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EFFECT OF HIGHER ORDER TAYLOR SERIES EXPANSION TERMS OF THE NI-RPIM ON THE SOLUTION ACCURACY OF 2D ELASTIC PROBLEMS

YÜKSEK DERECELİ NI-RPIM TAYLOR SERİSİ AÇILIMI TERİMLERİNİN 2 BOYUTLU ELASTİK PROBLEMLERİN ÇÖZÜM HASSASİYETİNE ETKİSİ

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Abstract

In this study, effects of higher order Taylor series expansion terms in the nodal integration scheme of radial point interpolation method (NI-RPIM) are investigated on the solution accuracy of 2D elastic problems. The nodal integration scheme is proposed by Liu et al. [1] and based on the Taylor series expansion. It is used with increasing the order of terms up to 4th order in this study. 3 different case studies are applied and the results are compared with analytical, FEM and RPIM with Gaussian integration solutions. Also the effect of number of nodes is investigated. It can be accepted that the usage of Taylor series expansion and Gaussian method in integration of RPIM give similar solution times. However NI-RPIM with higher order Taylor series expansion terms has better solution speed than using Gaussian integration, especially in the solutions of model which has higher number of nodes. It is detected that 2nd order terms of nodal integration give sufficient results. If stress values are investigated, 4th order terms of nodal integration can be used for accuracy of the solution.

Keywords: NI-RPIM, RBF, Nodal integration, Taylor expansion

Öz

Bu çalışmada, yüksek mertebeden Taylor serisi açılım terimlerinin radyal nokta interpolasyon yöntemi düğüm entegrasyon şeması (NI-RPIM) açısından 2 boyutlu elastik problemlerin çözüm doğruluğuna etkileri incelenmiştir. Düğüm integrasyon şeması Liu ve diğ. [1] tarafından tasarlanmıştır ve Taylor serisi açılımı üzerinedir. Bu çalışmada 4. mertebeye kadar terimleri arttırılarak kullanılmıştır. 3 farklı alan çalışması yapılmış ve sonuçları analitik, sonlu elemanlar yöntemi ve Gauss integrasyonlu RPIM sonuçları ile kısaylanmıştır. Ayrıca nokta sayısının etkisi araştırılmıştır. RPIM integrasyonu içerisinde kullanılan Taylor serisi açılımı ve Gauss metodu benzer çözüm zamanları verdiği kabul edilebilir. Bununla birlikte özellikle yüksek sayıda nokta içeren modellerin çözümünde, NI-RPIM ile yüksek mertebeden Taylor serisi açılımı terimleri Gauss integrasyonundan daha iyi çözüm hızına sahiptir. 2. mertebeden düğüm integrasyon terimlerinin yeterli sonuç verdiği belirlenmiştir. Eğer gerilme değerleri incelenmekteyse, çözüm hassasiyeti için 4. mertebe düğüm integrasyon terimleri kullanılabilinir.

Anahtar kelimeler: NI-RPIM, RBF, Noktasal integrasyon, Taylor açılımı

1 Introduction

In the usage of numerical methods, numerical integration has an important role and it is widely used in the most of the solution of engineering problems. These problems are either concepts of mechanics or heat transfer, fluid mechanics and other study areas. Their solutions need fully definite mathematical representations, if numerical solution techniques are used. On the contrary to use experimental methods, numerical techniques are easily adapted and they do not require any experimental setup or apparatus. If a numerical model provides satisfied and logical results, other related investigations are adapted quickly for research. But it is hard to handle to describe the investigated conditions with a suitable mathematical representation.

Lots of different conditions and dependent or independent parameters are available in engineering problems. Some concepts of engineering problems have been easily solved. They can behave linear, elastic and steady. However in most of the cases, mechanical systems have nonlinear, time dependent and unpredictable responses. For prevention of deadlock in these cases, some simplifications and assumptions are accepted without losing the systems' behaviour. Besides of that, some assumptions are not enough for solving the problem. The applied numerical solution technique and its adaptability are also important.

Besides of the solution and its accuracy, researchers are trying to develop easily adapted numerical methods. Some methods like BEM (boundary element method), FDM (finite difference method), FEM (finite element method) have been developed and widely used with respect to developed computational technology. In commercial usage, finite element method may be the most used and common, especially in solid mechanics problems. The solution has been progressed by small divided elements, which is called finite elements. The representation of model with finite elements is very important and directly influences the results. Hence most of the investigation time is spent on the construction of the finite element models of the investigated problems.

To remove the dependency of finite elements and their construction, developers are trying to develop different solution methods. Meshfree methods are mainly developed for prevention of finite element formation and supply a suitable numerical model. It can also be used with FEM in different cases [2]-[6].

The study of meshfree methods initially begins with the study of SPH (smoothed particle hydrodynamics) [7]. In the further stage of SPH, different meshfree techniques are also developed.

Several meshfree methods have been reported in literature, such as DEM (diffuse element method) [8], EFG (element free Galerkin method) [9], RKPM (reproducing kernel particle method) [10], MLPG (meshless local Petrov-Galerkin) [11], PIM (point interpolation method) [12] and RPIM (radial point interpolation method) [13] and so on. These methods are mainly used for prevention of predefined mesh construction for interpolation and integration of numerical model. Some of them use strong forms (such as SPH) and the others use weak form (such as EFG and PIM), especially Galerkin weak form, on the solutions of differential equations.

Researchers are aimed to prevent the dependency of numerical integration with background cells. Different tries are available in the literature. LC-PIM (linearly conforming point interpolation method) [14] has been extended from PIM and RPIM for increasing the accuracy in numerical integration. Nodal integration technique is used in NI-RPIM (nodal integration radial point interpolation method) [1] by using Taylor series expansion. 3D heat transfer problems are investigated by developed NS-PIM (node-based smoothed point interpolation method) [15]. CS-RPIM (cell-based smoothed radial point interpolation method) [16] uses triangular small divided background cells with RPIM and is used in static and vibration analysis of solids.

These studies also include different shape parameters, construction rule of shape functions and numerical integration methods. Hence, their effects on the accuracy of the solution must be investigated carefully. Taylor series expansion is used in NI-RPIM [1] and the degree of series expansion has been effective. In this study, the effect of order of Taylor series expansion in NI-RPIM [1] on the solution of 2D elastic problems is investigated. The results are compared with RPIM with Gaussian integration technique, FEM solution and analytical results. All solutions are performed in FORTRAN for RPIM with nodal and Gaussian integrations.

