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An optimization model for designing acceptance sampling plan based on cumulative count of conforming run length using minimum angle method

Ahmad Ahmadi Yazdi* and Mohammad Saber Fallah Nezhad†

Abstract

The purpose of this article is to present an acceptance sampling plan based on cumulative count of conforming using minimum angle method. In this plan, if the number of inspected items until r_{th} defective items is greater than an upper control threshold then lot is accepted and if it is less than a lower control threshold then the lot is rejected and if it is between control thresholds, process of inspecting the items continues. To design this model, we considered some important concepts like number of inspected items until r_{th} nonconforming item in inspection, first and second type of error, average number inspected (ANI), AQL and LQL. Also derivative of (ANI) function in point AQL is used for optimization. The objective function of this model was constructed based on minimum angle method. Also a comparison study is carried out to evaluate the performance of proposed methodology in 50 different data sets.

Keywords: Quality control, Markovian model, Conforming run length, Acceptance sampling plan, Minimum angle method.

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*Ph.D Student of Industrial Engineering, Isfahan University of Technology, Email: Ahmad_ahmadi_yazdi@yahoo.com Corresponding Author.
†Associate Professor of Industrial Engineering, Yazd University, Email:Fallahnezhad@yazd.ac.ir

1. Introduction

Acceptance sampling plan is a statistical quality control technique. In such plans, a sample is taken from a lot and the lot will either be rejected or accepted or inspection continues upon the results of the sample taken. The purpose of acceptance sampling plan is to determine the quality level of an incoming lot or the end production and also ensure that the quality level of the lot satisfies the predetermined requirement. Many types of acceptance sampling plans have been proposed. One approach to design acceptance sampling plans is minimum angle method (Fallahnezhad [7]). In this research, a new acceptance sampling plan is developed based on minimum angle method using cumulative conforming run length. This idea is based on the concept of cumulative conforming control charts. Design of cumulative conforming control charts is a favorable issue for many authors. Cumulative conforming control charts (CCC-charts) usually are constructed by using geometric and negative binomial variables (Chan et al. [5]). Calvin [7] presented a control chart by using run-length of successive conforming items. Goh [12] presented a method to control the production with low-nonconformity by (CCC-charts). Lai [15] proposed a discrete time renewal event process when a success is preceded by a failure and introduced modified CCC-chart. Also he calculated ANI (average number inspected) and other indicators for this modified chart. Some authors also refer CCC-charts as CRL-type (conforming run length) control charts or SCRL (sum of CRL) chart (Wu et al. [17]). A CCC-chart which is based on number of inspected items until detection of r_{th} defective item is called CCC_r-charts. Calvin [4], Goh [12], Xie and Goh [18] and many other authors have applied CCC_1-charts. Chan et al. [5] denoted that CCC_r-chart is more reliable than CCC_1-chart but it takes more time and inspection items than CCC_1-charts for detecting change in fraction of non-conforming. He also presented a two-stage decision procedure for monitoring processes with low fraction of nonconforming and introduced $CCC_1 + \gamma$ chart for this purpose and presented an economical model for minimizing total cost of the system. Di Bucchianico et al. [6] presented a case study for monitoring the packing process in coffee production based on choosing optimal value of r when using CCC_r-charts. Aslo Bourke [2] has applied the concept of conforming run length in designing the acceptance sampling plans. In this research, we used Markov model in designing the sampling plan based on the concept of conforming run length. An absorbing Markov model is developed for this sampling system (Bowling et al. [3]). In this subject, Fallahnezhad et al. [9] developed a Markov model based on sum of run-lengths of successive conforming items. Fallahnezhad and Niaki [11] proposed a sampling plan using Markov model based on control threshold policy. They considered the run-lengths of successive conforming items as a measure for process performance. Fallahnezhad et al.[10] proposed an economical model for sampling based on decision tree. Fallahnezhad and Hosseininasab [8] proposed a one stage economical acceptance sampling model based on the control threshold policy. In our sampling plan we used the concept of minimum angle method that its purpose is to reach ideal OC curve in order to decrease the risk of sampling plan. Bush et al. [1] analyzed the sampling systems by comparing operation characteristic (OC) curve against the ideal OC curve. His study was a motivation for constructing the concept of minimum angle method. Soundararajan and Christina [16] proposed a method for the selection of optimal single stage sampling plans based on the minimum angle method. They were first authors who used minimum angle method for designing a sampling plan. But little studies have been done on designing a sampling plan based on minimum angle method. Soundararajan and Christina [16] used the tangent of angle between the lines that joins $[AQL, P_a(AQL)]$ to $[LQL, P_a(LQL)]$ in order to reach ideal OC curve. $P_a(AQL)$ is the probability of acceptance when the percentage of the defective items of the lot is AQL. This angle (θ) is denoted in Figure 1 It is obvious that minimizing (θ) is favorable because the OC curve approaches to ideal OC curve. $\tan(\theta)$ is obtained as follows,

