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Comparing the Performance of Stochastic Loss Reserving Methods for Different Loss Severity Distributions

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Abstract

In this paper, stochastic loss reserving methods are compared using the mean absolute percentage error criterion. For this purpose, loss development squares (upper-left development triangles and lower-right goal triangles) are simulated using individual losses with changing severity under the scenarios where various distributional assumptions are made. In this loss square generating method, the distribution and the development process of a loss are considered from its occurrence to the closure based on the reporting and settlement delays. After simulating the loss development squares, loss reserves are estimated with the inflation-adjusted chain-ladder method and three different logarithmic regression models. The performance of the loss reserving methods is examined by comparing the estimated and actual loss reserves for different distributions commonly used for individual loss amount modeling in non-life insurance. It is seen that logarithmic regression models generally perform better than chain-ladder method for the specified scenarios.

Keywords: Loss development triangle, changing loss severity, inflation-adjusted loss, chain-ladder method, logarithmic regression models.

Öz

Farklı Hasar Şiddeti Dağılımları için Stokastik Hasar Rezerv Yöntemlerinin Tahmin Performansı

Bu çalışmada, ortalama mutlak yüzde hata kriteri ile stokastik hasar rezerv yöntemleri karşılaştırılmıştır. Bu amaçla, değişen tutarlı bireysel hasarlar kullanılarak çeşitli dağılım varsayımları yapılan senaryolar altında hasar gelişim kareleri (sol-üst gelişim üçgenleri ve sağ-alt hedef üçgenler) benzetimle üretilmiştir. Bu hasar karesi üretme yönteminde, bir hasarın dağılımı ile bildirilme ve kapatılma gecikmelerine bağlı olarak bir hasarın meydana gelmesinden kapatılmasına kadar geçen süredeki gelişim süreci ele alınır. Hasar gelişim karelerinin benzetim ile üretilmesinden sonra, hasar rezervleri enflasyon-düzeltmeli zincir merviden yöntemi ve üç farklı logaritmik regresyon modeliyle tahmin edilmiştir. Rezerv yöntemlerinin performansı, hayat dışı sigortalarda bireysel hasar tutarlarının modellenmesinde sıklıkla kullanılan çeşitli dağılımlar için tahmini ve gerçek hasar rezervleri karşılaştırılarak incelenmiştir. Logaritmik regresyon modellerinin, belirlenen senaryolar için çoğunlukla zincir merdiven yönteminden daha iyi performans gösterdiği görülmüştür.

Anahtar Sözcükler: *Hasar gelişim üçgeni, değişen hasar tutarı, enflasyon-düzeltmeli hasar, zincir merdiven yöntemi, logaritmik regresyon modelleri.*

Introduction

The solvency of insurance companies does not only depend on their paid losses, but also an accurate estimation of their expected future loss payments. It is essential to allocate loss reserves sufficiently. The estimation of future claims has become crucial to insurance industry especially after the Solvency Capital Requirement (SCR). The SCR consists of various risks one of which is non-life underwriting risks including reserve risk [11]. In this context, it can be said that one of the most important responsibilities of an insurance company is to estimate unpaid claims and to allocate an adequate reserve in order to compensate these claims. Choosing the most suitable method among all deterministic and stochastic reserving methods is crucial. In order to do that, the loss reserving process should be understood properly. This process includes defining the model structure for the claim, preparing the loss data in accordance with the loss development triangle (upper-left triangle) and obtaining the goal triangle (lower-right triangle) by means of a suitable loss reserving method [10].

The Undersecretariat of Treasury is an institution which regulates the rules for technical and actuarial calculations of insurance companies in Turkey. For non-life loss reserving, the techniques which actuaries are allowed to use are limited to chain-ladder (CL) methods for the sake of simplicity and practibility. These methods are based on loss development triangles. In this study, we use inflation-adjusted CL method and loglinear regression models using run-off triangles for loss reserving. We propose that logarithmic regression models generally give better results in comparison with traditional CL methods. Providing standard errors for estimated parameter values in addition to estimating reserves using development triangles make regression models become advantageous choices when compared with the basic CL techniques.

