Hacettepe Journal of Mathematics and Statistics
Volume 44 (3) (2015), 513 – 519

Generalized statistical convergence and some sequence spaces in 2-normed spaces

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Abstract

In this work, we first define the concepts of A-statistical convergence and $A^{\mathcal{I}}$-statistical convergence in a 2-normed space and present an example to show the importance of generalized form of convergence through an ideal. We then introduce some new sequence spaces in a 2-Banach space and examine some inclusion relations between these spaces.

2000 AMS Classification: 40A35, 40C05, 40G15.

Keywords: Statistical convergence, Ideal convergence, 2-normed space, Modulus function, Sequence space.

Received 03/02/2012 : Accepted 24/02/2014 Doi : 10.15672/HJMS.2015449421

1. Introduction

The idea of statistical convergence was first introduced by Fast [6] and also independently by Buck [2] and Schoenberg [22] for real and complex sequences, but the rapid developments started after the papers of Šalát [18], Fridy [8] and Connor [3].

Let $K \subseteq \mathbb{N}$ and $K_n = \{k \leq n : k \in K\}$. Then the natural density of K is defined by $\delta(K) = \lim_n n^{-1} |K_n|$ if the limit exists, where $|K_n|$ denotes the cardinality of K_n.

The number sequence $x = (x_k)$ is said to be statistically convergent to the number L provided that for every $\varepsilon > 0$ the set $K(\varepsilon) := \{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\}$ has natural density zero. In this case we write $st - \lim x = L$.

Let X, Y be two sequence spaces and $A = (a_{nk})$ be an infinite matrix. If for each $x \in X$ the series $A_n(x) = \sum_{k=1}^{\infty} a_{nk} x_k$ converges for all n and the sequence $Ax = (A_n(x)) \in Y$, then we say that A maps X into Y. By (X, Y) we denote the set of all matrices which maps X into Y, and in addition if the limit is preserved then we denote the class of such matrices by $(X, Y)_{reg}$. A matrix A is called regular if $A \in (c, c)_{reg}$, where c denotes the space of all convergent sequences.

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The well-known Silverman-Toeplitz theorem asserts that A is regular if and only if
(R_1) $\|A\| = \sup_n \sum_k |a_{nk}| < \infty$;
(R_2) $\lim_n a_{nk} = 0$, for each k;
(R_3) $\lim_n \sum_k |a_{nk}| = 1$.

Following Freedman and Sember [7], we say that a set $K \subset \mathbb{N}$ has A-density if

$$\delta_A(K) = \lim_n \sum_{k \in K} a_{nk}$$

exists, where $A = (a_{nk})$ is nonnegative regular matrix.

The idea of statistical convergence was extended to A-statistical convergence by Connor [3] and also independently by Kolk [12]. A sequence x is said to be A-statistically convergent to L if $\delta_A(K(\varepsilon)) = 0$ for every $\varepsilon > 0$. In this case we write $st_A - \lim x = L$.

Let $X \neq \emptyset$. A class $\mathcal{I} \subset 2^X$ of subsets of X is said to be an ideal in X provided; **(i)** $\emptyset \in \mathcal{I}$; **(ii)** $A, B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$; **(iii)** $A \in \mathcal{I}$, $B \subset A$ implies $B \in \mathcal{I}$. $\mathcal{I}$ is called a nontrivial ideal if $X \notin \mathcal{I}$, and a nontrivial ideal $\mathcal{I}$ in X is called admissible if $\{x\} \in \mathcal{I}$ for each $x \in X$.

Let $\mathcal{I} \subset 2^{\mathbb{N}}$ be a nontrivial ideal. Then the sequence $x = (x_k)$ of real numbers is said to be ideal convergent or $\mathcal{I}$-convergent to a number L if for each $\varepsilon > 0$ the set $\{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\} \in \mathcal{I}$ (see [15]).

Note that if $\mathcal{I}$ is an admissible ideal in $\mathbb{N}$, then usual converges implies $\mathcal{I}$-convergence.