2 RPIM Shape Functions

Construction of shape functions are one of the main concept of FEM and meshfree methods. Accomplish of the solution, accuracy and simplicity can be directly influenced with respect to effects of shape functions. Relation of numerical model components is provided with constructed shape functions.

Different numerical methods and their shape functions are available in the literature. PIM [12] has been developed, which uses polynomial basis functions. It is developed and added radial basis functions (RBF) in RPIM to increase accuracy. The shape functions in NI-RPIM are constructed by using RPIM, which has been defined by Liu and Gu [13]. This technique has been used in the construction of interpolation functions at NI-RPIM models and it has been applied and defined in detail in the study of Liu and Gu [17].

Main concept of this technique [1],[17] is adding to radial basis functions to polynomial functions for representing the assumed function. The assumed function or field function u(x) can be represented as Eq. 1 for local support domain.

$$u(x) = \sum_{i=1}^{n} R_i(x)a_i + \sum_{j=1}^{m} P_j(x)b_j = R^T(x)a + P^T(x)b \tag{1}$$

In here, $R_i(x)$ represents radial basis and $P_i(x)$ represents polynomial basis functions. a_i and b_j are corresponding constants, n is the number of field nodes in the local support domain and m is the number of polynomial terms. Different kinds of radial basis functions are available in the literature. Multiquadric basis (MQ) is one of RBF and is used in the solutions of partial differential equations. It is used as radial basis function [18] in Eq. 2.

$$R_i(x) = \left(r_i^2 + (\alpha_c d_c)^2\right)^q \tag{2}$$

Where d_c is the average nodal spacing near the point of interest at x; α_c and q are two arbitrary real numbers of dimensionless parameters, which are called shape parameters. q is [13],[17]-[19] not recommended to equal to use as 1.00 with respect to singularity problem of moment matrix in RPIM-MQ. The effect of shape parameters are also investigated in the study of Wang and Lui [19]. It is recommended [19]-[21] to use q as 1.03 and α_c as 3 for MQ basis. The radial distance is given in Eq. 3. Also the used polynomial terms are given in Eq. 4 which are mainly derived from binomial expansion.

$$r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2} \tag{3}$$

$$P^T(x) = \{1, x, y, x^2, xy, y^2, \dots\} \tag{4}$$

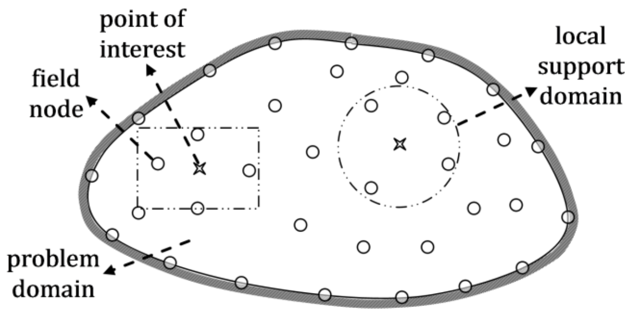

Figure 1: Different types of local support domains for point of interests [17].

A support domain determination is needed to prepare interpolation. Point of interest, x is used for the centre of support domains and interpolations are applied for field nodes in the local support domain, which is given in Figure 1. Different types of support domain geometries can be used, like circular, elliptical, triangular or rectangular. A circular local support domain is used and its covered area is given by radius of circle (d_s), which is given in Eq. 5.

$$d_s = \alpha_s d_c \tag{5}$$

Where d_c is average nodal spacing and α_s is a positive real number of dimensionless size of the local support domain. Its value is commonly used between 2 and 3 [17].

The unknown constants of field function of a_i and b_j in Eq. 1 can be determined by enforcing the field function pass through all n field nodes in the local support domain. At the k^{th} point or last point in local support domain, field function can be written as:

$$u(x_k, y_k) = \sum_{i=1}^{n} R_i(x_k, y_k)\, a_i + \sum_{j=1}^{m} P_j(x_k, y_k)\, b_j, \tag{6}$$

The matrix form of the above equation can be expressed as

$$U_e = R_q a + P_m b = \{u_1\, u_2 \dots u_n\}^T \quad (7)$$

Where U_e the vector of function values at the nodes in the local support domain. R_q is the moment matrix of RBF and P_m is the polynomial moment matrix, which are given in Eq. 8 and 9 respectively.

$$R_q = \begin{bmatrix} R_1(r_1) & R_2(r_1) & \dots & R_n(r_1) \\ R_1(r_2) & R_2(r_2) & \dots & R_n(r_2) \\ \dots & \dots & \dots & \dots \\ R_1(r_n) & R_2(r_n) & \dots & R_n(r_n) \end{bmatrix}_{(n\times n)} \quad (8)$$

$$P_m = \begin{bmatrix} 1 & x_1 & y_1 & \cdots & p_m(x_1) \\ 1 & x_2 & y_2 & \cdots & p_m(x_2) \\ \dots & \dots & \dots & \dots & \dots \\ 1 & x_n & y_n & \cdots & p_m(x_n) \end{bmatrix}_{(n\times m)} \quad (9)$$

a is the vector of unknown coefficients for RBF and b is the vector of unknown coefficients for polynomial basis functions. They are given in Eq. 10 and 11.

$$a^T = \{a_1\, a_2 \dots a_n\} \quad (10)$$

$$b^T = \{b_1\, b_2 \dots b_m\} \quad (11)$$

For solution of the field function, unknown parameter a in Eq. 7 must satisfy in polynomial function,

$$\sum_{i=1}^{n} p_j(x_i)\, a_i = P_m^T a = 0, \;\; j = 1,2,\dots,m \quad (12)$$

Combination of Eq. 7 and Eq. 1 yields the following equations in the matrix form:

$$\widetilde{U_e} = \begin{bmatrix} U_e \\ 0 \end{bmatrix} = \begin{bmatrix} R_q & P_m \\ P_m^T & 0 \end{bmatrix} \begin{Bmatrix} a \\ b \end{Bmatrix} = G a_0 \quad (13)$$

Where

$$\widetilde{U_e} = \begin{bmatrix} U_e \\ 0 \end{bmatrix} = \{a_1\, a_2 \dots a_n\; 0\; 0 \dots 0\}^T \quad (14)$$

