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# The portfolio optimization with simulated annealing algorithm: An application of Borsa Istanbul

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**ABSTRACT**

One of the key concepts in finance is Markowitz's constrained mean-variance model, the number of assets to be included in the portfolio is restricted. The solution of this generalized problem, which belongs to the quadratic and integer programming problem class, as the number of dimensions increases, is difficult to obtain with standard methods. In this study, the simulated annealing (SA) algorithm, which is one of the local search-based meta-heuristic methods, was preferred. The developed SA algorithm was applied to the Hang-Seng benchmark data set, and the results were compared with pioneering studies. According to the experimental results, upon the performance of the algorithm was found to be sufficient, the SA algorithm was applied for the Borsa Istanbul 30 index. The results of the experiments based on the Markowitz mean-variance model demonstrate that, while more assets must be maintained at lower risk levels to converge an unconstrained efficient frontier and the number of assets needed to do so decreases as risk rises

# Tavlama benzetim algoritmasıyla portföy optimizasyonu: Borsa İstanbul uygulaması

**MAKALE BİLGİSİ**

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**ÖZ**

Finans alanının önemli konularından Markowitz'in kısıtlı ortalama-varyans modelinde, portföye dahil edilecek varlık sayısı sınırlandırılır. Kuadratik ve tamsayılı programlama problem sınıfına ait genelleştirilmiş bu problemin, boyut sayısının artmasıyla çözümünün standart yöntemlerle elde edilmesi zordur. Bu çalışmada yerel arama tabanlı

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meta-sezgisel yöntemlerden olan tavlama benzetim (TB) algoritması tercih edilmiş, geliştirilen TB algoritması Hang-Seng benchmark veri setine uygulanmış, sonuçlar öncü çalışmalarla kıyaslanmıştır. Markowitz kısıtlı ortalama-varyans modeline dayanarak elde edilen kısıtsız etkin sınıra yaklaşabilmek için, düşük risk düzeyinde varlık sayısının daha fazla, yüksek risk seviyesinde varlık sayısının daha az olması gerektiği sonucuna ulaşılmıştır.

**Anahtar Kelimeler:** Portföy optimizasyonu, markowitz ortalama-varyans modeli, tavlama benzetim, sezgisel optimizasyon.

## 1. Introduction

The aim of national economies is to provide sustainable economic growth. In order to achieve this goal, the Neo-Classical School argues that savings should turn into investments. In the dominant paradigm, there is the phenomenon of attracting the savings of both individuals and institutional structures to the market through the financial system and converting them into investments and increasing production. Businesses, which are an element of economic development, can either provide the funds they need to produce through loans or through share markets by going public. However, investors have certain earning expectations that take into account the inherent risks in the share market, wishing to maximize the future values of their individual wealth while trading in the share markets. The stock market, which is one of the most important markets used by investors, is a market that should be treated cautiously due to its variable structure. Market risk is one of the systematic risks that should be analyzed before making an investment decision (Jadhav and Ramanathan, 2018). Traditional Portfolio Theory claims that investors could maximize their expected return by investing in multiple securities rather than investing in a single asset in order to increase their wealth. The emphasis on "multiple assets" in this claim refers to the word portfolio. According to the financial terms dictionary, portfolio refers to "the sum of assets owned". However, in time, it has been observed that the increase in the number of assets in the portfolio does not prevent loss of earnings. Markowitz's mean-variance model, which forms the basis of modern finance theory, argues that the risk can be reduced at the expected return level by considering the relationship between the returns of different financial assets, not the number of assets in the portfolio, and the assets, that do not have a positive relationship between them, being in the same portfolio. Due to the fact that investor actions will influence the portfolios they choose, the theory is a cornerstone of contemporary finance theory and among the most significant theories in asset allocation, investment, and behavioral finance (Wong, 2020). However, the standard mean-variance model does not include any cardinality and importance level assumptions in portfolio investments. According to Markowitz's assumption, asset returns follow a multivariate normal distribution. Accordingly, the return of a portfolio can be completely defined by expected return and variance. The normal distribution assumption gives rise to some problems in the mean variance model. Compared to the distribution associated with normality, the asset return distribution typically exhibits a higher probability of outliers (Chang, Meade, Beasley and Sharaiha, 2000). In other words, the distribution of asset returns is leptokurtic.