If we take $\mathcal{I}$=$\mathcal{I}_f$, the ideal of all finite subsets of $\mathbb{N}$, then $\mathcal{I}_f-$convergence coincides with usual convergence. We also note that the ideals $\mathcal{I}_\delta = \{B \subset \mathbb{N} : \delta(E) = 0\}$ and $\mathcal{I}_{\delta_A} = \{B \subset \mathbb{N} : \delta_A(B) = 0\}$ are admissible ideals in $\mathbb{N}$, also $\mathcal{I}_\delta$-convergence and $\mathcal{I}_{\delta_A}$-convergence coincide with statistical convergence and A-statistical convergence respectively.

Savaş et al. (see [21]) have generalized A -statistical convergence by using ideals. Let $A = (a_{nk})$ be a nonnegative regular matrix. A sequence $x = (x_k)$ is said to be $A^{\mathcal{I}}$-statistically convergent (or $S_A(\mathcal{I})$ -convergent) to L if for any $\varepsilon > 0$ and $\delta > 0$,

$$\left\{ n \in \mathbb{N} : \sum_{k \in K(\varepsilon)} a_{nk} \geq \delta \right\} \in \mathcal{I}.$$

In this case we shall write $S_A(\mathcal{I}) - \lim x = L$.

Note that if we take $\mathcal{I}$=$\mathcal{I}_f$, then $A^{\mathcal{I}}$-statistical convergence coincides with A-statistical convergence. Furthermore, the choice of $\mathcal{I}$=$\mathcal{I}_f$ and $A = C_1$, the Cesàro matrix of order one, give us $\mathcal{I}$-statistical convergence introduced in [5] and [20].

Let X be a real vector space of dimension d, where $2 \leq d < \infty$. A 2-norm on X is a function $\|.,.\| : X \times X \to \mathbb{R}$ which satisfies **(i)** $\|x, y\| = 0$ if and only if x and y are linearly dependent; **(ii)** $\|x, y\| = \|y, x\|$; **(iii)** $\|\alpha x, y\| = |\alpha| \|x, y\|$, $\alpha \in \mathbb{R}$; **(iv)** $\|x, y + z\| \leq \|x, y\| + \|x, z\|$. The pair $(X, \|.,.\|)$ is then called a 2-normed space [9]. As an example of a 2-normed space we may take $X = \mathbb{R}^2$ being equipped with the 2-norm $\|x, y\| :=$ the area of parellelogram spanned by the vectors x and y, which may be given explicitly by the formula

$$\|x, y\| = |x_1 y_2 - x_2 y_1|, \quad x = (x_1, x_2),\ y = (y_1, y_2). \tag{1.1}$$

Recall that $(X, \|.,.\|)$ is a 2-Banach space if every Cauchy sequence in X is convergent to some x in X.

The concept of statistical convergence in 2-normed spaces has been introduced and examined by Gürdal and Pehlivan [10]. Let (x_n) be a sequence in 2-normed space $(X, \|.,.\|)$. The sequence (x_n) is said to be statistically convergent to L if for every $\varepsilon > 0$

$$\lim_n \frac{1}{n} |\{n : \|x_n - L, z\| \geq \varepsilon\}| = 0$$

for each nonzero z in X. In this case we write $st-\lim_n \|x_n,z\| = \|L,z\|$.

Finally, we recall that a modulus f is a function from $[0,\infty)$ to $[0,\infty)$ such that **(i)** $f(x)=0$ if and only if $x=0$; **(ii)** $f(x+y)\le f(x)+f(y)$ for all $x\ge 0$ and $y\ge 0$; **(iv)** f is increasing and (iv) f is continuous from the right at 0.

2. $A^{\mathcal{I}}$-statistical convergence in 2-normed spaces

In this section we introduce the concepts of A-statistical convergence and $A^{\mathcal{I}}$-statistical convergence in a 2-normed space when $A=(a_{nk})$ is a nonnegative regular matrix and $\mathcal{I}$ is an admissible ideal of $\mathbb{N}$.

