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Değişken kesitli sıcaklığa bağımlı çubuğun boyuna titreşimi

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Longitudinal Vibration of Temperature Dependent Bar with Variable Cross-Section

Araştırma Makalesi / Research Article

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ABSTRACT

Vibration behavior of a bar with variable cross-section, which its material properties vary with temperature, is investigated in this study. In the analysis, not only theoretical solution but also numerical solution is performed for validation. The numerical analysis is overcome by SolidWorks program based on finite element method. Four types of effects on the bar are investigated. These are effects of temperature variation, geometric ratio, slenderness ratio and mode numbers variation. The temperature is increased from 22 °C to 250 °C. The geometric ratio is varied from 0 to -1/L at intervals of 0.25/L. The slenderness ratio is varied from 1/10 to 1/20 by increasing the length of bar from 200 mm to 400 mm. As for the mode numbers, the first three mode shapes are examined in the analysis. The boundary condition of the bar is taken as clamped-free. According to the results, the natural frequency decreases with increasing the temperature. The natural frequency also decreases with decreasing the slenderness ratio. But, it increases with decreasing the geometric ratio and also increases with increasing the mode number. When the theoretical and numerical results are examined, it is seen that the results are in harmony.

Anahtar Kelimeler: Bar, temperature, vibration, variable cross-section.

Değişken Kesitli Sıcaklığa Bağımlı Çubuğun Boyuna Titreşimi

ÖZ

Bu çalışmada malzeme özellikleri sıcaklıkla değişen, değişken kesitli bir çubuğun titreşim davranışları araştırılmıştır. Analizde, doğrulama için sadece teorik çözüm değil aynı zamanda sayısal çözümde yapılmıştır. Sayısal analiz, sonlu elemanları esas alan SolidWorks programı ile yapılmıştır. Çubuğa etkiyen dört tip etki araştırılmıştır. Bunlar, sıcaklık değişiminin, geometrik oranın, narinlik oranının ve mod sayısının değişimin etkileridir. Sıcaklık 22 °C'den 250 °C'ye kadar artırılmıştır. Geometrik oran, 0.25/L aralıklarla 0'dan -1/L'ye değiştirilmiştir. Narinlik oranı, çubuğun boyu 200 mm'den 400 mm'ye artırılarak, 1/10'dan 1/20'ye değiştirilmiştir. Mod sayılarına gelince, analizde ilk üç mod şekli incelenmiştir. Çubuğun sınır koşulları ankastre-serbest olarak alınmıştır. Sonuçlara göre sıcaklığın artmasıyla doğal frekans düşer. Doğal frekans, narinlik oranının düşmesiyle de düşer. Fakat geometrik oranın düşmesiyle ve mod sayısının artmasıyla artar. Teorik ve sayısal sonuçlar incelendiğinde, sonuçların uyum içinde olduğu görülür.

Keywords: Çubuk, sıcaklık, titreşim, değiken kesit.

1. INTRODUCTION (GİRİŞ)

The structural elements such as bars, beams, plates, are widely used in the engineering applications. Therefore, many studies have been made on the vibrations of structural elements [1, 2]. Among these structural elements, the bar element has a wide application area in the machine and civil engineering such as mechanisms and frame structures. Some of the studies on vibration of the bar elements are given below.

Li [3] has studied on the free longitudinal vibration analysis of multi-step non-uniform bars. He has reduced the differential equations of the free longitudinal vibrations of bars with variable distributed mass and stiffness to Bessel's equations and he has derived an analytical solution to determine the longitudinal natural frequencies and mode shapes. Bert and Zeng [4] have applied the differential transform method to the axial vibration problems of compound bars. They have showed the accuracy, simplicity and efficiency of the differential transform method. Ma [5] has presented a new finite element formulation and algorithm for exact solutions of undamped axial vibration problems of elastic bars. They have determined the natural frequencies and the associated vibration mode shapes with their iterative procedure. Arndt *et al.* [6] have applied the adaptive Generalized Finite Element Method to free longitudinal vibration analysis of straight bars and trusses. They have shown the efficiency and convergence of the proposed method in vibration analysis of uniform and non-uniform straight bars in their studies. Velasco *et al.* [7] have recorded the sound wave in a cylindrical steel bar by using a microphone and they have obtained resonance spectrum of the bar by using a sound analysis software. Ranjbaran *et al.* [8] have proposed a new method for computation of longitudinal vibrations of multi-cracked