Unique solution is obtained if the inverse of matrix G exists:

$$a_0 = \begin{Bmatrix} a \\ b \end{Bmatrix} = G^{-1} \widetilde{U_e} \quad (15)$$

Substituting Eq. 15 into Eq. 1, the interpolation with respect to field function can be expressed as,

$$u(x) = \{R^T(x) \quad P^T(x)\} G^{-1} \widetilde{U_e} = \tilde{\varphi}(x) \widetilde{U}_e \quad (16)$$

Finally, the RPIM shape functions for the corresponding n field nodes can be obtained as

$$\varphi^T(x) = \{\varphi_1(x) \quad \varphi_2(x) \quad \dots \quad \varphi_n(x)\} \quad (17)$$

The approximation function can be written as

$$u(x) = \varphi^T(x) U_e = \sum_{i=1}^{n} \varphi_i u_i \quad (18)$$

The derivatives of $u(x)$ can be easily obtained as

$$u_{i,k}(x) = \varphi_{,k}^T(x) U_e \quad (19)$$

Where k denotes the coordinates x or y. Partial differentiation is taken with respect to that defined coordinated by k.

The usage of radial basis functions in interpolation becomes widely. Dinis et al. investigate analysis of 3D solids [22] and plates [23] by NNR-PIM (natural neighbour radial point interpolation method) with using RBF. Static and free vibration [24] analysis of thick plate is investigated by using LRPIM (local radial point interpolation method) and radial basis functions. Quadratic polynomial basis functions are used as trial function. Polynomial and radial basis functions support Kronecker Delta Function property in shape functions.

3 Nodal Integration Based on Taylor Series Expansion

Representations of function with serial expansions are another powerful mathematical operation, used on mainly to describe hard handle functions. It is generally used on the determination of function values from known values to unknown values. The usage area of series is wide, especially in the numerical analysis. The methodology of series converge the related solution with step by step. Integrals and limits can be defined and solved by using series.

Taylor series are one of the concepts of numerical methods and its usage in numerical solutions can be seen in finite difference method. General representation of FDM is given in Eq. 20. $x_0 + h$ value of function can be defined serial expansion of x_0 value of functions. R_n is the total error between value of $f(x_0 + h)$ and its Taylor expansion results. In general, the degree of used terms in FDM increases the accuracy.

$$f(x_0 + h) = f(x_0) + \frac{f'(x_0)}{1!} h + \frac{f''(x_0)}{2!} h^2 + \cdots + \frac{f^n(x_0)}{n!} h^n + R_n \quad (20)$$

This serial representation can be applied in numerical integration. The numerical integration is applied [1] on Galerkin weak formulation of governing equations of the solid mechanics problems. Equilibrium equation is given in Eq. 21, which is valid in the domain. Also applied natural and essential boundary conditions are given in Eq. 22 and 23, respectively.

$$L^T \sigma + b = 0 \quad (21)$$

$$\sigma \cdot n = \bar{t} \quad on\ \tau_i \quad (22)$$

$$u = \bar{u} \quad on\ \tau_u \quad (23)$$

L^T is [1] differential operator, σ is the stress vector, u is the displacement vector, b is the body force vector, $\bar{t}$ is prescribed traction on the natural boundies, $\bar{u}$ is prescribed displacement on the essential boundaries and n is the vector of unit outward normal on the natural boundary. They are given in Eq. 24, respectively.

$$L^T = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \sigma = \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix}; \;\; u = \begin{bmatrix} u \\ v \end{bmatrix}; \;\; b = \begin{bmatrix} b_x \\ b_y \end{bmatrix} \quad (24)$$

The equilibrium equation (Eq. 21) can be defined as in Galerkin weak formulation in Eq. 25,

$$\int (L\delta u)^T (DLu)\, d\Omega - \int \delta u^T b\, d\Omega - \int \delta u^T t\, d\Gamma = 0 \quad (25)$$

D matrix is material coefficient matrix and it is given for linear elasticity plane stress problems in Eq. 26, where E is Young's modulus and ϑ is Poisson's ratio.

$$D=\frac{E}{1-\vartheta^2}\begin{bmatrix}1 & \vartheta & 0\\ \vartheta & 1 & 0\\ 0 & 0 & \frac{1-\vartheta}{2}\end{bmatrix} \tag{26}$$

When substituting the approximated Eq. 18, into Eq. 25, we have

$$Ku=f \tag{27}$$

Where

$$K_{ij}=\int B_i^T D B_j\, d\Omega \tag{28}$$

$$f_i=\int \varphi_i t\, d\Gamma+\int \varphi_i b\, d\Omega \tag{29}$$

$$B_i=\begin{bmatrix}\varphi_{i,x} & 0\\ 0 & \varphi_{i,y}\\ \varphi_{i,y} & \varphi_{i,x}\end{bmatrix} \tag{30}$$

Taylor series expansion can be used in different areas, like developing [25] 2D weight functions for investigation of cracks. Both FDM [26] (which includes Taylor series expansion) and radial interpolation is used for the solutions of mechanics problems.

$$\begin{aligned}f(x,y)\cong f(x_0,y_0)&+(x-x_0)\frac{\partial f(x_0,y_0)}{\partial x}\\&+(y-y_0)\frac{\partial f(x_0,y_0)}{\partial y}\\&+\frac{1}{2!}\left((x-x_0)^2\frac{\partial f^2(x_0,y_0)}{\partial x^2}\right.\\&+2(x-x_0)(y-y_0)\frac{\partial f^2(x_0,y_0)}{\partial x\partial y}\\&\left.+(y-y_0)^2\frac{\partial f^2(x_0,y_0)}{\partial y^2}\right)\\&+\frac{1}{3!}\left((x-x_0)^3\frac{\partial f^3(x_0,y_0)}{\partial x^3}\right.\\&+3(x-x_0)^2(y-y_0)\frac{\partial f^3(x_0,y_0)}{\partial x^2\partial y}\\&+3(x-x_0)(y-y_0)^2\frac{\partial f^3(x_0,y_0)}{\partial x\partial y^2}\\&\left.+(y-y_0)^3\frac{\partial f^3(x_0,y_0)}{\partial y^3}\right)+\frac{1}{4!}\dots\end{aligned} \tag{31}$$