$$\tan(\theta) = \frac{LQL - AQL}{P_a(AQL) - P_a(LQL)}$$

Since (θ) should be minimized, thus the value of $\tan(\theta)$ should be minimized also since $LQL - AQL$ is constant thus the value of $[P_a(AQL) - P_a(LQL)]$ should be maximized. In this paper, a nonlinear model for acceptance sampling plans by developing a Markov

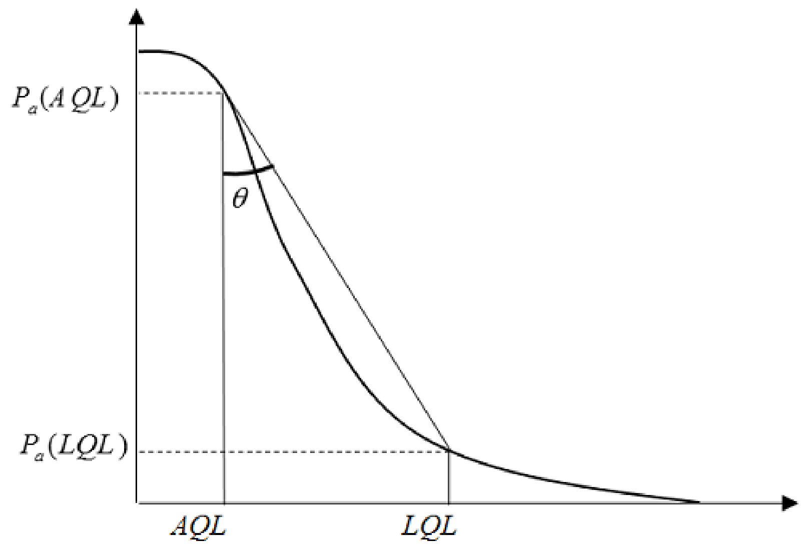

Figure 1. Tangant angle minimizing using AQL, LQL [16]

model is presented. To design this model, we considered some important concepts like number of conforming items until r_{th} nonconforming item in inspection, first and second type of error, average number inspected (ANI), AQL and LQL. Also derivative of (ANI) function in point AQL is used for optimization. The objective function of this model was constructed based on minimum angle method. The model has been solved for 4 scenarios in the cases $r = 1$ or $r = 2$ or $r = 3$ by using visual basic 6 in Microsoft excel 2013. Then the optimal solutions have been collected and analyzed in order to determine which one of these sampling plans is more desirable in practical environment. The rest of the paper is organized as follows. We present the model in Section 2. A case study is solved in Section 3. Section 4 provides a sensitivity analysis for illustrating the effect of different parameters on the objective function. In section 5, a comparison study is carried out in 50 different data sets.

2. Model Development

The purpose of this model is to develop an optimization model for determining the optimum value of thresholds of an acceptance sampling design. This acceptance sampling design is based on run length of conforming items. Assume that in an acceptance sampling plan, Y is defined as the number of inspected items until detecting r_{tk} nonconforming item. It is obvious that Y follows negative binomial distribution.

