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# A Stocking diagram for silvicultural implications in

# Scots Pine (Pinus sylvestris L.) stands

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#### Abstract

*Aim of study*: Residual stand densities are commonly defined using basal area ( $m^2 ha^{-1}$ ) (BA) and number of trees per hectare (TPH) when using even-aged silvicultural methods such as clearcutting and shelterwood methods in Scots pine (*Pinus sylvestris* L.) forests. However, it has been stated that absolute density measures such as BA and TPH are not the most useful indexes because growing space at a given BA or TPH varies with average tree diameter. Therefore, silvicultural management tools such as stocking charts (SC) and density management diagrams (DMD) have seemed to be more useful when allocating growing space to achieve a broad range of silvicultural objectives in even-aged stands. Given the importance of stand density on the regeneration success and growth of Scots pine, and given the shortages of absolute density measures (i.e. BA and TPH), a density diagram would be a useful tool for this tree species as well. Thus, in this study, a stand stocking diagram (SD) was developed for Scots pine stands.

*Material and Methods*: Two published equations developed from open grown Scots pine trees, and from normal Scots pine stands were utilized to determine the minimum density of full site occupancy and the average maximum density, respectively. The form of the diagram follows Gingrich stocking chart.

*Main results*: The A-line on the diagram represents the average maximum density where trees, on average, have the minimum growing space needed to survive. The B-line represents the lowest density where canopy closure can occur and is the minimum stocking necessary for full site occupancy or canopy closure. Consistent with a published Scots pine DMD, there is biological relevance and utility to the stocking diagram created in this study.

*Highlights:* With the stocking diagram presented in this paper, regeneration and tending applications will be more practical in Scots pine forests because available growing space will be well-determined and utilized using the diagram.

Keywords: Even-aged, Management tools, Silviculture, Stocking diagram

## Sarıçam (Pinus sylvestris L.) ormanlarındaki silvikültürel uygulamalar

# için yapılmış bir meşcere sıklık diyagramı

#### Özet

*Çalışmanın amacı:* Sarıçam (*Pinus sylvestris* L.) ormanlarında tıraşlama ve siper-altı gibi aynı yaşlı silvikültürel yöntemler kullanıldığında, nihai meşcere sıklığı genellikle göğüs yüzeyi alanı (m<sup>2</sup> ha<sup>-1</sup>) (BA) ve hektardaki ağaç sayısı (TPH) gibi mutlak sıklık ölçü birimleri kullanılarak ifade edilmektedir. Fakat belli bir BA ve TPH'daki sıkışıklık ortalama ağaç çapına göre değişiklik göstereceğinden, BA ve TPH'nın kullanımında bazı sakıncalar olduğu belirtilmiştir. Bu sebeple, aynı yaşlı ormanlarda belirlenen silvikültürel amaçlara ulaşmak maksadıyla stok çizelgesi (SC) ve sıklık idare diyagramı (DMD) gibi silvikültürel araçların çok daha faydalı olduğu belirtilmiştir. Meşcere sıklığının sarıçamın gençleştirilmesi ve büyümesi üzerine olan etkisi göz önüne alındığında, ayrıca BA ve TPH gibi mutlak sıklık ölçü birimlerinin eksiklikleri dikkate alındığında, sıklık diyagramlarının bu tür için de faydalı olacağı düşünülmektedir. Bu sebeple, bu çalışmada sarıçam ormanları için bir meşcere stok diyagramı (SD) oluşturulmuştur.

*Materyal ve Yöntem:* Kapalılığın oluştuğu minimum sıklığı ve ortalama maksimum sıklığı belirlemek amacıyla, serbest büyüyen sarıçam ağaçları ve normal sarıçam meşcereleri kullanılarak geliştirilen iki adet formül kullanılmıştır. Bu diyagram Gingrich'in stok diyagramını temel alınılarak oluşturulmuştur.

*Sonuçlar*: Diyagram üzerindeki A-line meşcerenin ortalama maksimum sıklığını gösterirken, B-line kapalılığın oluştuğu minimum sıklık derecesini göstermektedir. Bu çalışmada oluşturulan SD ile mevcut DMD arasındaki tutarlılık, elde edilen SD'nin biyolojik uygunluk ve kullanışlılığını göstermektedir.

