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THERMAL STRESS ANALYSIS OF AXIALLY LAYERED FUNCTIONALLY GRADED BEAMS USING FINITE ELEMENT AND TAGUCHI METHODS

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ABSTRACT

The aim of this paper is to investigate the thermal stress analysis of axially layered functionally graded beams under clamped-clamped (C-C) boundary conditions using finite element software which named ANSYS V13 parametric design language (APDL). The beams were made as three layers using functionally graded materials with Zirconia (ceramic) and Aluminum (metal) systems. The order of the layers of the beams was designed using L_9 orthogonal array depending on Taguchi Method and thus nine different beam configurations were used. Analysis of signal-to-noise (S/N) ratio was performed to determine the effects of optimum levels of layers on the thermal stress characteristic. Analysis of Variance (ANOVA) was evaluated to carry out the meaningful layers and the percentage contributions of the layers on the thermal stress response. According to the results, the most effective layers on responses were found to be Layer 1 with 75.70 % contribution, Layer 2 with 21.05 % contribution, and Layer 3 with 3.23 % contribution respectively. In addition, the increase of Young's modulus and thermal expansion values of the layers lead to the thermal stress results of axially layered functionally graded beams. This paper can be determined as a reference for thermal stress analysis of the axially layered beams produced functionally graded materials consists of ceramic and metal contents under clamped-clamped boundary conditions.

Keywords: Thermal stress, Functionally graded materials (FGMs), Beam, Finite element approach

1. INTRODUCTION

Thermal stress can be performed based on any change of temperature of a material such as metal or ceramic and thus the plastic deformation of the material may occurred based on the level of the increase of temperature. In the various applications of mechanical engineering, thermal stress analyses of columns and beams can generally have important effects in terms of environmental conditions in order to improve the utilization efficiency. The columns and beams can be produced using various materials such as metal, composite, ceramic, functionally graded materials etc. Recently years, the functionally graded materials have been investigated for various applications of different engineering areas. Functionally graded materials (FGMs) named as new class among composite materials [1], was offered as a means in order to design thermal barrier materials for high temperatures by some scientists working in the material fields [2]. In other words, the concept of FGMs were primarily produced and improved to resist high temperature gradients [3]. An ordinary FGM is performed using a mixture including the ceramic and the metal materials [4]. In FGMs made of ceramic and metal materials, area with ceramic content and the area consisting of metal content provide good thermal resistance and superior fracture toughness, respectively [5]. In recently years, several studies including thermal analyses using FGM have been made. Sankar and Tzeng [6] analyzed the thermal stresses of beams designed using functionally graded materials. Giunta, Belouettar and Carrera [7] evaluated the thermal stresses of 3D beams including functionally graded materials and they used the refined one-dimensional models and strong form solutions for analyses. Noda [8] presented a research including thermal stress analyses of functionally graded materials. Eslami, Babaei and Poultangari [9] derived a general solution and they analyzed steady state thermal and mechanical stresses based on one-dimensional using the FGM hollow sphere with thick structure under general types of boundary

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conditions. Cho and Oden [10] reported a parametric research with thermal-stress analyses of functionally graded materials and they used the Crank–Nicolson–Galerkin scheme. Jin and Paulino [11] studied the thermal stress characteristic of a FGM with an edge crack and they used transient thermal loading conditions for analyses. In this study, thermal stress analyses of axially layered functionally graded (FG) beams including ceramic and metal materials were investigated under clamped-clamped (C-C) boundary condition based on uniform temperature. Numerical thermal stress analyses of the axially layered FG beams were solved using the finite element software ANSYS parametric design language (APDL). The order of the layers of the beam configurations was achieved using L_9 orthogonal array according to Taguchi Method and thus nine beam types with various layers contents including different mechanical and thermal properties were used. Each beam configuration is different from others. The material properties, such as mechanical and thermal properties, of the axially layered FG beams were changed from an end to the other end in axial direction and so increase of the contents in the layers is different with each other. Predicted result obtained using Taguchi Method was compared with result determined using ANSYS software. This study provides a different contribution on thermal stress analysis of axially layered functionally graded beams under clamped-clamped (C-C) boundary conditions using finite element software ANSYS and Taguchi Method. Finite element approach were carried out in order to observe the maximum thermal stress results for each axially layered functionally graded beam according to von Mises stress.