Taylor series expansion is used in nodal integration in the study [1]. Eq. 28 represents stiffness matrix and will be used as approximate function $f(x,y)$, which is given in Eq. 31. The nodal integration of Eq. 31 can be written as Eq. 32,

$$\begin{aligned}\int f(x,y)d\Omega\cong\int\Bigg(f(x_0,y_0)&+(x-x_0)\frac{\partial f(x_0,y_0)}{\partial x}\\&+(y-y_0)\frac{\partial f(x_0,y_0)}{\partial y}\\&+\frac{1}{2!}\left((x-x_0)^2\frac{\partial f^2(x_0,y_0)}{\partial x^2}\right.\\&+2(x-x_0)(y-y_0)\frac{\partial f^2(x_0,y_0)}{\partial x\partial y}\\&\left.+(y-y_0)^2\frac{\partial f^2(x_0,y_0)}{\partial y^2}\right)\\&+\frac{1}{3!}\left((x-x_0)^3\frac{\partial f^3(x_0,y_0)}{\partial x^3}\right.\\&+3(x-x_0)^2(y-y_0)\frac{\partial f^3(x_0,y_0)}{\partial x^2\partial y}\\&+3(x-x_0)(y-y_0)^2\frac{\partial f^3(x_0,y_0)}{\partial x\partial y^2}\\&\left.+(y-y_0)^3\frac{\partial f^3(x_0,y_0)}{\partial y^3}\right)+\frac{1}{4!}\dots\Bigg)d\Omega\end{aligned} \tag{32}$$

Regulating the Eq. 32, we have

$$\begin{aligned}\int f(x,y)d\Omega\cong\int f(x_0,y_0)\,d\Omega&+\int(x-x_0)\frac{\partial f(x_0,y_0)}{\partial x}d\Omega\\&+\int(y-y_0)\frac{\partial f(x_0,y_0)}{\partial y}d\Omega\\&+\frac{1}{2}\int(x-x_0)^2\frac{\partial f^2(x_0,y_0)}{\partial x^2}d\Omega\\&+\int(x-x_0)(y-y_0)\frac{\partial f^2(x_0,y_0)}{\partial x\partial y}d\Omega\\&+\frac{1}{2}\int(y-y_0)^2\frac{\partial f^2(x_0,y_0)}{\partial y^2}d\Omega\\&+\frac{1}{6}\int(x-x_0)^3\frac{\partial f^3(x_0,y_0)}{\partial x^3}d\Omega\\&+\frac{1}{2}\int(x-x_0)^2(y\\&-y_0)\frac{\partial f^3(x_0,y_0)}{\partial x^2\partial y}d\Omega\\&+\frac{1}{2}\int(x\\&-x_0)(y-y_0)^2\frac{\partial f^3(x_0,y_0)}{\partial x\partial y^2}d\Omega\\&+\frac{1}{6}\int(y-y_0)^3\frac{\partial f^3(x_0,y_0)}{\partial y^3}d\Omega+\cdots\end{aligned} \tag{33}$$

$$\begin{aligned}\int f(x,y)\,d\Omega \cong f(x_0,y_0)\int 1\,d\Omega \\ + f_x(x_0,y_0)\int (x-x_0)\,d\Omega \\ + f_y(x_0,y_0)\int (y-y_0)\,d\Omega \\ + \frac{f_{xx}(x_0,y_0)}{2}\int (x-x_0)^2\,d\Omega \\ + f_{xy}(x_0,y_0)\int (x-x_0)(y-y_0)\,d\Omega \\ + \frac{f_{yy}(x_0,y_0)}{2}\int (y-y_0)^2\,d\Omega \\ + \frac{f_{xxx}(x_0,y_0)}{6}\int (x-x_0)^3\,d\Omega \\ + \frac{f_{xxy}(x_0,y_0)}{2}\int (x-x_0)^2(y-y_0)\,d\Omega \\ + \frac{f_{xyy}(x_0,y_0)}{2}\int (x-x_0)(y-y_0)^2\,d\Omega \\ + \frac{f_{yyy}(x_0,y_0)}{6}\int (y-y_0)^3\,d\Omega + \cdots\end{aligned} \tag{34}$$

Eq. 33 can be regulating more and it is given in Eq. 34, where dΩ represent Taylor integration cell domain or area ($d\Omega = da = dx.dy$). The function represents K_{ij} in Eq. 28. Derivatives of the function causes to take more derivatives of B matrix, which contains shape functions. Shape functions also have radial basis functions, whose more derivatives are also necessary for increased order terms of Taylor series expansion. The derivatives [18] of Eq. 2, where r depends on x and y (given in Eq. 3), are given in Eq. 35 and 36 for x and y coordinates with respect to used order of Taylor Series expansion. Further derivatives are also taken for 2nd, 3rd, 4th and further degrees of x and y.

$$\frac{\partial R_i(x,y)}{\partial x} = 2q\left(r_i^2 + (\alpha_c d_c)^2\right)^{q-1}(x-x_i) \tag{35}$$

$$\frac{\partial R_i(x,y)}{\partial x} = 2q\left(r_i^2 + (\alpha_c d_c)^2\right)^{q-1}(y-y_i) \tag{36}$$

4 Solutions and Discussion

Besides of numerical solution for approximate results, in some cases analytical solutions are available in engineering problems. Axial loaded, cantilever and simply supported beam with distributed loading problems have analytical solutions. Also numerical techniques like FEM can be applied on these types of problems. Representation of nodal integration by using Taylor series expansion is mentioned. Increasing the orders of Taylor series expansions terms on these types of problems are examined in these cases.

Three different loading conditions are formed on a beam. Beam models have same geometrical and material properties. The only changed parameters are applied boundary conditions. Beam is modelled unit thickness and in the calculations, plane stress assumption is used. Young modulus of 200 GPa and Poisson's ratio of 0.0 are used in material properties. The selection of Poisson's ratio as 0.0 is mainly used for providing the similar condition as analytical solutions, although solutions are ideal. The behaviour of material is assumed fully elastic.

Beam models are given for analytical solution in Figure 2. The beam has a length of 1.00 m and a height of 0.1 m. P is applied as 1000 N and w is applied as 1000N/m. In axial loaded beam, deformation and normal stress have more dominant effect on mechanical robustness. Results are taken at the natural axis of the beam in x-direction. The deformation and x-stress (axial stress) are given in Eq. 37 and 38, respectively for analytical solution.

