Iğdır Üniversitesi Fen Bilimleri Enstitüsü Dergisi, 12(1): 435-445, 2022
Journal of the Institute of Science and Technology, 12(1): 435-445, 2022
ISSN: 2146-0574, eISSN: 2536-4618
Matematik / Mathematics **DOI: 10.21597/jist.975119**
Araştırma Makalesi / ***Research Article***
Geliş tarihi / Received: 27.07.2021 **Kabul tarihi / Accepted: 15.10.2021**

Atıf İçin: Karta M, 2022. Fisher Denkleminin Yaklaşık Çözümü için Sayısal Metod. Iğdır Üniversitesi Fen Bilimleri Enstitüsü Dergisi, 12(1): 435-445.

To Cite: Karta M, 2022. Numerical Method for Approximate Solution of Fisher's Equation. Journal of the Institute of Science and Technology, 12(1): 435-445.

Numerical Method for Approximate Solution of Fisher's Equation

Melike KARTA[1]

ABSTRACT: In this paper, Fisher's reaction diffusion equation has been solved numerically by Strang splitting technique depending on collocation method with cubic B-spline. For our purpose, the initial and boundary value problem consisting of Fisher's equation is split into two sub-problems to be one linear and the other nonlinear such that each one contains the derivative in terms of time. Then, the whole problem is reduced to the algebraic equation system using finite element collocation method combined with the cubic B-spline for spatial discretization and the convenient classical finite difference approaches for time discretization. The effective and efficiency of the newly given method have been shown on the four examples. In addition, the newly obtained numerical results are shown in formats graphical profiles and tables to compare with studies available in the literature.

Keywords: Fisher's equation, B-splines, Collocation method

[1] Melike KARTA (**Orcid ID:** 0000-0003-3412-4370), Ağrı İbrahim Çeçen Üniversitesi, Fen Edebiyat Fakültesi, Matematik Bölümü, Ağrı, Türkiye
***Sorumlu Yazar/Corresponding Author:** Melike KARTA, e-mail: mkarta@agri.edu.tr

INTRODUCTION

In this manuscript, we are going to consider one dimensional non-linear Fisher's equation

$$U_t = \gamma U_{xx} + \mu U(1-U), \ x_L \le x \le x_R, \quad t \ge 0 \tag{1}$$

with conditions given at the boundaries and the initial time

$$\begin{aligned} &U(x,0) = U_0(x), \quad x_L \le x \le x_R \\ &U(x_L,t) = h_0(t), \quad U(x_R,t) = h_1(t) \\ &U_x(x_L,t) = f_0(t), \quad U_x(x_R,t) = f_1(t) \end{aligned} \tag{2}$$

Fisher's equation has influential implementations in many fields such as science and engineering. Firstly, Fisher's equation is investigated theoretically by (Kalmogoroff et.al., 1937; Canosa, 1973). Outside of theoretical works, the approximate solution of Fisher's equation has been handled by lots of authors. (Gazdag and Canosa, 1974) used a pseudo-spectral approach for equation. The numerical work of Fisher's equation has been described by a moving mesh method by (Qiu and Sloan, 1998). (Zhao and Wei, 2003) solved equation by discrete singular convolution (DSC) algorithm. The wavelet-Galerkin approach using complex harmonic wavelets has been presented by (Cattani and Kudreyko, 2008). (Mittal and Arora, 2010) applied equation finite difference method with cubic B-spline. The approximate solution of the equation has been investigated using Galerkin method with quadratic B-spline by (Dağ et al., 2010). (Mittal and Jain, 2012) proposed finite element collocation method with cubic B-spline to approximate the non-linear parabolic partial differential eqation with Neumann's boundary conditions. The numerical approach of equation has been given via collocation method with modified cubic B-spline by (Mittal and Jain, 2013). Also, to find solutions of the equation, collocation method with the extended cubic B-spline has been used by (Ersoy and Dag, 2015). (Dag and Ersoy, 2016) applied exponential B-spline collocation method for the equation. The approximate solution of equation with a new method "extend modified cubic B-spline differential quadrature method "(EMCB-DQM) has been introduced by (Shukla and Tamsir, 2016). For Fisher's equation, (Tamsir et al., 2018) suggested an exponential modified cubic B-spline differential quadrature algorithm. They used Runge-Kutta method for this purpose. (Kapoor et al., 2020) proposed Hyperbolic B- spline based on differential quadrature method for the nonlinear Fisher's equation.

In this paper, we employ Strang splitting technique using collocation method with cubic B-spline for the numerical approach of given equation. For this purpose, firstly, in section 2, the finite element collocation method with cubic B-spline is explained and Fisher's equation split into two sub-equations and then the obtained sub-equations are applied Strang splitting technique with help of collocation method utilizing cubic B- spline with the proper conditions given at the boundaries and the initial time of problem. After that, the initial vector is formed using the condition at initial time and the conditions at the boundaries. In section 3, Fisher's equation is applied to four test problems and the error norms L_2 and L_∞ are computed and then compared with existing studies in literature. In section 4, a brief conclusion is given.

