GU J Sci 31(2): 544-553 (2018)

Gazi University

Journal of Science

http://dergipark.gov.tr/gujs

Bernstein Series Approximation for Dirichlet Problem

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Article Info

Received: 30/06/2016
Accepted: 28/11/2017

Keywords

Dirchlet problem
Collocation method
Bernstein series solution
Error analysis

Abstract

The basic aim of this paper is to present a novel efficient matrix approach for solving the Dirichlet problem. The method converts the Dirichlet problem to a matrix equation, which corresponds to a system of linear algebraic equations. Error analysis is included to demonstrate the validity and applicability of the technique.

1. INTRODUCTION

The Dirichlet problem is to find a function $U(z)$ that is harmonic in a bounded domain $D \subset R^2$, is continuous up to the boundary ∂D of D, assumes the specified values $U_0(z)$ on the boundary ∂D, where $U_0(z)$ is a continuous function on ∂D.

Laplace's equation is one of the most significant equations in physics. It is the solution to problems in a wide variety of fields including thermodynamics and electrodynamics. Today, the theory of complex variables is used to solve problems of heat flow, fluid mechanics, aerodynamics, electromagnetic theory and practically every other field of science and engineering. A broad class of steady-state physical problems can be reduced to find the harmonic functions that satisfy certain boundary conditions. The Dirichlet problem for the Laplace equation is one of the above mentioned problems.

The Chebyshev Tau technique was studied for the solution of Laplace's equation by Ahmadi and Adibi [1]. The Dirichlet problem was solved for some regions by Kurt et al. [9-11]. Also, Chebyshev polynomial approximation was employed for Dirichlet problem in [12]. The analytic solution for two-dimensional heat equation in some regions was expressed by Baykus Savasaneril et al. [2-4,6]. Chebyshev tau matrix method for Poisson-type equations in irregular domain and error analysis of the Chebyshev collocation method for linear second-order partial differential equations were given by Kong et al. [7] and by Yuksel et al. [13,14]. Gas Dynamics equation arising in shock fronts and solution of conformable fractional partial differential equations by using the reduced differential transform method were studied in [8].

In this paper, we have employed a matrix method, which is based on Bernstein polynomials and collocation points. Let us consider the Dirichlet problem on $D = [0,a] \times [0,b]$,

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$$\nabla^2 U = 0, \quad z \in D, \quad U \big|_{z\in\partial D} = U_0(z). \tag{1}$$

Here, for a point (x, y) in the plane R^2, one takes the complex notation $z = x + yi$, $U(z) = U(x, y)$ and $U_0(z) = U_0(x, y)$ are real functions and $\nabla^2 U = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$ is the Laplace operator. Similarly the Dirichlet problem for the Poisson equation can be formulated as

$$\nabla^2 U = G(x, y) \tag{2}$$

with the conditions defined at the points $x = \alpha_k, \quad y = \beta_k$ for $(\alpha_k, \beta_k) \in \partial D, \quad a^k_{i,j}$, $k = 1,...,t$ and λ_t are constants,

$$\sum_{k=1}^{t}\sum_{i=0}^{1}\sum_{j=0}^{1} a^k_{i,j}\, U^{(i,j)}(\alpha_k, \beta_k) = \lambda_t .$$

Here, $G(x, y)$ are functions defined on D. We will find an approximated solution, namely Bernstein series solution of (2) such that

$$U_{n,n}(x, y) = \sum_{i=0}^{n}\sum_{j=0}^{n} a_{ij}\ \boldsymbol{B}_{i,n}\ (x)\ \boldsymbol{B}_{j,n}(y) \tag{3}$$

where $B_{k,n}, (0 \le k \le n)$ are Bernstein polynomials.

2. FUNDAMENTAL RELATION

Let $P_{n,n}$ be Bernstein series solution of (2). Let us find the matrix form of $p_{n,n}$ and $p^{(i,j)}_{n,n} = \dfrac{\partial^{i+j} p_{n,n}}{\partial x^i \partial y^j}$.