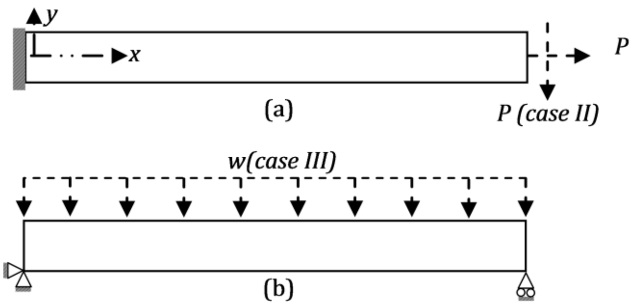

Figure 2: The models are used in analytical solution for;
a) Axial loaded beam (case I), cantilever beam (case II),
b) Simply supported beam problem with distributed loaded (case III.

$$\delta = \frac{Px}{AE} \tag{37}$$

$$\sigma = \frac{P}{A} \tag{38}$$

P represents applied force, A is area, E is Young's modulus in the notifications. X-stress is also dominant effect on cantilever and simply supported beam problem. However, they occur at the maximum values on the surfaces. Hence the x-stress results are taken on the upper surface of the beams. They are given in Eq. 39 for cantilever and simply supported beam models. c represents the distance between natural axis and upper surface of the beam.

$$\sigma = \frac{Mc}{I} \tag{39}$$

On the contrary to check x-direction deformation, y-direction deflection is considered in cantilever and simply supported beams. Deflection is given in Eq. 40 for cantilever beam problem. I is inertia of the beam. Deflection of distributed loaded simply supported beam model is given in Eq. 41, where w is applied distributed force.

$$y = \frac{P}{6EI}(x^3 - 3Lx^2) \tag{40}$$

$$y = -\frac{w}{24EI}(x^4 - 2Lx^3 - L^3x) \tag{41}$$

Numerical models of FEM are given in Figure 3 for 10 elements (a) and 20 elements (b). 10 element model have 18 nodes and 20 element model have 33 nodes. Same boundary conditions as analytical models are applied on FEM models. Plane182 element, which has 4 nodes for rectangular shape, is used in ANSYS.

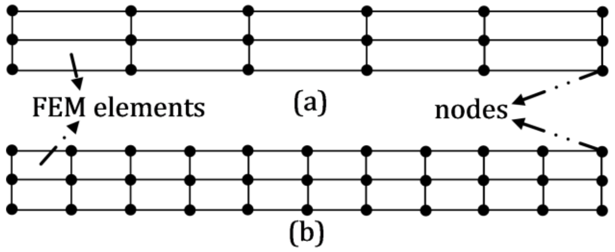

Figure 3: The models are used in FEM solutions;
a) 10 elements model, b) 20 elements model.

In Figure 4, NI-RPIM models are given for 18 nodes and 33 nodes. These NI-RPIM models are both used for Gaussian and Taylor integration. In 18 nodes model, a nodal interval of 0.05 m in height and a nodal interval of 0.2 m in length are used. In 33 nodes model, a nodal interval of 0.05 m in height and a nodal interval of 0.1 m in length are used. The used Taylor series expansion terms are up to 4th and if Gaussian integration is used, 2*2 gauss points are available in each background cell.

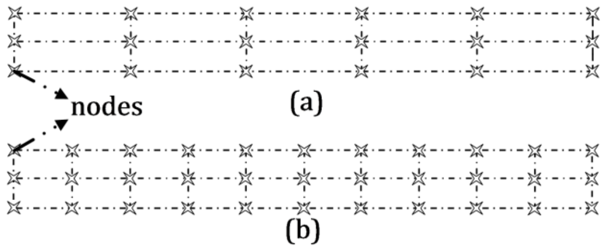

Figure 4: The models are used in NI-RPIM solutions;
a) 18 nodes model, b) 33 nodes model.

4.1 Axial Loaded Beam Model

In Figure 5, analytical, FEM (10 elements) RPIM (18 nodes) with Gaussian integration and Taylor integration (2nd order NI-RPIM) results are given for axial loaded beam problem. Whole compared numerical results have good agreements with analytical solution. However, results of NI-RPIM (2nd order terms) have larger deformation than analytical results at the force application location of the beam. But, general characteristics of compared results behave similar and the difference can be accepted. Further increasing the number of nodes to 33, the results of NI-RPIM (2nd order terms) get better and are drawn to analytical results, especially the force applied location in Figure 6. FEM and Gaussian int. of PRIM results have good agreement with analytical results.

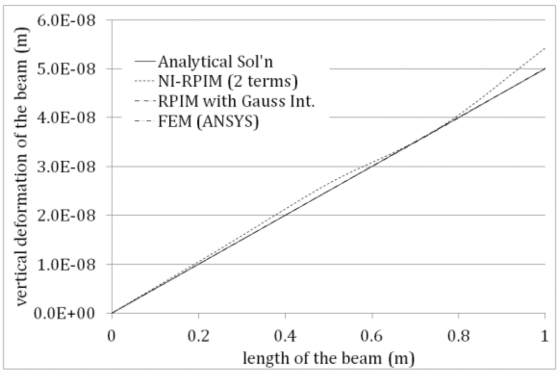

Figure 5: Comparison of different solution techniques on vertical deformation of 18 nodes axial loaded beam.

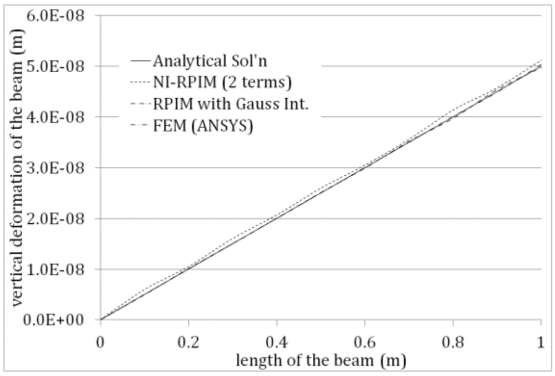

Figure 6: Comparison of different solution techniques on vertical deformation of 33 node axial loaded beam.

The effect of used terms in Taylor Series expansion is given in Figure 7 for 18 nodes and they are compared with only analytical results. The usage of 1st order term in Taylor series expansion in NI-RPIM causes to a fluctuation at the vertical deformation results of the beam. The fluctuation disappears and the vertical deformation results are improved with increasing the used order of Taylor series expansion terms. However, the vertical deformations of tip locations of beam are same in all used terms of NI-RPIM scheme.

