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# Stronger Forms of $\alpha$GS-Continuous Functions in Topology

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**Abstract** In this paper, a new stronger forms of continuity called strongly $\alpha$gs-continuous and perfectly $\alpha$gs-continuous functions are introduced. The aim of this paper is to characterize strongly $\alpha$gs-continuous and perfectly $\alpha$gs-continuous functions via $\alpha$gs-closed sets and relate these concepts to the classes of $\alpha$gs-compact and $\alpha$gs-connected spaces.

**Keywords** *$\alpha$gs-closed set, Strongly $\alpha$gs-continuous, Perfectly $\alpha$gs-continuous, $\alpha$gs-continuous, $\alpha$gs-connected, $\alpha$gs-compact.*

## 1 Introduction

General topology plays an important role in mathematics and in applied science. In analysis the concepts like continuity, separation axioms, compactness, connectedness etc are generalized by many topologists using generalized forms of open and closed sets. Recently, Rajamani and Vishwanathan[8] introduced the notion of $\alpha$gs-closed set using $\alpha$-closure operator.
In this paper the new classes of continuous functions called strongly $\alpha$gs-continuous functions and perfectly $\alpha$gs-continuous functions which are stronger than $\alpha$gs-continuous functions is presented. Also we apply these continuous functions to the classes of $\alpha$gs-compact and $\alpha$gs-connected spaces which are defined in [7].

## 2 Preliminary

Throughout this paper (X, $\tau$), (Y, $\sigma$)(or simply $X$, $Y$)denote topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset $A$ of a space $X$ the closure and interior of $A$ with respect to $\tau$ are denoted by $Cl(A)$ and $Int(A)$ respectively.

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**Definition 2.1.** *A subset A of a space X is called*
*(1) semi open set [2] if $A \subset Cl(Int(A))$.*
*(2) semi closed set [1] if $Int(Cl(Int(A))) \subset A$.*
*(3) $\alpha$-open [5]if $A \subset Int(Cl(Int(A)))$*

**Definition 2.2.** [8] *A subset A of X is $\alpha$generalized semi-closed(briefly, $\alpha$gs-closed)set if $\alpha Cl(A) \subset U$ whenever $A \subset U$ and U is semi open in X. The complement of $\alpha$gs-closed set is $\alpha$generalized-semi open (briefly,$\alpha$gs-open).The family of all $\alpha$gs-closed sets of X is denoted by $\alpha GSC(X,\tau)$ and $\alpha$gs-open sets by $\alpha GSO(X,\tau)$.*

**Definition 2.3.** [4]:A topological space X is called *$\alpha$gs-$T_2$ if for each pair of distinct points x and y of X, there exist disjoint $\alpha$gs-open sets, one containing x and the other containing y .*

**Definition 2.4.** [9] *A function $f : X \rightarrow Y$ is said to be*
*(i) $\alpha$gs-continuous if the inverse image of evey closed set in Y is a $\alpha$gs-closed set in X.*
*(ii) $\alpha$gs-irresolute if the inverse image of $\alpha$gs-closed set in Y is a $\alpha$gs-closed set in X.*

**Definition 2.5.** [9] *A space X is said to be $T_{\alpha gs}$-space if every $\alpha$gs-closed set in it is closed set.*

# 3 STRONGLY $\alpha$GS-CONTINUOUS FUNCTIONS

In this section, the notion of a new class of functions called strongly $\alpha$gs-continuous functions is introduced and obtained some of their characterizations and properties. Also, the relationships with some other related functions are discussed.

**Definition 3.1.** *A function $f : X \rightarrow Y$ is said to be strongly $\alpha$gs-continuous if $f^{-1}(V)$ is closed in X for every $\alpha$gs-closed set V of Y.*

**Theorem 3.2.** *A function $f : X \rightarrow Y$ is strongly $\alpha$gs-continuous if and only if $f^{-1}(V)$ is open in X for every $\alpha$gs-open set V in Y.*

**Proof**. Let $f : X \rightarrow Y$ is strongly $\alpha$gs-continuous and V be a $\alpha$gs-open set in Y. Then $V^c$ is $\alpha$gs-closed set in Y. Therefore, $f^{-1}(V^c)$ is closed set in X. But $f^{-1}(V^c) = (f^{-1}(V))^c$ and hence $f^{-1}(V)$ is open in X. Converse is obvious.

