level When the Normality Assumption in Hypothesis Tests is Violated

Abdullah YALÇINKAYA[1], Mehmet Niyazi ÇANKAYA[2,*], Ömer ALTINDAĞ[3], Yetkin TUAÇ[1]

[1]*Department of Statistics, Faculty of Science, Ankara University, Ankara, Turkey.*

[2]*Department of Statistics, Faculty of Arts&Science, Uşak University, Uşak, Turkey.*

[3]*Department of Statistics, Faculty of Arts&Science, Bilecik Şeyh Edebali University, Bilecik, Turkey.*

Article Info

Received: 27/09/2016
Accepted: 23/03/2017

Keywords

Robustness
Contaminated distribution
Hypothesis tests
Significance level

Abstract

Many of the test statistics which are used to test $\mathrm{H}_0: \theta = \theta_0$ are constructed under a postulated model. However, when the postulated model is not correct, the true significance level α' will be different from that of the postulated model. The significance level α is used whether or not the hypothesis will be rejected. So, determining the true significance level is important in hypothesis tests. In this paper, the true significance level for testing hypothesis about location and scale parameters under different contaminated distributions is obtained when the normality assumption is violated. As a result, the robustness of significance levels is investigated.

1. INTRODUCTION

The most of the hypothesis used to test $H_0: \theta = \theta_0$ vs $H_1: \theta > \theta_0$ is constructed under a postulated model. However, when the postulated model is wrong, the significance level (α) will be different than that of the postulated model. For instance, in hypothesis tests, the test statistics are generally constructed under normal distribution assumption. However, in real life cases the data set may not be normally distributed but it may be distributed as near to normal. In this situation, when the hypothesis is tested under normality assumption with specified significance level, the true significance level of the test may be different depending on the wrong model selection. So, investigating how the significance level is effected under wrong model selection is very crucial. Such a case may arise when the data set has contaminated distribution. Assume that the random variables $X_1, X_2, \dots, X_n$ are mutually independent and have identically contaminated distribution function as $F_X = \rho F_W + (1-\rho)F_Y$ with $0 \le p \le 1$, see [6]. Here, W denotes the normally distributed random variable. We consider the random variable Y has the following distributions:

1) $Y \sim N(\mu, \tau^2)$,
2) $Y \sim SCN(\mu, \alpha, \eta)$,
3) $Y \sim GT(\mu, \delta, \lambda, \beta, q)$.

Here, N shows the normal distribution, SCN denotes the symmetric component normal distribution [4] and GT shows the generalized t distribution [1, 3]. In this study, it is investigated how the asymptotic level of the test changes with the change of ρ and random variable Y while the postulated model is chosen as normal despite the fact that the true model has contaminated distribution. This procedure is done for the hypotheses of location and scale parameters.

*Corresponding author: e-mail:mehmet.cankaya@usak.edu.tr

The rest of the paper is organized as follows. In Section 2, the distributions we consider and some of their properties are briefly reminded. Next, general information for contaminated distributions are provided in Section 3. Robustness of the significance level for testing location and scale parameters is investigated in Section 4. Finally, the results are discussed and the study is concluded in Section 5.

2. DISTRIBUTIONS AND SOME PROPERTIES

Distributions considered for the random variable Y are given in the following subsections.

2.1. Normal Distribution

The random variable Y with $N(\mu, \tau^2)$ has the probability density function (pdf) as

$$f(y) = \frac{1}{\sqrt{2\pi\tau^2}} exp\left\{-\frac{(y-\mu)^2}{2\tau^2}\right\}, \qquad y, \mu \in \mathbb{R}, \tau > 0, \tag{1}$$

where μ and τ are location and scale parameters, respectively. Some of moments of the distribution are

$$\begin{aligned} E(Y) &= \mu, \\ E(Y^2) &= \mu^2 + \tau^2, \\ E(Y^3) &= \mu^3 + 3\mu\tau^2, \\ E(Y^4) &= \mu^4 + 6\mu^2\tau^2 + 3\tau^4. \end{aligned} \tag{2}$$

2.2. Symmetric Component Normal Distribution

The random variable Y with $SCN(\mu, a, \eta)$ has the following pdf

$$f(y) = \frac{1 + a\left(\frac{y-\mu}{\eta}\right)^2}{(1+a)\eta} \phi\left(\frac{y-\mu}{\eta}\right), \quad y, \mu \in \mathbb{R}, a \geq 0, \eta > 0, \tag{3}$$

where $\phi(\cdot)$ denotes the pdf of the standard normal distribution. Here, a is a parameter that produces the bimodality in the normal distribution. μ and η are location and scale parameters, respectively. If $a = 0$, it reduces to the normal distribution with variance η^2. Some moments of the distribution are

$$\begin{aligned} E(Y) &= \mu, \\ E(Y^2) &= \mu^2 + \eta^2 \frac{1+3a}{1+a}, \\ E(Y^3) &= \mu^3 + 3\eta^2\mu \frac{1+3a}{1+a}, \\ E(Y^4) &= \mu^4 + 6\eta^2\mu^2 \frac{1+3a}{1+a} + 3\eta^4 \frac{1+5a}{1+a}. \end{aligned} \tag{4}$$

2.3. Generalized t Distribution

The random variable Y with $GT(\mu,\delta,\lambda,\beta,q)$ has the following pdf

$$f(y)=\frac{\beta q^q\Gamma\left(\frac{1}{2}\right)}{(\lambda\pi)^{\frac{1}{2}}\delta\mathrm{B}\left(q,\frac{1}{2\beta}\right)}\left[q+\left(\frac{(y-\mu)^2}{\lambda\,\delta^2}\right)^{\beta}\right]^{-(q+\frac{1}{2\beta})},\qquad y\in\mathbb{R}\,. \tag{5}$$

Here, $\lambda,\delta>0$ are scale parameters, $\beta,q>0$ are shape parameters and $\mu\in\mathbb{R}$ is location parameter. Some moments of the distribution are

$$\begin{aligned} E(Y) &= \mu,\\ E(Y^2) &= \mu^2+V_{GT}(Y),\\ E(Y^3) &= \mu^3+3\mu V_{GT}(Y),\\ E(Y^4) &= \mu^4+(6\mu^2+C)V_{GT}(Y)+3V_{GT}^2(Y),\qquad q\beta>2, \end{aligned} \tag{6}$$

where $V_{GT}(Y)=\frac{q^{1/\beta}\lambda\delta^2\Gamma\left(\frac{3}{2\beta}\right)\Gamma\left(q-\frac{1}{\beta}\right)}{\Gamma\left(\frac{1}{2\beta}\right)\Gamma(q)}$, $q\beta>1$ and $C=\frac{q^{1/\beta}\Gamma\left(\frac{5}{2\beta}\right)\Gamma\left(q-\frac{2}{\beta}\right)}{\Gamma\left(\frac{3}{2\beta}\right)\Gamma\left(q-\frac{1}{\beta}\right)}$.