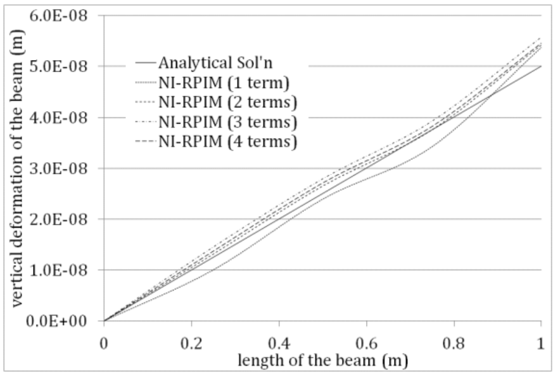

Figure 7: Effect of Taylor series expansion terms on vertical deformation of 18 nodes axial loaded beam.

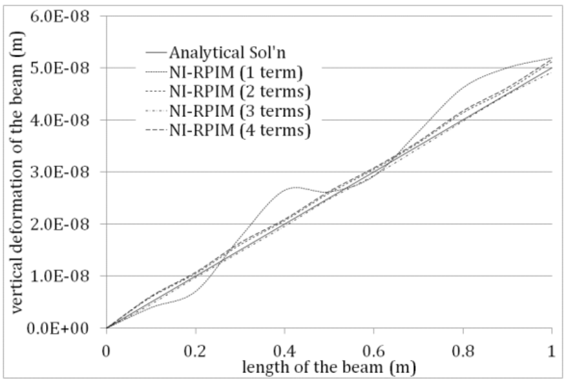

Figure 8: Effect of Taylor series expansion terms on vertical deformation of 33 nodes axial loaded beam.

The increased node results in NI-RPIM are given in Figure 8. It has seen that 2nd and greater terms of NI-RPIM results have good agreements with compared analytical solution. However, the used 1st term in NI-RPIM gives greater fluctuation results than 1st term of 15 node NI-RPIM results. Besides of that the tip vertical deformation of the beam approximately overlaps with analytical result.

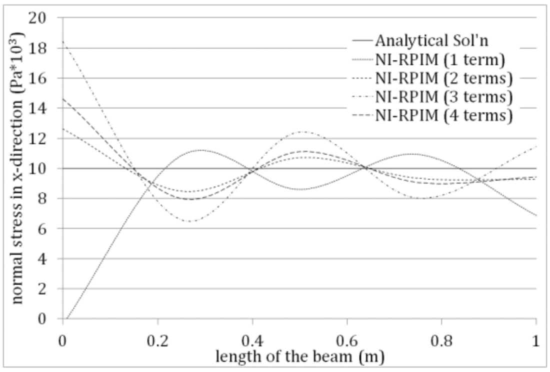

Figure 9: Effect of Taylor series expansion terms on axial stress of 18 nodes axial loaded beam.

The effect of used terms can be seen more easily in stress results. Normal stress results in x-direction are given for 18 nodes in Figure 9. NI-RPIM stress results at the support location of the beam have a great difference from analytical solution. This location influences the further sections results of the beam. But the fluctuation decreases at the far sections of support location. This local high stresses may be related with Saint Venant's principle. The results are improved with increasing the order of Taylor series expansion terms. In Figure 10, increasing the number of nodes provide more suitable and better normal stress results. 4th order terms of NI-RPIM results are the best results and have the least fluctuation. Besides of that, using only 1st term in NI-RPIM gives the worst stress results.

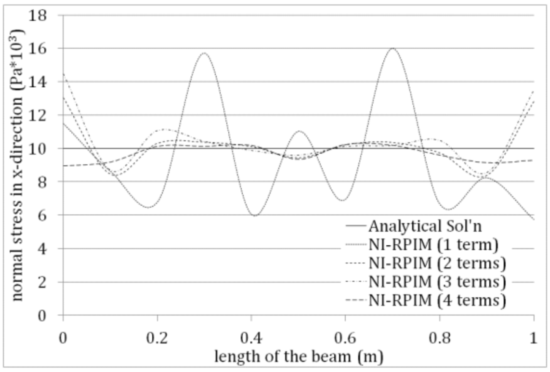

Figure 10: Effect of Taylor series expansion terms on axial stress of 33 nodes axial loaded beam.

4.2 Cantilever Beam Model

The deflection results of cantilever beam for different methods are given in Figure 11. The compared methods give sufficient results with respect to analytical solution. However, there is a small gap between analytical and NI-RPIM results, which have 2nd order Taylor series expansion terms. This gap between the compared results is eliminated with increasing the number of nodes in Figure 12. Nearly whole of the compared methods have perfectly agreement with analytical solution.

The effect of used terms in horizontal deformation of the beam is given in Figure 13. 1st order term of NI-RPIM gives insufficient results. When increasing the order of terms, the results become close enough to analytical results.

But increasing the number of nodes to 33, the results are approximately same as analytical solution without 1st order term results in Figure 14.

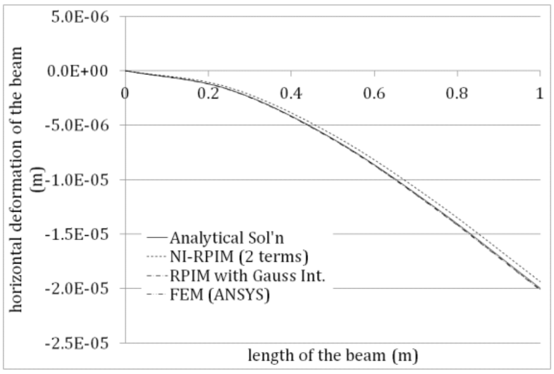

Figure 11: Comparison of different solution techniques on vertical deformation of 18 nodes cantilever beam.

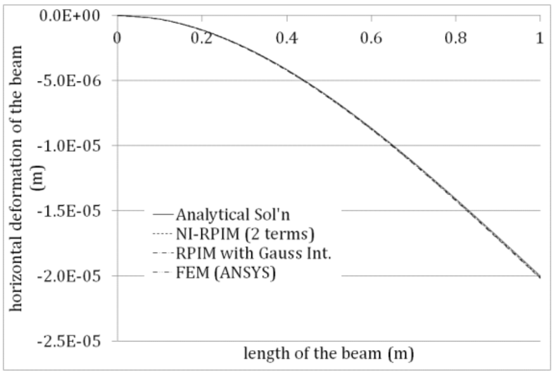

Figure 12: Comparison of different solution techniques on vertical deformation of 33 nodes cantilever beam.