$p_{n,n}$ can be written as

$$p_{n,n}(x, y) = \boldsymbol{B}_n(x)\ \boldsymbol{Q}_n(y)\ \boldsymbol{A} \tag{4}$$

where

$$\boldsymbol{B}_n(x) = \begin{bmatrix} B_{0,n}(x) & B_{1,n}(x) & ... & B_{n,n}(x) \end{bmatrix},$$

$$\boldsymbol{Q}_n(y) = \begin{bmatrix} B_n(y) & 0 & ... & 0 \\ 0 & B_n(y) & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & ... & B_n(y) \end{bmatrix},$$

and

$$\boldsymbol{A} = \begin{bmatrix} a_{00} & a_{01} & \cdots & a_{0n} & a_{10} & a_{11} & \cdots & a_{1n} & \cdots & a_{n0} & a_{n1} & \cdots & a_{nn} \end{bmatrix}.$$

Hence, $P^{(i,j)}_{n,n}$ can be written as

$$P_{n,n}^{(i,j)}(x,y) = \boldsymbol{B}_n^i(x)\ \boldsymbol{Q}_n^j(y)\ \boldsymbol{A}\,.$$

On the other hand, $\boldsymbol{B}_n^i(x)$ can be written as [5]

$$\boldsymbol{B}_n^i(x) = \boldsymbol{X}^{(i)}(x)\ \boldsymbol{D}^T \tag{5}$$

where

$$\boldsymbol{D} = \begin{bmatrix} d_{00} & d_{01} & \cdots & d_{0n} \\ d_{10} & d_{11} & \cdots & d_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{n0} & d_{n1} & \cdots & d_{nn} \end{bmatrix} \qquad d_{ij} = \begin{cases} \dfrac{(-1)^{j-i}}{R^j}\dbinom{n}{j}\dbinom{n-i}{j-i} & ,i \le j \\ 0 & ,i > j \end{cases}$$

$$\boldsymbol{X}\ (x) = \begin{bmatrix} 1 & x & \cdots & x^n \end{bmatrix},$$

for $\boldsymbol{X}^{(i)}(x)$, the relation

$$\boldsymbol{X}^{(k)} = \boldsymbol{X}\ (x)\ \boldsymbol{B}^k \tag{6}$$

is obtained where

$$\boldsymbol{B} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 2 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & n \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

By substituting (6) into (5), we get

$$\boldsymbol{B}_n^{(i)}(x) = \boldsymbol{X}\ (x)\ \boldsymbol{B}^i\ \boldsymbol{D}^T\,. \tag{7}$$

If a similar prodecude is carried out, the relation $\boldsymbol{Q}_n(y) = \overline{\boldsymbol{Y}}\ (y)\ \overline{\boldsymbol{D}}$ will be obtained as

$$\overline{\boldsymbol{Y}}(y) = \begin{bmatrix} Y(y) & 0 & \cdots & 0 \\ 0 & Y(y) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Y(y) \end{bmatrix},\ \boldsymbol{Y}\ (y) = \begin{bmatrix} 1 & y & \cdots & y^n \end{bmatrix},\ \overline{\boldsymbol{D}} = \begin{bmatrix} D^T & 0 & \cdots & 0 \\ 0 & D^T & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & D^T \end{bmatrix}.$$

Thus, $\overline{\boldsymbol{Y}}^{(j)}(y)$ can be written as

$$\overline{\boldsymbol{Y}}^{(j)}(y) = \overline{\boldsymbol{Y}}\ (y)\ \overline{\boldsymbol{B}}^{\,j}, \tag{8}$$

where

$$\overline{\boldsymbol{B}} = \begin{bmatrix} B & 0 & \cdots & 0 \\ 0 & B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & B \end{bmatrix}.$$