MATERIALS AND METHODS

For the numerical behavior of Equation (1), we consider the solution domain $[x_L, x_R]$ and define $x_L = x_0 < x_1 < \cdots < x_N = x_R$ as uniform partition of the solution range by the nodal points x_m with $h = x_{m+1} - x_m = \frac{x_R - x_L}{N}$, $m = 0,1,\dots,N$. An approximate solution corresponding to the analytical solution $\mathrm{U(x,t)}$ can be given as

$$U_N(x,t)=\sum_{m=j-1}^{j+1}\delta_m(t)\varphi_m \tag{3}$$

where $\delta_m(t)$ are unknown time-dependent parameters obtained using the boundary conditions and equation (1). It is presented cubic B-spline functions on the domain $[x_L,x_R]$ in terms of nodal points x_m by (Prenter , 1975) as follows

$$\varphi_m(x)=\frac{1}{h^3}\begin{cases}(x-x_{m-2})^3\,, & [x_{m-2},x_{m-1}]\\ h^3+3h^2(x-x_{m-1})+3h(x-x_{m-1})^2-3(x-x_{m-1})^3, & [x_{m-2},x_{m-1}]\\ h^3+3h^2(x_{m+1}-x)+3h(x_{m+1}-x)^2-3(x_{m+1}-x)^3, & [x_{m-2},x_{m-1}]\\ (x_{m+2}-x)^3, & [x_{m-2},x_{m-1}]\\ 0, & otherwise\end{cases} \tag{4}$$

where $\{\varphi_1,\varphi_0,\dots,\varphi_N,\varphi_{N+1}\}$ is a base on the domain $[x_L,x_R]$. Equation (1) contains the term U_m, the first and second derivatives of U_m. So, we need the values of the 1st U_m' , the 2nd U_m'' with respect to space variable x and the values U_m in terms of cubic B-spline functions using the approximations (3), (4) and time-dependent parameters $\delta(t)$. These values are obtained as follows

$$\begin{aligned}U_m&=U(x_m)=\delta_{m-1}+4\delta_m+\delta_{m+1}\\ U_m'&=U'(x_m)=(3/h)(-\delta_{m-1}+\delta_{m+1}\,)\\ U_m''&=U''(x_m)=\left(\frac{6}{h^2}\right)(\delta_{m-1}-2\delta_m+\delta_{m+1}).\end{aligned} \tag{5}$$

The time split form of Equation (1) is as follows

$$U_t-\gamma U_{xx}-\mu U=0, \tag{6}$$

$$U_t+\mu UU=0 \tag{7}$$

By substituting the values U_m, U_m' and U_m'' in system (5) in equations (6) and (7), we obtain the 1st order system of ODE as follows:

$$\dot{\delta}_{m-1}+4\dot{\delta}_m+\dot{\delta}_{m+1}-\frac{6}{h^2}\gamma(\delta_{m-1}-2\delta_m+\delta_{m+1})-\mu(\delta_{m-1}+4\delta_m+\delta_{m+1})=0 \tag{8}$$

$$\dot{\delta}_{m-1}+4\dot{\delta}_m+\dot{\delta}_{m+1}+\mu z_m(\delta_{m-1}+4\delta_m+\delta_{m+1})=0. \tag{9}$$

Here ‘’.’’ denotes derivative in terms of time variable t and for linearization form, the value of z_m is taken as

$$z_m=(\delta_{m-1}+4\delta_m+\delta_{m+1}).$$

When it is written $\frac{\delta_m^{n+1}+\delta_m^n}{2}$ instead of the parameter δ_m and $\frac{\delta_m^{n+1}-\delta_m^n}{\Delta t}$ instead of the $\dot{\delta}_m$, we have the following equations

$$v_1\delta_{m-1}^{n+1}+v_2\delta_m^{n+1}+v_3\delta_{m+1}^{n+1}=v_4\delta_{m-1}^n+v_5\delta_m^n+v_6\delta_{m+1}^n \tag{10}$$

$$z_1\delta_{m-1}^{n+1}+z_2\delta_m^{n+1}+z_3\delta_{m+1}^{n+1}=z_4\delta_{m-1}^n+z_5\delta_m^n+z_7\delta_{m+1}^n \tag{11}$$

respectively and here

$$v_1=1-\frac{3\gamma\Delta t}{h^2}-\frac{\mu\Delta t}{2},\quad v_2=4+\frac{6\gamma\Delta t}{h^2}-2\mu\Delta t,\quad v_3=1-\frac{3\gamma\Delta t}{h^2}-\frac{\mu\Delta t}{2},$$

$$v_4=1+\frac{3\gamma\Delta t}{h^2}+\frac{\mu\Delta t}{2},v_5=4-\frac{6\gamma\Delta t}{h^2}+2\mu\Delta t,\quad v_6=1+\frac{3\gamma\Delta t}{h^2}+\frac{\mu\Delta t}{2},$$

$$z_1 = 1 + \frac{\mu z_m \Delta t}{2}, \quad z_2 = 4 + 2\mu z_m \Delta t, \quad z_3 = 1 + \frac{\mu z_m \Delta t}{2},$$
$$z_4 = 1 - \frac{\mu z_m \Delta t}{2}, \quad z_5 = 4 - 2\mu z_m \Delta t, \quad z_6 = 1 - \frac{\mu z_m \Delta t}{2}.$$