2.1. Definition. Let (x_k) be a sequence in 2-normed space $(X,\|.,.\|)$. Then (x_k) is said to be A-statistically convergent to L if for every $\varepsilon>0$

$$\lim_n \sum_{k:\|x_k-L,z\|\ge\varepsilon} a_{nk} = 0$$

for each nonzero z in X, in other words, (x_k) is said to be A-statistically convergent to L provided that $\delta_A(\{k\in\mathbb{N}:\|x_k-L,z\|\ge\varepsilon\})=0$ for every $\varepsilon>0$ and each nonzero z in X. In this case we write $st_A-\lim_k \|x_k,z\|=\|L,z\|$.

We remark that if we take $A=C_1$ in Definition 2.1, then A-statistical convergence coincides with the concept of statistical convergence introduced in [10].

Now we introduce the concept of $A^{\mathcal{I}}$-statistical convergence in a 2-normed space.

2.2. Definition. A sequence (x_k) in 2-normed space $(X,\|.,.\|)$ is said to be $A^{\mathcal{I}}$-statistically convergent to L provided that for every $\varepsilon>0$ and $\delta>0$

$$\left\{ n\in\mathbb{N} : \sum_{k:\|x_k-L,z\|\ge\varepsilon} a_{nk}\ge\delta \right\}\in\mathcal{I}$$

for each nonzero z in X. In this case we write $S_A(\mathcal{I})-\lim_k\|x_k,z\|=\|L,z\|$.

We shall denote the space of all A-statistically convergent and $A^{\mathcal{I}}$-statistically convergent sequences in a 2-normed space $(X,\|.,.\|)$ by $S_A(\|.,.\|)$ and $S_A(\mathcal{I},\|.,.\|)$, respectively. It is clear that if $\mathcal{I}=\mathcal{I}_f$, then the space $S_A(\mathcal{I},\|.,.\|)$ is reduced to $S_A(\|.,.\|)$.

Example. Let $X=\mathbb{R}^2$ be equipped with the 2-norm by the formula (1.1). Let $\mathcal{I}\subset 2^{\mathbb{N}}$ be an admissible ideal, $C=\{p_1<p_2<\ldots\}\in\mathcal{I}$ be an infinite set and define the matrix $A=(a_{nk})$ and the sequence (x_k) by

$$a_{nk}=\begin{cases} 1 & ;\text{if } n=p_i,\ (i\in\mathbb{N}),\ k=2p_i \\ 1 & ;\text{if } n\ne p_i,\ k=2n+1 \\ 0 & ;\text{otherwise.}\end{cases}$$

and

$$x_k=\begin{cases} (0,k) & ;\text{ if } k \text{ is even} \\ (0,0) & ;\text{ otherwise}\end{cases}$$

respectively. Also let $L=(0,0)$ and $z=(z_1,z_2)$. If $z_1=0$ then

$$\{k:\|x_k-L,z\|\ge\varepsilon\}=\emptyset$$

for each z in X. Then $\delta_A(\{k\in\mathbb{N}:\|x_k-L,z\|\ge\varepsilon\})=0$. Hence we have $z_1\ne 0$. For each $\varepsilon>0$

$$\{k:\|x_k-L,z\|\ge\varepsilon\} \overset{\text{if } k \text{ is even}}{=} \left\{k: k\ge\frac{\varepsilon}{|z_1|}\right\},$$

hence for each $\delta > 0$ we obtain

$$\left\{ n \in \mathbb{N} : \sum_{k : \|x_k - L, z\| \geq \varepsilon} a_{nk} \geq \delta \right\} = \{ n \in \mathbb{N} : n = p_i \} = C \in \mathcal{I}.$$

This means that $S_A(\mathcal{I}) - \lim_k \|x_k, z\| = \|(0,0), z\|$, but $st_A - \lim_k \|x_k, z\| \neq \|(0,0), z\|$ since

$$\lim_n \sum_{k : \|x_k - L, z\| \geq \varepsilon} a_{nk} = 1 \neq 0.$$

This example also shows that $A^{\mathcal{I}}$-statistical convergence is more general than A-statistical convergence in a 2-normed space.

3. Some New Sequence Spaces

Following the study of Maddox [16], who introduced the notion of strongly Cesàro summability with respect to a modulus, several authors used modulus function to construct some new sequence spaces by using different methods of summability. For instance, see [4], [19] and [1]. Also in [11, 13, 14, 17] some new sequence spaces are defined in a Banach space by means of sequence of modulus functions $\mathcal{F} = (f_k)$.