By putting (7) and (8) into (4), we obtain the matrix form of $P_{n,n}^{(i,j)}(x,y)$ as

$$P_{n,n}^{(i,j)}(x,y) = \boldsymbol{X}(x)\,\boldsymbol{B}^{i}\,\boldsymbol{D}^{T}\,\overline{\boldsymbol{Y}}(y)\,\overline{\boldsymbol{B}}^{j}\,\overline{\boldsymbol{D}}\,\boldsymbol{A}. \tag{9}$$

By substituting (4) and (9) to (2), we obtain fundamental matrix equation as

$$[P(x,y)\,\boldsymbol{X}(x)\,\boldsymbol{B}^{2}\,\boldsymbol{D}^{T}\,\overline{\boldsymbol{Y}}(y)\,\overline{\boldsymbol{D}}\,\boldsymbol{A} + R(x,y)\,\boldsymbol{X}(x)\,\boldsymbol{B}\,\boldsymbol{D}^{T}\,\overline{\boldsymbol{Y}}(y)\,\overline{\boldsymbol{B}}\,\overline{\boldsymbol{D}}$$

$$+S(x,y)\,\boldsymbol{X}(x)\,\boldsymbol{D}^{T}\,\overline{\boldsymbol{Y}}(y)\,\overline{\boldsymbol{B}}^{2}\,\overline{\boldsymbol{D}} + T(x,y)\,\boldsymbol{B}\,\boldsymbol{D}^{T}\,\overline{\boldsymbol{Y}}\,\overline{\boldsymbol{D}}$$

$$+U(x,y)\,\boldsymbol{X}(x)\,\boldsymbol{D}^{T}\,\overline{\boldsymbol{Y}}(y)\,\overline{\boldsymbol{B}}\,\overline{\boldsymbol{D}} + V(x,y)\,\boldsymbol{X}(x)\,\boldsymbol{D}^{T}\,\overline{\boldsymbol{Y}}(y)\,\overline{\boldsymbol{D}}]\,\boldsymbol{A} = G(x,y). \tag{10}$$

Bu using the collocation points $\{(x_i, y_j) : 0 \le i, j \le n\}$ in (9), one obtains a matrix $\mathbf{W}_{(n+1)^2 \times (n+1)^2}$ whose m-th row, $1 \le m \le (n+1)^2$, comes from $(x_k, y_l), k = \left[\left|\frac{m}{n+1}\right|\right], l = m - k(n+1) - 1$. Similarly, $\mathbf{G}$ is column matrix such as $[\mathbf{G}]_{1m} = G(x_t, y_l), t = \left[\left|\frac{m}{n+1}\right|\right], l = m - t(n+1) - 1$. Thus, a linear system is obtained as

$$\mathbf{WA} = \mathbf{G}. \tag{11}$$

We investigate the matrix forms for the conditions in three parts. By using (4) and (9), matrix relations are obtained for the conditions

$$\boldsymbol{C}\;\boldsymbol{A}\;\sum_{k=0}^{t}\sum_{i=0}^{1}\sum_{j=0}^{1} a_{i,j}^{k}\,\boldsymbol{X}(\alpha_t)\,\boldsymbol{B}^{i}\,\boldsymbol{D}^{T}\,\overline{\boldsymbol{Y}}(\beta_t)\,\overline{\boldsymbol{B}}^{j}\,\overline{\boldsymbol{D}}\,\boldsymbol{A} = \lambda_t \tag{12}$$

respectively. Let us write (11) as

$$\boldsymbol{C}\;\boldsymbol{A} = \boldsymbol{G}_1$$

where $[\boldsymbol{G}_1]_{t1} = \lambda_t$. By combining $[\boldsymbol{W}, \boldsymbol{G}]$ and $[\boldsymbol{C}, \boldsymbol{G}_1]$, it follows a new system $[\tilde{\boldsymbol{W}}; \tilde{\boldsymbol{G}}]$:

$$[\tilde{\boldsymbol{W}}; \tilde{\boldsymbol{G}}] = \begin{bmatrix} \boldsymbol{W} & , & \boldsymbol{G} \\ \boldsymbol{C} & , & \boldsymbol{G}_1 \end{bmatrix}. \tag{13}$$