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bars. They have modelled the cracks by equivalent axial springs. They have showed the accuracy, efficiency and robustness of their method in their study. Akgoz and Civalek [9] have investigated the longitudinal free vibration analysis of axially functionally graded microbars. They have used Rayleigh-Ritz solution technique in their study. Bui *et al.* [10] have obtained the optimal configurations of circular bars under free torsional and longitudinal vibration. They have used Pontryagin's maximum principle in their study. II'gamov [11] has investigated the longitudinal vibrations of a bar with incipient transverse cracks. He has proposed a model based on the assumption that the crack size is small compared with the bar cross-section area. Lee *et al.* [12] have proposed an enhanced spectral element model to solve the axial vibration problems of functionally graded bars.

As the literature survey is examined, it is seen that the vibration analysis of bars with variable or constant cross-section is examined with different methods. Because of the difficulties in the calculation of natural frequency of a bar which has variable material properties due to the temperature, the material properties are assumed to be constant in earlier studies. A study on the effect of the temperature on the longitudinal vibration of the bars is almost never encountered in the open literature. In this study, free longitudinal vibration of a bar, which its material properties vary with temperature is performed. The effects of temperature, geometric ratio, slenderness ratio and mode numbers on the natural frequency of the bar are investigated. Moreover, the mode shapes of bar obtained from theoretical and numerical solutions are also given in the study.

2. MATERIAL AND GEOMETRIC PROPERTIES OF THE BAR (ÇUBUĞUN MALZEME VE GEOMETRİK ÖZELLİKLERİ)

2.1. Geometric Properties (Geometrik Özellikler)

An isotropic rectangular bar as shown in Figure 1 is considered in this study. It is assumed that the width of the bar varies along *x* direction.

As shown in Figure 1 that b_o is the half width of the clamped end of the bar and it is varied exponentially as follow.

$$b(x) = b_o e^{\delta x} \tag{1}$$

In this equation, δ is geometric ratio, and the width of the bar is constant when the δ is equal to zero. The length (L) and the thickness (h) of the bar are taken as constant.

The boundary condition is assumed as clamped at one end and free at other end.

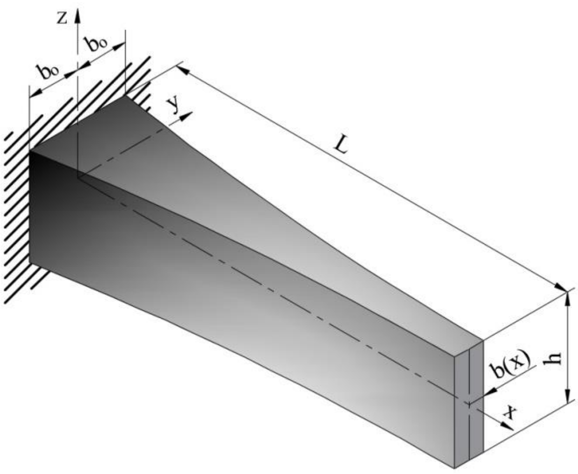


Figure 1. Rectangular bar with variable cross-section (Değişken kesili dikdörtgen çubuk)

2.2. Material Properties (Malzeme Özellikleri)

As in real life, it is assumed that the material properties of the bar change with temperature in this study. A Titanium alloy (Ti-6Al-4V) is preferred as bar material. This material has high strength, and corrosion resistance and low weight. So, this material is widely used especially in medical and space industry [13]. Variation of the material properties of some materials with temperature is given by the following formula by Shen [14].

$$P_j = P_0(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3) \tag{2}$$

In this equation, P_0, P_{-1}, P_1, P_2 and P_3 are constants related to the material and given in Table 1. T is temperature in Kelvin and P_j is material properties.