The decision making method is as follows,

If $Y \geq U$ then the lot is accepted and if $Y \leq L$ then the lot is rejected. If $U > Y > L$ then inspection of the items continues where U is an upper control threshold and L is a lower control threshold. Thus states of the decision making method are as follows,
State 1: $U > Y > L$, continue inspecting.
State 2: $Y \geq U$, the lot is accepted.
State 3: $Y \leq L$, the lot is rejected.
If p_{kl} denotes the probability of going from state k to state l then transition probabilities are obtained as follows, [7]

$$p_{11} = P\left\{U > Y > L\right\}, p_{12} = P\left\{Y \geq U\right\}, p_{13} = P\left\{Y \leq L\right\} \tag{2.1}$$

where $P\left(Y\left|r,p\right.\right) = \left(\begin{array}{c} i-1 \\ r-1 \end{array}\right)(1-p)^{i-r}\, p^r$; for $i = r, r+1, ...$ is the negative binomial distribution and p denotes the proportion of nonconforming items in the lot.
Fallahnezhad [7] proposed a new optimization model for designing sampling plans based on minimum angle method and run length of inspected items with considering minimum angle method and average number of inspection (ANI) in the optimization model. He tried to solve his model by search procedure just for $r = 1$ (r is number of nonconforming items in inspection process). In the proposed model, we try to optimize some important criteria of sampling plans simultaneously. The objective function is constructed using minimum angle method which optimizes the producer risk and consumer risk simultaneously. Also the constraints of average number inspected (ANI) and first derivative of ANI function and risks are included in the model. Then we tried to solve the proposed model by search method for $r = 1, 2, 3$ with considering all mentioned concepts.
The transition probability matrix is as follows (Fallahnezhad [7]),

$$P = \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \left[\begin{array}{ccc} \overset{1}{p_{11}} & \overset{2}{p_{12}} & \overset{3}{p_{13}} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right] \tag{2.2}$$

States 2 and 3 are absorbing state and state 1 is transient. The transition probability matrix should be rewritten in the following form in order to calculate long run probabilities of absorption:

$$\left[\begin{array}{cc} A & O \\ R & Q \end{array}\right] \tag{2.3}$$

where Q is transition probability matrix among non-absorbing states and R is the matrix containing probabilities of going from non-absorbing states to absorbing states and A is an identity matrix and O is matrix of zeros. Thus following matrix is obtained (Fallahnezhad [7]),

$$\begin{array}{c} 2 \\ 3 \\ 1 \end{array} \left[\begin{array}{ccc} \overset{2}{1} & \overset{3}{0} & \overset{1}{0} \\ 0 & 1 & 0 \\ p_{12} & p_{13} & p_{11} \end{array}\right] \tag{2.4}$$

The fundamental matrix M can be determined as follows (Bowling et al [3]):

$$M = m_{11}(p) = (I - Q)^{-1} = \frac{1}{1 - p_{11}} = \frac{1}{1 - P\left\{U > Y > L\right\}} \tag{2.5}$$

where I is the identity matrix. The value $m_{11}(p)$ denotes the expected number of visiting the transient state 1 until absorption occurs. The absorption probability matrix, F is calculated as follows (Bowling et al. [3]):

$$F = M \times R = 1 \left[\begin{array}{cc} \overset{2}{f_{12}(p)} & \overset{3}{f_{13}(p)} \end{array}\right] = 1 \left[\begin{array}{cc} \overset{2}{\frac{p_{12}}{1-p_{12}}} & \overset{3}{\frac{p_{13}}{1-p_{13}}} \end{array}\right] \tag{2.6}$$

where $f_{12}(p)$ and $f_{13}(p)$ denote the probabilities of accepting and rejecting the lot, respectively.
The objective function of this model is written by using minimum angle method. In this approach, our goal is to maximize the value of $\{P_a(AQL) - P_a(LQL)\}$ where $P_a(LQL)$ and $P_a(AQL)$ are the probabilities of accepting the lot when the proportion of nonconforming items in the lot is respectively LQL and AQL. It is obvious that $1 - P_a(AQL)$ is risk of producer thus maximizing $P_a(AQL)$ is favorable. Also $P_a(LQL)$ is the risk of consumer thus minimizing $P_a(LQL)$ is favorable. Consequently maximizing $\{P_a(AQL) - P_a(LQL)\}$ for a sampling system would be desired. The values of $P_a(LQL)$ and $P_a(AQL)$ are determined as follows,

$$p = AQL \rightarrow P_a(AQL) = f_{12}(AQL) = \frac{P\{U \leq Y\}}{1 - P\{U > Y > L\}} \tag{2.7}$$

$$p = LQL \rightarrow P_a(LQL) = f_{12}(LQL) = \frac{P\{U \leq Y\}}{1 - P\{U > Y > L\}} \tag{2.8}$$