By using the Gauss elimination method and removing zero rows of augmented matrix $[\tilde{\boldsymbol{W}}; \tilde{\boldsymbol{G}}]$, if $\tilde{\boldsymbol{W}}$ is a square matrix, then the unknown matrix $\boldsymbol{A}$ is obtained as

$$\boldsymbol{A} = \tilde{\boldsymbol{W}}^{-1}\tilde{\boldsymbol{G}}. \tag{14}$$

The collocation points should be changed such that $\dim(\tilde{\boldsymbol{W}}) = (n+1)^2$. Also, if the columns of $\tilde{\boldsymbol{W}}$ are linearly independent, then the matrix $\boldsymbol{A}$ can be calculated by the pseudoinverse method; that is,

$$\tilde{\boldsymbol{W}}^{+} = \left(\tilde{\boldsymbol{W}}^{*}\tilde{\boldsymbol{W}}\right)^{-1}\tilde{\boldsymbol{W}}^{*} \tag{15}$$

where $\tilde{\boldsymbol{W}}^{*}$ is the transpose of $\tilde{\boldsymbol{W}}$.

3. ACCURACY OF THE SOLUTION AND ERROR ANALYSIS

We can easily check the accuracy of the solution. When the function $P(x,y)$ and its derivatives are substituted in Eq. (1), the obtained equation satisfy approximately;
for $(x,y)=(x_q,y_q)\in\left\{0\le x_q\le a,\quad 0\le y_q\le b\right\}$ $\left(q=0,1,2,...\right)$,

$E(x_q,y_q)=\left|D(x_q,y_q)-\lambda I(x_q,y_q)\right|\cong 0$ and $E(x_q,y_q)\le 10^{-k_q}$ (k_q positive integer).

If $\max 10^{-k_q}=10^{-k}$ (k positive integer) is prescribed, then the truncation limit N is increased, $E(x_q,y_q)$ becomes smaller than the prescribed 10^{-k}. The obtained error can be estimated by the function

$$E_N\left(x,y\right)=\sum_{r=0}^{N}\sum_{s=0}^{N}a_{r,s}T_{r,s}(x,y),-g(x,y)-I(x,y).$$

As $E_N(x,y)\to 0$ and N is sufficiently large enough, then the error decreases. Hence, we can determine the accuracy of the solution.

4. NUMERICAL EXAMPLE

In this section, a numerical example has been given to illustrate the efficiency of the method. Also, we have performed a computer program written on Maple, in order to solve this example.

4.1. Example

Let us consider the following Laplace equation with Dirichlet boundary conditions

$$\begin{aligned}&\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0\\ &u(x,0)=0\,,\ u(x,K)=1 \qquad 0<y<K\\ &u(0,y)=u(K,y)=0 \qquad 0<x<K\end{aligned} \tag{16}$$

the fundamental matrix equation for (16) is obtained as

$$[\,\boldsymbol{X}\ (x)\ \boldsymbol{B}^{\,2}\ \boldsymbol{D}^{\,T}\ \overline{\boldsymbol{Y}}\ (y)\ \overline{\boldsymbol{D}}\ +\ \boldsymbol{X}\ (x)\ \boldsymbol{D}^{\,T}\ \overline{\boldsymbol{Y}}\ (y)\ \overline{\boldsymbol{B}}^{\,2}\ \overline{\boldsymbol{D}}\,]\,\boldsymbol{A}=0\,.$$