**Remark 3.3.** *Every strongly $\alpha$gs-continuous is $\alpha$gs-continuous function. But the converse need not to be true from the following example.*

**Example 3.4.** *Let X $=\{a, b, c\}=$ Y, $\tau = \{X, \phi, \{a\}, \{b, c\}\}$. We have $\alpha$gs-closed sets in X are $\{\{b\}, \{c\}, \{a, b\}, \{a, c\}, \}$. $\sigma = \{Y, \phi, \{a\}, \{a, b\}\}$.We have $\alpha$gs-closed sets in Y are $\{\{c\}, \{b, c\}, \}$. Define a function $f : X \rightarrow Y$ by f(a) = b, f(b) = c, f(c) = a. Then f is $\alpha$gs-continuous but not strongly $\alpha$gs-continuous as $f^{-1}(\{b, c\}) = \{a, b\}$ is not closed set in X.*

Recall that a function $f : X \rightarrow Y$ is strongly continuous [3] if $f^{-1}(V)$ is clopen in X for every subset V of Y.

**Remark 3.5.** *Every strongly contiuous function is strongly $\alpha$gs-continuous function. But the converse need not to be true from the following example.*

**Example 3.6.** *Let X* $=\{a, b, c\}=Y$, $\tau = \{X, \phi, \{c\}, \{b, c\}\}$, $\sigma = \{Y, \phi, \{a\}, \{a, b\}\}$. *We have* $\alpha$*gs-closed sets in* $Y$ *are* $\{\{c\}, \{b, c\}, \}$. *Define a function* $f : X \to Y$ *by f(a) = c, f(b) = b, f(c) = a. Then f is strongly* $\alpha$*gs-continuous but not strongly continuous as* $f^{-1}(\{a, b\}) = \{b, c\}$ *is open set but not closed set in X.*

**Theorem 3.7.** *The composition of two strongly* $\alpha$*gs-continuous functions is strongly* $\alpha$*gs-continuous.*

**Proof**. $f : X \to Y$ and $g : Y \to Z$ be two stronly $\alpha$gs-continuous functions. Let V be a $\alpha$gs-closed set in Z. Since g is strongly $\alpha$gs-continuous, $g^{-1}(V)$ is closed in Y. Then $g^{-1}(V)$ is $\alpha$gs-closed in Y. Since f is strongly $\alpha$gs-continuous, $f^{-1}(g^{-1}(V))$ is closed in X. That is $(g \circ f)^{-1}(V)$ is closed in X. Hence $g \circ f$is strongly $\alpha$gs-continuous.

**Theorem 3.8.** *Let* $Y$ *be* $T_{\alpha gs}$*-space and* $f : X \to Y$ *be any function. Then following are equivalent*
*(i)* $f$ *is strongly* $\alpha$*gs-continuous function.*
*(ii)* $f$ *is continuous.*

**Proof:** $(i) \Rightarrow (ii)$ Obvious because every open set is $\alpha$gs-open set.
$(ii) \Rightarrow (i)$ Suppose $F$ is $\alpha$gs-closed set in $Y$ and $Y$ is $T_{\alpha gs}$-space. Therefore $F$ is closed in $Y$. Since $f$ is continuous, $f^{-1}(F)$ is closed in $X$. Hence $f$ is strongly $\alpha$gs-continuous function.