3. CONTAMINATED DISTRIBUTIONS AND SOME PROPERTIES

Consider that a researcher wants to test a hypothesis about analyzing a data set, it can be thought that the data set has normal distribution in sight. But, although it looks similar to normal distribution, it can be different from normal distribution in real. This situation can be considered as contaminated distribution. To formalize the idea of contaminated distributions, the following form is described by

$$F_X=\rho F_W+(1-\rho)F_Y,\ \ 0\le\rho\le 1. \tag{7}$$

If F_X, F_W and F_Y have pdfs such as f_X, f_W and f_Y, respectively, it is well known that

$$f_X=\rho f_W+(1-\rho)f_Y,\ \ 0\le\rho\le 1. \tag{8}$$

It can be considered that the proportion $(1-\rho)$ of the observations is generated by the F_Y while a proportion ρ is generated by unknown functions and/or distribution function F_W. The distribution function F_X is defined to be contaminated distribution. In general setting, F_X is called as mixture of F_W and F_Y, see [6]. In this section, the contaminated distributions which will be considered throughout the study are introduced and some of their properties are given.

3.1. Contaminated Distribution of $N(\mu,\sigma^2)$ and $N(\mu,\tau^2)$ (C-N-N)

Consider the random variables W and Y with distributed as $N(\mu,\sigma^2)$ and (μ,τ^2), respectively and let $F_X=\rho F_W+(1-\rho)F_Y$ with $0\le\rho\le 1$. In this situation some moments of the distribution F_X are given in the following way:

$$\begin{aligned} E(X) &= \mu,\\ E(X^2) &= \mu^2+\rho\sigma^2+(1-\rho)\tau^2,\\ E(X^3) &= \mu^3+3\mu[\rho\sigma^2+(1-\rho)\tau^2],\\ E(X^4) &= \mu^4+3[\rho\sigma^4+(1-\rho)\tau^4]+6\mu^2[\rho\sigma^2+(1-\rho)\tau^2]. \end{aligned} \tag{9}$$

3.2. Contaminated Distribution of $N(\mu,\sigma^2)$ and $SCN(\mu,a,\eta)$ (C-N-SCN)

Consider the random variables W and Y with distributed as $N(\mu,\sigma^2)$ and $SCN(\mu,a,\eta)$, respectively and let $F_X=\rho F_W+(1-\rho)F_Y$ with $0\le\rho\le 1$. In this situation some moments of the distribution F_X are given as

$$\begin{aligned}
E(X) &= \mu,\\
E(X^2) &= \mu^2+\rho\sigma^2+(1-\rho)\eta^2\frac{1+3a}{1+a},\\
E(X^3) &= \mu^3+3\mu\rho\sigma^2+3\mu(1-\rho)\eta^2\frac{1+3a}{1+a},\\
E(X^4) &= \mu^4+3\left[\rho\sigma^4+(1-\rho)\eta^4\frac{1+5a}{1+a}\right]+6\mu^2\left[\rho\sigma^2+(1-\rho)\eta^2\frac{1+3a}{1+a}\right].
\end{aligned} \tag{10}$$

3.3. Contaminated Distribution of $N(\mu,\sigma^2)$ and $GT(\mu,\delta,\lambda,\beta,q)$ (C-N-GT)

Consider the random variables W and Y with distributed as $N(\mu,\sigma^2)$ and $GT(\mu,\delta,\lambda,\beta,q)$, respectively and let $F_X=\rho F_W+(1-\rho)F_Y$ with $0\le\rho\le 1$. In this situation some moments of the distribution F_X are given as follows:

$$\begin{aligned}
E(X) &= \mu,\\
E(X^2) &= \mu^2+\rho\sigma^2+(1-\rho)V_{GT}(Y),\\
E(X^3) &= \mu^3+3\mu[\rho\sigma^2+(1-\rho)V_{GT}(Y)],\\
E(X^4) &= \mu^4+3\rho\sigma^4+6\mu^2\rho\sigma^2+(1-\rho)[CV_{GT}(Y)+6\mu^2V_{GT}(Y)+3V_{GT}^2(Y)],
\end{aligned} \tag{11}$$

where, C and $V_{GT}(Y)$ are as given in equation (6).

4. ROBUSTNESS OF THE SIGNIFICANCE LEVEL

When the normality assumption is violated, the theorem 3.5.1. of [5] is restated as follows. Also, the definition of robustness is given.

Theorem 4.1. Let T_n be a sequence of test statistics which satisfies $\sqrt{n}\big(T_n-\mu(\theta_0)\big)/\tau(\theta_0)\to N(0{,}1)$, under the postulated model. So that the test $\sqrt{n}\big(T_n-\mu(\theta_0)\big)/\tau(\theta_0)\ge z_\alpha$ for $H_0\colon\theta=\theta_0$ versus $H_1\colon\theta>\theta_0$ has nominal asymptotic level α. Here z_α denotes the critical point of standard normal distribution with level α, and $\mu(\theta_0)$, $\tau(\theta_0)$ denotes the expected value and standard deviation of T_n under H_0, respectively. Also α is assumed to be less than $1/2$. Suppose that the postulated model is wrong and that under the true model $\sqrt{n}\big(T_n-\mu(\theta_0)\big)/\tau'(\theta_0)\to N(0{,}1)$ satisfies. If α_n' denotes the actual level of the test $\sqrt{n}\big(T_n-\mu(\theta_0)\big)/\tau(\theta_0)\ge z_\alpha$ then $\alpha'=\lim\alpha_n'$ exists and is given by $\alpha'=1-\Phi\big(z_\alpha\tau(\theta_0)/\tau'(\theta_0)\big)$. Therefore $\alpha'\le(\ge)\alpha$ as $\tau'(\theta_0)\le(\ge)\tau(\theta_0)$.