Stanard [13] and Pentikäinen and Rantala [9] used simulation methods for comparing loss reserving methods. Verrall [14] applied logarithmic linear models for the reserve estimation. Renshaw and Verrall [10] associated the chain-ladder method directly with generalized linear models. Antonio and Plat referred to underlying data set as ‘micro-level’ data because a loss triangle is actually the summary of the claim data set. The temporal information such as the occurance time, the reporting and settlement delays as well as the amounts of the claims are used in order to represent the over-all data [1]. Likewise, we use a simulation procedure taking into account this temporal information of claims and that the amounts of claims change in time until they are paid.

Studies about the comparison of loss reserving methods are mostly focused on ‘backtesting’ or ‘simulation’. Performance of the loss reserving methods can be compared stochastically with simulation [7, 8]. Choy et al. [4] defined the ‘growing triangle’ approach in order to compare the different methods used for reserve estimation. Jing et al. [6] applied the ‘cross-validation’ technique which measures estimation errors for performance testing of actuarial projection methods.

Nevruz and Gençtürk [8] used four different loss square generating methods one of which assumes “individual losses with changing severity” and concluded that this method is more effective when one considers stochastic characteristics of the loss and the related reporting and settlement delays. In their paper, the individual claim amounts are assumed to have Pareto distribution for this method. Here, we extend this assumption by obtaining and applying the solutions of developed loss amount estimations for lognormal and gamma distributions which are also commonly used for modeling loss severities in the actuarial literature. Also, the CL method is adjusted by taking the inflation rate into account. The scenarios are generated in line with the inflation rates, the distributions of the individual claim amount, and mean and dispersion of the individual claim amount random variable.

This paper is organized as follows: In Section 2, the simulation and reserving methods are given after the claim process is explained. The scenarios for the simulations are introduced and results are discussed in Section 3. Conclusion remarks are made in the last section.

1. Materials and Methods

The loss development triangle, which is used to summarize the loss data, is an important tool for various loss reserving methods. It is a table representing changes in values of different data groups related to the loss [5]. Consider that each claim of a risk portfolio is settled in the accident year or within the following n development years. Let $S_{i,j}$ and $L_{i,j}$ denote the incremental loss and the cumulative loss random variables, respectively for the i-th accident year and j-th development year. It is assumed that incremental and cumulative losses are observable for $i+j-1 \leq n$. The loss development triangles on an accident year basis are represented in Figure 2.1.

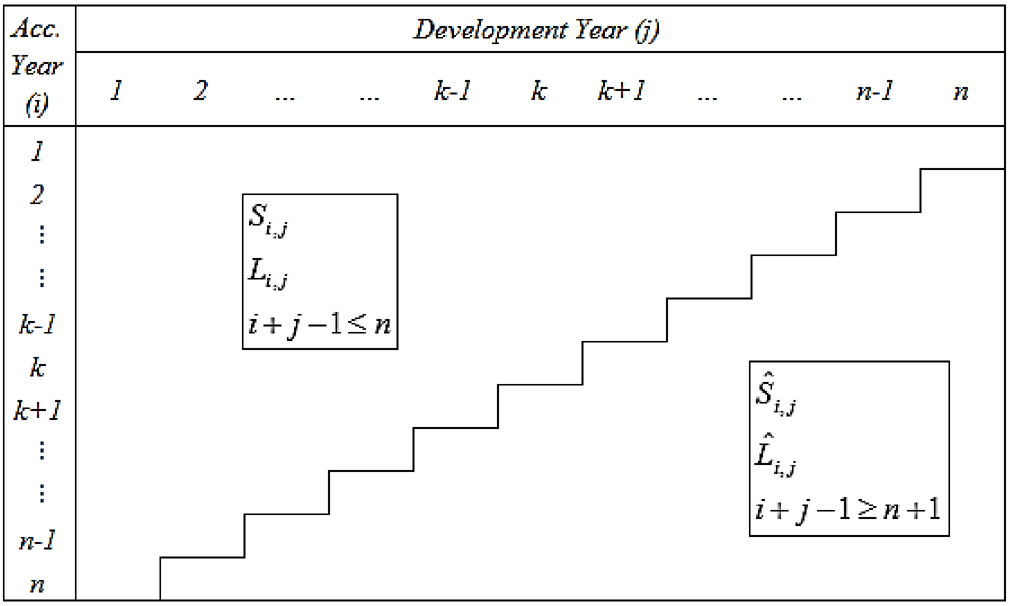

Figure 2.1. Upper-left and lower-right loss development triangles

If incremental losses are known, the cumulative loss is given by $L_{i,j} = \sum_{k=1}^{j} S_{i,k}$. Let $\hat{S}_{i,j}$ be the estimated incremental loss random variable. Then, the estimated cumulative loss is given by