The objective function in minimum angle method is as follows, (Fallahnezhad [7])

$$Z = \underset{L,U}{Max}\,\{P_a(AQL) - P_a(LQL)\} \tag{2.9}$$

An important performance measure of sampling plans is the average number inspected (ANI) . Since sampling and inspecting has cost, therefore designs with minimum ANI are preferred. Therefore we try to consider the ANI in constraint of optimization model so that its value does not get more that a control threshold. These constraints are written for both cases of acceptable and unacceptable lots where the proportion of nonconforming items in lot is equal to AQL and LOL, respectively. This constraint is written based on the value of $m_{11}(p)$. As mentioned, $m_{11}(p)$ is the expected number of times that the transient state 1 is visited until absorption occurs, since in each visit to transient state, the average number of inspections is $\frac{r}{p}$ which is the mean value of negative binomial distribution, consequently the value of ANI is given by $\frac{r}{p}m_{11}(p)$. Now these constraints are obtained for both cases of acceptable lot $(p = AQL)$ and unacceptable lot $(p = LQL)$ respectively,

$$ANI(AQL) \leq W \tag{2.10}$$

$$ANI(LQL) \leq M \tag{2.11}$$

where W and M are upper control limits for these constraints and,

$$ANI(AQL) = \frac{r}{AQL}m_{11}(AQL) \tag{2.12}$$

$$ANI(LQL) = \frac{r}{LQL}m_{11}(LQL) \tag{2.13}$$

It is very important that acceptance sampling plans satisfy the constraints of first and second type errors. These two types of errors are important performance measure of acceptance sampling plans. First type error probability is the probability of rejecting an acceptable lot and Second type error probability is the probability of accepting an unacceptable lot. So we have included these two concepts as the constraints of optimization model.
Thus we added following constraints to the optimization model for both cases of acceptable lot $(p = AQL)$ and unacceptable lot $(p = LQL)$ respectively,

$$P_a(AQL) \geq 1 - \alpha \tag{2.14}$$

$$(2.15) \quad P_a(LQL) \leq \beta$$

where α is the value of first type error probability and β value of second type error probability. According to the ANI graph, when the percentage of the defectives in lot is equal to the AQL, the ideal is that the derivation of the function at this point be equal to zero, or in other words, reaches its minimum value. We try to consider this concept as a constraint and examine its impact on the optimal solution of the model. The first derivative of ANI function is written as follows (Chen [13]),

$$(2.16) \quad ANI_p(p) = \frac{\partial}{\partial p}\frac{r}{k(p)} = \frac{-rk_p(p)}{k^2(p)}$$

where

$$(2.17) \quad \begin{array}{l} k(p) = p\{1 - [F(U-1\,|r,p) - F(L\,|r,p)]\} \\ k_p(p) = 1 - F(U-1\,|r,p) - F(L\,|r,p) + (L-1)f(L-1\,|r,p) - Uf(U\,|r,p) \end{array}$$

We considered upper and lower limits for derivative of ANI when the percentage of the defective in lot is equal to AQL in order to apply this constraint in the model. Since AQL is an important parameter in decision making about the lot thus this value is selected as reference value in constraint of ANI derivative. It is obvious that lower limit is negative and upper limit is positive. As much as the interval of these limits would be tighter then it will be closer to zero that is more favorable for us. This constraint is obtained as follows,

$$(2.18) \quad \lambda_1 \leq ANI_p(AQL) \leq \lambda_2$$

where λ_1 and λ_2 are lower and upper limits for the first derivation of ANI function, respectively. Now the optimization problem can be defined as follows,

$$(2.19) \quad \begin{array}{l} \underset{L,U}{Max}\, Z \\ s.t. \\ ANI(AQL) \leq W \\ ANI(LQL) \leq M \\ P_a(AQL) \geq 1-\alpha \\ P_a(LQL) \leq \beta \\ \lambda_1 \leq ANI_p(AQL) \leq \lambda_2 \end{array}$$