Let the collocation points be the Chebyshev interpolation nodes

$$\left\{\left(x_i,y_j\right):\quad 0\le i,\quad j\le n,\quad x_i=\frac{1}{2}+\frac{1}{2}\cos\left(\frac{2i-1}{2n}\right)\pi\,,\quad y_i=\frac{1}{2}+\frac{1}{2}\cos\left(\frac{2i-1}{2n}\right)\pi\right\}$$

or equidistant nodes. Then, **W** is a matrix rows are

$$\boldsymbol{X}\ \boldsymbol{B}^{2}\ \boldsymbol{D}^{T}\ \overline{\boldsymbol{Y}}\ (y_j)\ \overline{\boldsymbol{D}} + \boldsymbol{X}\ (x_i)\ \boldsymbol{D}^{T}\ \overline{\boldsymbol{Y}}\ (y_j)\ \overline{\boldsymbol{B}}^{2}\ \overline{\boldsymbol{D}}$$

and **G** is a zero matrix. The condition matrices for $u(x,0)=0$, $u(x,K)=1$, $u(0,y)=0$, and $u(K,y)=0$ are obtained as

$$p_{n,n}(x_i,0) = \boldsymbol{X}\ (x_i)\ \boldsymbol{D}^{T}\ \overline{\boldsymbol{Y}}\ (0)\ \overline{\boldsymbol{D}}\ \boldsymbol{A} = 0 \qquad i=0,1,...,n$$

$$p_{n,n}(x_i,K) = \boldsymbol{X}\ (x_i)\ \boldsymbol{D}^{T}\ \overline{\boldsymbol{Y}}\ (K)\ \overline{\boldsymbol{D}}\ \boldsymbol{A} = 1 \qquad i=0,1,...,n$$

$$p_{n,n}(0,y_i) = \boldsymbol{X}\ (0)\ \boldsymbol{D}^{T}\ \overline{\boldsymbol{Y}}\ (y_i)\ \overline{\boldsymbol{D}}\ \boldsymbol{A} = 0 \qquad i=0,1,...,n$$

$$p_{n,n}(K,y_i) = \boldsymbol{X}\ (K)\ \boldsymbol{D}^{T}\ \overline{\boldsymbol{Y}}\ (y_i)\ \overline{\boldsymbol{D}}\ \boldsymbol{A} = 0 \qquad i=0,1,...,n$$

Combining these matrices gives the augmented matrix $\left[\tilde{\boldsymbol{W}};\tilde{\boldsymbol{G}}\right]$. By calculating the coefficient matrix $[\boldsymbol{A}]$, Bernstein series solutions are obtained for different n values. For N=5, we obtain

$$\boldsymbol{X}\ (x) = \begin{bmatrix} 1 & x & x^2 & x^3 & x^4 & x^5 \end{bmatrix}_{1x6},$$

$$\boldsymbol{B} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{6x6}, \quad \overline{\boldsymbol{B}} = \begin{bmatrix} B & 0 & 0 & 0 & 0 & 0 \\ 0 & B & 0 & 0 & 0 & 0 \\ 0 & 0 & B & 0 & 0 & 0 \\ 0 & 0 & 0 & B & 0 & 0 \\ 0 & 0 & 0 & 0 & B & 0 \\ 0 & 0 & 0 & 0 & 0 & B \end{bmatrix}_{36x36},$$

$$\boldsymbol{D} = \begin{bmatrix} 1 & \frac{(-1)^1}{(K)^1}\binom{5}{0}\binom{5}{1} & \frac{(-1)^2}{(K)^2}\binom{5}{0}\binom{5}{2} & \frac{(-1)^3}{(K)^3}\binom{5}{0}\binom{5}{3} & \frac{(-1)^4}{(K)^4}\binom{5}{0}\binom{5}{4} & \frac{(-1)^5}{(K)^5}\binom{5}{0}\binom{5}{5} \\ 0 & \frac{(-1)^0}{(K)^1}\binom{5}{1}\binom{4}{0} & \frac{(-1)^1}{(K)^2}\binom{5}{1}\binom{4}{1} & \frac{(-1)^2}{(K)^3}\binom{5}{1}\binom{4}{2} & \frac{(-1)^3}{(K)^4}\binom{5}{1}\binom{4}{3} & \frac{(-1)^4}{(K)^5}\binom{5}{1}\binom{4}{4} \\ 0 & 0 & \frac{(-1)^0}{(K)^2}\binom{5}{2}\binom{3}{0} & \frac{(-1)^1}{(K)^3}\binom{5}{2}\binom{3}{1} & \frac{(-1)^2}{(K)^4}\binom{5}{2}\binom{3}{2} & \frac{(-1)^3}{(K)^5}\binom{5}{2}\binom{3}{3} \\ 0 & 0 & 0 & \frac{(-1)^0}{(K)^3}\binom{5}{3}\binom{2}{0} & \frac{(-1)^1}{(K)^4}\binom{5}{3}\binom{2}{1} & \frac{(-1)^1}{(K)^1}\binom{5}{4}\binom{2}{2} \\ 0 & 0 & 0 & 0 & \frac{(-1)^0}{(K)^4}\binom{5}{4}\binom{1}{0} & \frac{(-1)^1}{(K)^5}\binom{5}{4}\binom{1}{1} \\ 0 & 0 & 0 & 0 & 0 & \frac{(-1)^0}{(K)^5}\binom{5}{5}\binom{0}{0} \end{bmatrix}_{6x6}$$