$\hat{L}_{i,j} = L_{i,n-i+1} + \sum_{k=n-i+2}^{j} \hat{S}_{i,k}$ [12].

1.1. Loss Square Generating Method: Individual Losses with Changing Severity

In order to compare loss reserving methods, simulation techniques are preferred because simulated loss development triangles are stochastic and they do not give any considerable advantage to a specific loss reserving method [7]. Since incremental losses are assumed to be positive under logarithmic regression models, the loss square generating method is set to generate positive incremental losses.

It is considered that there are n accident and n development years ($i, j = 1,2, \dots, n$). The claims for each accident year are developed to the ultimate claims (n-th calendar year). The lower-right triangle, which includes the last column showing the projections of the ultimate claims, is estimated from the observable upper-left triangle. The ultimate claim of an accident year is the total claim amount to be paid for this year. The unpaid amount of the total claim (total claim amount minus paid claim amount) is the reserve to be allocated.

The cumulative losses $L_{i,j}$ are simulated for $i, j = 1,2, \dots, n$. Since claims are assumed to be settled at the end of the n-th year, the ultimate loss for i-th accident year is $L_i = L_{i,n}$.

Stanard [13] and Bühlmann et al. [3] proposed the idea behind the loss square generating method assuming individual losses with changing severity. The algorithm for this method is as follows:

Step 1. Generate claim numbers N_i and individual claim amounts $C_{i,k}$; $k = 1, 2, \dots, N_i$ for accident year i. Here, parameters are estimated by "method of moments".

i. Claim number random variable is assumed to be Poisson distributed.
ii. Individual claim amount random variable is assumed to have lognormal, Pareto and gamma distributions.

Step 2. Obtain the percentile of each individual claim amount as $U_{i,k} = F(C_{i,k})$ where F is the distribution function of the individual claim amount random variable.

Step 3. For each $C_{i,k}$; $k = 1, 2, \dots, N_i$, generate the occurrence date $X_{i,k,1}$, the reporting delay $X_{i,k,2}$ and the settlement delay $X_{i,k,3}$.

Here, the occurrence date $X_{i,k,1}$ is simulated from Uniform(0,1) distribution since the claims occur within the accident year. The reporting delay $X_{i,k,2}$ and the settlement delay $X_{i,k,3}$ are assumed to have exponential distribution. It is considered that the higher the mean and the variance of the individual claim amount random variable, the longer it takes to report and settle the claim. Therefore, the means of reporting and settlement delays are taken smaller for the scenarios where the claims amount random variable is assumed to have low mean-low variance in comparison with the scenarios where the claims amount random variable is assumed to have high mean-high variance.

The following random variables are defined based on the assumption that claims are settled in the accident year or within the following n development years:

i. $r_{i,k} = \min\{\lfloor X_{i,k,1} + X_{i,k,2} \rfloor, n\}$, and
ii. $R_{i,k} = \min\{\lfloor X_{i,k,1} + X_{i,k,2} + X_{i,k,3} \rfloor, n\}$

where $\lfloor x \rfloor$ is the greatest integer that is less than or equal to x. Thus, the k-th individual loss in the i-th accident year is reported and settled in the calendar years $i + r_{i,k}$ and $i + R_{i,k}$, respectively [7].