Definition 4.1. Let $\mathcal{F}$ be the postulated and $\mathcal{F}'$ be the true model. A test which has asymptotic level α under the model $\mathcal{F}$, is said to be *Conservative* if $\alpha'\le\alpha$, *Liberal* if $\alpha'\ge\alpha$ and *Robust* if $\alpha'=\alpha$ when $\theta=\theta_0$ under the model $\mathcal{F}'$.

In this section, robustness of the significance level for the hypothesis of the location parameter μ and the variance parameter σ^2 under three different contaminated models of normal distribution is investigated. For this purpose, the contaminated distributions with different parameter settings, N with $\tau=0.8$, 0.9, 1, 1.1 and 1.2, SCN with $(\alpha=1,\eta=0.45)$, $(\alpha=0.6,\eta=0.5)$, $(\alpha=0,\eta=1)$, $(\alpha=0.8,\eta=0.53)$, $(\alpha=0.01,\eta=1.2)$ and GT with $(q=2,\beta=2,\lambda=1.25)$, $(q=2.3,\beta=3,\lambda=1.5)$, $(q=2.5,\beta=5,\lambda=2)$, $(q=3,\beta=10,\lambda=2.5)$ will be considered.

4.1. Robustness of the Significance Level for the Hypothesis of Location Parameter

Consider the hypothesis test of the location parameter, say μ, under the normality assumption. If $X_1, X_2, \dots, X_n$ have normal distribution with the location μ and the variance σ^2, the rejection region at asymptotic level α to test $H_0: \mu = \mu_0$ versus $H_1: \mu > \mu_0$ is

$$\frac{\sqrt{n}\left(\bar{X} - \mu_0\right)}{\sigma} \geq z_\alpha \tag{12}$$

if σ is known, see [2, 5]. However, when the data set is not normally distributed in real, but if it is still assumed to be normally distributed, the true asymptotic level of the above rejection region, say α', will be different from α.

Let $X_1, X_2, \dots, X_n$ be independent and identically any distributed random variables with $E(X_1) = \mu$, $Var(X_1) = \kappa^2 < \infty$. In this situation,

$$\frac{\sqrt{n}\left(\bar{X} - \mu_0\right)}{\kappa} \rightarrow N(0,1) \tag{13}$$

holds and the true asymptotic level α' of the test is obtained with the aid of above theorem as

$$\alpha' = P_{H_0}\left(\frac{\sqrt{n}\left(\bar{X} - \mu_0\right)}{\kappa} \geq \frac{z_\alpha \sigma}{\kappa}\right) = 1 - \Phi\left(\frac{z_\alpha \sigma}{\kappa}\right) \tag{14}$$

where Φ is the cumulative distribution function of standard normal distribution.

In this subsection, when it is assumed that the random sample $X_1, X_2, \dots, X_n$ comes from normal distribution, the true level of the test (12) is investigated though $X_1, X_2, \dots, X_n$ have contaminated distribution in real.

4.1.1 C-N-N distribution case

Let $X_1, X_2, \dots, X_n$ have contaminated distribution as $F_X = \rho\, F_W + (1 - \rho) F_Y$, where $W \sim N(\mu, \sigma^2)$ and $Y \sim N(\mu, \tau^2)$. But assume that the random sample comes from $N(\mu, \sigma^2)$. In this situation when testing $H_0: \mu = \mu_0$ versus $H_1: \mu > \mu_0$ the true asymptotic level of the test (12) will be

$$\alpha' = 1 - \Phi\left(\frac{z_\alpha\, \sigma}{(\rho\sigma^2 + (1 - \rho)\tau^2)^{1/2}}\right). \tag{15}$$

The pdfs of true and postulated models and the behavior of α' are given in below figures. Green lines in all figures show the postulated model and the others show the true model with different parameters.

It should be noted that the parameters μ and σ^2 are chosen as 0 and 1, respectively, for the sake of simplicity. The parameters of N, SCN and GT distributions are used to produce the contaminated normal distributions with different values of proportion ρ which are given in Figures 1-6.