Table 1. The constants for Ti-6Al-4V [14] (Ti-6Al-4V için sabitler [14])

Material Properties	P_0	P_{-1}	P_1	P_2	P_3
Young's modulus [Pa]	122.56e+9	0	-4.586e-4	0	0

It is assumed that the bar is affected by the thermal effect in this study. The temperature is increased from 100 °C to 250 °C at intervals of 50 °C. Moreover, the room temperature (22 °C) is also taken into consideration. Then, the Young's modulus of the material is calculated for each temperature value according to Equation (2). According to the results obtained from the calculation, the variation of the Young's modulus with temperature is shown in Figure 2.

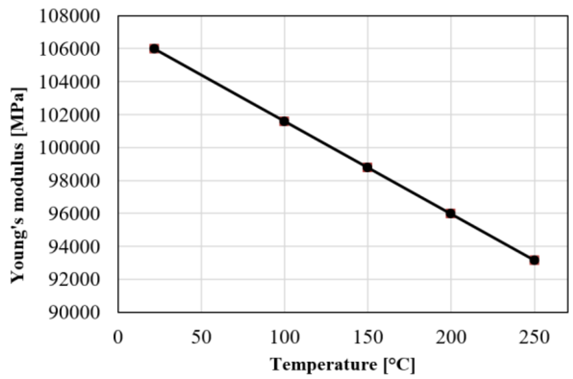


Figure 2. Variation of Young's modulus (Elastisite modülündeki değişim)

The density of bar is taken as constant since one varies slightly with temperature and it equals to 4429 kg/m^3 [13].

3. THEORETICAL AND NUMERICAL SOLUTIONS (TEORİK VE SAYISAL ÇÖZÜMLER)

3.1. Theoretical Solution (Teorik Çözüm)

The equation of motion of a longitudinally vibrating bar is written as follows [15],

$$\rho A(x)\frac{\partial u(x,t)}{\partial t^2}=\frac{\partial}{\partial x}\left(A(x)E(T)\frac{\partial u(x,t)}{\partial x}\right) \tag{3}$$

where *u(x,t)* is longitudinal displacement, ρ is density, *E(T)* is variable Young's modulus and it is calculated from Equation (2) according to temperature, *A(x)* is variable cross-section area and equal to multiply the variable width by constant thickness.

The longitudinal displacement *u(x,t)* can be taken as exponential function to find the natural frequency of the bar as follows,

$$u(x,t)=U(x)e^{i\omega t} \tag{4}$$

where *U(x)* is mode shape, ω is circular frequency and *t* is time. Substituting Equation (4) into Equation (3), reduces to,

$$\frac{d^2U(x)}{dx^2}+\delta\frac{dU(x)}{dx}+\mu^2U(x)=0 \quad 0\le x\le L \tag{5}$$

where

$$\mu^2=\frac{\rho\omega^2}{E} \tag{6}$$

The solution of the Equation (5) is,

$$U(x)=C_1e^{\left(-\frac{1}{2}\delta+\frac{1}{2}\sqrt{\delta^2-4\mu^2}\right)x}+C_2e^{\left(-\frac{1}{2}\delta-\frac{1}{2}\sqrt{\delta^2-4\mu^2}\right)x} \tag{7}$$

where C_1 and C_2 are constants and determined from boundary conditions.

Because the boundary conditions of the bar are clamped-free, the following equations are applied to the Equation (7).

$$U=0 \qquad at \qquad x=0 \tag{8}$$

$$dU/dx=0 \qquad at \qquad x=L \tag{9}$$

Applying these two conditions to Equation (7) yields the following vector form.

$$[K]\{C\}=\{0\} \tag{10}$$

By equalizing the determinant of the coefficient matrix *[K]* to zero, the following characteristic equation is obtained:

$$-\frac{1}{2}e^{-\frac{1}{2}\left(\delta+\sqrt{\delta^2-4\mu^2}\right)L}\delta-\frac{1}{2}e^{-\frac{1}{2}\left(\delta+\sqrt{\delta^2-4\mu^2}\right)L}\sqrt{\delta^2-4\mu^2}+\frac{1}{2}e^{\frac{1}{2}\left(-\delta+\sqrt{\delta^2-4\mu^2}\right)L}\delta-\frac{1}{2}e^{\frac{1}{2}\left(-\delta+\sqrt{\delta^2-4\mu^2}\right)L}\sqrt{\delta^2-4\mu^2}=0 \tag{11}$$