Step 4. Estimate the developed loss amounts $\hat{C}_{i,k,j}$ for $j = 1,2, \dots, n$ for each individual claim amount by means of the inverse of the distribution function, i.e. $F^{-1}(U_{i,k})$. Here, $\hat{C}_{i,k,j}$ increases as j increases. In this step, the solutions of $\hat{C}_{i,k,j}$ are obtained for lognormal, Pareto and gamma distributions, respectively as follows:

i. Let $C_{i,k}$ has lognormal distribution with parameters μ_j and σ_j^2. $\hat{C}_{i,k,j}$ is obtained by

$$\hat{C}_{i,k,j} = \begin{cases} 0 & ; j = 1,2, \dots, r_{i,k} \\ \exp\{\sqrt{2}\sigma_j [\mathrm{erf}^{-1}(2U_{i,k} - 1)] + \mu_j\} & ; j = r_{i,k} + 1, \dots, R_{i,k} \\ \exp\{\sqrt{2}\sigma_{R_{i,k}+1} [\mathrm{erf}^{-1}(2U_{i,k} - 1)] + \mu_{R_{i,k}+1}\} & ; j = R_{i,k} + 1, \dots, n \end{cases}, \quad (1)$$

where erf^{-1} is the inverse of the error function $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$.

ii. Let $C_{i,k}$ has Pareto distribution with parameters α_j and θ_j. $\hat{C}_{i,k,j}$ is obtained by

$$\hat{C}_{i,k,j} = \begin{cases} 0 & ; j = 1,2,\dots,r_{i,k} \\ \theta_j \left[\frac{1}{(1-U_{i,k})^{1/\alpha_j}} - 1 \right] & ; j = r_{i,k}+1,\dots,R_{i,k} \\ \theta_{R_{i,k}+1} \left[\frac{1}{(1-U_{i,k})^{1/\alpha_{R_{i,k}+1}}} - 1 \right] & ; j = R_{i,k}+1,\dots,n \end{cases} . \tag{2}$$

iii. Let $C_{i,k}$ has gamma distribution with parameters θ_j and α_j. $\hat{C}_{i,k,j}$ is obtained by

$$\hat{C}_{i,k,j} = \begin{cases} 0 & ; j = 1,2,\dots,r_{i,k} \\ \theta_j P^{-1}(\alpha_j, U_{i,k}) & ; j = r_{i,k}+1,\dots,R_{i,k} \, , \\ \theta_{R_{i,k}+1} P^{-1}(\alpha_{R_{i,k}+1}, U_{i,k}) & ; j = R_{i,k}+1,\dots,n \end{cases} \tag{3}$$

where $P(a,x) = \frac{\gamma(a,x)}{\Gamma(a)} = \frac{\int_0^x t^{a-1} e^{-t} dt}{\Gamma(a)}$ is the lower regularized gamma function.

Step 5. Calculate the cumulative losses $\hat{L}_{i,j} = (1+e)^{i-1} \sum_{k=1}^{N_i} \hat{C}_{i,k,j}$ where e is the inflation rate. Since the cumulative losses are obtained increasingly, the incremental losses are positive.

1.2. Loss Reserving Methods

1.2.1. Inflation-Adjusted Chain-Ladder (IACL) Method

Although the reserve calculation is based on the experience in past years, one should be cautious about the impact of the inflation. The values of the claims will change in the development years if the claims are not settled in the accident year when they occur. Therefore, CL method is adjusted using the inflation-adjusted cumulative losses $L_{i,j}$, which is obtained as the summation of the inflation-adjusted incremental losses $S_{i,j}(1+e)^{n-i-j+1}$. According to the IACL method, the reserve is estimated for the accident year $i = 2,3,\dots,n$ (since all claims are developed for the first accident year, $\hat{R}_1 = 0$) such that

$$\hat{R}_i = \hat{L}_i - L_{i,n-i+1} = L_{i,n-i+1} \prod_{j=n-i+2}^{n} \hat{f}_j - L_{i,n-i+1} \, , \tag{4}$$

where $\hat{L}_i$ is the estimated ultimate loss and $\hat{f}_j$ is the estimated development factor such that

$$\hat{f}_j = \frac{\sum_{i=1}^{n-j+1} L_{i,j}}{\sum_{i=1}^{n-j+1} L_{i,j-1}} ; j = 2,3,\dots,n \text{ [7]}. \tag{5}$$