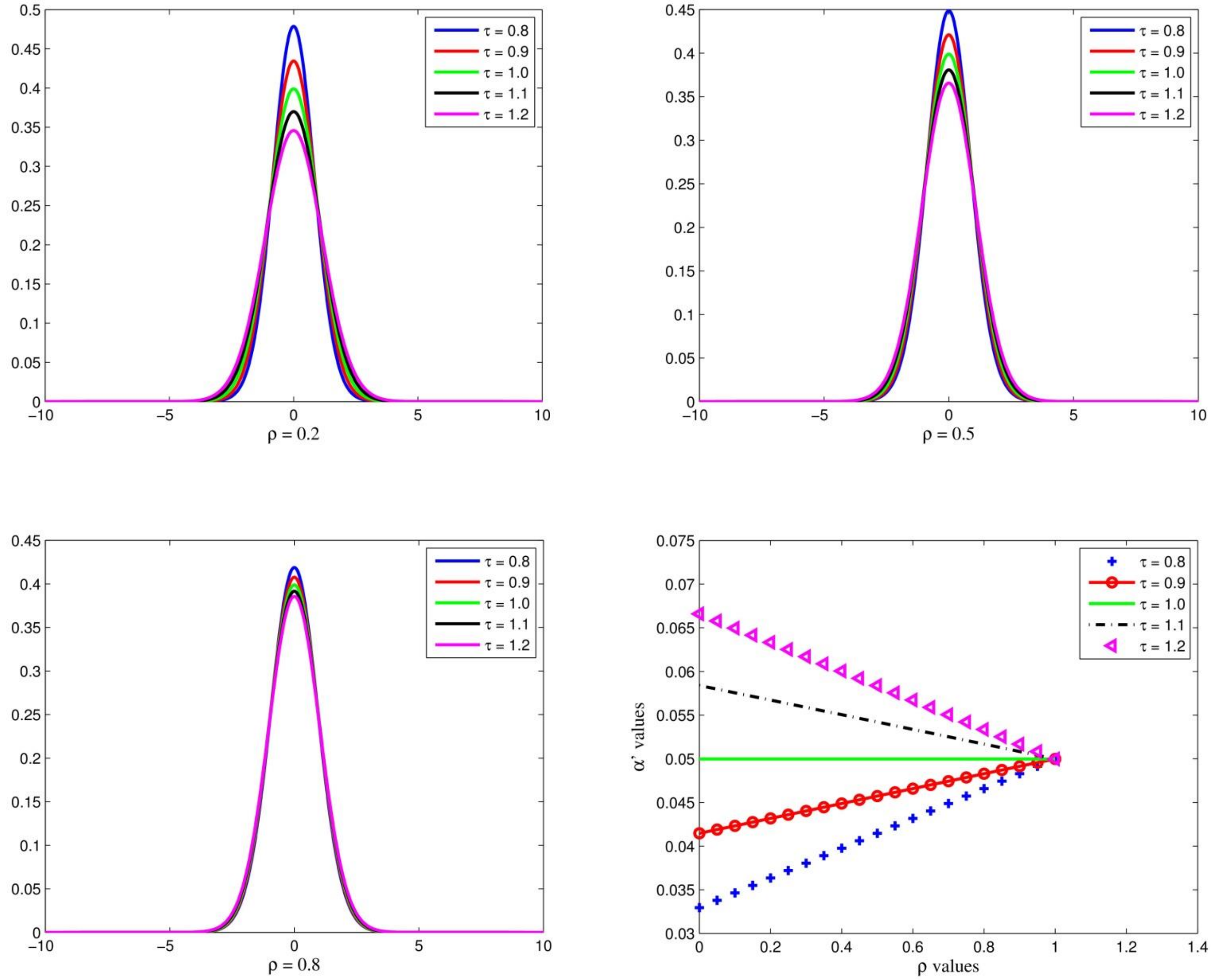

Figure 1. *True Level α' for Hypothesis of the Location Parameter at C-N-N Case.*

4.1.2 C-N-SCN distribution case

Let $X_1, X_2, \dots, X_n$ have contaminated distribution as $F_X = \rho\, F_W + (1-\rho)F_Y$, where $W \sim N(\mu, \sigma^2)$ and $Y \sim SCN(\mu, a, \eta)$. But assume that the random sample comes from $N(\mu, \sigma^2)$. In this situation when testing $H_0\!: \mu = \mu_0$ versus $H_1\!: \mu > \mu_0$, the true asymptotic level of the test (12) will be

$$\alpha' = 1 - \Phi\left(\frac{z_\alpha\, \sigma}{\left(\rho\sigma^2 + (1-\rho)\eta^2 \frac{1+3a}{1+a}\right)^{1/2}}\right). \tag{16}$$

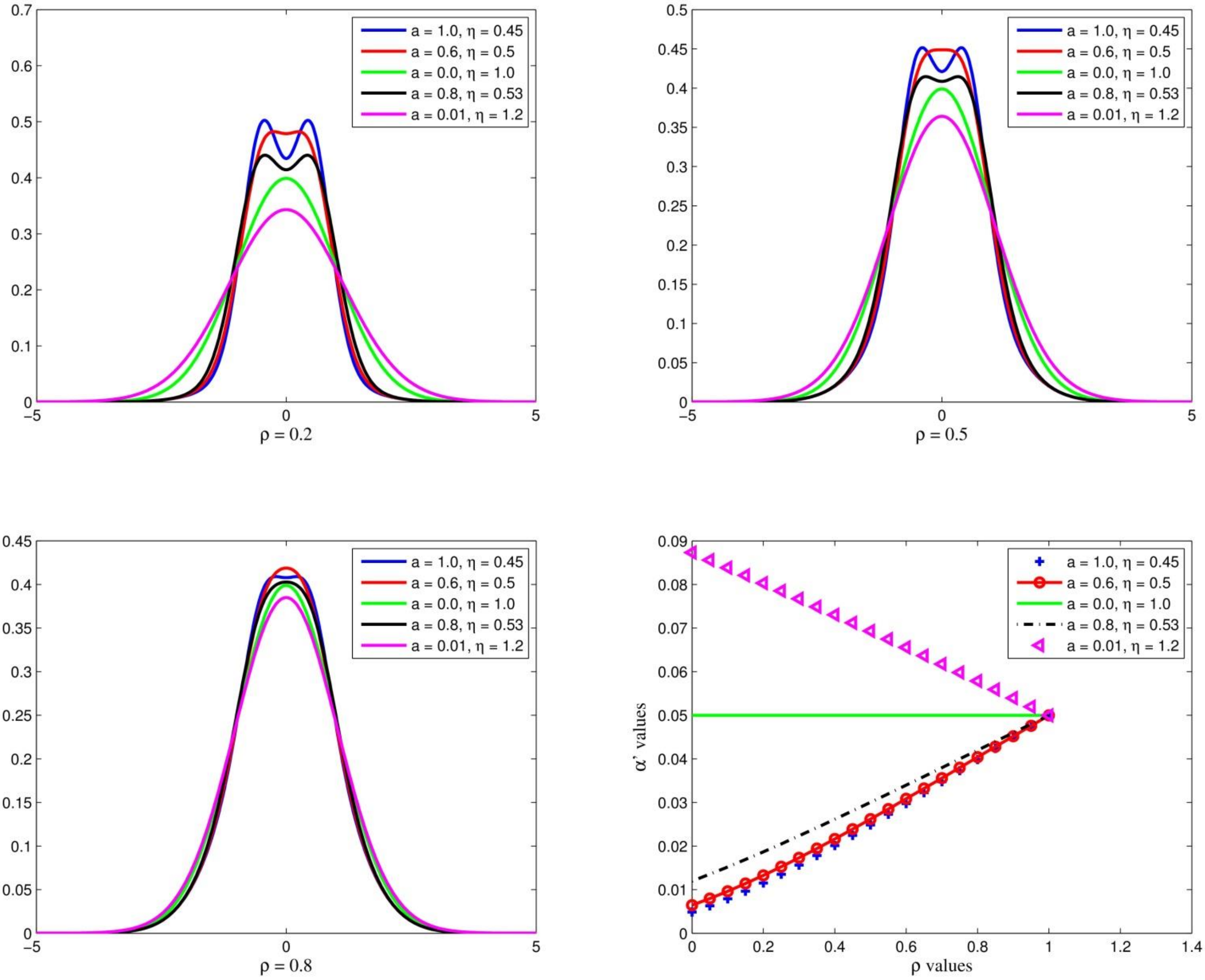

Figure 2. *True Level α' for Hypothesis of the Location Parameter at C-N-SCN Case.*