Markowitz mean-variance model does not contain any conditions related to the number of asset constraints while creating a portfolio, which can be considered as another weakness. The increase in the number and conditions of today's financial assets has led to the necessity of adding a restriction on the number of assets to the model. Considering the abundance of investment options in financial markets, the rapid functioning of the markets and their complex structure, it is clear that new methods will be needed to create an optimal portfolio. In other words, risk–return dynamics continuously change with time in modern markets (Meghwani and Thakur, 2018). Integer constraints that limit a portfolio to a certain number of assets or impose limits on the proportion of the portfolio held in a particular asset cannot be easily applied. It has been shown by Shaw, Liu and Kopman (2008) that the constrained model is included in the NP-Hard problem class. For this reason, the researchers proposed heuristic search methods for this difficult problem type.

Heuristic approach was first developed by Speranza (1996). To solve the portfolio optimization problem, Schaerf (2002) developed local search heuristics, but it was realized that these intuitions were stuck to local optimums. Therefore, metaheuristic methods, which make broader searches than heuristic methods, have started to be preferred. The best-known metaheuristic methods are genetic algorithms, particle swarm optimization, simulated annealing technique and tabu search technique. Genetic algorithm technique, which is the most preferred method among metaheuristic methods, was first used by Shoaf and Foster (1998) for the solution of portfolio optimization.

Chang et al. (2000), compared the Hang Seng, DAX100, FTSE 100, S&P 100 and Nikkei 225 data with genetic algorithm, simulated annealing and tabu search techniques for portfolio optimization solution for March 1992 and September 1997 period (Chang et al., 2000). As a result, they found that each algorithm showed different advantages over different data sets, and they showed that all these algorithms can be applied for efficient portfolio selection. These techniques have also been used by Moral-Escudero, Ruiz-Torrubiano and Suarez (2006), Chang, Yang and Chang (2009), Özdemir (2011), Eshlaghy, Abdolahi, Moghadasi and Maatofi (2011), Gorgulho, Neves and Horta (2011), Chen, Mabu, and Hirasawa (2011), Woodside-Oriakhi, Lucas, and Beasley (2011), Bermúdez, Segura and Vercher (2012), Pandari, Azar and Shavazi (2012), Ackora-Prah, Gyamerah, Andam and Gyamfi (2014), Hsu (2014), Yakut and Çankal (2016), Zeren and Bayğın (2015), Chen, Lin, Zeng, Xu, and Zhang (2017), Jalota and Thakur (2018), Sasaki, Laamrani, Yamashiro, Alehegn and Kamoyedji (2018), Garcia, Guijarro and Oliver (2018), Vasiani, Handari and Hertono (2020).

This field has attracted a lot of attention due to its practical benefits and its literature is quite extensive. For comprehensive and recent studies examining the proposed deterministic models for mean-variance portfolio optimization and various variants of this model, as well as additional real-life constraints, the researchers can draw upon the comprehensive survey of Metaxiotis and Liagkouras (2012), Kalaycı, Ertenlice, Akyer and Aygören (2017a), Kalaycı, Ertenlice, Akyer and Aygören (2017b) and Kalaycı, Ertenlice, Akbay (2019).

Particle swarm optimization was first used by Cura (2009) for portfolio optimization solution. Cura (2009) compared the particle swarm optimization, genetic algorithms, simulated annealing and tabu search techniques for March 1992 and September 1997 with data from Hang Seng, DAX100, FTSE 100, S&P 100, and Nikkei, and as a result, it has been discovered that no technique performed better than another and particle swarm optimization works better in a portfolio with low-risk investments. This technique is also used by Zhu and Wang (2011), Sun, Fang, Wu, Lai and Xu (2011), Golmakani and Fazel (2011), Deng, Lin, and Lo (2012), Corazza, Fasano and Gusso (2013), Cesarone, Scozzari, and Tardella (2013), Kamali (2014), Wang, Chen, Zhang and Lin (2015), Çelenli, Eğrioğlu and Çorba (2015), Yin, Ni and Zhai (2015), Ni, Yin, Tian and Zhai (2017), Abuelfadl (2017) and Akyer, Kalaycı and Aygören (2018).