1.2.2. Logarithmic Regression Models

For the logarithmic regression models, the reserve is estimated using the estimated expected value of the incremental losses for the accident year $i = 2,3,\dots,n$ ($\hat{R}_1 = 0$) such that

$$\hat{R}_i = \sum_{j=n-i+2}^{n} \hat{\theta}_{i,j} \, , \tag{6}$$

where $\hat{\theta}_{i,j}$ is the unbiased estimation of $\theta_{i,j} = \mathbb{E}(S_{i,j}) = \exp\left(\boldsymbol{X}_{i,j}\boldsymbol{\beta} + \frac{1}{2}\sigma^2\right)$. Here, $\boldsymbol{X}_{i,j}$ is the row vector of explanatory variables and $\boldsymbol{\beta}$ is the column vector of parameters. Since the incremental loss random variables are assumed to be independent and lognormally distributed, the dependent variable of the regression model defined as $Z_{i,j} = \ln(S_{i,j})$ has normal distribution with the expectation $\mathbb{E}(Z_{i,j}) = \boldsymbol{X}_{i,j}\boldsymbol{\beta}$ and the variance $\mathbb{V}(Z_{i,j}) = \sigma^2$.

The investigated models, which differ based on the idea that the accident year or/and development year is/are significant for loss reserving, are as follows:

Model 1: $Z_{i,j} = \mu + \alpha_i + \beta_j + \varepsilon_{i,j} \Rightarrow \boldsymbol{\beta} = [\mu, \alpha_2, \dots, \alpha_n, \beta_2, \dots, \beta_n]$
Model 2: $Z_{i,j} = \mu + (i-1)\alpha + \beta_j + \varepsilon_{i,j} \Rightarrow \boldsymbol{\beta} = [\mu, \alpha, \beta_2, \dots, \beta_n]$
Model 3: $Z_{i,j} = \mu + (i-1)\alpha + (j-1)\beta + \gamma \ln(j) + \varepsilon_{i,j} \Rightarrow \boldsymbol{\beta} = [\mu, \alpha, \beta, \gamma]$

where the error terms $\varepsilon_{i,j}$ are independent and normally distributed with parameters 0 and σ^2. Here, it is assumed that $\alpha_1 = 0$ and $\beta_1 = 0$ to make the model full rank [14].

After $\widehat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{z}$ is obtained by the method of least squares, the variance of the error is calculated as $\hat{\sigma}^2 = \frac{1}{r-p}(\boldsymbol{z} - \boldsymbol{X}\widehat{\boldsymbol{\beta}})'(\boldsymbol{z} - \boldsymbol{X}\widehat{\boldsymbol{\beta}})$. Here, $r = \frac{1}{2}n(n+1)$ is the number of observations, p is the number of parameters, $\boldsymbol{X}$ is the $(r \times p)$ design matrix having the rows $\boldsymbol{X}_{i,j}$ and $\boldsymbol{z} = [Z_{1,1}, Z_{1,2}, \dots, Z_{1,n}, Z_{2,1}, \dots, Z_{n,1}]'$ is the vector of observed losses.

Lastly, the unbiased estimation of the expected value of the incremental losses is obtained for $i = 2,3, \dots, n$ and $j = n - i + 2, \dots, n$ such that

$$\hat{\theta}_{i,j} = \exp(\boldsymbol{X}_{i,j}\widehat{\boldsymbol{\beta}})\, g_m\left[\frac{1}{2}\left(1 - \boldsymbol{X}_{i,j}(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}_{i,j}'\right)s^2\right], \tag{7}$$

where $g_m(t) = \sum_{k=0}^{\infty} \frac{m^k(m+2k)}{m(m+2)\dots(m+2k)} \frac{t^k}{k!}$. Here, $m = r - p$ is the degree of freedom of $\hat{\sigma}^2$ and $s^2 = \frac{r}{r-p}\hat{\sigma}^2$ is the unbiased estimate of σ^2 [2].

2. Application Results and Discussion

In order to compare the loss reserving methods through simulation, firstly different scenarios are generated. In the scenarios, the assumptions about characteristics of a risk portfolio are specified as in Table 3.1.