Systems (10) and (11) contain $(\mathrm{N}+1)$ equations and $(\mathrm{N}+3)$ unknowns δ_m parameters, $\mathrm{m} = 0,1,\dots,\mathrm{N}+1$. We have to exterminate δ_{-1} and δ_{N+1} to obtain a solvable system using the boundary conditions $U_x(x_R,\ t) = f_0(t),\ \ U_x(x_R,\ t) = f_1(t)$ for the systems (10) and (11) . Thus, we obtain the following equations

$$\delta_{-1} = \delta_1 - \tfrac{h}{3} U_x(1), \quad \delta_{N+1} = \delta_{N-1} + \tfrac{h}{3} U_x(N+1) \tag{12}$$

The parameters δ_{-1} and δ_{N+1} are eliminated in systems (10) and (11) using the equation (16) and eventually obtained a three diagonal $(\mathrm{N}+1)\mathrm{x}(\mathrm{N}+1)$ band matrix. A unique solution of these systems is obtained using Thomas algorithm. We need the initial parameters δ_m^0 to solve ones. For this, the initial parameters $\mathrm{U}(\mathrm{x},0) = U_0(x)$ are firstly obtained. The initial vector δ_m^0 can be found out using the following IC and BCs

$$\mathrm{U}(\mathrm{x},0) = U_0(x),$$
$$U_x(x_L,\ t) = f_0(t), \qquad U_x(x_R,\ t) = f_1(t)$$

Thus, the initial vector δ_m^0 is obtained as follows

$$A^* = \begin{bmatrix} 4 & 2 & & & & \\ 1 & 4 & 1 & & & \\ & & \ddots & & & \\ & & & 1 & 4 & 1 \\ & & & & 2 & 4 \end{bmatrix}, \quad \delta^0 = \begin{bmatrix} \delta_0^0 \\ \delta_1^0 \\ \vdots \\ \delta_{N-1}^0 \\ \delta_N^0 \end{bmatrix}, \quad B^* = \begin{bmatrix} U(x_0) + \frac{h}{3} f_0 \\ U(x_0) \\ \vdots \\ U(x_{N-1}) \\ U(x_N) - \frac{h}{3} f_1 \end{bmatrix}$$

Namely, it is written in the form of $A^*\delta^0 = B^*$. From here, the dimensional band matrix $(N+1)x(N+1)$ for parameters δ_m^0 is found.

RESULTS AND DISCUSSION

In the present section, we will consider four numerical examples to determine the effectiveness of the proposed approach using Strang splitting technique via collocation method with cubic B-spline for Fisher's equation. For our goal, we compute the error norms L_2 and L_∞ determined as

$$L_2 = \|U - U_N\|_2 = \sqrt{h \sum_{j=0}^{N} (U - U_N)^2},$$
$$L_\infty = \|U - U_N\|_\infty = \max_j |U - U_N|$$

Relative error (Madzvamuse, 2006)= $\dfrac{\sqrt{\sum_{j=0}^{N} \left|U_j^{n+1} - U_j^n\right|^2}}{\sqrt{\sum_{j=0}^{N} \left|U_j^{n+1}\right|^2}}$.

Example 1

In the present example, we are going to deal with Equation (1) with BCs $U(x_L,\ t) = U(x_R,\ t) = 0$ and IC given by

$U_0(x) = sech^2(10x).$

For this problem, discretization parameters are chosen as $\mathrm{h} = 0.025, \Delta\mathrm{t} = 0.05$ on the domain $[-50,50]$ for $\gamma = 0.1$, μ=1 as in the studies (Dağ et al., 2010) and (Dağ and Ersoy, 2016). Physcial behaviour of equation (1) has been drawn in graphical profiles. In Figure 1, for different time level $\mathrm{t} = 0$ to $\mathrm{t} = 0.5$, we have seen that near $\mathrm{x} = 0$, $\mathrm{U(x,t)}$ reachs maximum value $\mathrm{U} = 1$. However, the peak rapidly comes down since diffusion term U(1-U) dominates over reaction. Because of the reaction influence, Figure 2 indicates that the peak value is gradually increasing in time from $\mathrm{t} = 0$ to 5. Also it is seen that the peak value reaches until the top $\mathrm{U} = 1$ in at the time levels $0,5,10,15,20,25,30,40$ in Figure 3. Tables 1 presents a comparison of the relative error at various times and shows that our results are much better.