4.1.3 C-N-GT distribution case

Let $X_1, X_2, \dots, X_n$ have contaminated distribution as $F_X = \rho\, F_W + (1-\rho)F_Y$, where $W \sim N(\mu, \sigma^2)$ and $Y \sim GT(\mu, \delta, \lambda, \beta, q)$. But assume that the random sample comes from $N(\mu, \sigma^2)$. In this situation when testing $H_0\colon \mu = \mu_0$ versus $H_1\colon \mu > \mu_0$, the true asymptotic level of the test (12) will be

$$\alpha' = 1 - \Phi\left(\frac{z_\alpha\, \sigma}{(\rho\sigma^2 + (1-\rho)V_{GT}(Y))^{1/2}}\right) \tag{17}$$

where V_{GT} is the variance of random variable Y distributed as GT distribution.

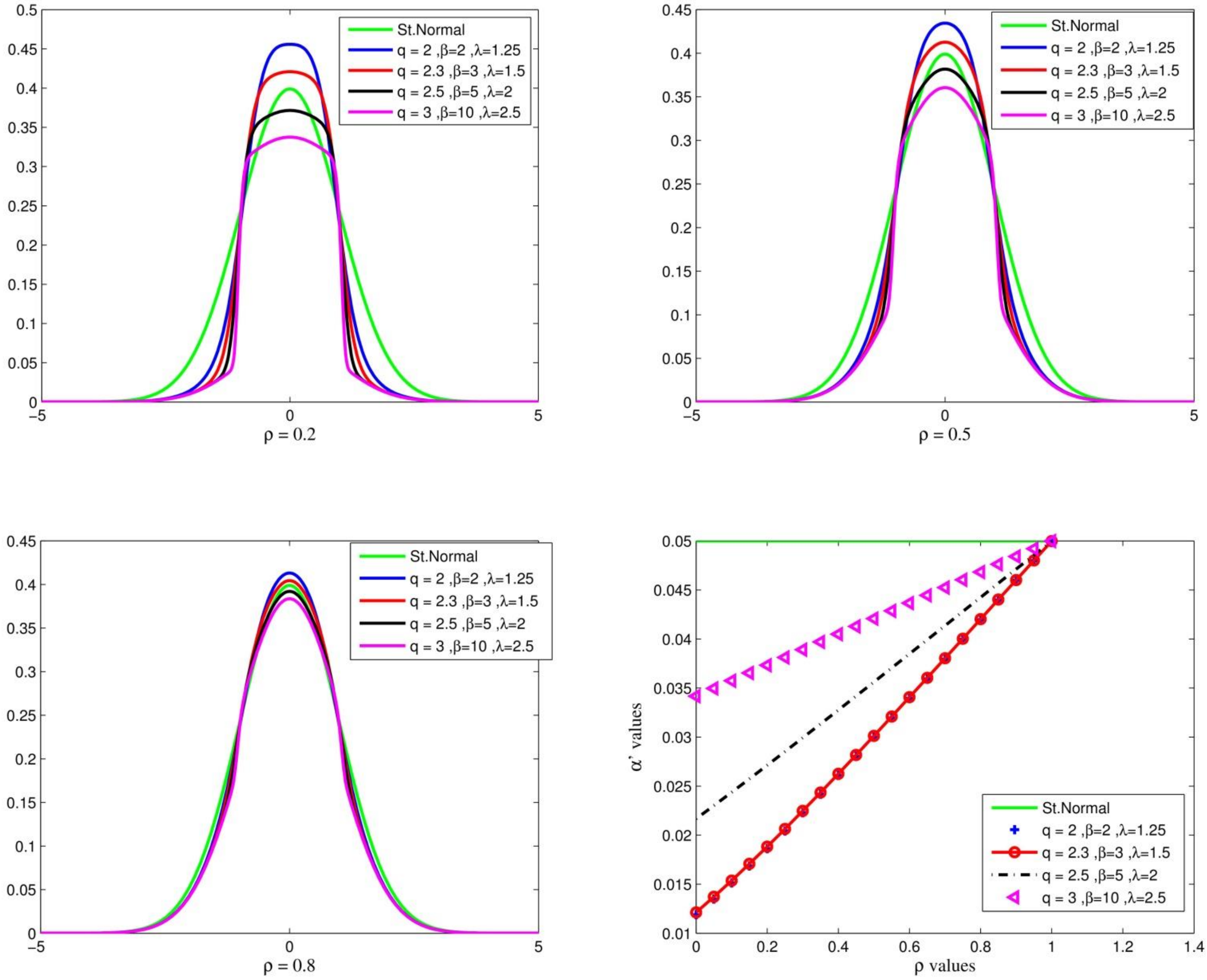

Figure 3. *True Level α' for Hypothesis of the Location Parameter at C-N-GT Case.*

4.2. Robustness of the Significance Level for Hypothesis of Variance Parameter

Consider the hypothesis test of the variance parameter, say σ^2, under the normality assumption. If $X_1, X_2, \ldots, X_n$ have normal distribution with the location μ and the variance σ^2, the rejection region at asymptotic level α to test $H_0\colon \sigma^2 = \sigma_0^2$ versus $H_1\colon \sigma^2 > \sigma_0^2$ is

$$\frac{\sqrt{n}\left(\frac{\sum (X_i - \bar{X})^2}{n} - \sigma_0^2\right)}{\sqrt{2}\sigma_0^2} \geq z_\alpha \tag{18}$$

see [2, 5].