The assumptions of the scenarios in Table 3.1 are given in detail as follows:

i. The inflation rate: Low (6%), Medium (8%), High (10%)
ii. The distribution of the individual claim amount random variable: Lognormal, Pareto, gamma.
iii. The mean and variance of the individual claim amount random variable: Low mean (500), low variance (150^2); low mean (500), high variance (1000^2); high mean (5000), low variance (1500^2); high mean (5000), high variance (10000^2)

In the application part, it is considered that $n = 11$, i.e. $i = 1,2, \dots, 11$ and $j = 1,2, \dots, 11$. Firstly, the ultimate losses and the actual reserves are calculated from the generated loss development squares with 10,000 trials. The reserves are estimated by the loss reserving methods from the upper-left triangles. Lastly, the deviations of the estimated reserves from the actual reserves are calculated.

Table 3.1. The summary of the scenarios

Scenarios	**Inflation Rate**			**Distribution of the Individual Claim Amount**			**Descriptives of the Individual Claim Amount**				**Scenarios**	**Inflation Rate**			**Distribution of the Individual Claim Amount**			**Descriptives of the Individual Claim Amount**			
	Low: %6	Medium: %8	High: %10	Lognormal	Pareto	Gamma	Mean: Low/Var: Low	Mean: Low/Var: High	Mean: High/Var: Low	Mean: High/Var: High		Low: %6	Medium: %8	High: %10	Lognormal	Pareto	Gamma	Mean: Low/Var: Low	Mean: Low/Var: High	Mean: High/Var: Low	Mean: High/Var: High
sce1	X			X			X				sce19		X			X				X	
sce2	X			X				X			sce20		X			X					X
sce3	X			X					X		sce21		X				X	X			
sce4	X			X						X	sce22		X				X		X		
sce5	X				X		X				sce23		X				X			X	
sce6	X				X			X			sce24		X				X				X
sce7	X				X				X		sce25			X	X			X			
sce8	X				X					X	sce26			X	X				X		
sce9	X					X	X				sce27			X	X					X	
sce10	X					X		X			sce28			X	X						X
sce11	X					X			X		sce29			X		X		X			
sce12	X					X				X	sce30			X		X			X		
sce13		X		X			X				sce31			X		X				X	
sce14		X		X				X			sce32			X		X					X
sce15		X		X					X		sce33			X			X	X			
sce16		X		X						X	sce34			X			X		X		
sce17		X			X		X				sce35			X			X			X	
sce18		X			X			X			sce36			X			X				X

The studies comparing the performance of loss reserving methods mostly focus on the performance criteria such as bias, root mean square error (RMSE), mean absolute error (MAE), mean percentage error (MPE), and correlation between the actual and estimated reserves. When one needs to measure errors, the magnitude of the difference between actual and estimated values gives more reliable information. In addition, the relative error, which is calculated as the error relative to the magnitude of the actual value, is more useful for comparing various data samples having different means and dispersions. In this study, the performance of the loss reserving methods is tested by mean absolute percentage error (MAPE) examining the prediction accuracy of the estimated reserves. The MAPE results for loss reserving methods are represented in Figure 3.1.

This figure helps to investigate the performance of each method among scenarios. Since our main aim is to examine the estimation performance of the IACL method and log-regression models for different loss severity distributions, we control the performance of each loss reserving method separately for all scenarios in this figure. In order to compare the loss reserving methods, the numerical MAPE results for each scenario are also examined separately for all loss reserving methods in Table 3.2.