Fernandez and Gomez (2007), Hang Seng, DAX100, FTSE 100, S&P 100 and Nikkei 225 data, used artificial neural networks, genetic algorithm, tabu search and simulated annealing methods in portfolio optimization solution for March 1992 and September 1997 period and they concluded that the artificial neural network, which is one of the techniques used for portfolio optimization solution, gives better results in low-risk investments. Coutino-Gomez, Torres-Jimenez and Villarreal-Antelo (2003) compared the genetic algorithm, greedy algorithm, simulated annealing methods in portfolio optimization and found that the simulated annealing method gives better results in low risk. Simulated annealing method was also used by Crama and Schyns (2003), Maringer and Kellerer (2003), Woodside-Oriakhi, et al. (2011). There are many other studies on simulated annealing in literature. Some of these studies are Fastrich and Winker (2012), Sen, Saha, Ekbal and Laha (2014), Qodsi, Tehrani and Bashiri (2015), Lukovac, Pamucar, Popovic and Dorovic (2017), Kumar, Doja and Baig (2018), Moradi, Kayvanfar and Rafiee (2021).

When the literature is examined, solving the resource-constrained portfolio optimization problem with meta-heuristic algorithms has attracted the attention of many researchers since the study of Chang et al. (2000). Most of these studies have been on comparing the performance of different meta-heuristic methods. When the current literature is examined, there is no meta-heuristic method that has a great advantage in all of the frequently used test data sets. For this reason, this study aims to obtain a good solution on the test data set and to show a real-time application with up-to-date data, rather than comparing the methods.

In this study, Simulated Annealing algorithm is proposed to obtain optimal/near-optimal solutions to our research problem. SA algorithm is a powerful method for solving many optimization problems and its flexibility and ability to approach global optimality is higher than other local search methods (Delahaye, Chaimatanan,and Mongeau, 2018). Considering the uncertainty and fluctuations in Turkey's financial market, the ability of the SA algorithm to produce solutions in a wide range of areas can make a significant contribution. On the other hand, the simulated annealing algorithm has flexible parameter settings and requires less CPU time than evolutionary algorithms (Crama, 2003).

Considering these advantages in practice, the SA algorithm is preferred for this study. Firstly, the algorithm developed in the study was tested on the Hang-Seng benchmark data set, and its result was compared with Cura (2009), Mozafari, Jolai and Tafazzoli (2011), Sadigh, Mokhtari, Iran poor and Fatemi-Ghomi (2012), Baykasoğlu, Yunusoglu and Özsoydan (2015), Kalaycı et al. (2017b), Kalaycı, Polat and Akbay (2020), which are pioneering and very competitive results in the literature. Once the performance of the algorithm was found sufficient, an application on the BIST -30 index in a real-time environment was demonstrated using the existing parameters. The current outcomes of the study will contribute to the portfolio optimization literature and market practitioners.

The theoretical foundations of the Markowitz mean-variance model will be briefly explained in the continuing section of this study. In the third section, simulated annealing algorithm will be presented and a special application of SA for portfolio optimization will be applied for both Hang-Seng and Borsa Istanbul 30 (BIST30) index average 100-day return data set. In the fourth section, application results will be discussed.