*Araştırma Vurguları*: Bu diyagram yardımıyla, daha başarılı ve pratik sarıçam gençleştirme ve bakım müdahalelerinin yapılabilmesi amaçlanmıştır. Çünkü bu diyagram mevcut meşcere sıkışıklığının daha doğru şekilde belirlenmesi ve kullanılmasına imkân vermektedir.

Anahtar kelimeler: Eşit-yaşlı, Silvikültürel araçlar, Silvikültür, Stok di	/agrami
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### Introduction

Scots pine (Pinus sylvestris L.) is one of the main native and economically important forest tree species in Turkish forestry due to its high quality timber (Bozkurt, 1971; Boydak et al., 2011; Yurtseven, 2017). Scots pine has a wide area in Turkey (Gezer et al., 2002), making up 7% (1.5 million ha) of the total forested land in the country forming pure and mixed stands with other tree species (Anonymous, 2014). This pine is among the most planted trees throughout Turkey (Oner, 2003). In addition, Scots pine forests constitute 20% of total forested area in Europe (Mason and Alia, 2000), this species also plays important ecological roles in several parts of Europe as well (Bialobok, 1975).

Stand density is an important ecological feature of forest structure (Sprintsin et al., 2009) because establishment, diameter growth, productivity and wood quality of trees are all affected by it (Zeide, 2005), as are light regime beneath the canopy, respiration, evapotranspiration, and water consumption (Sprintsin et al., 2009). Manipulation of stand density is a powerful tool for forest managers to meet specific silvicultural objectives (Farnden, 1996). Stand density manipulation is crucial in Scots pine forests because stand density affects regeneration success, seedling growth, cone production as well as tree growth in these forests (Odabaşı et al., 2004). Thus, stand density management and growing space allocation play an important role in effective management of Scots pine forests (Odabası et al., 2004).

Scots pine is a shade intolerant forest tree species (Richardson, 1998) and usually even-aged silvicultural managed using clearcutting methods such as and shelterwood which require relatively low stand densities. In these even-aged methods, the target stand densities are usually described by basal area (m<sup>2</sup> ha<sup>-1</sup>) (BA) or number of trees per hectare (TPH) (Odabaşı et al., 2004). However, it has been suggested that growing space at a given BA or TPH varies with average tree diameter; stands with a larger average tree diameter have more growing space than stands with smaller average tree diameter at a given BA

(Gingrich 1967; Goelz, 1995; Martin, 1996). On the other hand, relative stand density measures such as percent stocking and stand index (SDI) improve density can management efforts when allocating growing space to achieve silvicultural objectives (Reineke, 1933; Gingrich, 1967; Kara, 2016). Thus, it has been recommended that silvicultural management tools are effective methods in planning stand density management (Farnden, 1996; Martin, 1996).

Silvicultural management tools such as density management diagram (DMD) (Jack and Long, 1996) and stocking charts (Gingrich, 1967) graphically depict the relationships among stand density, average tree diameter (cm), mean tree volume (m<sup>3</sup> ha<sup>-</sup> <sup>1</sup>), basal area  $(m^2 ha^{-1})$  and tree height (m) in even-aged stands. Given the importance of stand density in the regeneration success and growth of Scots pine, and given the shortages of absolute density measures (i.e. BA and TPH), a stocking diagram would be a useful and alternative silvicultural tool for this tree species. Therefore, in this study, a stocking diagram was developed for Scots pine forests the published equations using and approaches in the literature. With this stocking diagram, regeneration and tending activities would be more practical and effective in Scots pine forests.

#### **Materials and Methods**

The stocking diagram formation used in this paper is similar to that proposed by Gingrich (1967). To develop a Gingrich style stocking diagram, two key reference curves need to be defined. The first reference curve is the B-line stocking line which refers to the onset of canopy closure (Larsen et al., 2010). The second reference curve is the A-line stocking line which refers to the average maximum density ( $D_{AM}$ ) (Larsen et al., 2010).