However, when the data set is not normally distributed in real, but it is still assumed to be normally distributed, the true asymptotic level of the above rejection region, say α', will be different from α.

Let $X_1, X_2, \dots, X_n$ be independent and identically any distributed random variables with $Var(X_i) = \sigma^2$, $Var(X_i^2) = \zeta^2 < \infty$. In this situation,

$$\frac{\sqrt{n}\left(\frac{\sum(X_i-\bar{X})^2}{n} - \sigma_0^2\right)}{\zeta} \rightarrow N(0,1) \tag{19}$$

holds and the true asymptotic level α' of the test is obtained with the aid of above theorem as

$$\alpha' = P_{H_0}\left(\frac{\sqrt{n}\left(\frac{\sum(X_i-\bar{X})^2}{n} - \sigma_0^2\right)}{\zeta} \geq \frac{z_\alpha\sqrt{2}\sigma_0^2}{\zeta}\right) = 1 - \Phi\left(\frac{z_\alpha\sqrt{2}\sigma_0^2}{\zeta}\right). \tag{20}$$

In this subsection, when it is assumed that the random sample $X_1, X_2, \dots, X_n$ comes from normal distribution, the true level of the test (18) is investigated though $X_1, X_2, \dots, X_n$ have contaminated distribution in real.

4.2.1 C-N-N distribution case

Let $X_1, X_2, \dots, X_n$ have contaminated distribution as $F_X = \rho\, F_W + (1-\rho)F_Y$, where $W \sim N(\mu, \sigma^2)$ and $Y \sim N(\mu, \tau^2)$. But assume that the random sample comes from $N(\mu, \sigma^2)$. In this situation when testing $H_0: \sigma^2 = \sigma_0^2$ versus $H_1: \sigma^2 > \sigma_0^2$, the true asymptotic level of the test (18) will be

$$\alpha' = 1 - \Phi\left(\frac{z_\alpha\sqrt{2}\sigma_0^2}{\zeta}\right) \tag{21}$$

where

$$\zeta^2 = 4\mu^2(\rho{\sigma_0}^2 + (1-\rho)\tau^2) + 3(\rho{\sigma_0}^4 + (1-\rho)\tau^4) - (\rho{\sigma_0}^2 + (1-\rho)\tau^2)^2.$$

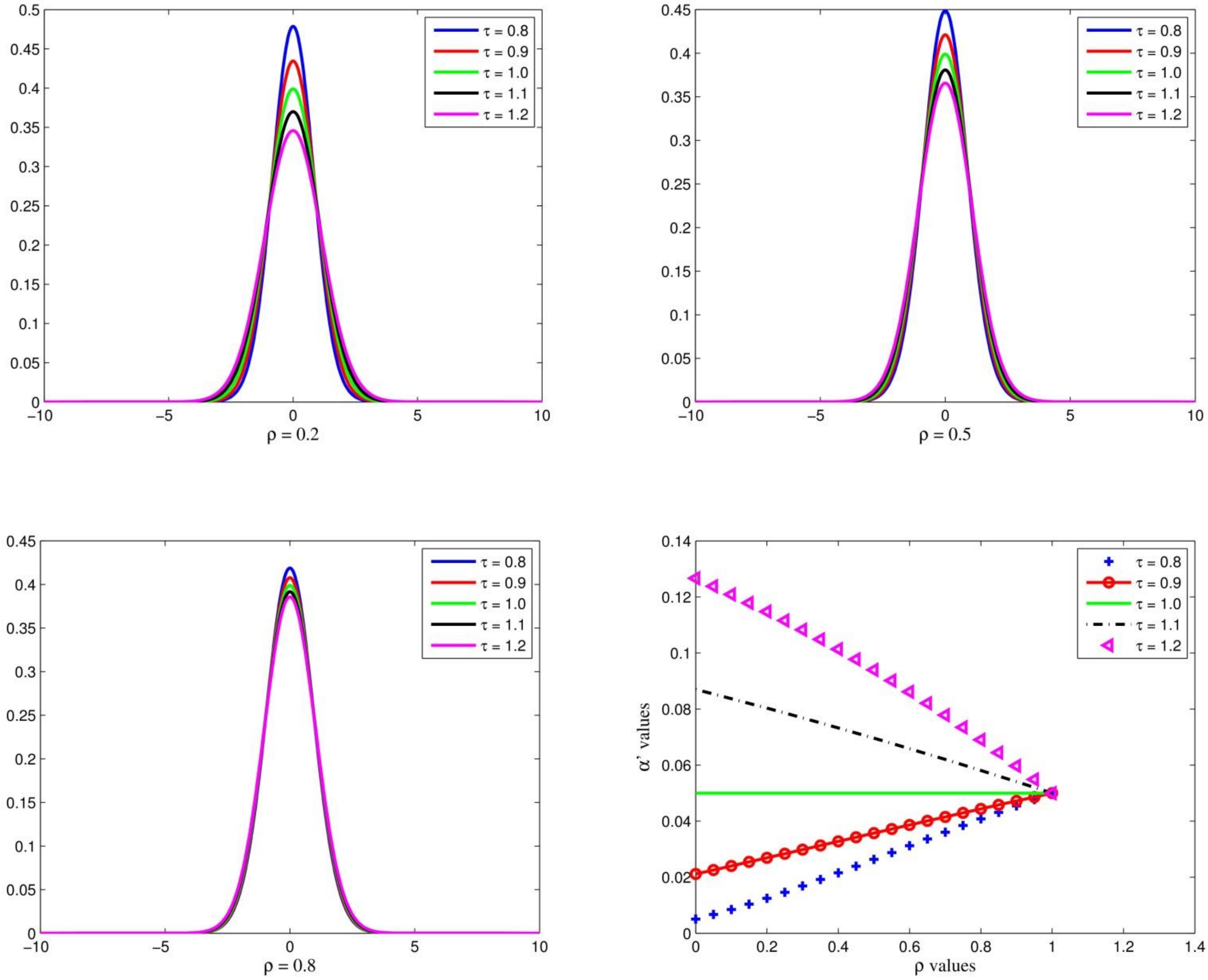

Figure 4. *True Level α' for Hypothesis of the Variance Parameter at C-N-N Case.*