Optimal values of L, U, r can be determined by solving above nonlinear optimization problem using search procedures or other optimization tools. The parameters like W, M, α, β, $\lambda_1, \lambda_2, AQL, LQL$ are predetermined for solving the model in order to reach the optimal values of L, U, r. The advantage of this sampling system is to consider most important critical factors affecting on performance of sampling methods in an optimization model which optimizes them simultaneously.

3. Case Study

A case study is solved using Visual basic codes in Microsoft excel 2013 in order to demonstrate the application of the proposed methodology in designing acceptance sampling models. The following example is intended to provide illustrations about application of the model in a juice factory. The quality engineer tries to design an acceptance sampling plan for accepting or rejecting an incoming lot received from suppliers. The values of AQL and LQL and other important parameters are specified as required quality standards by both sides (consumer and producer).

This case is solved and the values in intervals $L = [0, 20]$ and $U = [1, 90]$ and $r = [1, 2, 3]$ are searched for optimal solution in each scenario, while L and U are integer. In the other words, we restricted our search space in order to reach optimal value of L and U

Table 1. Optimal solution of case study

r	L	U	Z	$ANI(AQL)$	$ANI(LQL)$	$ANI_p(AQL)$	$P_a(AQL)$	$P_a(LQL)$
2	3	35	0.94	83.45	37.59	52.44	0.98011	0.00574

Table 2. Input Parameters of Different scenarios

Scenarios	M	W	λ_2	λ_1	β	α	LQL	AQL
1	70	80	250	-250	0.2	0.15	0.3	0.06
2	100	130	200	-200	0.2	0.1	0.2	0.04
3	60	70	400	-400	0.1	0.05	0.2	0.05
4	50	105	100	-100	0.1	0.05	0.2	0.05

Table 3. Number of feasible solutions for each scenario

Scenarios	$r = 1$	$r = 2$	$r = 3$
Scenario 1	10	48	11
Scenario 2	1	33	28
Scenario 3	0	6	0
Scenario 4	0	3	6

and r. It is observed that optimal solution lies in the specified intervals in all considered practical cases. Thus first the feasible value of L and U will be determined and the optimal solution which maximizes the objective function is determined among them. It is assumed that $AQL = 0.05$, $LQL = 0.2$, $\lambda_1 = -80$, $\lambda_2 = 80$, $M = 50$, $W = 90$, $\alpha = 0.05$, $\beta = 0.1$. We solved the proposed model with these input parameters. The results show that there are just 3 feasible solutions in the solution space. Table 1 shows the optimal solutions. It is obvious that the result of the proposed model is applicable in any production environment.
In the cases that required sample size is limited then we can easily consider this limitation in the constraints of the model. It is observed that ANI of proposed method is large for $r = 3, 4, ...$ but when small sample size is an important criterion, we may apply $r = 1, 2$ for sampling system. It is obvious that optimal solution of optimization model for $r = 1$ or $r = 2$ with tighter intervals for ANI function would result in smaller values for required number of inspected items.

4. Sensitivity Analysis

In this section, a sensitivity analysis is done for illustrating the effect of different parameters on the results of the model. This model was solved in several scenarios with different assumptions. Table 2 shows the input parameters of different scenarios.
Each scenario is solved in the cases, $r = 1$, $r = 2$ and $r = 3$ and the number of feasible solutions are summarized in Table 3.
As can be seen in Table 2, the number of feasible solutions for each scenario is not the same in cases, $r = 1$, $r = 2$ and $r = 3$. For example, case $r = 1$ will not have any feasible solutions in Scenario 3 and 4. Also case $r = 3$ will not have any feasible solutions in Scenario 3. Table 4 shows the optimal solution of the model for each scenario.