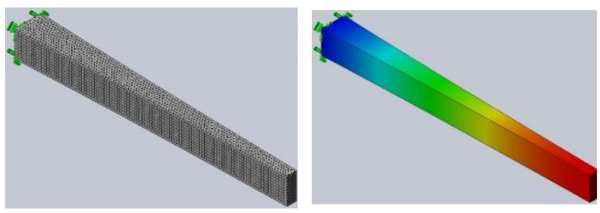
Solution of the above characteristic equation yields the natural frequencies.

Maple software is used to solve the above mathematical equations.

3.2. Numerical Solution (Sayısal Çözüm)

The numerical vibration analysis of the bar is performed by using SolidWorks commercial program based finite element method. The SolidWorks is a program that can do both design and engineering calculations of structural elements.

Firstly, the bar element is modelled in the drawing section of the program. In order to do this, two-dimensional wireframe bar model is drawn. The width of the bar is drawn according to the Equation (1). The three-dimensional model is obtained by extruding the wireframe model along to thickness direction. Then the simulation section of the program is run. In this section, the solver type is selected as FFESolver from the Options menu. Because the material properties of the bar vary with temperature, a new material is defined in the program according to the Section 2.2. Then the clamped-free boundary condition is applied to the bar. Since the program is based on the finite element method, the model is meshed. The mesh density is selected as fine mesh. The bar model after both meshing and analysis are given in Figures 3 (a) and (b), respectively.



(a) after meshing (ağlama sonrası) (b) after analysis (analiz sonrası)

Figure 3. The bar model (Çubuk model)

As an example, the element size, numbers of total nodes and total elements of the bar with δ= -1/L are obtained as 1.84961 mm, 77411 and 51975, respectively.

4. RESULTS AND DISCUSSION (SONUÇLAR VE TARTIŞMA)

In this study, the longitudinal vibration analysis of a bar with variable cross-section is investigated. It is assumed that the material properties of the bar vary with temperature.

The thickness of the bar is constant and is assumed as 20 mm. The width of the clamped end of the beam ($2b_o$) is also constant and is also assumed as 20 mm. But the width of the bar is decreasing towards the free end according to the Equation (1). The geometric ratio of the bar δ is varied from 0 to -1/L (=-1/200) with 0.25/L. In order to see the effects of slenderness ratio, the length of the beam is varied from 200 mm to 400 mm at intervals of 50 mm. Five models for geometric ratio and five models for slenderness ratio are examined in the study.

The abovementioned dimensions of clamped end of the bar are summarized in Table 2.

Table 2. The dimensions of clamped end of the bar (Çubuğun ankastre ucunun boyutları)

	Length (L)[mm]	*Width ($2b_o$)[mm]*	*Thickness (h)[mm]*
Values	200	20	20

The boundary condition of the bar is assumed as clamped-free. Furthermore, as for the temperature which affected the bar, it is increased from 22 °C to 250 °C. Moreover, first three mode shapes are taken into consideration for natural frequencies.

Both theoretical and numerical calculations have been made to see these effects. It can be seen from the results that the numerical and the theoretical results are in good agreement. The above mentioned effects are explained in detail below.

4.1 The Effects of Temperature (Sıcaklığın Etkileri)

In order to see the effects of temperature on the natural frequency of the bar with variable cross-section, the geometric and the slenderness ratios are taken -0.25/L (= -0.25/200) and 1/10, respectively. The results obtained from theoretical solution are compared with those obtained from numerical solution in Figure 4.