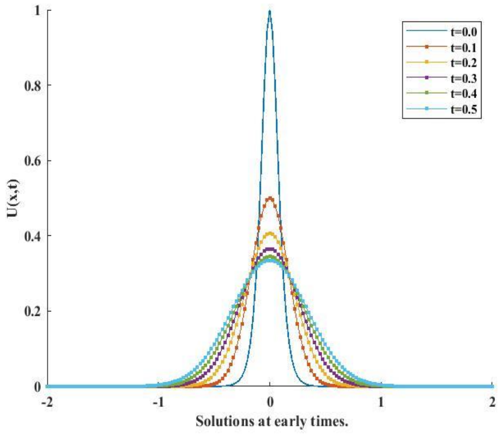

Figure 1. The numerical approaches of Example 1 for $t = 0(0.1)5$

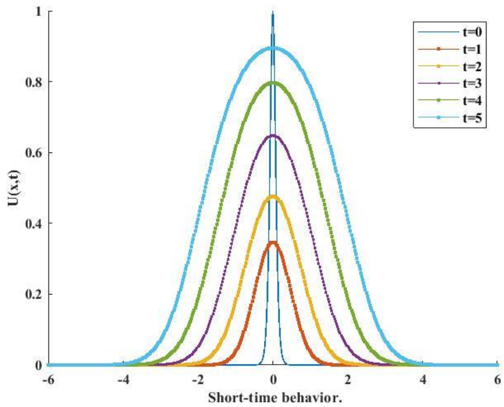

Figure 2. The numerical approaches of Example 1 for $t = 0(1)5$

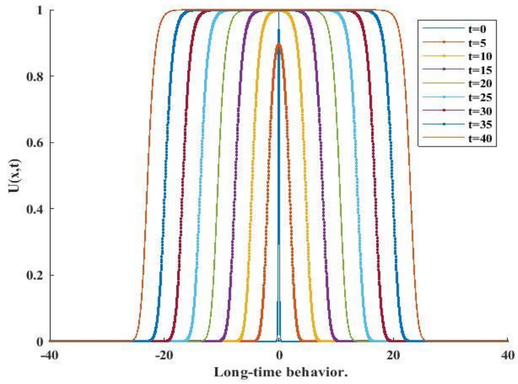

Figure 3. The numerical approaches of Example 1 for $t = 0(5)40$

Tablo 1. Comparison of relative errors for Example 1 at various times.

Relative Error	t=5	t=10	t=15	t=20	t=40
Present	1.383E − 2	7.835E − 3	6.029E − 3	5.067E − 3	3.417E -3
(Dağ et al., 2010)	1.386E − 2	7.860E − 3	6.054E − 3	5.090E − 3	3.434E -3

Example 2

In this example, Fisher's equation is taken with local boundary condition and initial condition as follows:

$$\begin{cases} e^{10(x+1)}, & x < 1 \\ 1, & -1 \leq x \leq 1 \\ e^{-10(x+1)}, & x > 1 \end{cases}$$

or this problem, we use coefficients $\alpha = 0.1,\ \beta = 1$ and parameters $\mathrm{h} = 0.025, \Delta\mathrm{t} = 0.05$ as in the first problem over domain $[-50,50]$ until time 40 considering to studies (Dağ et al., 2010) and (Dağ and Ersoy, 2016). In Figure 4 and 5, it is graphically performed the solutions at early times. In these figures, the reaction-diffusion effective is quite minor. Because the reaction effect is more effective than the diffusion effect.Thus, they become smooth from having sharp. Also, figure 6 shows that the top of the wave have risen and shown that it is getting more and more flat. Table 2 submits a comparison of the relative error at various times and indicates that our results are much better.

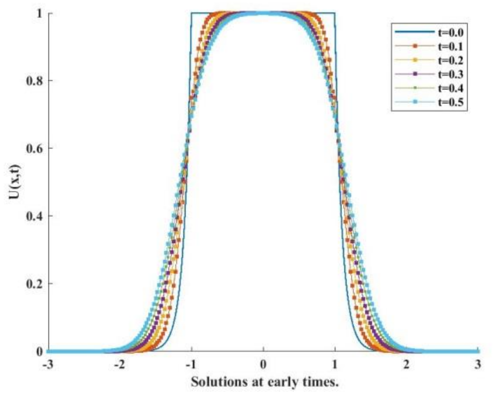

Figure 4. The numerical approaches of Example 2 for $t = 0(0.1)0.5$

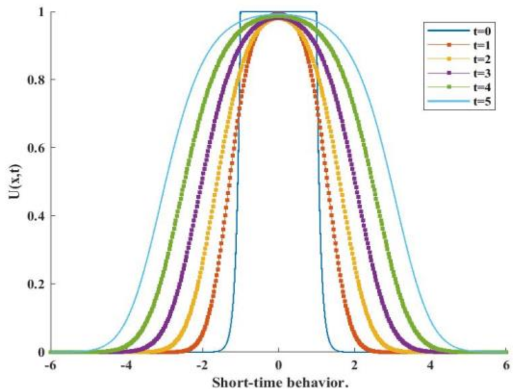

Figure 5. The numerical approaches of Example 2 for $t = 0(1)5$

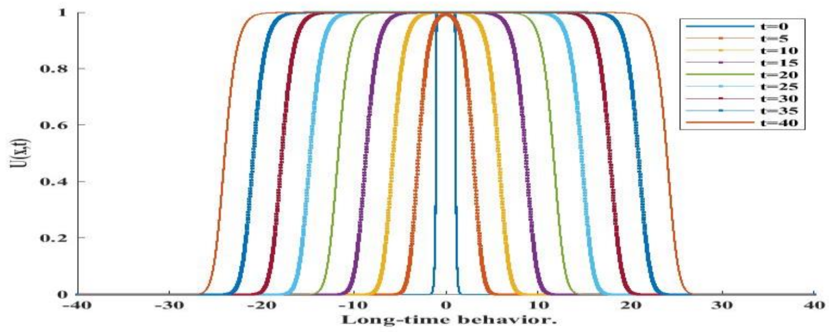

Figure 6. The numerical approaches of Example 2 for $t = 0(5)40$

Tablo 2. Comparison of relative errors for Example 2 at various times.