#### **B-line** stocking level

The B-line reference curve represents the minimum number of trees per hectare ( $M_{TPH}$ ) for the onset of canopy closure (Krajicek et al., 1961). In order to determine the onset of canopy closure, the maximum tree area that a tree can occupy under open-grown conditions is used (Krajicek et al., 1961).

Diameter at breast height (DBH) (cm) and crown width (CW) (m) of an open-grown tree are measured to calculate the maximum tree area. Because the crown of a tree will occupy the maximum area allocated to it up to the species' specific physiological limit, the measurement of crown width is the best predictor of the number of trees per unit area (Yang and Titus, 2002). In addition, the correlation between crown width and diameter is known to be very strong (Zeide, 1987), and their measurements are less subject to measurement errors (Yang and Titus, 2002).

In order to fit B-line reference curve, a published equation (I) developed by Hasenauer (1997) was utilized. Hasenauer (1997) measured DBH and CW of opengrown Scots pine trees (Table 1) in Austria, and determined the relationships between the two variables. Trees were located at elevations ranging from 200 to 1400 m. DBH ranged from 1.4 to 76 cm while it was between 1 and 13 m for CW (Hasenauer, 1997). Dominant soil types were carbonate and silicate while slope ranged from 0 to 125% within the areas where open-grown trees were measured (Hasenauer, 1997). Open-grown species were within dry to wet climatic regions (Hasenauer, 1997). It should be noted that age and site quality have negligible influence on the relationship between DBH and CW (Chisman and Schumacher, 1940; Gingrich, 1967).

$$\log(CW) = b_0 + b_1 \log(DBH)$$
(I)

Using formula 1 for each 5-cm DBH class (ranging from 15 to 55 cm), a CW was calculated in meters. Next, using formula (II), the  $M_{TPH}$  was determined for each CW value.

$$M_{\rm TPH} = \frac{10000}{CW_{\rm max^2} \,(\Pi/4)}$$
 (II)

Then, BA  $(m^2 ha^{-1})$  for each M<sub>TPH</sub> was calculated using formula (III).

$$BA = QMD^2 (0.00007854)M_{TPH}$$
 (III)

## A-line stocking level

The A-line reference curve represents the minimum growing space required for a tree to survive under fully-stocked conditions (Johnson et al., 2009). In order to determine

the  $D_{AM}$ , the most fully-stocked plots are used. It has been suggested that the relationship between TPH and quadratic mean diameter (QMD) is linear on a log-log scale (Reineke, 1933). QMD is defined as the diameter of the tree with mean BA, and has a better correlation to stand volume than DBH.

In order to fit A-line stocking level, the equation developed by Pretzsch and Biber (2005) was utilized. Pretzsch and Biber (2005) determined the relationship between TPH and QMD from fully-stocked Scots pine stands (Table 1) in Germany. Plots were located on meager Jurassic and cretaceous sites, and size of the stands ranged from 0.25 to 0.3 ha (Pretzsch and Biber, 2005). Tree ages were between 26-138 while TPH ranged from 358 to 5100 trees, and BA ranged from 16 to 45 m<sup>2</sup> ha<sup>-1</sup> across the study plots Pretzsch and Biber (2005).

In forestry studies, fitting a line through a data set using regression techniques is a common means of analysis for prediction of one variable when another is known (Leduc 1987). Pretzsch and Biber (2005) used ordinary least square (OLS) regression to determine the relationship between TPH and QMD from fully-stocked Scots pine stands. Their slope (-1.550) of the relationship between TPH and QMD was close to Reineke's (1933) universal slope of -1605. This published formula (IV) from Pretzsch and Biber (2005) was used to fit the A-line reference curve. D<sub>AM</sub> for each 5-cm DBH class (ranging from 15 to 55 cm) was calculated using formula 4 from Pretzsch and Biber (2005) (Table 1).

$$log(D_{AM}) = b_0 + b_1 (log(QMD)) \quad (IV)$$

Next, BA for each 5-cm DBH class was calculated using formula (V).

$$BA = QMD^2 (0.00007854) D_{AM}$$
 (V)

## Composition of the stocking diagram

Following the calculation of BA values of  $M_{TPH}$  and  $D_{AM}$  for each 5-cm DBH class, Bline and A-line reference levels were drawn on the same diagram using the  $M_{TPH}$  and  $D_{AM}$  for average stand diameters between 15 and 55 cm. SigmaPlot 12.0 statistical software was used to draw the reference curve.