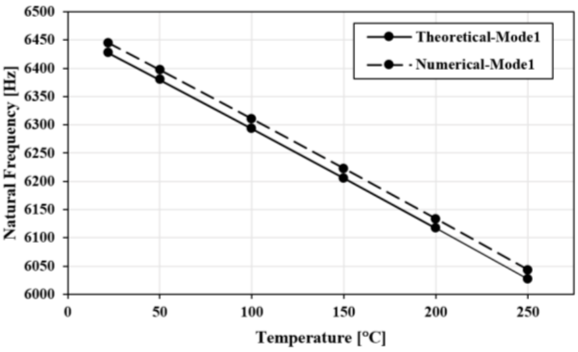


Figure 4. The effect of temperature on natural frequency (Doğal frekans üzerine sıcaklığın etkisi)

It can be seen from Figure 4 that the natural frequency of the bar with variable cross-section (δ=0.25) decreases with increasing the temperature. It is also seen that this decline is about linear. When the natural frequencies obtained at room temperature and 250 °C are compared, the difference is almost 400 Hz. Therefore, it is clear that the effect of temperature on the material properties must be taken into account in vibration calculations. However, in literature, it is generally assumed that the material properties do not change with temperature in the vibration calculations.

When the theoretical and numerical results are examined, it is seen that the results are in harmony.

4.2 The Effects of Geometric Ratio (Geometrik Oranın Etkileri)

The variation of the natural frequency of the bar with geometric ratio is represented in Figure 5. In this figure, the temperature and the slenderness ratio are taken 22 °C, 1/10, respectively,

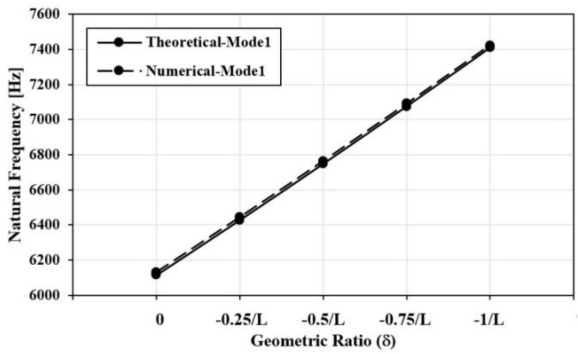


Figure 5. The effect of geometric ratio on natural frequency (Doğal frekans üzerine geometric oranın etkisi)

The geometric ratio of the bar is varied from 0 to -1/L (=-1/200) with 0.25/L. This means that the width of the free end of the bar is gradually decreased. But width of the clamped end of the bar is always taken as constant. When δ is equal to zero, the cross-sectional area of the bar does not change. It can be seen from the figure that the natural frequency of the bar increases gradually with decreasing the width of the free end of the bar. If high natural frequency is desired, it is more advantageous to use the narrowing bar. Another advantage of the narrowing bar is that less material is used in the bar. The theoretical and numerical results are shown in Figure 5, and they are also in good agreement.

4.3 The Effects of Slenderness Ratio (Narinlik Oranının Etkileri)

The ratio of the width to the length of the bar is defined as slenderness ratio in this study. To see the effect of slenderness ratio of the bar, it is taken 1/10, 1/12.5, 1/15, 1/17.5 and 1/20. In order to get these ratios, the length of the bar is increased from 200 mm to 400 mm at intervals of 50 mm while the width was fixed at 20 mm. The geometric ratio is 0 and the temperature is 22 °C. The variation of the natural frequency of the bar with slenderness ratio is shown in Figure 6.

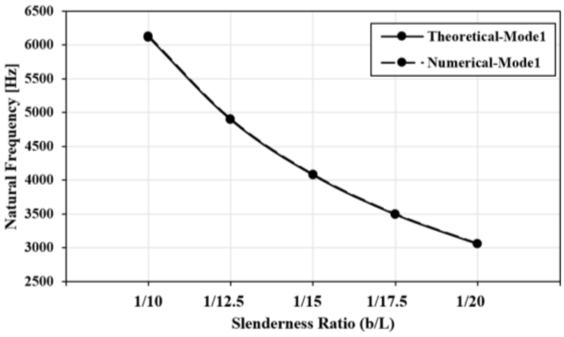


Figure 6. The effect of slenderness ratio on natural frequency (Doğal frekans üzerine narinlik oranın etkisi)

It can be seen from the Figure 6 that the natural frequency of the bar decreases with decreasing the slenderness ratio. It can also seen from the figure that the curve decreases exponentially. In the other words, the natural frequency decreases with increasing the length of the bar.