**Theorem 3.9.** *The following are equivalent for the function* $f : X \to Y$.
*(i) The function* $f$ *is strongly* $\alpha$*gs-continuous.*
*(ii) For each* $x \in X$ *and each* $\alpha$*gs-open set* $V$ *in* $Y$ *with* $f(x) \in V$*, there exist an open set* $U$ *in* $X$ *such that* $x \in U$ *and* $f(U) \subset V$.
*(iii)* $f^{-1}(V) \subset Int(f^{-1}(V))$ *for each* $\alpha$*gs-open set* $V$ *of* $Y$.
*(iv)* $f^{-1}(F)$ *is closed in* $X$ *for every* $\alpha$*gs-closed set* $F$ *of* $Y$.

**Proof:** (i)⇒(ii) Suppose (i) holds. Let $x \in X$ and $V$ be a $\alpha$gs-open set in $Y$ containing $f(x)$. Since $f$ is strongly $\alpha$gs-continuous, $f^{-1}(V)$ is an open set in $X$ such that $x \in f^{-1}(V)$. Put $U = f^{-1}(V)$, then $x \in U$ and $f(U) = f(f^{-1}(V)) \subset V$. Thus (ii) holds.
(ii)⇒(iii) Suppose (ii) holds. Let $V$ be any $\alpha$gs-open set in $Y$ and $x \in f^{-1}(V)$. By (ii), there exists an open set $U$ in $X$ such that $x \in U$ and $f(U) \subset V$. This implies $x \in U \subset Int(U) \subset Int(f^{-1}(V))$, which implies $x \in Int(f^{-1}(V))$. Therefore, $f^{-1}(V) \subset Int(f^{-1}(V))$
(iii)⇒(iv). Suppose (iii) holds. Let $F$ be any $\alpha$gs-closed set of $Y$. Set $V = Y - F$, then $V$ is $\alpha$gs-open set in $Y$. By (iii) $f^{-1}(V) \subset Int(f^{-1}(V))$. That is $f^{-1}(Y - F) \subset Int(f^{-1}(Y - F))$. This implies $X - f^{-1}(F) \subset X - Cl(f^{-1}(F))$. This implies $Cl(f^{-1}(F)) \subset f^{-1}(F)$. But $f^{-1}(F) \subset Cl(f^{-1}(F))$ is always true. Therefore, $f^{-1}(F) = Cl(f^{-1}(F))$. This shows that, $f^{-1}(F)$ is closed in $X$.
(iv)⇒(i) Suppose (iv) holds. Let $V$ be any $\alpha$gs-open set of $Y$. Set $F = Y - V$. Then $F$ is $\alpha$gs-closed set of $Y$. By (iv), $f^{-1}(F)$ is closed in $X$. But $f^{-1}(F) = f^{-1}(Y - V) = X - f^{-1}(V)$. This implies $f^{-1}(V)$ is an open set in $X$. Therefore $f$ is strongly $\alpha$gs-continuous.

**Theorem 3.10.** *If* $f : X \to Y$ *is injective strongly* $\alpha$*gs-continuous and* $Y$ *is* $\alpha$*gs-*$T_2$ *space, then* $X$ *is* $T_2$ *space.*

**Proof:** Suppose $f : X \to Y$ is injective strongly $\alpha$gs-continuous and $Y$ is $\alpha$gs-$T_2$. Let $x$ and $y$ be any two distinct points in $X$. Since $f$ is injective $f(x)$ and $f(y)$ are distinct points in $Y$. Since $Y$ is $\alpha$gs-$T_2$, there exist disjoint $\alpha$gs-open sets $G$ and $H$ in $Y$ such that $f(x) \in G$ and $f(y) \in H$. This implies, $x \in f^{-1}(G)$ and $y \in f^{-1}(H)$. Again, since $f$ is strongly $g\delta s\alpha$gs-continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint open sets in $X$. Therefore $X$ is $T_2$ space.