2. MATERIALS AND METHODS

The FGMs are generally produced using ceramic and metal materials. In this study, two different material as Zirconia (ceramic) and Aluminum (metal) were used for analyses of thermal stress. The Poisson ratio for each material was held constant as 0.3 value. Thermal and mechanical properties of ceramic and metal materials were shown in Table 1.

Table 1. Material properties of Zirconia and Aluminium [12]

Property	Symbol	Unit	Zirconia	Aluminum
Young's modulus	E	GPa	151	70
Thermal expansion	α	$1/^0C$	$10x10^{-6}$	$23x10^{-6}$
Poisson's ratio	ν	-	0.3	0.3

The material properties of the layered FG beam, except Poisson's ratio, were assumed to vary gradually in axial direction according to a simple rule of mixture of composite materials, and thus the beams with non-homogeneous were designed. Layer contents were calculated using Al and ZrO_2 materials according to a simple rule of mixture of composite materials. The layers were determined depending on the increase of 4% ZrO_2 material. Content in each layer has different mechanical and thermal properties and so the effective material properties (P_{ef}) of layers for axially layered FG beams may be expressed as Equation 1[1].

$$\boldsymbol{P_{ef}} = \sum_{\boldsymbol{j=1}} \boldsymbol{P_j V_{f_j}} \tag{1}$$

in which, Pj and Vfj are defined as the material properties named as mechanical and thermal, and volume fraction for the constituent material j respectively. In addition, the sum of the volume fractions for all the constituent materials can be defined as one according to Equation 2 [1],

$$\sum_{\boldsymbol{j=1}} \boldsymbol{V_{f_j}} = \boldsymbol{1} \tag{2}$$

The material properties of the contents in the layers are determined using Equation 1 and 2. Thus the layers with various material properties according to ceramic and metal materials were assumed as control factors. The control factors and their levels were presented as showing in Table 2.

Table 2. Layers and Levels

Layer	Symbol	Level 1	Level 2	Level 3
Layer 1	L1	4% Zirconia 96% Aluminum	8% Zirconia 92% Aluminum	12% Zirconia 88% Aluminum
Layer 2	L2	16% Zirconia 84% Aluminum	20% Zirconia 80% Aluminum	24% Zirconia 76% Aluminum
Layer 3	L3	28% Zirconia 72% Aluminum	32% Zirconia 68% Aluminum	36% Zirconia 64% Aluminum

The order of the layers for the axially layered FG beam configurations was employed according to Taguchi L9 orthogonal array. This orthogonal array has three control factors. Each control factor occurs three levels and so nine beam configurations with different layers were used for analyses. The beam configurations were demonstrated in Table 3. Each beam configuration was modelled using finite element software named ANSYS V13 and so thermal stress results obtained were analyzed using Minitab 15 software according to "higher is better" quality characteristic as shown in Equation 3 [13].

$$(S/N)_{HB} \text{ for } \sigma_T = -10.\log\left(n^{-1}\sum_{i=1}^{n}(y_i^2)^{-1}\right) \quad (3)$$

in which, n is number of thermal stress analyses of the beams in a trial and yi shows ith data evaluated.

3. FINITE ELEMENT APPROACH

The axially layered FG beams were designed using three different layers and thermal stress analyses of the beams were investigated using finite element software named ANSYS V13 parametric design language. The thermal stress analyses of the beams were determined according to von Mises stress. The BEAM189 element depending on Timoshenko beam theory is used for analyses and is a quadratic 3-node beam element in three dimension [14]. In addition, the element is appropriate for investigating slender to moderately stubby/thick beam structures [14]. Beams has a circular cross-section and 30 mm in diameter. Length of beam is 300 mm and each layer is equal to the others. Reference and uniform temperatures were considered as 27 ^{0}C and 135 ^{0}C respectively. Temperature of the beam is uniformly changed and temperature change (ΔT) is calculated as 108 ^{0}C. The beam has clamped-clamped boundary condition and so UX, UY, UZ, ROTX, ROTY, and ROTZ for each boundary condition are held constant as 0. The axially layered FG beams with clamped-clamped (C-C) boundary condition is demonstrated in Figure 1.