In this section, we introduce some new sequence spaces in a 2-Banach space by using sequence of modulus functions and ideals. We further examine the inclusion relations between these sequence spaces.

Let $A = (a_{nk})$ be a nonnegative regular matrix, $\mathcal{I}$ be an admissible ideal of $\mathbb{N}$ and let $p = (p_k)$ be a bounded sequence of positive real numbers. By $s(2 - X)$ we denote the space of all sequences defined over $(X, \|.,.\|)$. Throughout the paper $\mathcal{F} = (f_k)$ is assumed to be a sequence of modulus functions such that $\lim_{t \to 0^+} \sup_k f_k(t) = 0$ and further let $(X, \|.,.\|)$ be a 2-Banach space. Now we define the following sequence space:

$$w^{\mathcal{I}}(A, \mathcal{F}, p, \|.,.\|) = \Big\{ x \in s(2 - X) : \big\{ n \in \mathbb{N} : \sum_k a_{nk} \left[f_k \left(\|x_k - L, z\| \right) \right]^{p_k} \geq \delta \big\} \in \mathcal{I}$$
$$\text{for each } \delta > 0 \text{ and } z \in X, \text{ for some } L \in X \Big\}.$$

If $x \in w^{\mathcal{I}}(A, \mathcal{F}, p, \|.,.\|)$ then x is said to be strongly $(A, \mathcal{F}, \|.,.\|)$-summable to $L \in X$.

Note that if $0 < p_k \leq \sup_k p_k =: H$, $D := \max(1, 2^{H-1})$, then

$$|a_k + b_k|^{p_k} \leq D \left\{ |a_k|^{p_k} + |b_k|^{p_k} \right\} \tag{3.1}$$

for all k and $a_k, b_k \in \mathbb{C}$.

3.1. Theorem. $w^{\mathcal{I}}(A, \mathcal{F}, p, \|.,.\|)$ *is a linear space.*

Proof. *Assume that the sequences x and y are strongly $(A, \mathcal{F}, \|.,.\|)$-summable to L and L', respectively and let $\alpha, \beta \in \mathbb{C}$. By using the definitions of modulus function and 2-norm and also from (3.1), we have*

$$\sum_{k=1}^{\infty} a_{nk} \left[f_k \left(\|(\alpha x_k + \beta y_k) - (\alpha L + \beta L'), z\| \right) \right]^{p_k} \leq DM_\alpha^H \sum_{k=1}^{\infty} a_{nk} \left[f_k \left(\|x_k - L, z\| \right) \right]^{p_k}$$
$$+ DM_\beta^H \sum_{k=1}^{\infty} a_{nk} \left[f_k \left(\|y_k - L, z\| \right) \right]^{p_k}$$

where M_α and M_β are positive numbers such that $|\alpha| \leq M_\alpha$ and $|\beta| \leq M_\beta$. From the last inequality, we conclude that $\alpha x + \beta y \in w^{\mathcal{I}}(A, \mathcal{F}, p, \|.,.\|)$.

If we take $f_k(t) = t$ for all k and t, then the space $w^{\mathcal{I}}(A, \mathcal{F}, p, \|.,.\|)$ is reduced to

$$w^{\mathcal{I}}(A, p, \|.,.\|) = \Big\{ x \in s(2-X) : \big\{ n \in \mathbb{N} : \sum_k a_{nk} \left(\|x_k - L, z\|\right)^{p_k} \geq \delta \big\} \in \mathcal{I} \text{ for each } \delta > 0 \text{ and } z \in X, \text{ for some } L \in X \Big\}.$$

If $x \in w^{\mathcal{I}}(A, p, \|.,.\|)$ then we say that x is strongly $(A, \|.,.\|)$-summable to $L \in X$.

3.2. Lemma. *Let f be any modulus function and $0 < \delta < 1$. Then for each $t \geq \delta$ we have $f(t) \leq 2f(1)\delta^{-1}t$ [16].*

3.3. Theorem. *If x is strongly $(A, \|.,.\|)$-summable to L then x is strongly $(A, \mathcal{F}, \|.,.\|)$-summable to L, i.e. the inclusion*

$$w^{\mathcal{I}}(A, p, \|.,.\|) \subset w^{\mathcal{I}}(A, \mathcal{F}, p, \|.,.\|)$$

holds.