4.4 The Effects of Variation in Mode Numbers (Mod Sayılarındaki Değişimin Etkileri)

The first three mode shapes are taken into consideration in the study. The normalized mode shapes of the bar with uniform cross-section are given in Figure 7.

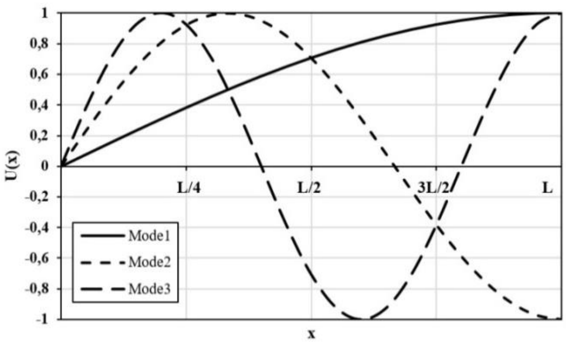


Figure 7. Mode shapes of the bar (Çubuğun mod şekilleri)

These three longitudinal mode shapes are also obtained from SolidWorks program and given in Figures 8, 9 and 10. It can be seen from the Figure 7 that there is only one peak in the curve of Mode 1. But, there are one peak and one trough in the curve of Mode 2. As for the curve of Mode 3, there are two peaks and one trough in it. The SolidWorks results shown in Figures 8-10 are supported the graph shown in Figure 7.

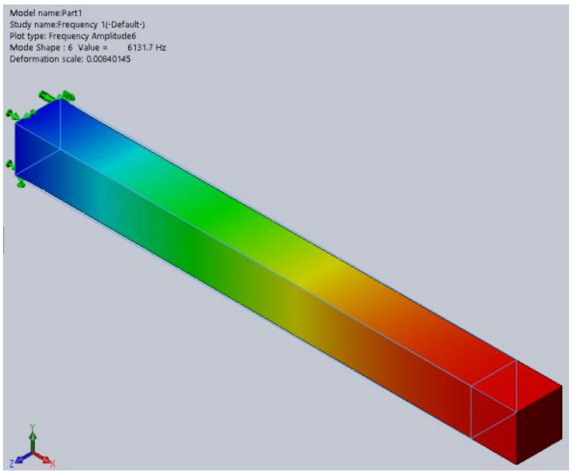


Figure 8. First mode shape (Birinci mod şekli)

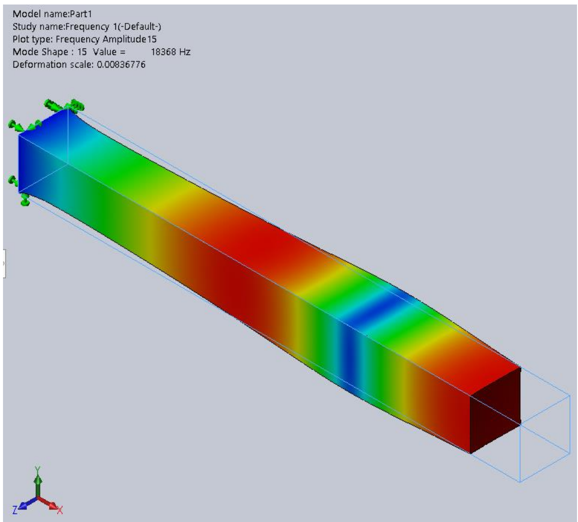


Figure 9. Second mode shape (İkinci mod şekli)