Figure1. Axially layered FG beam with C-C boundary conditions

4. RESULTS AND DISCUSSIONS

In order to investigate the maximum thermal stress results of the axially layered FG beams under uniform temperature, the numerical results were calculated and S/N ratio values of numerical results was calculated. The numerical von Mises stress results obtained based on finite element approach and their S/N ratio values for all the beam configurations are given in Table 3.

Table 3. Numerical thermal stress results and their S/N ratios

Run	Beam Configurations			Results	
				Thermal Stress σ_T (MPa)	**S/N Ratio η (dB)**
1	$(L1)_1$	$(L2)_1$	$(L3)_1$	185.713	45.3768
2	$(L1)_1$	$(L2)_2$	$(L3)_2$	186.821	45.4285
3	$(L1)_1$	$(L2)_3$	$(L3)_3$	187.675	45.4681
4	$(L1)_2$	$(L2)_1$	$(L3)_2$	187.479	45.4591
5	$(L1)_2$	$(L2)_2$	$(L3)_3$	188.529	45.5076
6	$(L1)_2$	$(L2)_3$	$(L3)_1$	188.599	45.5108
7	$(L1)_3$	$(L2)_1$	$(L3)_3$	188.995	45.5290
8	$(L1)_3$	$(L2)_2$	$(L3)_1$	189.199	45.5384
9	$(L1)_3$	$(L2)_3$	$(L3)_2$	190.257	45.5868
Overall Mean ($\overline{\boldsymbol{T}}$)				188.141	

4.1. Evaluation of Layers with Optimum Levels

In order to observe the optimum levels of layers of the axially layered FG beam configurations on the thermal stress analyses, average data and their S/N ratio values for all the layers at the each level using numerical results analyzed was calculated. The average results obtained using Minitab 15 software were demonstrated in Table 4.

Table 4. Responses for average thermal stress results and S/N ratios

Level	S/N ratio (dB)			Means (MPa)		
	L1	**L2**	**L3**	**L1**	**L2**	**L3**
1	45.42	45.45	45.48	186.70	187.40	187.80
2	45.49	45.49	45.49	188.20	188.20	188.20
3	45.55	45.52	45.50	189.50	188.80	188.40
Delta	0.13	0.07	0.03	2.70	1.40	0.60
Rank	1	2	3	1	2	3

According to Table 4, the optimum levels were obtained as third level for each layer of the beams. In other words, the beam configuration designed using third levels of all the layers has the maximum thermal stress value. In addition, Delta is difference between maximum and minimum level of each layer and thus the maximum result were calculated for L1. It was followed by L2 and L3, respectively.

4.2. Influence towards Thermal Stress

In order to study the effects of layers including content with different properties such as mechanical and thermal, average S/N ratio data for all the layers at Level 1, Level 2, and Level 3 were used. These data were taken from Table 4. Main effects plot for S/N ratios based on average thermal stress results was illustrated in Figure 2.