Fable	1.	Parameter	estimates	and	tıt	stat1st1c	of	A-line	stocking	"log( $N_{AMax}$ )	$= b_0$	+
b <sub>1</sub> log	(Q)	MD)" and B	-line stocki	ng le	vels	"log(CW	, max	$(x) = b_0 - b_0$	$+ b_1 \log(1)$	DBH)"		

Stocking level	Source of data	Ν	Slope	Intercept	$\mathbb{R}^2$		
A-line	Pretzsch and Biber,	9 <sup>a</sup>	-1.550	11.508	0.96		
	2005						
B-line	Hasenauer, 1997	53 <sup>b</sup>	0.259	0.113	0.96		
*a refers to number of plots while b refers to number of trees.							

**Results and Discussion** 

The stocking diagram for Scots pine is presented in Figure 1. The A-line on the minimum growing space needed to survive. A-line on the Scots pine's stocking diagram ranges from 30 to 56 m<sup>2</sup> ha<sup>-1</sup> of BA (Figure 1). The stocking levels below the A-line reference curve were determined as a proportional of A-line.

The B-line represents the lowest density where canopy closure can occur and is the minimum stocking necessary for full site occupancy or canopy closure. B-line on the Scots pine's stocking diagram ranges from 11 to 29 m<sup>2</sup> ha<sup>-1</sup> of BA (Figure 1). Although the B-line curves for other species have been closely parallel to a given stocking level (Gingrich, 1967; Larsen et al., 2010), the Bline reference curve of Scots pine is getting steeper with the increasing tree diameter. This suggests that the relationship between DBH and CW of open-grown Scots pine trees is not linear (Hasenauer, 1997). In another words, the rate of the increase in CW declines with the larger DBH trees which makes the B-line curve steeper.

The stocking diagram illustrates relationships between BA, TPH and average tree diameter. The diagram can be used to determine average growing space of a Scots pine stand based on any two of these three measurements (i.e. BA, TPH and DBH). For example, a stand with an average DBH of 20 cm and TPH of 700 should have a BA of 20  $m^2$  ha<sup>-1</sup> and stocking percent of 65 (Figure 1). In general terms, stands falling above the Aline reference level are considered overstocked (Gingrich, 1967), and they tend toward the A-line as additional growth and density dependent mortality occur. Stands falling anywhere between the A-line and Bline are considered fully-stocked which diagram represents the average maximum density where trees, on average, have the means that all of the available growing space is being utilized (Johnson et al., 2009). Stands below **B**-line stocking are understocked (Gingrich, 1967). In addition, Williams (2003) noted that density-induced mortality begins at 70% stocking. Above this stocking level, individual tree growth is slower. The growing space that becomes available following density-induced mortality is occupied by the neighboring trees.

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Figure 1. Stand stocking diagram for Scots pine forests. The blue colored stocking level curve represents the average maximum density while the red colored one shows the minimum density of full site occupancy.

BA alone is a commonly used measure of density when allocating growing space in even-aged stands of intolerant species. However, as stated before, growing space at a given BA varies with average tree diameter; stands with a larger average tree diameter represent more growing space than stands with smaller average tree diameter (Gingrich, 1967; Goelz, 1995; Martin, 1996). Using figure 2 as an example, stand X has a basal area of 22  $m^2 ha^{-1}$  with average tree diameter of 15 cm. Stand Y has same basal area (i.e. 22  $m^2 ha^{-1}$ ), but, its average tree diameter is 45 cm. A forest manager may

treat these two stands using the same silvicultural prescriptions because he/she may think that these stands have the same stand densities (i.e. same BA). However, using the diagram created for Scots pine, we can easily determine that stocking of stand X is 75% while it is 45% for stand Y (Figure 2). In other words, density-related mortality may start in stand X as suggested by Williams (2003), while stand Y has the maximum growing space available because it is below the B-line (Figure 2). Consequently, the amount of available growing space, intertree competition, cone production, seedling growth, micro-climate conditions, litter decomposition etc. will be different for stand X and stand Y. Therefore, it seems that stocking diagram can improve management efforts when allocating growing space through silvicultural manipulation of a forest stand. On the diagram, curved stocking lines may span a substantial range of BAs, especially as stocking percentage increases, thus, the diagram should be very useful when allocating growing space to manage a trait that is tied to some specific level of stocking.