Table 4. Optimal solution for each scenario

Scenarios	r	L	U	Z	$ANI(AQL)$	$ANI(LQL)$	$ANI_p(AQL)$	$p_a(AQL)$	$p_a(LQL)$
Scenario 1	3	4	34	0.98	78.31	28.30	-200.67	0.99014	0.00099
Scenario 2	3	5	58	0.99	131.42	73.79	190.35	0.9946	0.00095
Scenario 3	2	3	29	0.92	69.43	36.60	-286.97	0.96	0.04
Scenario 4	3	7	46	0.97	103.70	46.28	14.74	0.98011	0.00574

Table 5. Input Parameters

Scenarios	M	W	λ_2	λ_1	β	α	LQL	AQL
1	70	80	250	0	0.2	0.15	0.3	0.06
2	100	130	200	0	0.2	0.1	0.2	0.04

Table 6. Optimal Solution

Scenarios	r	L	U	Z	$ANI(AQL)$	$ANI(LQL)$	$ANI_p(AQL)$	$p_a(AQL)$	$p_a(LQL)$
Scenario 1	2	1	28	0.97	68.82	30.80	49.60	0.978592	0.00199
Scenario 2	3	5	57	0.99	129	73.77	85.93	0.9947	0.00115

According to Table 4, the case $r = 3$ will be optimal in most of the scenarios and case $r = 2$ will be optimal in scenario 3. Since we saw that the model could not find any feasible solution in case $r = 3$ for scenarios 3 thus this result was justified. Also the case $r = 1$ has not been optimal in any of the scenarios. So we can say that the case $r = 3$ is suitable for practical real world problems. But since we have not investigated the cases with the values of $r > 3$, this is suggested as future studies but in general, it seems that the value of $r > 4$ need so much more inspections and may not be feasible as can be seen in Table 3, where the number of feasible solution has decreased significantly by changing $r = 2$ to $r = 3$.
The first derivative of ANI function is included in the model to minimize the number of inspected items. It is needed to analyze the effect of lower limit and upper limit for first derivative of ANI function in order to investigate the behavior of optimal solution by changing them. It is obvious that when the first derivative of a convex function at a point is zero then that point is minimum value of a convex function. Thus considering negative and positive bounds for first derivative is logical which results in finding near optimal solution. We used this concept for monitoring the ANI value by calculating its first derivative. Then we defined an interval for the first derivative of $ANI(ANI_p(AQL))$. We defined two scenarios for $\lambda_1 = 0$ and $\lambda_2 > 0$ inorder to check the effects of λ_1 and λ_2. Table 5 shows the input parameters and Table 6 shows the optimal solutions.

The results shows that when we consider $\lambda_1 = 0$ and $\lambda_2 > 0$, then the variations of objective function is negligible. In this state, a better optimal solution is obtained according to the values of $ANI(AQL)$, $ANI(LQL)$ and $ANI_p(AQL)$.

5. Comparison Study

After constructing proposed method optimization model, it is very beneficial to compare this new model with traditional single stage sampling method. For illustrating the effect of different data sets on the results of the proposed model and discussion about the application of the model in the different practical environments, we carried out a simulation study with 50 different random data sets. Then we compared the proposed model withtraditional single stage sampling methodassuming the same constraints. It is tried to search all feasible points of solution space in order to obtain general optimal values for L, U, r. The optimization model for traditional single stage sampling method is as follows;

$$\begin{aligned} & Z' = \underset{n,c}{Max} \{P_a(AQL) - P_a(LQL)\} \\ & s.t. \\ & P_a(AQL) \geq 1 - \alpha \\ & P_a(LQL) \leq \beta \end{aligned} \tag{5.1}$$

where $P_a(p)$ denotes the probability of accepting the lot which is obtained by cumulative function of binomial distribution as follows;

$$P_a(p) = \sum_{x=0}^{c} \binom{n}{x} p^x (1-p)^{n-x} \tag{5.2}$$

It is obvious that the constraints regarding first derivation of ANI function, $ANI(AQL)$, and have not been considered in the optimization model because ANI in the traditional single stage sampling method is fixed ($ANI = n$).

50 different scenarios of parameters are randomly generated by uniform distribution. The results are summarized in Table 7. According to Table 7, proposed method has better value of objective function in 28% of cases but proposed model is worse than traditional method in 14% of cases and for the rest of the cases, the objective function of these two methods are equal.