Proof. *Let $x = (x_k) \in w^{\mathcal{I}}(A, p, \|.,.\|)$. Since a modulus function is continuous at $t = 0$ from the right and $\lim_{t \to 0^+} \sup_k f_k(t) = 0$, then for any $\varepsilon > 0$ we can choose $0 < \delta < 1$ such that for every t with $0 \leq t \leq \delta$, we have $f_k(t) < \varepsilon$ $(k \in \mathbb{N})$. Then, from Lemma 3.2, we have*

$$\begin{aligned} \sum_{k=1}^{\infty} a_{nk} \left[f_k \left(\|x_k - L, z\|\right)\right]^{p_k} &= \sum_{k : \|x_k - L, z\| \leq \delta} a_{nk} \left[f_k \left(\|x_k - L, z\|\right)\right]^{p_k} \\ &+ \sum_{k : \|x_k - L, z\| > \delta} a_{nk} \left[f_k \left(\|x_k - L, z\|\right)\right]^{p_k} \\ &\leq \max\left(\varepsilon^{\inf p_k}, \varepsilon^{\sup p_k}\right) \sum_{k=1}^{\infty} a_{nk} \\ &+ \max\left(M_1, M_2\right) \sum_{k=1}^{\infty} a_{nk} \left(\|x_k - L, z\|\right)^{p_k} \end{aligned}$$

where $M_1 = \left(2 \sup f_k(1)\delta^{-1}\right)^{\inf p_k}$ and $M_2 = \left(2 \sup f_k(1)\delta^{-1}\right)^{\sup p_k}$. Let $M := \max(M_1, M_2)$ and $N := \max\left(\varepsilon^{\inf p_k}, \varepsilon^{\sup p_k}\right)$. Now by considering the inequality $\sum_k a_{nk} \leq \|A\|$ for each $n \in \mathbb{N}$, choose a $\sigma > 0$ such that $\sigma - N\|A\| > 0$. Then we obtain

$$\begin{aligned} &\left\{ n \in \mathbb{N} : \sum_k a_{nk} \left[f_k \left(\|x_k - L, z\|\right)\right]^{p_k} \geq \sigma \right\} \\ &\quad \subset \left\{ n \in \mathbb{N} : \sum_k a_{nk} \left[f_k \left(\|x_k - L, z\|\right)\right]^{p_k} \geq \frac{\sigma - N\|A\|}{M} \right\} \end{aligned}$$

From the assumption we conclude that $x \in w^{\mathcal{I}}(A, \mathcal{F}, p, \|.,.\|)$.

3.4. Theorem. *Let $\mathcal{F} = (f_k)$ be the sequence of modulus functions such that $\lim_{t \to \infty} \inf_k \frac{f_k(t)}{t} > 0$. Then $w^{\mathcal{I}}(A, \mathcal{F}, p, \|.,.\|) \subset w^{\mathcal{I}}(A, p, \|.,.\|)$.*

Proof. *Let $x \in w^{\mathcal{I}}(A, \mathcal{F}, p, \|.,.\|)$. If $\lim_{t \to \infty} \inf_k \frac{f_k(t)}{t} > 0$ then there exists a $c > 0$ such that $f_k(t) > ct$ for every $t > 0$ and for all $k \in \mathbb{N}$. Thus, for each $\delta > 0$ we have*

$$\begin{aligned} &\left\{ n \in \mathbb{N} : \sum_{k=1}^{\infty} a_{nk} \left[f_k \left(\|x_k - L, z\|\right)\right]^{p_k} \geq \delta \right\} \\ &\quad \supset \left\{ n \in \mathbb{N} : \min\left(c^{\inf p_k}, c^{\sup p_k}\right) \sum_{k=1}^{\infty} a_{nk} \left(\|x_k - L, z\|\right)^{p_k} \geq \delta \right\}. \end{aligned}$$

Hence $x \in w^{\mathcal{I}}(A, p, \|.,.\|)$ *and this compltes the proof of theorem.*

Finally, we establish the relations betwen the spaces $S_A(\mathcal{I}, \|.,.\|)$ and $w^{\mathcal{I}}(A, \mathcal{F}, p, \|.,.\|)$.