## 2. Markowitz mean-variance model

According to Markowitz (1952), portfolio creation is a process that starts with observation and experience, ends with expectations about the future performance of existing securities, and a second phase that starts with expectations about the performance of future securities and ends with portfolio selection. Markowitz mean-variance model deals with the second stage. In this context, the Markowitz mean-variance model assumptions can be listed as follows (Markowitz, 1952):

- Rational investors prefer assets with the highest expected return and lowest risk when choosing their portfolio assets,
- Mean return and variances of assets are measured while considering risk and expected return,
- The rational investor prefers the asset with the highest expected return at the same risk level and the asset with the lowest risk at the same expected return level,
- The aim of the rational investor is to maximize the expected return and minimize the risk.

In the quantitative framework that Markowitz created for portfolio selection, the return of the portfolio can be fully defined by the expected return and variance (risk).

A set of asset portfolios that offer the minimum risk for a given level of return from the efficient frontier, and portfolios at the efficient frontier can be found by quadratic programming (QP). To be more precise; Markowitz proposes the mean-variance model, which is accepted as a quadratic programming problem in portfolio optimization (Adıgüzel-Mercangöz, 2019).

The parameters of the Markowitz mean-variance model are expressed as follows:

N: number of assets

$\mu_i$: expected return of the asset "i" (i= 1, …. N)

$\sigma_{ij}$: covariance value between i and j assets (i =1,…N; j=1,….N)

R*: expected return of the portfolio

Decision variable $w_i$: the weight of asset i in the portfolio ($0 \leq w_i \leq 1$) being;

The mathematical expression / objective function of the Markowitz mean variance model is given in Equation (1) - (4).

$$min\sum_{i=1}^{N}\sum_{j=1}^{N}w_i w_j \sigma_{ij} \tag{1}$$

$$\sum_{i=1}^{N}w_i \mu_i = R^* \tag{2}$$

$$\sum_{i=1}^{N}w_i = 1 \tag{3}$$

$$0 \leq w_i \leq 1, \quad i = 1,2,3,....N \tag{4}$$

Equation 1 expresses that the risk of the portfolio is minimized, equation 2 indicates the expected return of the portfolio, and equation 3 expresses that the sum of the weights of the assets in the portfolio must be equal to 1. Equation (1)-(4) is a quadratic programming problem and an unconstrained model. In the efficient frontier plot, a continuous increasing curve can be obtained according to different values of the expected value (R *). As Chang et al. (2000) pointed out, the objective function, with the weighting parameter λ (0 < λ <1) in the efficient frontier drawing without constraints, can be rewritten as follows:

$$\min \quad \lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N}w_i w_j \sigma_{ij}\right] - (1-\lambda)\left[\sum_{i=1}^{N}w_i \mu_i\right] \tag{5}$$

When the λ parameter is 0, the expected return is maximum, and when the λ parameter is 1, the risk is minimum. Parameters in the cardinality constrained model of Chang et al (2000):

K: the desired number of assets in the portfolio

$\varepsilon_i$: minimum weight of asset i in portfolio

$\delta_i$: maximum weight of asset i in portfolio

It should be $0 \leq \varepsilon_i \leq \delta_i \leq 1$ here.

And decision variable:

$$z_i \begin{cases} 1 \, an \, i \, asset \, is \, in \, the \, portfolio \\ 0 \, i \, asset \, is \, not \, in \, the \, portfolio \end{cases}$$

Hence, the objective function in the cardinality constrained model is given in equation (6)-(11).

$$min\sum_{i=1}^{N}\sum_{j=1}^{N}w_i w_j \sigma_{ij} \tag{6}$$

$$\sum_{i=1}^{N}w_i \mu_i = R^* \tag{7}$$

$$\sum_{i=1}^{N}w_i = 1 \tag{8}$$

$$\sum_{i=1}^{N}z_i = K \tag{9}$$

$$\varepsilon_i z_i \leq w_i \leq \delta_i z_i, \quad i = 1,2,3...N \tag{10}$$

$$z_i \in [0,1], \quad i = 1,2,3,...N \tag{11}$$

Due to the addition of the constraint that guarantees the number of assets to be included in the portfolio to be a certain K number and the increase in the dimensions of the redefined asset-constrained portfolio optimization problem, it has become difficult to obtain the exact solution with mathematical programming-based solution processes.