Relative Error	t=5	t=10	t=15	t=20	t=40
Present	9.397E − 3	6.892E − 3	5.590E − 3	4.804E − 3	3.335E -3
(Dağ et al., 2010)	9.435E − 3	6.917E − 3	5.614E − 3	4.825E − 3	3.352E -3

Example 3

In the present example, we handle Eq. (1) with BCs $U(x_L,\ t) = 1,\ U(x_R,\ t) = 0,\ t \geq 0$ and the analytical solution given as follows:

$$U(x,t) = \left[1 + \exp\left(\sqrt{\frac{\mu}{6}}x - \sqrt{\frac{5\mu}{6}}t\right)\right]^{-2}$$

In Table 3, we have firstly presented a comparison of the error norms L_2 and L_∞ of Equation (1) with discretization parameters h=1, Δt=0.01 on range $-10 \leq$ x ≤ 10 for reaction-diffusion coefficients γ=1 and μ=2 by considering the study (Mittal and Jain, 2012) and also, we have calculated the error norms L_2 and L_∞ for values $\mathrm{h} = 0.5,\ \Delta\mathrm{t} = 0.01$ at times $\mathrm{t} = 5,10,15,20$ in Table 4. For values h=0.25, Δt=0.01, t≤ 2, in Figure 7, it is shown graphically together a comparison of analytical and numerical scheme of Example 3. Secondly, in Table 5, we have presented a comparison of the error norms L_2 and L_∞ for the numerical approach of Example 3 by taking $\mathrm{N} = 64, 150$ and $\Delta\mathrm{t} = 0.000005$ at times $\mathrm{t} = 0.0005, 0.0015, 0.0025, 0.0035$ over region $[-0.2,0.8]$ with reaction-diffusion coefficient γ=1, μ =10000 taking into account some studies in literature. Table 3 shows that our results are very good and Table 4 indicates that we have achieved very low results. Figure 7 exhibits that the numerical scheme of the problem show fairly a good physical behaviours for h=0.25, Δt=0.01 at times t ≤2. Table 5 displays that results of the error norms L_2 and L_∞ computed by Strang splitting technique utilizing collocation method combined with cubic B-spline are better than in (Dağ and Ersoy, 2016), CN (Qiu and Sloan, 1998) and close to results in (Dağ et al., 2010), ASD (Qiu and Sloan, 1998) and it shows that the results of DSC (Qiu and Sloan, 1998) are better than the presented method. Also, it is seen that solution profiles and absolute error distributions in Figure 8 exhibit fairly accurate physical behaviors for parameters $\mathrm{N} = 200$ and $\Delta\mathrm{t} = 0.000005$ at times $\mathrm{t} = 0.0005, 0.001, 0.0015, 0.002, \dots, 0.0035$ as in (Dağ et al., 2010). So, the method presented can be recommended as alternative solution to other non-linear equations such as the Fisher's equation. Additionally, to indicate the effectiveness and performance of the suggested method, it is presented together the numerical and analytical solution graphically at different times in Figure 9 taking $\mu = 2000$ and 5000 for $\mathrm{N} = 200$ with $\Delta t = 0.00001$ on the solution region $[-0.2,0.8]$ as in studies (Mittal and Jain, 2013) and (Kappoor and Joshi, 2020).

Tablo 3. Comparison of the error norms L_2 and L_∞ for $\Delta t = 0.01, h = 1$ of Example 3.

	Present		(Mittal and Jain, 2012)	
t	L_2	L_∞	L_2	L_∞
0.5	1.77E-03	1.10E-03	1.76E-03	1.10E-03
1	2.93E-03	1.75E-03	2.92E-03	1.75E-03
1.5	3.65E-03	1.85E-03	3.67E-03	1.86E-03
2	4.28E-03	2.93E-03	4.50E-03	3.00E-03

Tablo 4. The error norms L_2 and L_∞ for $\Delta t = 0.01, h = 1$ of Example 3 at some various times.

Errors	t=5	t=10	t=15	t=20
L_2	1.66E-03	1.30E-03	1.76E-03	1.10E-03
L_∞	0.33E-03	0.08E-03	2.92E-03	1.75E-03

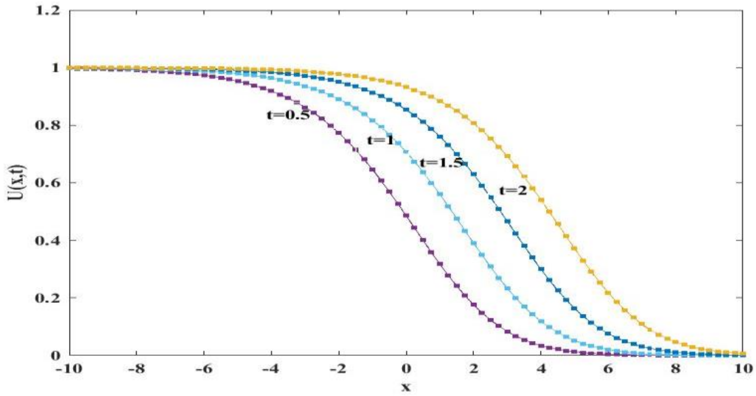

Figure 7. The numerical solutions of Example 3 for $t \le 2$ ($\Delta t = 0.01, h = 0.25$)