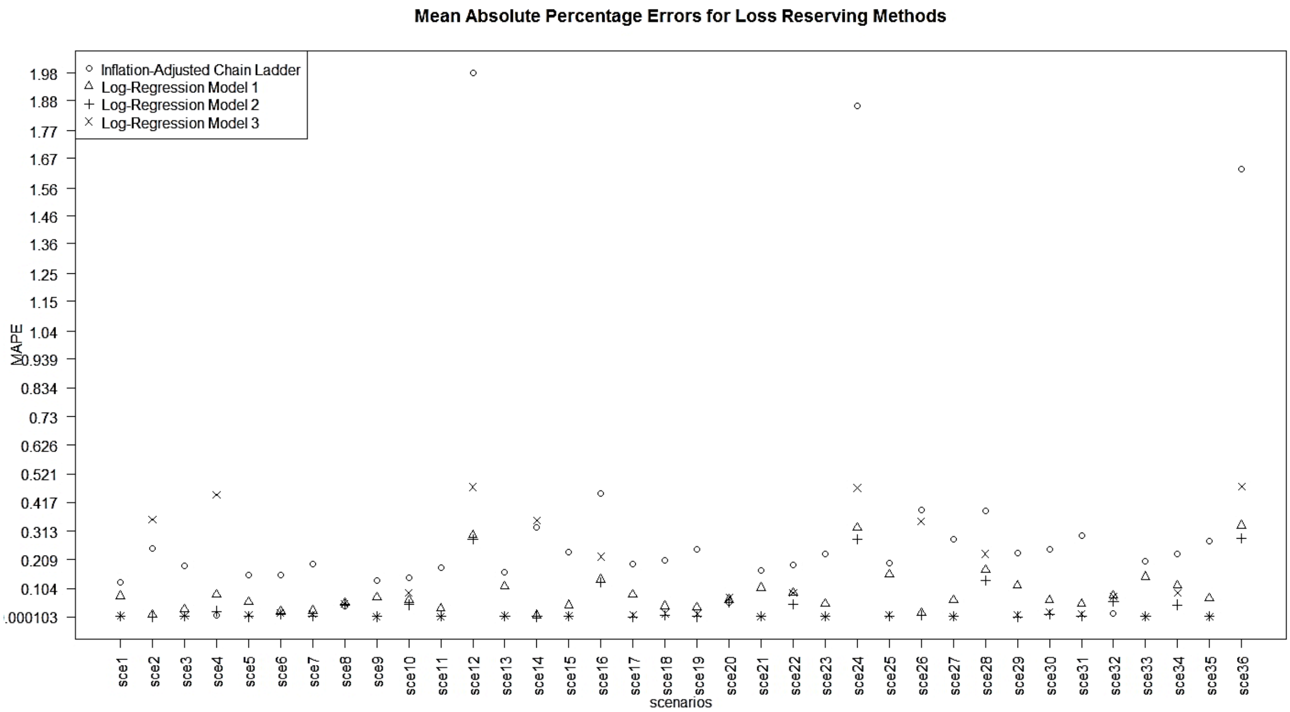

Figure 3.1. MAPE graph for IACL, Log-Regression Model 1, Log-Regression Model 2 and Log-Regression Model 3 loss reserving methods

The IACL method performs better for lognormal and Pareto distributed samples which have heavier tails than gamma. Specifically, the performance of the IACL method is relatively higher for Pareto distributed samples with high mean and high variance (Scenarios 8, 20, 32). This method does not generally work well for the scenarios where the individual claim amount has lognormal distribution with high variance (Scenarios 14, 16, 26 and 28) and gamma distribution with high mean and high variance (Scenarios 12, 24, 36).

This figure shows that the Log-Regression Model 1 works better for lognormal samples with low mean and high variance (Scenarios 2, 14 and 26). If one wants to investigate where this method performs poorly, it is concluded that the performance of the Log-Regression Model 1 is lower for the scenarios where the individual claim amount has lognormal and gamma distributions with low dispersion relative to the mean, i.e. low mean-low variance or high mean-high variance and the inflation rate is not low (Scenarios 12, 13, 16, 24, 25, 28, 36).

As seen from this figure, the performance of the Log-Regression Model 2 is not very good for Pareto and lognormal distributed samples with high mean and high variance (Scenarios 4, 8, 16, 20, 28 and 32), and gamma distributed samples with low mean and high variance (Scenarios 10, 22, 34).

It is also seen that the Log-Regression Model 3 does not perform well for lognormal samples with high variance (Scenarios 2, 4, 14, 16, 26, 28) and and gamma distribution with high mean and high variance (Scenarios 12, 24, 36).

Numerical MAPE results are presented in Table 3.2. It must be noted that the lower the MAPE, the better the performance of the loss reserving method. In this table, the lowest MAPE value among all reserving methods is shown in bold in order to see the best performing loss reserving method for each scenario.