3.5. Theorem. *Let* $\mathcal{F} = (f_k)$ *be a sequence of modulus functions such that* $\inf_k f_k(t) > 0$. *Then* $w^{\mathcal{I}}(A, \mathcal{F}, p, \|.,.\|) \subset S_A(\mathcal{I}, \|.,.\|)$.

Proof. *Let* $x \in w^{\mathcal{I}}(A, \mathcal{F}, p, \|.,.\|)$ *and* $\varepsilon > 0$. *If* $\inf_k f_k(t) > 0$ *then there exists* $c > 0$ *such that* $f_k(\varepsilon) > c$ *for all* k. *If we write* $K(\varepsilon) = \{k : \|x_k - L, z\| \geq \varepsilon\}$, *then*

$$\sum_{k=1}^{\infty} a_{nk}\left[f_k\left(\|x_k - L, z\|\right)\right]^{p_k} \geq \min\left(c^{\inf p_k}, c^{\sup p_k}\right) \sum_{k \in K(\varepsilon)} a_{nk}.$$

Let $C := \min\left(c^{\inf p_k}, c^{\sup p_k}\right)$. *Thus we have*

$$\left\{n \in \mathbb{N} : \sum_{k \in K(\varepsilon)} a_{nk} \geq \delta\right\} \subset \left\{n \in \mathbb{N} : \sum_{k=1}^{\infty} a_{nk}\left[f_k\left(\|x_k - L, z\|\right)\right]^{p_k} \geq \frac{\delta}{C}\right\}$$

for all $\delta > 0$. *Since the set on the right-hand of the above inclusion belongs to* $\mathcal{I}$, *we conclude that* $x \in S_A(\mathcal{I}, \|.,.\|)$. *This completes the proof.*

3.6. Theorem. *Let* $\mathcal{F} = (f_k)$ *be a sequence of modulus functions such that* $\sup_t \sup_k f_k(t) > 0$. *Then* $S_A(\mathcal{I}, \|.,.\|) \subset w^{\mathcal{I}}(A, \mathcal{F}, p, \|.,.\|)$.

Proof. *Let* $x \in S_A(\mathcal{I}, \|.,.\|)$ *and* $h(t) := \sup_k f_k(t)$, $M := \sup_t h(t)$. *Then for every* $\varepsilon > 0$, *we have*

$$\begin{aligned}
\sum_{k=1}^{\infty} a_{nk}\left[f_k\left(\|x_k - L, z\|\right)\right]^{p_k} &= \sum_{k:\|x_k - L, z\| \geq \varepsilon} a_{nk}\left[f_k\left(\|x_k - L, z\|\right)\right]^{p_k} \\
&+ \sum_{k:\|x_k - L, z\| < \varepsilon} a_{nk}\left[f_k\left(\|x_k - L, z\|\right)\right]^{p_k} \\
&\leq \max\left(M^{\inf p_k}, M^{\sup p_k}\right) \sum_{k:\|x_k - L, z\| \geq \varepsilon} a_{nk} \\
&\quad + h(\varepsilon) \sum_{k:\|x_k - L, z\| < \varepsilon} a_{nk} \\
&\leq M_0 \sum_{k:\|x_k - L, z\| \geq \varepsilon} a_{nk} + \varepsilon_1 \|A\|,
\end{aligned}$$

where $M_0 = \max\left(M^{\inf p_k}, M^{\sup p_k}\right)$ *and* ε_1 *is a positive number such that* $h(\varepsilon) < \varepsilon_1$, *which can be obtained from the condition* $\lim_{t \to 0^+} h(t) = 0$. *Hence, from the last inequality we obtain that* $x \in w^{\mathcal{I}}(A, \mathcal{F}, p, \|.,.\|)$.

Acknowledgment. The authors were supported by the Scientific Research Project Fund of Cumhuriyet University under the project number F334.

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