Table 5. Comparison of the error norms L_2 and L_∞ at various times t of Example 3 for $\alpha = 1, \beta = 10000$

Method	N	Error	t			
			0.0005	0.0015	0.0025	0.0035
Present	64	L_2	$1.50E-3$	$0.21E-1$	$4.99E-2$	$0.79E-1$
		L_∞	$6.41E-3$	$0.89E-1$	$2.12E-1$	$3.25E-1$
Present	150	L_2	$4.45E-4$	$0.36E-2$	$0.86E-2$	$1.40E-2$
		L_∞	$3.27E-2$	$1.52E-2$	$3.64E-2$	$5.90E-2$
Present	200	L_2	$0.35E-3$	$0.02E-1$	$0.48E-2$	$0.78E-2$
		L_∞	$2.84E-3$	$0.83E-2$	$2.00E-2$	$3.27E-2$
Dağ et.al., 2010	150	L_2	$6.89E-5$	$1.30E-2$	$1.55E-2$	$8.82E-3$
		L_∞	$2.57E-4$	$5.65E-2$	$6.63E-2$	$3.93E-2$
Dağ and Ersoy, 2016 (p = 1)	64	L_∞	$1.10E-2$	$1.49E-1$	$3.44E-1$	$5.08E-1$
CN(Zhao and Wei, 2003)	64	L_2	$1.92E-3$	$2.65E-2$	$6.18E-2$	$9.91E-1$
		L_∞	$1.03E-2$	$1.25E-1$	$2.80E-1$	$4.48E-1$
ASD(Zhao and Wei, 2003)	64	L_2	$2.09E-3$	$1.06E-2$	$2.02E-2$	$2.35E-2$
		L_∞	$1.07E-2$	$4.93E-2$	$9.37E-2$	$9.44E-1$
DSC(Zhao and Wei, 2003)	64	L_2	$1.24E-6$	$5.92E-7$	$1.16E-6$	$1.64E-6$
		L_∞	$6.28E-6$	$1.98E-6$	$4.46E-6$	$6.22E-6$

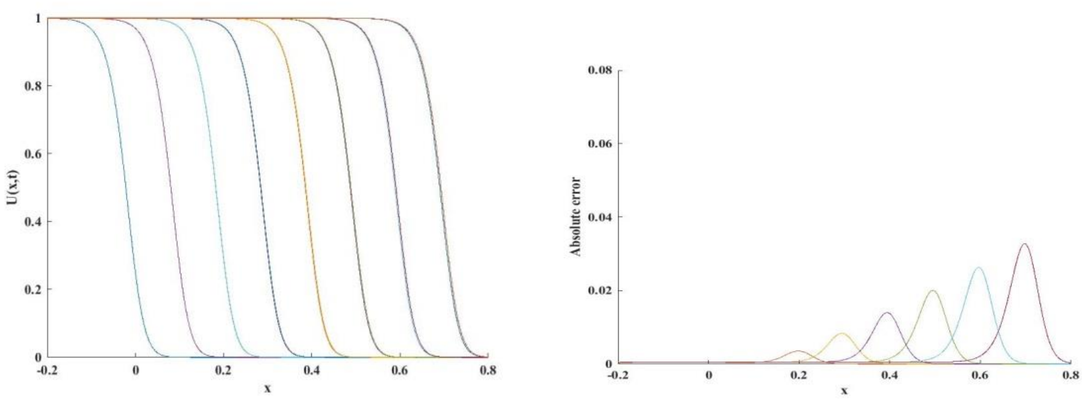

Figure 8. Solution profiles and absolute errors and of Example 3 for $N = 200$

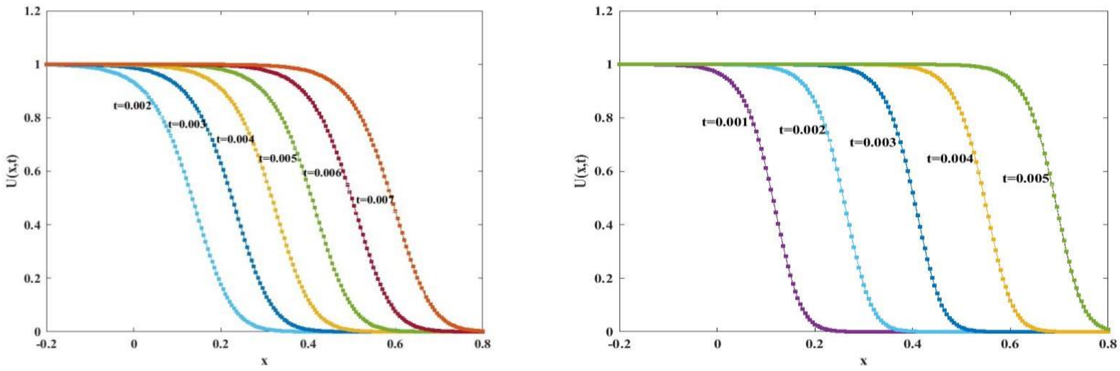

Figure 9. The approximate solutions for $\beta = 2000$ at times $t = 0.002, 0.003, 0.004, 0.005, 0.006, 0.007$ and $\beta = 5000$ at times $t = 0.001, 0.002, 0.003, 0.004, 0.005$ for $N = 200$ of Example 3