# 4 PERFECTLY $\alpha$GS-CONTINUOUS FUNCTIONS

**Definition 4.1.** *A function $f : X \to Y$ is said to be perfectly $\alpha$gs-continuous if $f^{-1}(V)$ is clopen in $X$ for every $\alpha$gs-closed set $V$ of $Y$.*

Note that $f : X \to Y$ is said to be perfectly $\alpha$gs-continuous if and only if inverse image of every $\alpha$gs-closed set of Y is clopen in X. T. Nori[6] introduced the notion of perfectly continuous function in topological spaces. Recall that a function $f : X \to Y$ is called perfectly continuous[6] if the inverse image of every open set of Y is clopen in X. Then we have

**Theorem 4.2.** *(i) If $f : X \to Y$ is said to be perfectly $\alpha$gs-continuous, then f is perfectly continuous.*
*(ii) If $f : X \to Y$ is said to be perfectly $\alpha$gs-continuous, then f is strongly $\alpha$gs-continuous.*

Note that the converses in the theorem above is not necessary true as shown by the following example.

**Example 4.3.** *(i)The function defined in Example 3.6 is strongly $\alpha$gs-continuous but not perfectly continuous, since for an open set $\{a\}$ $f^{-1}(\{a\}) = \{c\}$ is open set but not closed set in X.*
*(ii) The function defined in Example 3.6 is strongly $\alpha$gs-continuous but not perfectly $\alpha$gs-continuous as for $\alpha$gs-closed set $\{b, c\}$, $f^{-1}(\{b, c\}) = \{a, b\}$ is closed set but not an open set in X.*

**Theorem 4.4.** *For a function $f : X \to Y$ the following statements are eqivalent:*
*(i) f is perfectly $\alpha$gs-continuous.*
*(ii) f is strongly $\alpha$gs-continuous and inverse images of strongly $\alpha$gs-open sets are $\alpha$g-closed set.*

**Proof:**. Obvious.

**Theorem 4.5.** *A function $f : X \to Y$ is perfectly $\alpha$gs-continuous if the graph function $g : X \times X \to Y$, defined by $g(x) = (x, f(x))$ for each $x \in X$, is perfectly $\alpha$gs-continuous.*

**Proof:** Let $V$ be any $\alpha$gs-open set of $Y$. Then $X \times V$ is a $\alpha$gs-open set of $X \times Y$. Since $g$ is perfectly $\alpha$gs -continuous, $f^{-1}(V) = g^{-1}(X \times V)$ is clopen in $X$. Therefore $f$ is perfectly $\alpha$gs-continuous.

**Theorem 4.6.** *Let $A$ be any subset of $X$. If $f : X \to Y$ is perfectly $\alpha$gs-continuous, then the restriction function $f|_A : A \to Y$ is perfectly $\alpha$gs-continuous.*

**Proof:** Let $V$ be a $\alpha$gs-open set of $Y$. Since $f$ is perfectly $\alpha$gs-continuous, $f^{-1}(V)$ is clopen set in $X$. Then, $(f|_A)^{-1}(V) = A \cap f^{-1}(V)$ is clopen in $A$ and hence $f|_A$ is perfectly $\alpha$gs-continuous.

**Theorem 4.7.** *Let $f : X \to Y$ and $g : Y \to Z$ be two functions.*
*(i) If $f$, $g$ are perfectly $\alpha$gs-continuous functions, then $g \circ f$ is perfectly $\alpha$gs-continuous function.*
*(ii) If $f$ is perfectly $\alpha$gs-continuous function and $g$ is $\alpha$gs-irresolute, then $g \circ f$ is perfectly $\alpha$gs-continuous function.*
*(iii) If $f$ is perfectly continuous function and $g$ is strongly continuous, then $g \circ f$ is perfectly $\alpha$gs-continuous function.*
*(iv) If $f$ is perfectly $\alpha$gs-continuous function and $g$ is $\alpha$gs-continuous, then $g \circ f$ is perfectly continuous function.*
*(v) If $f$ is $\alpha$gs-continuous and $g$ is strongly continuous then $g \circ f$ is $\alpha$gs-continuous.*
*(vi) If $f$ is $\alpha$gs-irresolute and $g$ is perfectly $\alpha$gs-continuous, then $g \circ f$ is $\alpha$gs-irresolute function.*

**Proof:** Obvious.

**Theorem 4.8.** *Every perfectly $\alpha$gs-continuous function in to finite $T_1$ space is strongly continuous.*

**Proof:** Obvious because every finite $T_1$ space is discrete space. Therefore every subset of $X$ is open and hence $\alpha$gs-open. Since $f$ is perfectly $\alpha$gs-continuous function, $f^{-1}(A)$ is clopen for every subset of $Y$. Therefore $f$ is strongly continuous.