4.2.2 C-N-SCN distribution case

Let $X_1, X_2, \dots, X_n$ have contaminated distribution as $F_X = \rho\, F_W + (1-\rho)F_Y$, where $W \sim N(\mu, \sigma^2)$ and $Y \sim SCN(\mu, a, \eta)$. But assume that the random sample comes from $N(\mu, \sigma^2)$. In this situation when testing $H_0: \sigma^2 = \sigma_0^2$ versus $H_1: \sigma^2 > \sigma_0^2$, the true asymptotic level of the test (18) will be

$$\alpha' = 1 - \Phi\left(\frac{z_\alpha \sqrt{2}\sigma_0^2}{\zeta}\right) \tag{22}$$

where

$$\zeta^2 = 4\mu^2\left(\rho\sigma_0{}^2 + (1-\rho)\eta^2\frac{1+3a}{1+a}\right) + 3\left(\rho\sigma_0{}^4 + (1-\rho)\eta^4\frac{1+5a}{1+a}\right) - \left(\rho\sigma_0{}^2 + (1-\rho)\eta^2\frac{1+3a}{1+a}\right)^2.$$

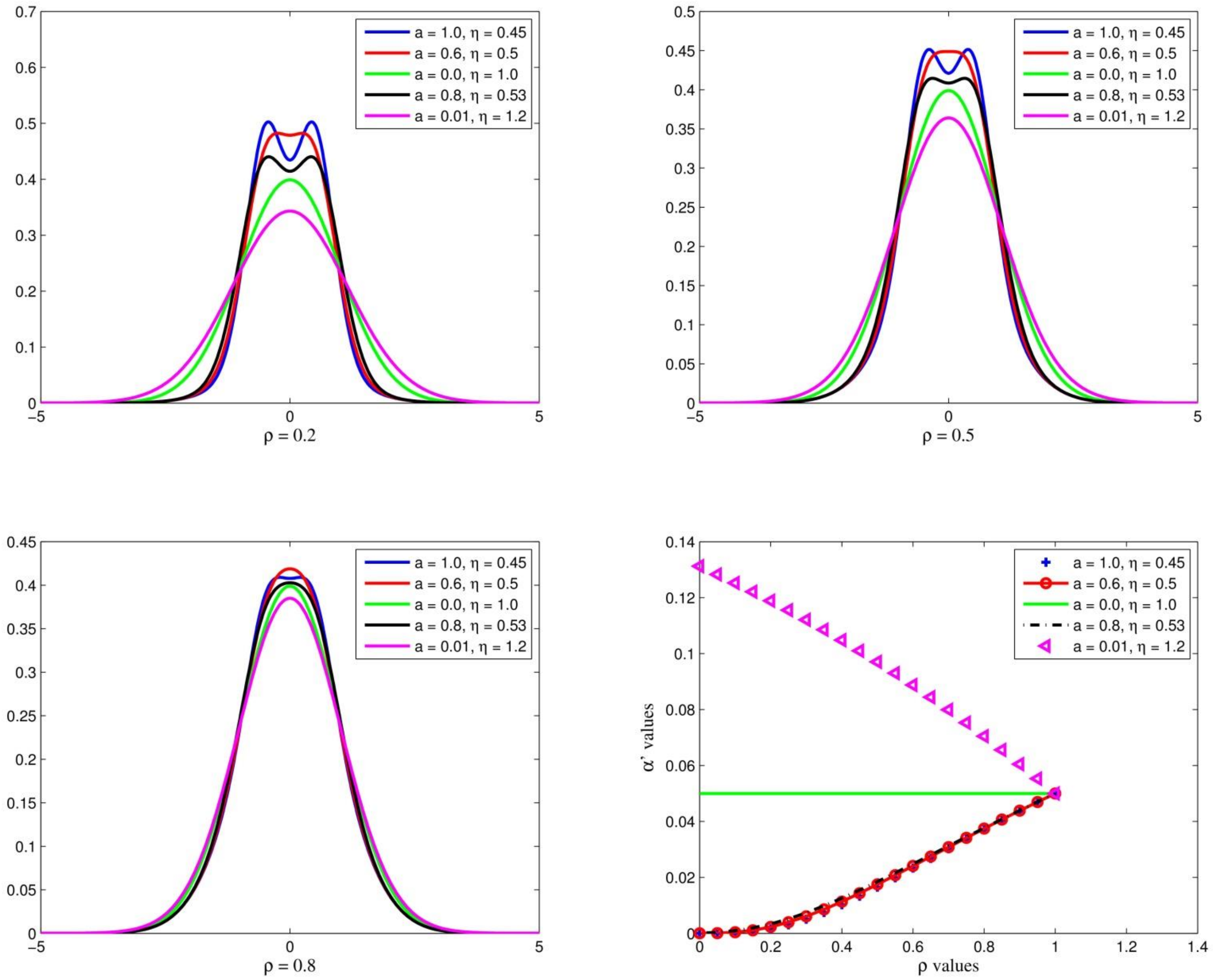

Figure 5. *True Level α' for Hypothesis of the Variance Parameter at C-N-SCN Case.*

4.2.3 C-N-GT distribution case

Let $X_1, X_2, \ldots, X_n$ have contaminated distribution as $F_X = \rho\, F_W + (1-\rho)F_Y$, where $W \sim N(\mu, \sigma^2)$ and $Y \sim GT(\mu, \delta, \lambda, \beta, q)$. But assume that the random sample comes from $N(\mu, \sigma^2)$. In this situation when testing $H_0\!: \sigma^2 = \sigma_0^2$ versus $H_1\!: \sigma^2 > \sigma_0^2$, the true asymptotic level of the test (18) will be

$$\alpha' = 1 - \Phi\left(\frac{z_\alpha \sqrt{2}\sigma_0^2}{\zeta}\right) \tag{23}$$

where

$$\zeta^2 = (1-\rho)CV_{GT}(Y) + 4\mu^2[\rho\sigma_0^2 + (1-\rho)V_{GT}(Y)] + 3[\rho\sigma_0^4 + (1-\rho)V_{GT}^2(Y)] - [\rho\sigma_0^2 + (1-\rho)V_{GT}(Y)]^2.$$