Table 3.2. Numerical MAPE results of the loss reserving methods for each scenario

Scenario	IACL	Logarithmic Regression Models			Scenario	IACL	Logarithmic Regression Models		
		Model 1	Model 2	Model 3			Model 1	Model 2	Model 3
sce1	0.1268	0.0761	**0.0019**	0.0037	sce19	0.2458	0.0343	**0.0014**	0.0117
sce2	0.2497	0.0076	**0.0011**	0.3537	sce20	0.0526	0.0626	**0.0523**	0.0714
sce3	0.1848	0.0292	**0.0018**	0.0048	sce21	0.1688	0.1062	0.0016	**0.0001**
sce4	**0.0051**	0.0810	0.0209	0.4439	sce22	0.1883	0.0876	**0.0455**	0.0877
sce5	0.1524	0.0550	**0.0012**	0.0072	sce23	0.2300	0.0475	0.0020	**0.0005**
sce6	0.1521	0.0211	**0.0089**	0.0138	sce24	1.8623	0.3244	**0.2819**	0.4703
sce7	0.1922	0.0227	**0.0018**	0.0125	sce25	0.1951	0.1554	**0.0019**	0.0067
sce8	**0.0391**	0.0511	0.0458	0.0470	sce26	0.3909	0.0148	**0.0052**	0.3489
sce9	0.1316	0.0721	0.0013	**0.0011**	sce27	0.2821	0.0613	**0.0022**	0.0023
sce10	0.1440	0.0606	**0.0447**	0.0869	sce28	0.3870	0.1718	**0.1340**	0.2296
sce11	0.1803	0.0316	0.0018	**0.0006**	sce29	0.2325	0.1140	**0.0011**	0.0070
sce12	1.9814	0.2987	**0.2840**	0.4732	sce30	0.2474	0.0615	**0.0095**	0.0172
sce13	0.1626	0.1120	**0.0020**	0.0051	sce31	0.2949	0.0475	**0.0019**	0.0101
sce14	0.3263	0.0068	**0.0013**	0.3527	sce32	**0.0143**	0.0782	0.0551	0.0750
sce15	0.2361	0.0430	**0.0018**	0.0038	sce33	0.2028	0.1470	0.0016	**0.0011**
sce16	0.4510	0.1361	**0.1257**	0.2200	sce34	0.2310	0.1144	**0.0444**	0.0871
sce17	0.1946	0.0813	**0.0006**	0.0066	sce35	0.2749	0.0676	0.0026	**0.0019**
sce18	0.2054	0.0405	**0.0064**	0.0129	sce36	1.6306	0.3339	**0.2854**	0.4752

From Table 3.2, it is seen that the Log-Regression Model 2 performs better for most of the scenarios in comparison with the other loss reserving methods. This result can also be derived from Figure 3.1. The performance of the Log-Regression Model 3 seems to be close to the Log-Regression Model 2 in Figure 3.1, however it is seen from Table 3.2 that the estimation performance of the Log-Regression Model 2 dominates other reserving methods for most of the scenarios. The Log-Regression Model 3 works better than other loss reserving methods for only the low-variance gamma samples (Scenarios 9,11, 21, 23, 33 and 35).

The IACL reserving method mostly gives the highest errors. It should be noted that none of the loss reserving methods performs well for the gamma distributed samples with high mean and high variance (Scenarios 12, 24 and 36) since all MAPE values for these scenarios are relatively high.

It is notable that Log-Regression Model 1 taking into account both accident year and development year together is not the best performing method for any scenarios whereas IACL is the best performing method for Scenarios 4, 8, and 32. These results indicate that the development year is significant in regression models. Thus, one should be careful about reporting and settlement delays of claims in loss reserving.

3. Conclusion

Based on the simulation results, it is concluded that logarithmic regression models considering the development year give better results. Although they do not provide the best results in all situations, they are consistent and give not only the point estimation but also a confidence interval. Unlike logarithmic regression models, the IACL reserving method mostly does not work well for the specified scenarios. Since the calculation of the development factors is very significant for chain-ladder methods, the performance of the IACL method could be better if the development factors are calculated taking into account of the changing severity structure of the individual losses. Hence, actuaries must be careful while applying the IACL method. Another result of this study is that both log-regression models and IACL perform poorly for the scenarios where individual losses are assumed to have gamma distribution with high mean and high variance. It can also be concluded that the inflation rate affects the performance of the loss reserving methods. Here, the inflation rate is taken constant over the accounting period. It could be modeled with time series analysis for future studies.

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