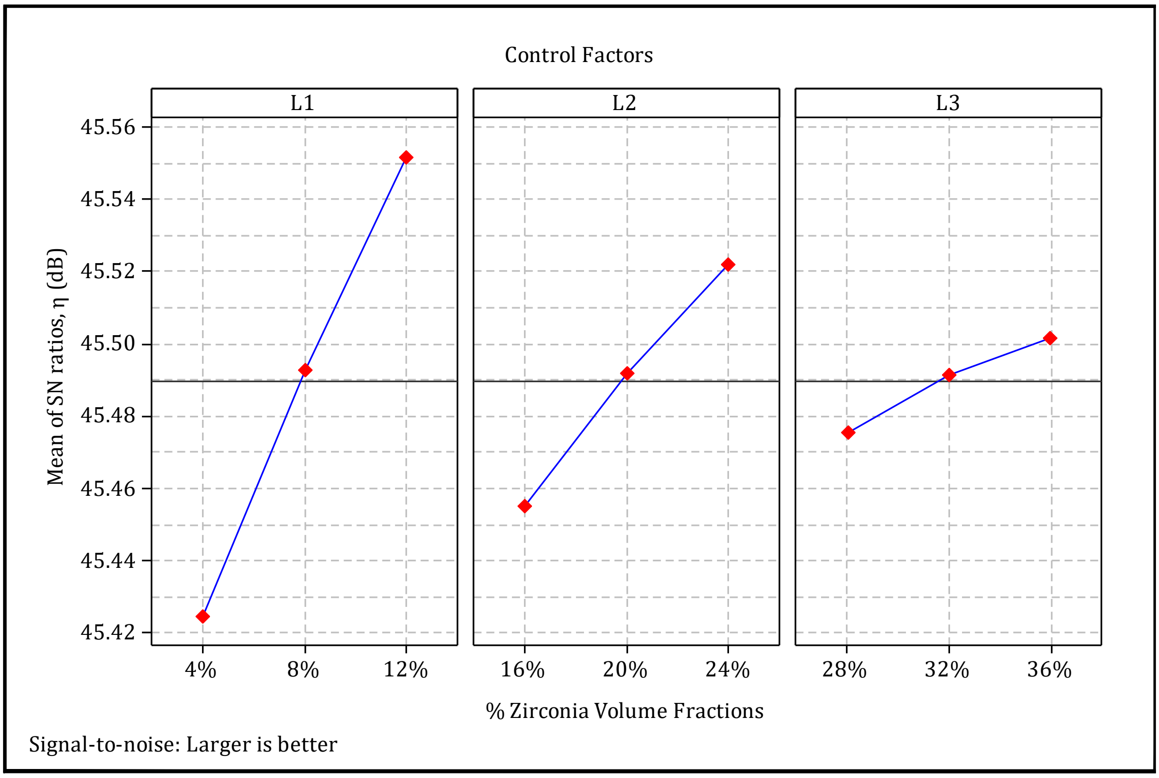

Figure 2. Main influences plot for S/N ratios of average thermal stress results

It is understood from Figure 2 that the layers have positive influences on the thermal stress. The increase of the levels of the layers increases the thermal stress of the axially layered FG beams. In other words, the increase of percent volume fractions of metal materials in the layers decreases the thermal stress. Contour plots for the effect of the interaction of layers such as L1, L2, and L3 on thermal stress analysis of axially layered FG beams are demonstrated in Figure 3.

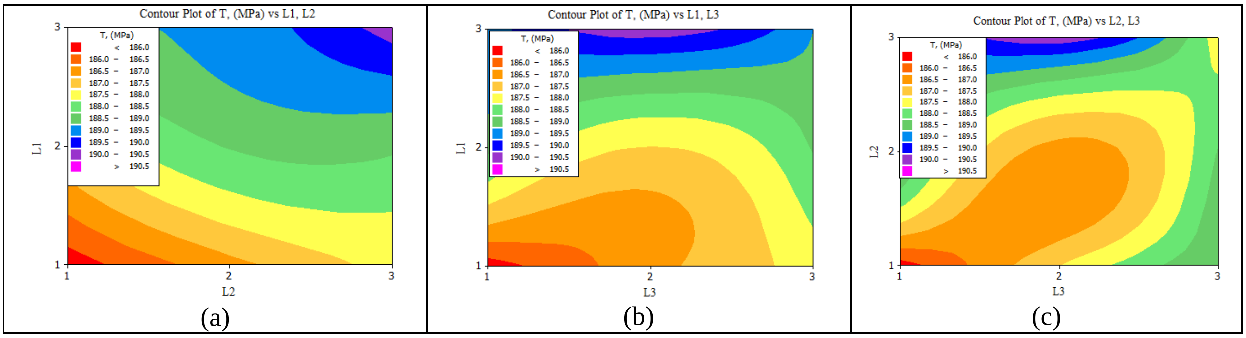

Figure 3. The effect of the interaction of layers for (a) L1 and L2, (b) L1 and L3, (c) L2 and L3

In Figure 3, thermal stress analyses were determined at eleven steps from T < 186.0 MPa to T >190.5 MPa. It is seen that the minimum thermal stress results are obtained using layers at the first level because of thermal stress (T) < 186.0 MPa. This situation is illustrated as red areas. That is why the increase of the metal materials in layers provides the decrease of thermal stress of the axially layered FG beams.