**Theorem 4.9.** *Let $X$ be a discrete topological space, $Y$ be any topological space and $f : X \to Y$ be a function. Then the following are equivalent.*
*(i) $f$ is perfectly $\alpha$gs-continuous.*
*(ii)$f$ is strongly $\alpha$gs-continuous.*

**Proof:** (i)$\Rightarrow$ (ii) Obvious because every clopen set is open.
(ii)$\Rightarrow$(i) Let $V$ is a $\alpha$gs-open in $Y$. By hypothesis, $f^{-1}(V)$ is open in $X$. Since $X$ is discrete space, $f^{-1}(V)$ is also closed set in $X$. Therefore $f$ is perfectly continuous.

**Theorem 4.10.** *If $f : X \to Y$ is perfectly $\alpha$gs-continuous injection and $Y$ is $\alpha$gs-$T_2$ space, then $X$ is ultra Hausdorff space.*

**Proof:** Suppose $f : X \to Y$ is perfectly $\alpha$gs-continuous injection and $Y$ is $\alpha$gs-$T_2$ space. Let $a$ and $b$ be any pair of distinct points of $X$. Since $f$ is injective $f(a)$ and $f(b)$ are distinct points in $Y$. Since $Y$ is $\alpha$gs-$T_2$ space, there exist disjoint $\alpha$gs-open sets $U$ and $V$ in $Y$ such that $f(a) \in U$ and $f(b) \in V$. This implies, $a \in f^{-1}(U)$ and $b \in f^{-1}(V)$. Since $f$ is perfectly $\alpha$gs-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint clopen sets in $X$. Therefore $X$ is ultra Hausdorff space.

**Remark 4.11.** *The following diagram is obtained from definitions.*

Strongly Continuity $\to$ Strongly $\alpha$gs-Continuity $\to$ Continuity $\to$ $\alpha$gs-Continuity
$\uparrow$
Perfectly Continuity $\leftarrow$ Perfectly $\alpha$gs-Continuity

# 5 $\alpha$GS-COMPACT SPACES

The concepts of $\alpha$gs-compact and $\alpha$gs-connnected spaces defined in [7]. In this section we discuss some of their characterizations and properties.

**Definition 5.1.** *A topological space $X$ is said to be $\alpha$gs-compact[7] if it cannot be written as the union of two non-empty disjoint $\alpha$gs-open sets.*

**Definition 5.2.** *A subset $A$ of a space $X$ is called $\alpha$gs-compact relative to $X$ if every collection $\{U_i : i \in I\}$ of $\alpha$gs-open subsets of $X$ such that $A \subset \bigcup\{U_i : i \in I\}$, there exists a finite subset $I_o$ of $I$ such that$A \subset \bigcup\{U_i : i \in I_o\}$.*

**Definition 5.3.** [7] *A subset $A$ of space $X$ is called $\alpha$gs-compact if $A$ is $\alpha$gs-compact as a subspace of $X$.*

**Theorem 5.4.** *Every $\alpha$gs-compact space is compact.*

**Proof:** Let $X$ be a $\alpha$gs-compact space and $\{A_i : i \in I\}$ be an open cover of $X$. Then $\{A_i : i \in I\}$ is a $\alpha$gs-open cover of $X$ as every open set is $\alpha$gs-open set. Since $X$ is $\alpha$gs-compact, the $\alpha$gs-open cover $\{A_i : i \in I\}$ of $X$ has a finite subcover say $\{A_i : i = 1...n\}$ for $X$. This shows that every open cover $\{A_i : i \in I\}$ of $X$ has a finite subcover.Therefore $X$ is compact.