$$\overline{\boldsymbol{D}} = \begin{bmatrix} D^T & 0 & 0 & 0 & 0 & 0 \\ 0 & D^T & 0 & 0 & 0 & 0 \\ 0 & 0 & D^T & 0 & 0 & 0 \\ 0 & 0 & 0 & D^T & 0 & 0 \\ 0 & 0 & 0 & 0 & D^T & 0 \\ 0 & 0 & 0 & 0 & 0 & D^T \end{bmatrix}_{36x36}, \overline{\boldsymbol{Y}}\ (y) = \begin{bmatrix} Y(y) & 0 & 0 & 0 & 0 & 0 \\ 0 & Y(y) & 0 & 0 & 0 & 0 \\ 0 & 0 & Y(y) & 0 & 0 & 0 \\ 0 & 0 & 0 & Y(y) & 0 & 0 \\ 0 & 0 & 0 & 0 & Y(y) & 0 \\ 0 & 0 & 0 & 0 & 0 & Y(y) \end{bmatrix}_{6x36}$$

$$\boldsymbol{Y}\ (y) = \begin{bmatrix} 1 & y & y^2 & y^3 & y^4 & y^5 \end{bmatrix}_{1x6}$$

and then, as a result, we get the following error analysis illustrated in figures and tables.

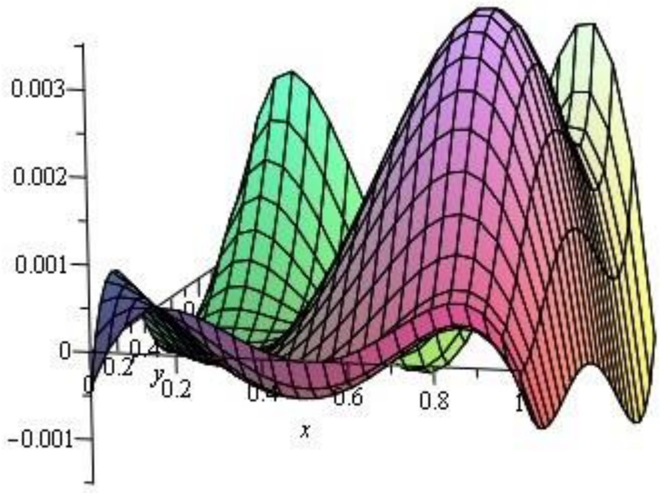

Figure 1. *Error analysis for N=5*

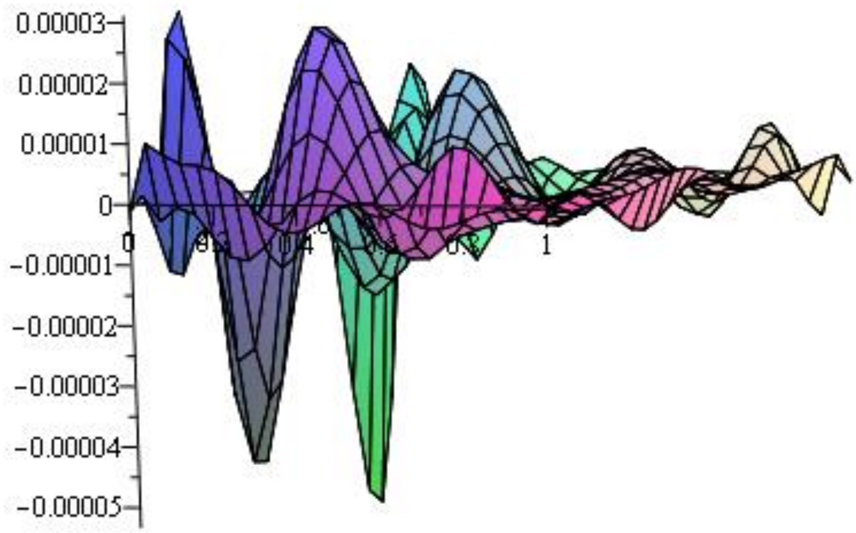

Figure 2. *Error analysis for N=10*

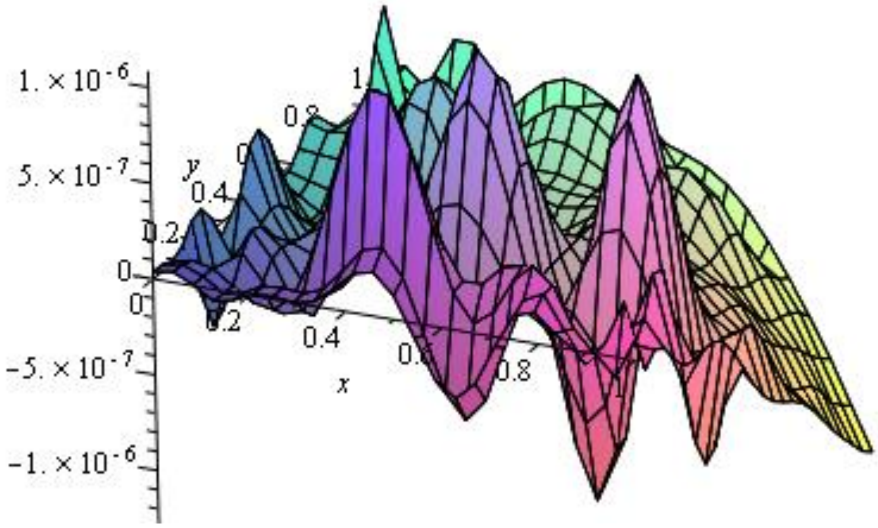

Figure 3. *Error analysis for N=12*

Table 1. *Comparison of the error analysis on* ∂D *that is the boundary of* D *for different values of N for example* $D:(0<x<K,\quad 0<y<K)$

x	y	N=5	N=7	N=9	N=10	N=12