Example 4

In the last example, we get non-linear Fisher's equation given as

$$U_t - \alpha U_{xx} = -\alpha_1 U^2 + \beta_1 U;\ -\infty \le x \le \infty,\ t \ge 0,$$

having the following initial and boundary conditions

$$U(x,t) = -\frac{\beta_1}{4\alpha_1}\left[sech^2\left(-\sqrt{\frac{\beta_1}{24c}}x\right) - 2tanh\left(-\sqrt{\frac{\beta_1}{24c}}x\right) - 2\right],$$

$$U(x_L,t) = 0.5\,,\quad U(x_R,t) = 0.$$

The analytical solution for the present problem is taken as

$$U(x,t) = -\frac{\beta_1}{4\alpha_1}\left[sech^2\left(\pm\sqrt{\frac{\beta_1}{24c}}x + \frac{5\beta_1}{12}t\right) - 2tanh\left(\pm\sqrt{\frac{\beta_1}{24c}}x + \frac{5\beta_1}{12}t\right) - 2\right].$$

The coefficients in this problem are choosen as α=1, α_1=1, β_1=0.5, c=1 for h=0.25, Δt=0.01 at times t=2 and t=4 on solution domain [-30,30] as in studies (Cattani and Kudreyko, 2008), (Mittal and Arora, 2010) and (Mittal and Jain, 2013). Table 6 and Table 8 report a comparison of the presented method solutions with those obtained in (Cattani and Kudreyko, 2008) and (Mittal and Arora, 2010). Also, Tables 7 and 9 give a comparation of the absolute error results found out by the presented method. From these tables, it is seen that our results are better than those of the previous studies. Figure 10 clearly illustrates a comparison between numerical and analytical solutions at times $t = 1,2,3,4,5$ and this figure displays that it can be found a good conformity with those given the earlier studies.

Tablo 6. Comparison of approximate solutions of Example 4 at various values of x for $t = 2$.

x	Cattani and Kudreyko,2008	Mittal and Arora, 2010	Present	Exact
-20	0.498681	0.498653	0.498650	0.498652
-16	0.495130	0.495745	0.495739	0.495740
-12	0.486758	0.486679	0.486668	0.486669
-8	0.459576	0.459478	0.459476	0.459478
-4	0.386681	0.386742	0.386787	0.386791
2	0.158878	0.159011	0.158859	0.158850
6	0.041822	0.041877	0.041852	0.041851
10	0.006455	0.006426	0.006465	0.006465
14	0.000750	0.000746	0.000754	0.000755
18	7.617E-05	7.79E-05	7.91E-05	7.92E-05

Tablo 7. Comparison of absolute error at various values of x for $t = 2$ of Example 4.

x	Mittal and Arora, 2010	Present
-20	1.52E −0 6	1.37E −0 6
-16	4.56E − 06	1.15E −0 6
-12	9.42E − 06	7.78E −0 7
-8	2.39E − 07	1.24E −0 7
-4	4.91E − 05	4.37E −0 6
2	1.61E − 04	8.77E −0 6
6	2.54E − 05	8.28E −0 6
10	3.92E − 05	2.65E −0 6
14	9.46E − 06	6.30E −0 7
18	1.23E − 06	8.24E −0 8

Tablo 8. Comparison of numerical approach at various values of x for $t = 4$ of Example 4.

x	Cattani and Kudreyko, 2008	Mittal and Arora, 2010	Present	Exact
-20	0.498678	0.499412	0.499411	0.499413
-16	0.498525	0.498146	0.498140	0.498142
-12	0.494757	0.494149	0.494139	0.494140
-8	0.481776	0.481763	0.481754	0.481756
-4	0.445508	0.445372	0.445394	0.445398
2	0.279025	0.280082	0.279947	0.279941
6	0.116980	0.117196	0.116975	0.116963
10	0.025927	0.025881	0.025967	0.025974
14	0.003695	0.003559	0.003618	0.003622
18	0.000409	0.000395	0.000405	0.000406

Tablo 9. Comparison of absolute error at various values of x for $t = 4$ of Example 4.

x	Mittal and Arora, 2010	Present
-20	1.35E −0 6	1.93E −0 6
-16	4.01E − 06	1.86E −0 6
-12	8.86E − 06	1.40E −0 6
-8	7.28E − 06	1.40E −0 6
-4	2.53E − 05	3.51E −0 6
2	1.41E − 04	5.96E −0 6
6	2.33E − 04	1.20E −0 5
10	9.30E − 05	7.02E −0 6
14	6.29E − 05	4.20E −0 6
18	1.12E − 05	7.45E −0 7