4.3. Analysis of Variance for Thermal Stress

Analysis of Variance (ANOVA) was employed at the 95 % confidence level. Raw data of numerical thermal stress was used and analysis was investigated for R-Sq = 99.98 % and R-Sq (adj) = 99.90 %. ANOVA results were given in Table 5.

Table 5. ANOVA for thermal stress results

Source	DF	Seq SS	Adj SS	Variance	F	P	% Contribution
L1	2	11.3388	11.3388	5.6694	3167.66	0.000	75.70
L2	2	3.1531	3.1531	1.5765	880.86	0.001	21.05
L3	2	0.484	0.484	0.2420	135.20	0.007	3.23
Error	2	0.0036	0.0036	0.0018			0.02
Total	8	14.9794					100

As can be seen from Table 5, the layers of the axially layered FG beams at the 95 % confidence level were detected as control parameters with significant influences due to $P < 0.05$ value. Analysis was determined based on significance of a factor for 95% confidence level. In addition, the most effective control parameters on thermal stress analyses are L1 for 75.70 % contribution, L2 for 21.05% contribution, and L3 for 3.23 % contribution, respectively.

4.4. Estimation of Optimum Thermal Stress

The optimum value of the thermal stress of the axially layered FG beams for the 95% confidence interval has been estimated. The optimum result of the thermal stress is determined considering the influence of the meaningful layers. The optimum result of the thermal stress is predicted using optimal levels of meaningful layers such as L1, L2, and L3. The predicted mean of the thermal stress can be calculated using Equation 4 [13].

$$\mu_{\sigma_T} = \overline{(L1)_3} + \overline{(L2)_3} + \overline{(L3)_3} - 2\overline{T} \quad (4)$$

where, $\overline{(L1)_3}$ = 189.50 , $\overline{(L2)_3}$ = 188.80 , and $\overline{(L3)_3}$ = 188.40 represent the average values of numerical thermal stress for the third level of Layer 1, Layer 2, and Layer 3, respectively. In addition, $\overline{T}$ = 188.141 refers to the overall mean of the thermal stress depending on L9 orthogonal array. According to Equation 4, μ_{σ_T} is calculated as 190.418 MPa. The 95 % confidence intervals of confirmation analyses (CI_{CA}) and population (CI_{POP}) were obtained according to Equation 5 [13] and 6 [13], respectively.

$$CI_{CA} = \left(F_{\alpha;1;n_2} V_{error}\left[\frac{1}{n_{eff}} + \frac{1}{R}\right]\right)^{1/2} \quad (5)$$

$$CI_{POP} = \left(\frac{F_{\alpha;1;n_2} V_{error}}{n_{eff}}\right)^{1/2} \quad (6)$$

$$n_{eff} = \frac{N}{(1 + T_{DOF})} \quad (7)$$

where, n2 = 2 is error value for degree of freedom and Verror = 0.0018 is error data for variance. These values such as n2 and Verror is taken from Table 5. In addition, $\alpha = 0.05$ is risk and so $F_{\alpha;1;n_2} = F_{0.05;1;2}$ is obtained as 18.5 [13] from F ratio values at the 95 % CI (α=0.05). R = 1 is the number of repetitions for confirmation analyses. n_{eff} is the influential number of replications and is determined using Equation 7 [13]. N = 9 is the number of numerical analyses for thermal stress based on L9 orthogonal array. TDOF = 6 is total degree of freedom for significant control parameters. Therefore, n_{eff} is solved as 1.286 and thus CI_{CA} and CI_{POP} are obtained as ± 0.243 and ± 0.161, respectively.

Therefore, the estimated confidence interval for confirmation analyses [13] is:

$$\boldsymbol{Mean}\ \mu_{\sigma_T} - \boldsymbol{CI}_{\mathbf{CA}} < \mu_{\sigma_T} < \boldsymbol{CI}_{\mathbf{CA}} + \boldsymbol{Mean}\ \mu_{\sigma_T}$$

The population at the 95 % confidence interval [13] is:

$$\boldsymbol{Mean}\ \mu_{\sigma_T} - \boldsymbol{CI}_{\mathbf{POP}} < \mu_{\sigma_T} < \boldsymbol{CI}_{\mathbf{POP}} + \boldsymbol{Mean}\ \mu_{\sigma_T}$$

Numerical and predicted optimal thermal stress results for confidence intervals were given in Table 6.