0	1	$-2.4403\,10^{-3}$	$1.5690\,10^{-4}$	$2.161\,10^{-5}$	$8.9\,10^{-8}$	$1.05\,10^{-7}$
0	0.9	$-2.0438\,10^{-2}$	$1.6914\,10^{-4}$	$-2.3880\,10^{-5}$	$-3.98\,10^{-7}$	$-3.90\,10^{-7}$
0	0.8	$-1.6157\,10^{-3}$	$-1.1162\,10^{-4}$	$-1.9550\,10^{-5}$	$3.7436\,10^{-8}$	$3.7436\,10^{-8}$
0	0.7	$-7.4720\,10^{-3}$	$-6.1234\,10^{-4}$	$2.195\,10^{-6}$	$1.584\,10^{-7}$	$1.60\,10^{-7}$
0	0.6	$-1.6648\,10^{-3}$	$-1.1537\,10^{-3}$	$5.6349\,10^{-6}$	$-1.40\,10^{-9}$	$9\,10^{-10}$
0	0.5	$-6.4541\,10^{-4}$	$-8.5508\,10^{-4}$	$-1.2969\,10^{-4}$	$-2.62\,10^{-9}$	$-2.6\,10^{-9}$
0	0.4	$-2.2257\,10^{-3}$	$-1.9680\,10^{-4}$	$-1.8373\,10^{-4}$	$-1.0338\,10^{-7}$	$-1.0362\,10^{-7}$
0	0.3	$-2.9730\,10^{-3}$	$6.9763\,10^{-5}$	$-4.1595\,10^{-5}$	$-9.3988\,10^{-8}$	$-9.387\,10^{-8}$
0	0.2	$-8.0039\,10^{-4}$	$-1.3218\,10^{-4}$	$4.7309\,10^{-5}$	$1.6025\,10^{-7}$	$1.6031\,10^{-7}$
0	0.1	$2.4436\,10^{-3}$	$-1.0054\,10^{-4}$	$-1.7810\,10^{-5}$	$1.1802\,10^{-8}$	$1.1826\,10^{-8}$
0	0	$-1.6157\,10^{-3}$	$-1.1162\,10^{-4}$	$-1.9550\,10^{-5}$	$3.7436\,10^{-8}$	$3.7436\,10^{-8}$