**Theorem 5.5.** *If $X$ is compact and $T_{\alpha gs}$-space, then $X$ is $\alpha$gs-compact.*

**Proof:** Let $\{A_i : i \in I\}$ be a $\alpha$gs-open cover of $X$. As $X$ is $T_{\alpha gs}$-space, $\{A_i : i \in I\}$ is an open cover of $X$. Since $X$ is compact, the open cover $\{A_i : i \in I\}$ of $X$ has a finite subcover say $\{A_i : i = 1, ..., n\}$. This shows that every $g\delta s$-open cover $\{A_i : i \in I\}$ of $X$ has a finite subcover. Therefore $X$ is $\alpha$gs-compact.

**Theorem 5.6.** *A topological space $X$ is $\alpha$gs-compact if and only if every family of $\alpha$gs-closed sets of $X$ having finite intersection property has a nonempty intersection.*

**Proof:** Suppose $X$ is $\alpha$gs-compact. Let $\{A_i : i \in I\}$ be a family of $\alpha$gs-closed sets with finite intersection property. To prove, $\cap_{i\in I}A_i \neq \phi$. Suppose $\cap_{i\in I}A_i = \phi$. Then, $X - \cap_{i\in I}A_i = X$. This implies, $\cup_{i\in I}(X - A_i) = X$. Thus the cover $\{X - A_i : i \in I\}$ is a $\alpha$gs-open cover of $X$. Since $X$ is $\alpha$gs-compact, the $\alpha$gs-open cover $\{X - A_i : i \in I\}$ has a finite subcover say $\{X - A_i : X = X - \cap_{i=1}^{n}A_i$ which implies $X - X = X - [X - \cap_{i=1}^{n}A_i]$ implies that $\cap_{i\in I}A_i = \phi$. This contradicts the hypothesis. Therefore, $\cap_{i\in I}A_i \neq \phi$.

Conversely, suppose every family of $\alpha$gs-closed sets of $X$ with finite intersection property has a nonempty intersection and if possible, let $X$ be not compact, then there exists a $\alpha$gs-open cover of $X$ say $\{G_i : i \in I\}$ having no finite subcover. This implies for any finite sub family $\{G_i : i = 1...n\}$ of $\{G_i : i \in I\}$, $\cup G_{i=1}^{n} \neq X$ which implies that $X - \cup_{i=1}^{n} \neq X - X$, this implies $\cap_{i=1}^{n}(X - G_i) \neq \phi$. Then the family $\{X - G_i : i \in I\}$ of $\alpha$gs-closed sets has a finite intersection property. Therefore $\cap(X - G_i) \neq \phi$, which implies, $\cap(X - G_i)$ is an infinite collection of $\alpha$gs-closed sets with f.i.p. Also, by hypothesis $\{G_i : i \in I\}$ being a $\alpha$gs-open covering of $X$. Therefore $X = \cup_{i\in I}G_i$. Taking complements, $\phi = X - \cup_{i\in I}G_i = \cap_{i\in I}(X - G_i)$, which is an infinite collection of $\alpha$gs-closed subsets of $X$ having f.i.p with empty intersection. This is a contradiction due to the fact that $X$ is not compact. Hence $X$ is $\alpha$gs-compact.

**Theorem 5.7.**
*(i) Every $\alpha$gs-closed subset of $\alpha$gs-compact space is $\alpha$gs-compact relative to X.*
*(ii) The surjective $\alpha$gs-continuous image of a $\alpha$gs-compact space is compact.*
*(iii) If $f : X \to Y$ is $\alpha$gs-irresolute and a subset A of X $\alpha$gs-compact relative to X, then its image f(A) is $\alpha$gs-compact relative to Y.*