The results shows that since proposed model has more constraints than the traditional single stage sampling method but it has better value for objective function in 28% of cases and both methods have equal objective function in 58% of cases. Also in most of cases, $ANI(LQL)$ in the proposed model is less value than the sample size, n in the traditional method but $ANI(AQL)$ of proposed model is often more than sample size, n in the traditional method. Thus we can assume tighter intervals for constraint regarding $ANI(AQL)$ in order to decrease the average number of inspected items. In general, the results show the advantages of proposed methodology over existing methods and this model can be efficiently applied in practical environment.

6. Conclusion

In this paper, we proposed a general nonlinear model for acceptance sampling based on cumulative count of conforming using minimum angle method. Number of inspected items until r_{th} defective items was selected as criteria for decision making. We presented our model using Markov model and derivative of ANI (average number inspected) in AQL point to ensure that ANI chart behavior is in desired level. It's ideal that the derivative of ANI in AQL point to be equal zero in order to ensure that ANI is minimized. This approach is suitable when our plan for accepting or rejecting a lot is based on number of inspected items until r_{th} nonconforming item. Also it is tried that constraint of first and second type of errors to be included in the model simultaneously. We concluded that the case $r = 3$ which denotes the method of sampling until the third defective item is suitable for practical real world problems. But since we have not investigated the cases with the values of $r > 3$, thus this is suggested as future studies but in general, it seems that the value of $r > 4$ needs so much more inspections and it may not be feasible. As can be seen in Table 3, the number of feasible solution has decreased significantly by changing $r = 2$ to $r = 3$. For analyzing the behavior of proposed model in different data sets, we solved the model for 50 different random scenarios and also we compared