It is obvious from Table 1 and Figures 1-3 that the error results get better, as N is increased.

Table 2. *Comparison of the error analysis in domain* $D:(0<x<K,\quad 0<y<K)$ *for N=5,7,9,10,12*

x	y	N=5	N=7	N=9	N=10	N=12
1	1	$2.9900\,10^{-4}$	$1.2359\,10^{-4}$	$-6.9849\,10^{-4}$	$7.5900\,10^{-7}$	$-1.1100\,10^{-5}$
0.5	0.5	$3.9486\,10^{-3}$	$-6.7268\,10^{-4}$	$1.5599\,10^{-5}$	$6.1000\,10^{-9}$	$-5.2350\,10^{-8}$
0.2	0.8	$1.1137\,10^{-3}$	$-1.4905\,10^{-4}$	$-6.6864\,10^{-6}$	$5.8962\,10^{-6}$	$4.2800\,10^{-10}$
0.1	0.7	$5.1846\,10^{-3}$	$-6.6201\,10^{-5}$	$-6.5836\,10^{-5}$	$-5.7451\,10^{-6}$	$1.3544\,10^{-7}$
0.6	0.6	$1.0404\,10^{-2}$	$-1.2613\,10^{-4}$	$1.1713\,10^{-6}$	$-3.3402\,10^{-6}$	$-1.4990\,10^{-7}$
0.3	0.2	$-3.8989\,10^{-3}$	$5.0266\,10^{-4}$	$6.1778\,10^{-6}$	$2.5621\,10^{-5}$	$-9.5388\,10^{-8}$
1	0.4	$-2.0282\,10^{-4}$	$1.9284\,10^{-4}$	$1.8854\,10^{-4}$	$3.1996\,10^{-6}$	$-4.8500\,10^{-7}$

0.8	0.3	$-6.5765\,10^{-3}$	$-3.0576\,10^{-4}$	$4.9226\,10^{-5}$	$-1.9480\,10^{-7}$	$4.5170\,10^{-7}$
0.2	0.9	$6.3258\,10^{-3}$	$1.6659\,10^{-5}$	$-2.6873\,10^{-5}$	$-9.7539\,10^{-7}$	$4.4977\,10^{-8}$
0.5	0.7	$6.1834\,10^{-4}$	$-3.5100\,10^{-4}$	$-2.7896\,10^{-5}$	$-3.7444\,10^{-6}$	$2.7140\,10^{-7}$

The some calculated values of the error analysis are given in Table 2. It is clearly seen that when the values of N increase, error function values rapidly decrease for N=5,7,9,10 and 12.

5. CONCLUSION

In this study, a technique has been developed for solving Laplace's equation with Dirichlet boundary condition. We introduce a new matrix method depending on Bernstein polynomials and collocation points. Present method provides two main advantes: it is very simple to construct the main matrix equations and it is very easy for computer programming.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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