**Proof:** (i) Let A be a $\alpha$gs-closed subset of a $\alpha$gs-compact space X. Let $\{U_i : i \in I\}$ be a cover of A by $\alpha$gs-open subsets of X. So $A \subset \bigcup\{U_i : i \in I\}$ and then $(X - A) \cup (\bigcup\{U_i : i \in I\}) = X$. Since X is $\alpha$gs-compact, there exists a finite subset $I_o$ of $I$ such that $(X - A) \cup (\bigcup\{U_i : i \in I_o\}) = X$. Then $A \subset \bigcup\{U_i : i \in I_o\}$. Hence A is $\alpha$gs-compact relative to X.
(ii) Let X be a $\alpha$gs-compact space and $f : X \to Y$ be surjective $\alpha$gs-continuous function. Let $\{U_i : i \in I\}$ be a cover of X by open sets. Then $\{f^{-1}(U_i) : i \in I\}$ is a cover of X by $\alpha$gs-open sets, since f is $\alpha$gs-continuous. By $\alpha$gs-compactness of X, there is fininte subset $I_o$ of $I$ such that $X = \bigcup\{f^{-1}(U_i) : i \in I_o\}$. Since f is surjective, $Y = \bigcup\{f^{-1}(U_i) : i \in I_o\}$ and hence Y is compact.
(iii) is similar to (ii).

**Theorem 5.8.** *If a function $f : X \to Y$ is strongly $\alpha$gs-continuous from a compact space X onto a topological space Y, then Y is $\alpha$gs-compact.*

**Proof:**. Let $\{A_i : i \in I\}$ be a $\alpha$gs-open cover of Y. Since f is strongly $\alpha$gs-continuous, $\{f^{-1}(A_i) : i \in I\}$ is an open cover of X. Again since X is compact space, the open cover $\{f^{-1}(A_i) : i \in I\}$ of X has a finite subcover say $\{f^{-1}(A_i) : i = 1...n\}$. Therefore $X = \bigcup\{f^{-1}(A_i) : i = 1, 2, ..., n\}$ which implies $f(X) = \bigcup A_i : i = 1, 2..., n$ so that $Y = \bigcup A_i : i = 1, 2, ..., n$. That is $\{A_1, A_2, .....A_n\}$ is a finite subcover of $\{A_i : i \in I\}$ for Y. Hence Y is $\alpha$gs-compact.

**Theorem 5.9.** *If a function $f : X \to Y$ is perfectly $\alpha$gs-continuous from a compact space X onto a topological space Y, then Y is $\alpha$gs-compact.*

**Proof:** Similar to the above proof.

**Theorem 5.10.** *Let $f : X \to Y$ be a perfectly $\alpha$gs-continuous surjection. If X is mildly compact, then $Y$ is $\alpha$gs-compact.*

**Proof:** Let $f : X \to Y$ be a perfectly $\alpha$gs-continuous function and let $\{A_i : i \in I\}$ be a $\alpha$gs-open cover of $Y$. Since $f$ is perfectly $\alpha$gs-continuous, $\{f^{-1}(A_i) : i \in I\}$ is clopen cover of $X$. Again since $X$ is mildly compact space, the clopen cover $\{f^{-1}(A_i) : i \in I\}$ of $X$ has a finite subcover say $\{f^{-1}(A_i) : i = 1..., n\}$. Therefore $X = \cup_{i=1}^{n} f^{-1}(A_i)$ which implies $f(X) = Y = \cup_{i=1}^{n} A_i$. That is $\{A_1, A_2..., A_n\}$ is a finite subcover of $\{A_i : i \in I\}$ for $Y$. Hence $Y$ is $\alpha$gs-compact.

# 6 $\alpha$GS-CONNECTED SPACES

**Definition 6.1.** *A topological space X is said to be $\alpha$gs-connected[7] if it cannot be written as the union of two non-empty disjoint $\alpha$gs-open sets.*

**Theorem 6.2.** *For a topological space X, the following are equivalent:*
*(i) X is $\alpha$gs-connected.*
*(ii) The only subsets of X which are both $\alpha$gs-open and $\alpha$gs-closed are the empty set $\phi$ and X.*
*(iii) Each $\alpha$gs-continuous function of X into a discrete space Y with at least two points is a constant function.*