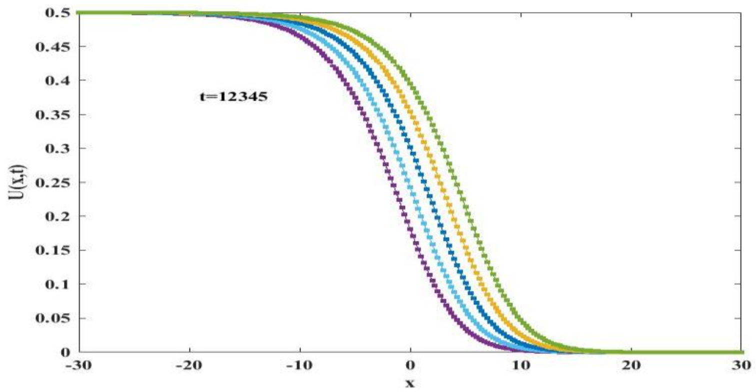

Figure 10. Approximate and exact solutions of Example 4 for $\Delta t = 0.01, h = 0.25.$ at $t = 1$ to$t = 5$

CONCLUSION

In the current study, the approximate results of nonlinear Fisher's equation have been obtained via Strang splitting technique using finite element collocation method combined with cubic B-spline. To display the correctness and validity of the presented method, the four examples given with suitable the initial-boundary condition available in literature have been considered and computed the error norms L_2 and L_∞. It has been seen that numerical results acquired with the presented method are very good. Consequently, we can say that the solutions of this study gotten Strang splitting technique can be both effectively implemented and considered as an alternative to obtain numerical results of these type of problems.

REFERENCES

Canosa J, 1973. On a nonlinear diffusion equation describing population growth, IBM J Res Dev 17: 307–313.

Cattani C, Kudreyko A, 2008. Mutiscale Analysis of the Fisher Equation, ICCSA , Part I, Lecture Notes in Computer Science, Springer-Verlag, Berlin/Heidelberg, Vol. 5072: 1171–1180.

Dag I, Sahin A, Korkmaz A, 2010. Numerical investigation of the solution of Fisher's equation via the B-spline Galerkin method. Numer Methods Partial Differ Equ 26(6): 1483–1503.

Dag I, Ersoy O, 2016. The exponential cubic B-spline algorithm for Fisher equation. Chaos Solitons Fractals 86: 101–106.

Dag I, 1994. Studies of B-spline finite elements, Ph.D. thesis, University College of North Wales, Bangor, Gwynedd.

Ersoy O, Dag I, 2015. The extended B-spline collocation method for numerical solutions of Fishers equation. AIP Conf Proc 1648: 370011.

Strang G. (1968) On the construction and comparison of difference schemes, SIAM J. Numer. Anal. 5: 506-517.

Gazdag J, Canosa J, 1974. Numerical solution of Fisher's equation, J Appl Prob 11: 445–457.Geiser J, Bartecki K, 2008. Additive,multiplicative and iterative splitting methods for Maxwell equations, Algorithms andapplications, AIP Conf. Proc. vol. 1978 p. 470002.

Hundsdorfer W, Verwer J, 2003. Numerical Solution of Time-Dependent Advection-Diffusion-Reaction Equations (First Edition), Springer-Verlag Berlin Heidelberg.

Kolmogoroff A, Petrovsky I, Piscounoff N, 1937. Study of the diffusion equation with growth of the quantity of matter and its application to biology problems, Bulletin de l'Université d'état à Moscou,Sére Internationale, Sec. A 1, 1–25.

Kapoor M, 2020. Solution of non-linear Fisher's reaction-diffusion equation by using Hyperbolic B-spline based differential quadrature method Journal of Physics: Conference Series 1531 -012064 IOP Publishing doi:10.1088/1742-6596/1531/1/012064.

Madzvamuse A, 2006. Time stepping schemes for moving grid finite elements applied to reaction-diffusion systems on fixed and growing domains, J Comput Phys 214, 239–263.

Mittal R.C, Arora G. 2010. Efficient numerical solution of Fisher's equation by using B-spline method Int. J. Comput. Math. 87 (13): 3039–51.

Mittal R.C, Jain R. (2012) Cubic B-splines collocation method for solving nonlinear parabolic partial differential equations with Neumann boundary conditions commun Nonlinear sci. Numer.Simulat 17: 4616-4625.

Mittal R.C, Jain R.K., (2013) Numerical solutions of nonlinear Fisher's reaction-diffusion equation with modified cubic B-spline collocation method Math. Sci. 7 (12): 1–10.

Qiu Y, Sloan D. M. (1998) Numerical solution of Fisher's equation using amoving mesh method, J Comput Phys 146: 726–746.

Prenter P. M. (1975) Spline sandvariational methods, Wiley, New York.

Shukla H.S, Tamsir M. (2016) Extended modified cubic B-spline algorithm for nonlinear Fisher's reaction-diffusion equation. Alexandria Engineering Journal 55(3): 2871-79.

Tamsir M, Srivastava V.K, Dhiman N. (2018) Chauhan, Numerical Computation of Nonlinear Fisher's Reaction–Diffusion Equation with Exponential Modified Cubic B-Spline Differential Quadrature Method.Int. J. Appl. Comput. Math 4-6.

Zhao S, Wei G.W. (2003) Comparison of the discrete singular convolution and three other numerical schemes for solving Fisher's equation, SIAM J Sci Comput 25: 127–147.