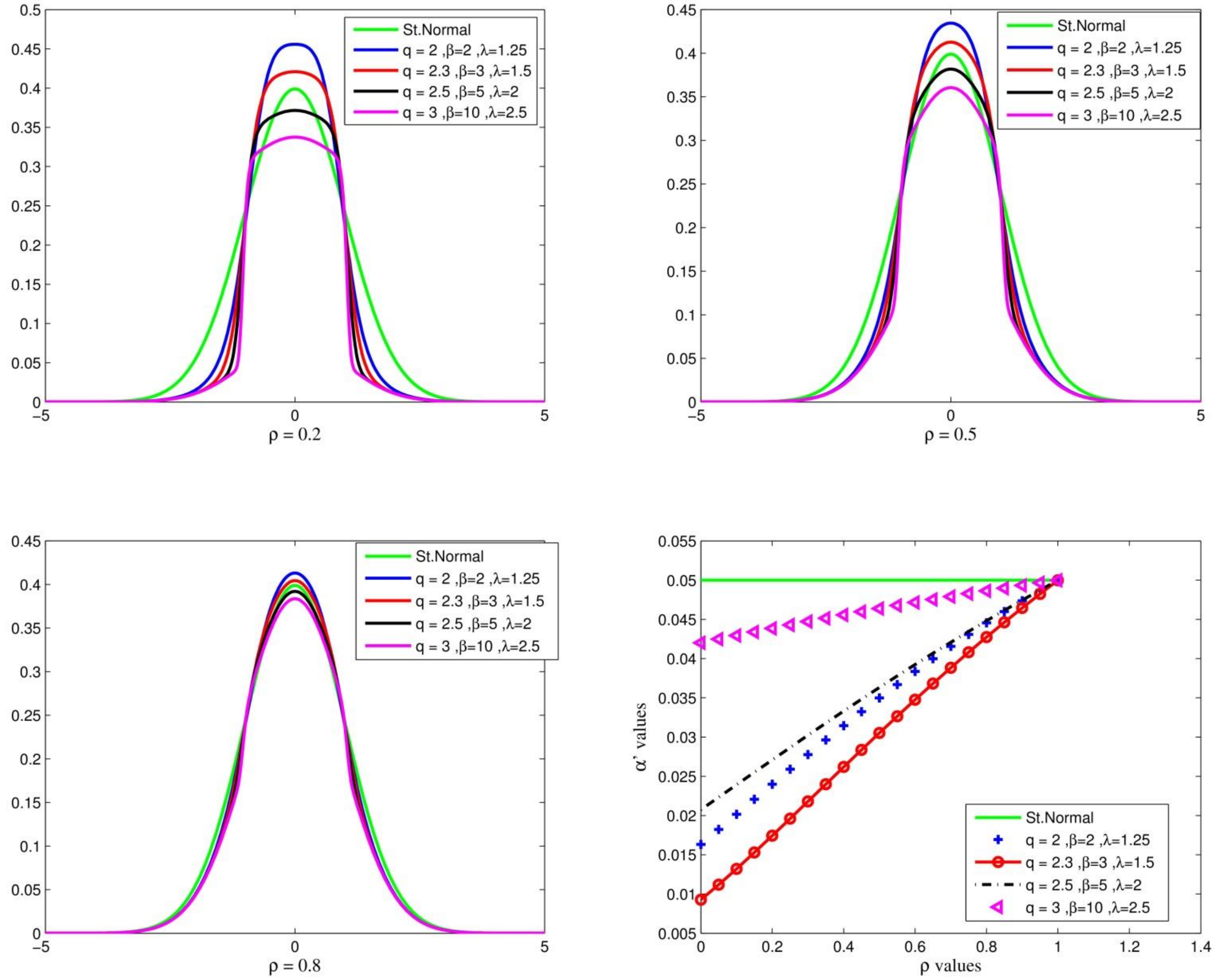

Figure 6. *True Level α' for Hypothesis of the Variance Parameter at C-N-GT Case.*

5. RESULTS AND CONCLUSIONS

This study has focused on the impact of the wrong model selection upon significance level in hypothesis tests. For this purpose, the normal distribution used frequently by researchers and some contaminated distributions which have similar form with normal are considered. The normal distribution is chosen as postulated model despite contaminated distributions are accepted as true model.

It is seen from Figures 1-6 for the contaminated distributions considered here, the true asymptotic levels of the test (12) and (18) for location and variance parameters are less or greater than α but not equal in general. This situation changes according to the standard deviation of true and postulated models as given in Theorem 4.1. So, if standard deviation of the postulated model is less (greater) than that of true model, the true asymptotic level α' of the test will be greater (less) than α. It can be concluded from the figures that if the values of scale parameters τ for *C-N-N* case and η for *C-N-SCN* case are smaller than 1 the tests are conservative, if the values are equal to 1 the tests are robust, and if the values are bigger than 1 the tests are liberal. However, in *C-N-GT* distribution case the tests are conservative for every selected values of the parameters.

As a conclusion, these tests are non-robust against violations of the normality assumptions, as emphasized by [5]. This shows that differentiation from normal distribution impacts the significance level even in asymptotic case.

ACKNOWLEDGEMENT

We thank to an anonymous referee for his/her valuable comments.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

REFERENCES

[1] Arslan O. (2004). Family of multivariate generalized t distribution. Journal of Multivariate Analysis, 89, 329-337.

[2] Casella G., Berger R.L. (2002). Statistical Inference (2nd ed.). Pacific Grove CA, Duxbury.

[3] Gomez E., Gomez-Villegas M.A., Marin J.M. (1998). A multivariate generalization of the power exponential family of distributions. Communication in Statistics: Theory and Methods, 27(3), 589-600.

[4] Gui W., Chen P.H., Wu H. (2013). A symmetric component alpha normal slash distribution. Journal of Statistical Theory and Applications, 12(1), 55-66.

[5] Lehmann E.L. (1999). Elements of Large-Sample Theory. New York, Springer-Verlag.

[6] Maronna, R., Martin, D., Yohai V. (2006). Robust Statistics: Theory and Methods. John Wiley & Sons, Chichester.