**Proof:** $(i) \rightarrow (ii)$ Suppose (i) holds and F is a proper subset of X, which is both $\alpha$gs-open and $\alpha$gs-closed. Then X-F is also both $\alpha$gs-open and $\alpha$gs-closed. Therefore $X = F \cup (X - F)$ is a disjoint union of two non empty $\alpha$gs-open sets. This contradicts the fact that X is $\alpha$gs-connected. Hence $F = \phi$ or $X$

$(ii) \rightarrow (i)$ Suppose (ii) holds. If possible X is not $\alpha$gs-connected, then $X = A \cup B$, where A and B are disjoint non empty $\alpha$gs-open sets in X. Since A=X-B, implies A is $\alpha$gs-closed set. But by assumption, $A = \phi$ or $X$, which is contradiction. Hence (i) holds.

$(ii) \rightarrow (iii)$ Let $f : X \rightarrow Y$ be a $\alpha$gs-continuous function, where Y is a discrete space with at least two points. Then $f^{-1}(\{y\})$is both $\alpha$gs-open and $\alpha$gs-closed for each $y \in Y$ and $X = \{f^{-1}(\{y\}) : y \in Y\}$. By assumptipon, $f^{-1}(\{y\}) = X$ or $\phi$. If $f^{-1}(\{y\}) = \phi$ for all $y \in Y$, then f will not be a function. Also there cannot exist more than one point $y \in Y$ such that $f^{-1}(\{y\}) = X$. Hence there exists only one $y \in Y$ such that $f^{-1}(y) = X$ and $f^{-1}(\{y_1\}) = \phi$ where $y \neq y_1 \in Y$. This shows that f is constant function.

$(iii) \rightarrow (ii)$ Let F be both $\alpha$gs-open and $\alpha$gs-closed in X. Suppose $F \neq \phi$. Let $f : X \rightarrow Y$ be a $\alpha$gs-continuous function defined by $f(F) = \{a\}$ and $f(X - F) = \{b\}$ for some distinct points a and b in Y. By assumption, f is constant function. Therefore $F = X$.

**Theorem 6.3.** *If $X$ is $T_{\alpha gs}$-space and connected, then $X$ is $\alpha$gs-connected.*

**Proof:** Suppose $X$ is not $\alpha$gs-connected. Then $X = A \cup B$ where $A$ and $B$ are disjoint nonempty $\alpha$gs-open sets in $X$. Since $X$ is $T_{\alpha gs}$-space, implies $A$ and $B$ are disjoint non empty open sets in $X$, implies $X$ is not connected space. This is contradiction to the hypothesis. Therefore $X$ is $\alpha$gs-connected.

**Theorem 6.4.** *If $f : X \rightarrow Y$ is a $\alpha$gs-irresolute, surjection and $X$ is $\alpha$gs-connected, then $Y$ is $\alpha$gs-connected.*

**Proof:** Suppose $Y$ is not $\alpha$gs-connected. Then $Y = A \cup B$ where $A$ and $B$ are disjoint nonempty $\alpha$gs-open sets in $Y$. Since $f$ is a $\alpha$gs-irresolute, surjection, $X = f^{-1}(A) \cup f^{-1}(B)$ are disjoint non empty $\alpha$gs-open subsets of $X$, implies $X$ is not $\alpha$gs-connected space. This is contradiction to the hypothesis. Therefore $Y$ is $\alpha$gs-connected.

**Theorem 6.5.** *If $f : X \rightarrow Y$ is a strongly $\alpha$gs-continuous surjection and X is connected, then Y is $\alpha$gs-connected.*

**Proof:**Suppose Y is not $\alpha$gs-connected. Then $Y = A \cup B$ where A and B are disjoint nonempty $\alpha$gs-open sets in Y. Since f is a strongly $\alpha$gs-continuous surjection, $X = f^{-1}(A) \cup f^{-1}(B)$ are disjoint non empty open subsets of X, implies X is not connected space. This is contradiction to the hypothesis. Therefore Y is $\alpha$gs-connected.

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