Table 6. ANSYS and predicted optimal thermal stress results

Optimum Configuration	ANSYS Results	Predicted Results	Predicted Confidence Intervals Based on 95% Confidence Level
$L1_3$-$L2_3$-$L3_3$	190.513	190.418	190.175<μ_{σ_T}<190.661 for CI_{CA}
			190.257<μ_{σ_T}<190.579 for CI_{POP}

4.5. Comparison of ANSYS and Predicted Data

In order to see the % differences and residuals between the numerical and predicted results of thermal stress characteristic, the axially layered FG beam configurations in Table 3were used. Numerical results were taken from Table 3 and the average value of each layer at the each level for raw data were taken from Table 4 in order to calculate predicted results. Predicted thermal stress data were solved using Equation 4. ANSYS and predicted thermal stress results are presented in Table 7.

Table 7. Numerical and predicted thermal stress

Beam Configurations			Thermal Stress σ_T (MPa)		% Difference	Residual
			ANSYS Results	Predicted Results		
$(L1)_1$	$(L2)_1$	$(L3)_1$	185.713	185.618	0.051	0.095
$(L1)_1$	$(L2)_2$	$(L3)_2$	186.821	186.818	0.002	0.003
$(L1)_1$	$(L2)_3$	$(L3)_3$	187.675	187.618	0.030	0.057
$(L1)_2$	$(L2)_1$	$(L3)_2$	187.479	187.518	-0.020	-0.039
$(L1)_2$	$(L2)_2$	$(L3)_3$	188.529	188.518	0.006	0.011
$(L1)_2$	$(L2)_3$	$(L3)_1$	188.599	188.518	0.043	0.081
$(L1)_3$	$(L2)_1$	$(L3)_3$	188.995	189.018	-0.010	-0.023
$(L1)_3$	$(L2)_2$	$(L3)_1$	189.199	189.218	-0.010	-0.019
$(L1)_3$	$(L2)_3$	$(L3)_2$	190.257	190.218	0.020	0.039

According to Table 7, the minimum % difference and residual between numerical and predicted thermal stress are obtained using beam configuration with $L1_1$-$L2_2$-$L3_2$ whereas the maximum % difference and residual are determined using beam configuration with $L1_1$-$L2_1$-$L3_1$.

5. CONCLUSIONS

In the study, thermal stress analyses of the axially layered beams having functionally graded materials including ceramic and metal materials were evaluated under uniform temperature using finite element software named ANSYS V13 parametric design language. The order of the layers was employed using L_9 orthogonal array based on Taguchi Method and so optimum levels of layers were determined to obtain maximum thermal stress. Significant layers of beams and their percent contributions on the thermal stress were found using ANOVA. The optimum value of the thermal stress response of the axially layered FG beams at the 95% confidence interval was estimated. The following points from this study can be summarized:

- According to a simple rule of mixture of composite materials, the increase of % ceramic volume fractions of layers increases the Young's modulus of layers, whereas thermal expansion values of layers decrease.
- The increase of % volume fractions of metal materials in layers decreases the thermal stress of the axially layered FG beams.
- The maximum thermal stress value of the axially layered FG beam with optimum levels is obtained using third levels of the layers.
- The most effective layers for thermal stress analyses of the axially layered FG beams are determined as L1 with 75.70% contribution, L2 with 21.05% contribution, and L3 with 3.23% contribution respectively.
- The layers for thermal stress analyses of the axially layered FG beams have significant influences for $P < 0.05$.
- Predicted maximum thermal stress values at 95 % confidence intervals of confirmation analyses (CI_{CA}) and population (CI_{POP}) are calculated as $190.175<\mu_{\sigma_T}<190.661$ and $190.257<\mu_{\sigma_T}<190.579$ respectively.
- Numerical and predicted thermal stress results of the axially layered FG beam configuration with optimum layer levels are investigated as 190.513 MPa and 190.418 MPa respectively.
- The thermal expansion and Young's modulus of the layers play meaningful role on thermal stresses the axially layered FG beams exactly.

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