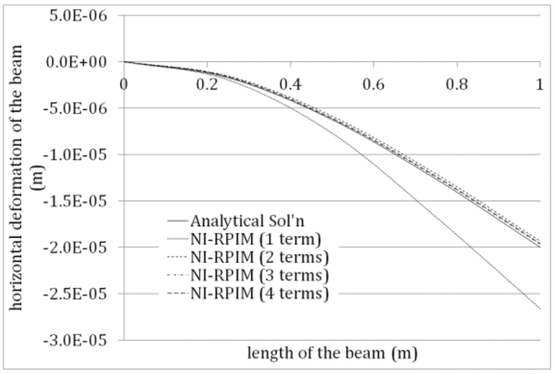

Figure 13: Effect of Taylor series expansion terms on vertical deformation of 18 nodes cantilever beam.

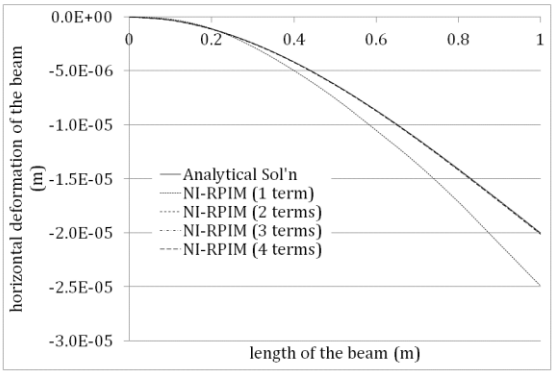

Figure 14: Effect of Taylor series expansion terms on vertical deformation of 33 nodes cantilever beam.

The stress results of cantilever beam for 18 nodes are given in Figure 15. The results of 1st order term give insufficient results. However increasing the order of NI-RPIM terms gives similar results as analytical solution. A high distortion on the results of 1st order terms is seen in Figure 16 and causes to not behave similar behaviour as analytical results. But 2nd and greater order of NI-RPIM terms nearly gives the same results as analytical solution.

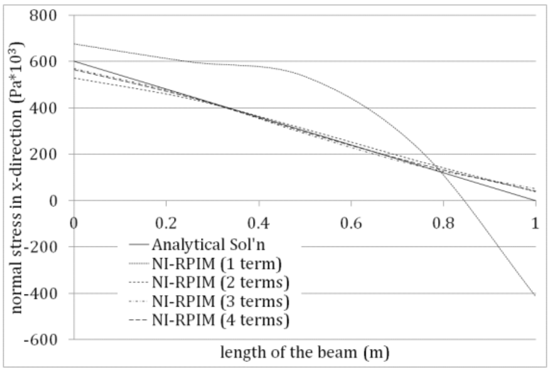

Figure 15: Effect of Taylor series expansion terms on axial stress of 18 nodes cantilever beam.

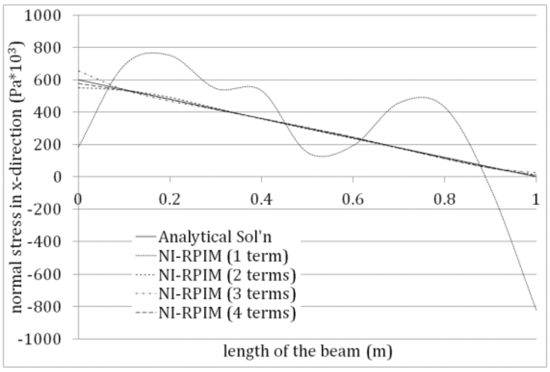

Figure 16: Effect of Taylor series expansion terms on axial stress of 33 node cantilever beam.

4.3 Simply Supported Beam Model

The horizontal deformation results of simply supported beam are given for 18 nodes in Figure 17. FEM results are the closest results to analytical solution. Besides, the results of 2nd order terms in NI-RPIM are the farthest results, general behaviour of the results has similar characteristics with analytical solution. When increasing the number of nodes, nearly all the compared methods overlap with the analytical solution in Figure 18.

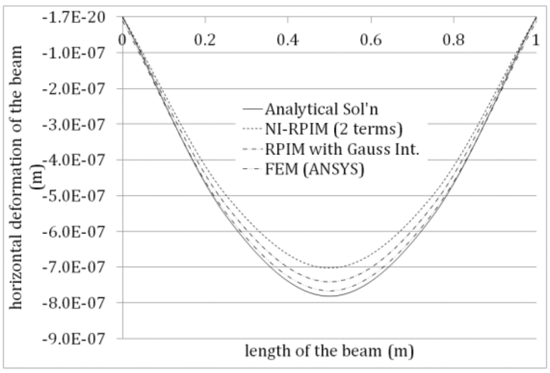

Figure 17: Comparison of different solution techniques on vertical deformation of 18 nodes simply supported beam.

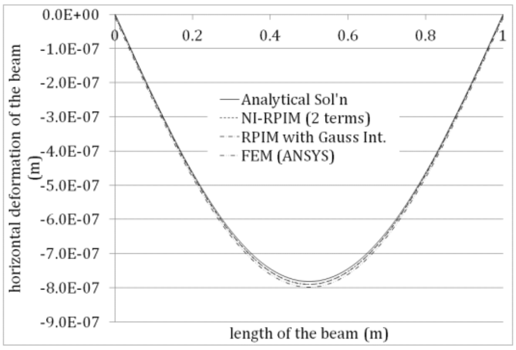

Figure 18: Comparison of different solution techniques on vertical deformation of 33 nodes simply supported beam.

The effect of terms of NI-RPIM is given in Figure 19 for 18 node results. It is seen that 2nd, 3rd and 4th order terms give similar and less deformation values than analytical solution. But 1st order term results have great difference from analytical solution and greater deformation values than analytical solution. When increasing number of nodes to 33, the results of 2nd, 3rd and 4th order terms approximately overlap with analytical solution in Figure 20. The difference between 1st order term and analytical results decreases, but it has not vanished.