Table 7. Proposed method VS. Traditional single sampling

Scenarios	Input parameters								Proposed Model						Traditional Single Sampling Method		
	AQL	LQL	W	M	λ_1	λ_2	$1-\alpha$	β	L	U	r	ANI(AQL)	ANI(LQL)	Z	n	c	Z'
1	0.04	0.27	241	99	-400	190	0.7	0.2	2	53	3	121	87	0.99	88	10	0.99
2	0.04	0.14	289	66	-311	218	0.72	0.11	12	64	3	141	65	0.95	90	6	0.91
3	0.04	0.31	237	130	-217	156	0.89	0.14	1	52	3	118	110	0.99	79	10	0.99
4	0.03	0.23	181	63	-357	443	0.82	0.25	1	60	2	145	61	0.99	90	8	0.99
5	0.03	0.31	128	68	-372	219	0.87	0.21	0	33	1	84	10	0.99	88	10	0.99
6	0.04	0.12	187	57	-246	464	0.79	0.14	16	61	3	130	57	0.92	90	6	0.85
7	0.03	0.16	216	86	-454	453	0.72	0.16	8	88	3	196	74	0.90	90	6	0.98
8	0.03	0.23	274	120	-319	179	0.80	0.21	3	67	3	151	92	0.99	90	9	0.99
9	0.02	0.15	135	78	-72	223	0.83	0.14	0	49	1	126	41	0.99	90	6	0.98
10	0.04	0.28	278	120	-90	352	0.73	0.11	2	62	3	140	76	0.93	88	10	0.99
11	0.05	0.16	135	113	0	266	0.76	0.17	7	52	3	117	87	0.99	90	8	0.94
12	0.03	0.27	286	112	-231	260	0.79	0.16	2	72	3	162	90	0.96	90	10	0.99
13	0.04	0.25	152	127	-324	161	0.74	0.22	2	59	3	133	109	0.99	90	10	0.99
14	0.03	0.11	284	57	-425	470	0.92	0.23	9	69	2	158	54	0.99	90	5	0.90
15	0.04	0.28	249	57	-46	331	0.88	0.23	3	56	3	127	50	0.93	86	10	0.99
16	0.03	0.21	122	86	-459	214	0.78	0.10	1	49	2	119	85	0.99	90	8	0.99
17	0.04	0.24	255	79	-494	154	0.90	0.12	1	46	3	105	71	0.99	71	10	0.99
18	0.04	0.27	128	124	0	438	0.70	0.13	2	51	2	116	85	0.99	84	10	0.99
19	0.05	0.21	276	67	-440	408	0.89	0.11	5	53	3	121	71	0.99	90	9	0.98
20	0.03	0.18	205	103	-236	182	0.85	0.22	5	73	3	164	85	0.99	90	7	0.98
21	0.04	0.2	170	61	-277	478	0.78	0.18	6	61	3	137	64	0.99	90	8	0.99
22	0.04	0.32	203	143	-355	410	0.88	0.12	1	56	3	127	97	0.99	79	10	0.99
23	0.04	0.33	194	104	-307	151	0.72	0.14	1	48	3	109	55	0.99	73	10	0.99
24	0.04	0.18	237	133	-268	221	0.71	0.12	5	61	3	138	96	0.99	90	8	0.97
25	0.04	0.15	261	57	-72	419	0.92	0.16	11	61	3	136	81	0.99	90	7	0.93
26	0.05	0.29	178	141	-436	265	0.82	0.23	2	45	3	103	105	0.95	78	10	0.99
27	0.02	0.1	169	103	-267	357	0.87	0.11	0	53	1	137	93	0.99	90	4	0.91
28	0.03	0.23	211	84	0	447	0.78	0.21	4	68	3	153	60	0.91	90	9	0.99
29	0.03	0.26	115	74	-358	354	0.85	0.23	0	33	3	84	14	0.99	90	10	0.99
30	0.05	0.28	191	66	-93	166	0.83	0.13	3	49	3	111	50	0.92	82	10	0.99
31	0.03	0.14	128	83	-256	177	0.88	0.15	0	39	3	99	49	0.99	90	6	0.94
32	0.02	0.31	195	66	-108	493	0.81	0.19	1	72	1	174	29	0.89	90	10	0.99
33	0.04	0.28	282	104	-457	161	0.91	0.22	2	48	1	109	74	0.99	81	10	0.99
34	0.03	0.34	239	94	-156	270	0.80	0.12	1	83	2	186	79	0.99	83	10	0.99
35	0.02	0.14	176	78	-177	335	0.79	0.22	0	52	3	135	52	0.99	90	5	0.97
36	0.03	0.26	233	135	-456	231	0.76	0.16	2	63	3	142	95	0.94	90	10	0.99
37	0.05	0.19	205	78	-257	369	0.91	0.22	6	49	3	111	66	0.99	90	9	0.96
38	0.03	0.15	130	110	-192	402	0.85	0.13	3	53	3	127	82	0.98	90	6	0.95
39	0.03	0.25	112	68	-368	228	0.74	017	0	29	1	74	15	0.98	90	10	0.99
40	0.03	0.31	122	92	-161	250	0.87	0.21	0	34	1	86	10	0.92	85	10	0.99
41	0.03	0.20	157	112	-289	369	0.79	0.15	4	67	1	151	107	0.91	90	8	0.99
42	0.05	0.22	158	60	-234	332	0.73	0.22	5	50	2	114	52	0.99	90	10	0.99
43	0.04	0.15	181	84	-113	334	0.79	0.14	9	60	3	134	257	0.99	90	7	0.94
44	0.02	0.30	221	91	-470	349	0.74	0.2	0	72	2	174	74	0.96	90	10	0.99
45	0.03	0.34	249	147	-243	124	0.75	0.2	1	64	3	144	72	0.99	76	10	0.99
46	0.05	0.26	115	147	-156	256	0.90	0.18	2	47	3	108	97	0.99	82	10	0.99
47	0.04	0.12	164	76	-199	441	0.91	0.24	14	71	3	156	72	0.94	90	6	0.88
48	0.04	0.25	205	145	-52	262	0.76	0.22	2	53	3	121	11	0.99	89	10	0.99
49	0.04	0.22	124	103	-297	497	0.72	0.1	1	45	2	110	72	0.99	90	9	0.99
50	0.04	0.15	226	92	-131	464	0.82	0.2	8	62	3	140	81	0.97	90	7	0.95

the results with traditional single sampling method. The results show that the proposed model has better performance.

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