During natural regeneration of Scots pine stands, canopy closure should be reduced to 60-80% with the preparation cut (if necessary), and to 40-60% for the seeding cut (Odabasi et al., 2004). Considering that the B-line reference curve refers to canopy closure on the Scots pine's stocking diagram created in this paper, density should be reduced to below the B-line reference curve when a preparation cut or a seeding cut is applied. For example; the stand X in the previous example had a basal area of 22 m<sup>2</sup> ha<sup>-1</sup> with average tree diameter of 15 cm (Figure 2). Based on the diagram, this stand has approximately 1250 trees per hectare. and 75% stocking. If regenerating stand X, TPH should be reduced to at least 600 (or BA should be reduced to  $10 \text{ m}^2 \text{ ha}^{-1}$ ) to fall below the B-line (i.e. canopy closure).

Vacchiano et al. (2008) generated a DMD for Scots pine forests. They stated that crown closure is indicated by 25% stand density index (SDI) on their DMD (Vacchiano et al., 2008). The B-line in the stocking diagram created in this study coincides with their 25% SDI. In addition, Vacchiano et al. (2008) stated that self-thinning starts at 65% SDI on their DMD. Their self-thinning line also coincides with the A-line reference on the stocking diagram created in this study. Consistent with the published Scots pine DMD (Vacchiano et al., 2008), there is biological relevance and utility to the stocking diagram created in this study. Unlike a stocking diagram, a DMD does not include an empirically based estimate of the minimum density of full site occupancy (i.e., B-line) (Johnson, 2009). In addition, a DMD does not depict basal area. Thus, omission of BA which is one of the most commonly used stand density measures in forestry can be considered one of the disadvantages of the DMDs (Farnden, 1996). However, the diagram is intended to stocking he complementary to the DMD developed for Scots pine (Vacchiano et al., 2008) and enhances the understanding of growing space relationships and its potential implications in regeneration and recruitment.

The dataset utilized to develop the equations cited (i.e. Hasenauer, 1997; Pretzsch and Biber, 2005) were collected in Germany and Austria which may represent different ecological conditions than Turkey. However, as it has been stated above, site characteristics and site quality have negligible or no influence on the size-density relationships and the relationship between DBH and CW (Chisman and Schumacher, 1940; Gingrich, 1967). Therefore, the use of equations developed using dataset collected outside Turkey would not hinder the applicability of the diagram for Scots pine forests within Turkey. The Scots pine stocking diagram created in this study should be successful in all Scots pine forests across its natural range. In addition, however, the data utilized to develop size-density and DBH-CW relationships were collected at similar elevations that Scots pine forests cover Turkey. in



Figure 2. An example of different stand stocking for Scots pine stands of same BA. Stand X and stand Y has a BA of 22 m<sup>2</sup> ha<sup>-1</sup>, however, their stand stockings are 45% and 75%, respectively.

#### Conclusions

A stocking diagram was developed for Scots pine forests. A stocking diagram was developed for Scots pine forests. Two published equations developed from open grown Scots pine trees (Hasenauer, 1997), and from normal Scots pine stands (Pretzsch and Biber, 2005) were used to determine the minimum density of full site occupancy and the average maximum density, respectively. In practice, foresters commonly use BA when describing residual stand density since it is a commonly used and understood measure. However, growing space at a given BA varies with average tree diameter. The stocking diagram created in this paper substantiate this assumption. Due to its ease of use, this diagram will be a handy tool for managing Scots pine forests within their natural range. The diagram can be used to determine average growing space of a stand based on any two of three measurements; BA, TPH and average tree diameter. As a result, growing space will be fully and more effectively used to achieve specific objectives including regeneration, timber production, thinning, and wildlife purposes.

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