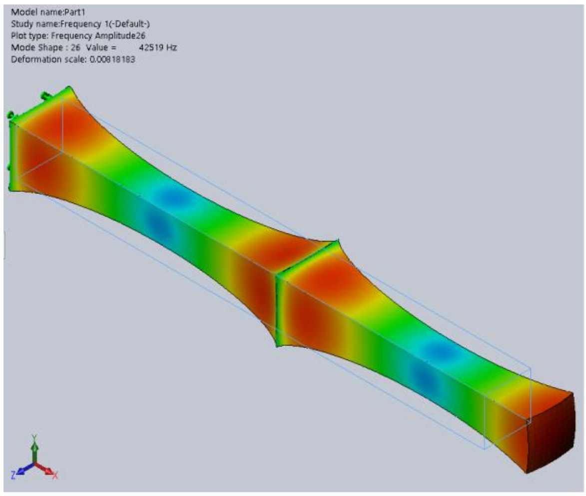


Figure 10. Third mode shape (Üçüncü mod şekli)

According to the results obtained from Figures 7-10, the results obtained from SolidWorks are in agreement with the theoretical results.

Moreover, the variation of the natural frequency of the bar with mode numbers is depicted in Figure 11.

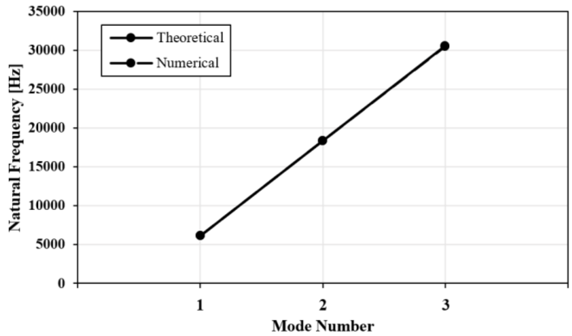


Figure 11. The effect of mode number on natural frequency (Doğal frekans üzerine mod sayısının etkisi)

In this figure, the geometric and the slenderness ratios of the bar are taken 0 and 1/10, respectively. Besides, the temperature is taken as 22 °C. As expected, the natural frequency increases gradually with increasing the mode number. It can be seen in the figure that the difference is very large when the results obtained for modes 1 and 3 are compared. Moreover, this increase appears to be almost linear.

In addition to Figure 11, Figure 12 shows the effect of temperature on the natural frequency as well as the number of modes.

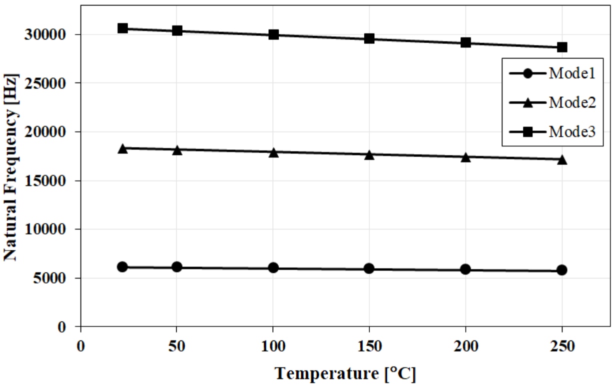


Figure 12. The effects of temperature and mode number on natural frequency (Doğal frekans üzerine sıcaklık ve mod sayısının etkileri)

In this figure, the geometric and the slenderness ratios of the bar are taken as 0 and 1/10, respectively. As mentioned before (in Section 4.1), the natural frequency of the bar decreases with increasing the temperature. But, it is seen in Figure 12 that this decrease is more for Mode 3. This means that the effect of temperature on natural frequency increases with increasing the number of modes.

The increase in natural frequency with the increase in the number of modes is also seen in Figure 12 similar to Figure 11.

5. CONCLUSIONS (SONUÇLAR)

The theoretical free axial vibration analysis of the temperature dependent bar with variable cross-section is investigated in this study. Furthermore, the results obtained theoretical solutions are supported by those obtained from the SolidWorks program. The effects of temperature, geometric and slenderness ratios and mode numbers on the natural frequency of the bar are studied and the following conclusions are drawn from the study.

- The natural frequency of the bar decreases with increasing the temperature.
- The natural frequency of the bar increases as the section of the bar narrows.
- The natural frequency of the bar decreases with decreasing the slenderness ratio.
- The natural frequency increases gradually with increasing the mode number and this increase is linear.

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