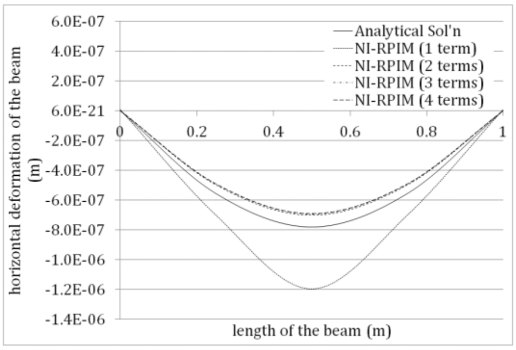

Figure 19: Effect of Taylor series expansion terms on vertical deformation of 18 nodes simply supported beam.

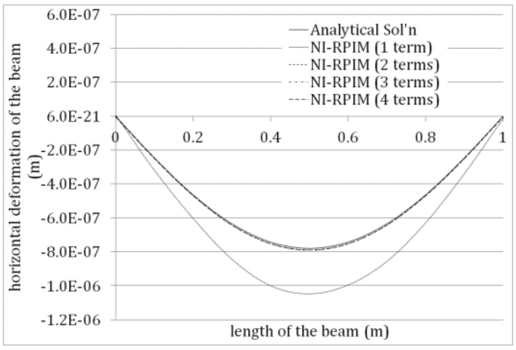

Figure 20: Effect of Taylor series expansion terms on vertical deformation of 33 nodes simply supported beam.

Similar condition can be seen in the stress results of simply supported beam. In Figure 21, the results show that 1st order term solution has insufficient results. However, 2nd and greater order terms have closed enough results to analytical solution. In Figure 22, nearly 2nd, 3rd and 4th order terms stress results are very close to analytical solution. The difference between 1st order terms results and analytical results decreases, but the stability of 1st order terms vanishes and a fluctuation occurs.

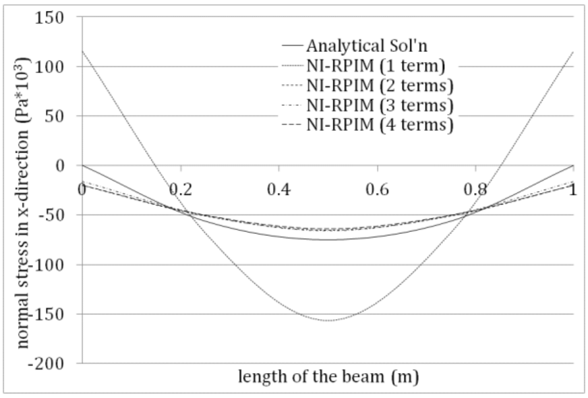

Figure 21: Effect of Taylor series expansion terms on axial stress of 18 nodes simply supported beam.

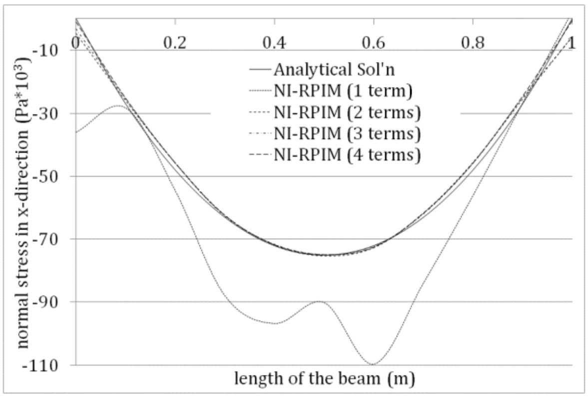

Figure 22: Effect of Taylor series expansion terms on axial stress of 33 nodes simply supported beam.

It has been reported that first-order finite difference scheme [27] doesn't give sufficient results in most of the computational fluid dynamics (CFD) analysis and second-order terms is suggested. Similar condition is detected in this study for 2D elastic problems.

Solution time and computational performance are also important subjects for a numerical analysis. These properties can affect the usage of numerical analysis, if their requirements in the solution need high solution time and cause high computational cost. Hence, solution times of RPIM with nodal and Gaussian integration are investigated and their results are given in Table 1. A computer has a memory of 3GB and a processor speed of 2.4 GHz, is used in the analysis. 2 different numbers of nodes (33 and 1111) are used for observing the solution times. It is seen that in the results of 33 nodes, Gaussian integration works faster than nodal integration, even if 4*4 number of Gauss points are used in the each background cell. But all solution cases are completed in less than one second.

However, when increasing the used number of nodes to 1111, the results of 1st order term of nodal integration and 2*2 gauss points of Gaussian integration have similar solution times. When increasing the number of gauss points (3*3 and 4*4), the solution time increases rapidly rather than the increment of terms of nodal integration. The increment of order of Taylor series expansion in nodal integration is not dominant as the used number of gauss points in Gaussian integration for solution time. Nearly, the solution of RPIM with 4th order terms of nodal integration needs half of solution time of 4*4 gauss point solutions of Gaussian integration.

Table 1: Comparison of solution times of RPIM with nodal and Gaussian integration techniques for axial loaded beam models of 33 and 1111 nodes.

RPIM with nodal integration		Sol'n time (in Secs)	Sol'n time (in Secs)	RPIM with Gauss integration	
33 nodes	1 term	0.442	0.187	2*2	33 nodes
	2 terms	0.474	0.249	3*3	
	3 terms	0.528	0.39	4*4	
	4 terms	0.73			
1111 nodes	1 term	22.58	22.259	2*2	1111 nodes
	2 terms	24.064	42.557	3*3	
	3 terms	27.033	70.98	4*4	
	4 terms	31.944			

5 Conclusion

In this study, the effect of higher order terms of Taylor series expansion is investigated up to 4th order within the method of NI-RPIM which is used by Liu et al. [1]. 3 different case studies for elastic problems are examined with plane stress assumption. Also the effect of number of nodes is observed by changing the used number of nodes.

Some findings can be summarized that;

Increasing the number of nodes gives better results.

It has been detected that 2nd order Taylor terms can satisfy suitable result with respect to analytical solution.

Fluctuation may occur in the 1st order terms in NI-RPIM.

In axial loaded beam results, higher order terms give more realistic results, especially on the investigation of stress conditions.

FEM, NI-RPIM with Gaussian and Taylor integration can give similar results as analytical solution.

On the contrary to high solution times in fewer nodes, NI-RPIM with Taylor series expansion has benefits and faster than Gaussian integration in the solution times, especially using lots of numbers in the analysis.

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