## 3. The heuristic approach to the constricted efficient frontier

In this section, firstly, the heuristic approach designed to find a restricted efficient frontier will be presented, then the basic principles of the proposed simulated annealing algorithm for the optimization problem discussed will be briefly explained and a special application will be shown.

### 3.1. Simulated annealing algorithm

The simulated annealing (SA) algorithm is a trajectory-based heuristic search method that performs stochastic neighborhood search independently of each other, proposed by Kirkpatrick, Gelatt and Vecchi (1983), Cerny (1985). The biggest advantage of SA, which provides good solutions for combinatorial optimization problems, compared to traditional local search methods, is that it does not get stuck at the local optimum while searching for a global optimum. SA's development was inspired by the physical annealing process of solids in physics. The annealing process consists of two steps. First, by increasing the temperature of the heat bath to the highest value a metal can melt, the atoms gain the necessary energy and have the freedom to move. In the next step, the appropriate crystal structure is obtained by cooling and freezing (annealing) the metal in a controlled manner until it enters a low energy state.

In the SA algorithm, the neighbor of the randomly selected initial solution is generated with a suitable mechanism of action. If the new solution creates an improvement in objective function, it is considered the current solution. This process continues until a better solution cannot be found among the neighbors of the current solution. This process is the descent algorithm and ends with a local best solution. However, there is an important step that characterizes the SA algorithm and removes this disadvantage of the descent algorithm. In SA, bad solutions are also allowed to be accepted in a controlled manner. In this way, it is ensured that it avoids local optimum. This action should be determined at random within a plan. The probability of acceptance of a move that causes a deterioration of the objective function  is calculated by the acceptance function. Metropolis, Rosenbluth, Rosenbluth, Teller and Teller (1953) suggested using the Boltzmann-Gibbs distribution as a probability function to make this decision. This acceptance function, inspired by thermodynamic models, is given in Equation 12.

$$P(acceptance)= e^{(-\Delta/T)} \tag{12}$$

Here, T is a control parameter corresponding to the temperature at the physical annealing and is the difference between the current solution and the neighboring solution. A fixed number of neighboring solutions are sought at each temperature value. After each stage, the temperature is reduced by a constant factor $\alpha \in (0.1)$. SA algorithms differ from each other according to various factors such as neighborhood search, cooling (annealing) schedule and termination criteria.

### 3.2. Simulated annealing for constrained portfolio optimization

Simulated annealing algorithm adapted for constrained portfolio model follows a common procedure with other heuristic optimization techniques. The problem is started with a random solution point coded as 0-1 and neighboring solutions of the current solution that can be reached with a single action are produced by operating the neighboring structures unique to the simulated annealing. The initial temperature value was generated from the initial solution value as in the study of Chang et al. (2000), and at each temperature, four times the total number of assets in the portfolio, neighboring solutions were sought. Chang et al (2000) determined the initial temperature value as the initial solution value / 10. Experiments were carried out with 80, 85, 90 and 95 values for the α parameter used for the cooling plan and 2 * N, 3 * N and 4 * N values for the number of neighbors. The best result was obtained when the number of neighbors was 4 * N and α was 95. Pseudo Code is given in Figure 1.

In Markowitz's unconstrained efficient frontier model, equation (1)-(4) defines the expected return of the portfolio as an equality constraint rather than an objective, and a single objective risk minimization problem is tried to be solved depending on the different levels of return desired. This problem can be easily solved by any solver that supports quadratic models. In this study, "qaprog" solver developed for quadratic programming problems is used.[1]

In order for feasible solutions to be obtained in the constrained model, the constraints in equations (5) - (7) must be met. Repair procedure in Chang (2000) has been applied to ensure these restrictions. In order to obtain the suitability value of the solutions that meet the constraints, it is necessary to calculate the objective function, which includes the risk and return of the portfolio with certain weights. Using the weighting parameter λ, which expresses the trade-off between risk and return, the sum of the mean return of the portfolio and the total risk calculated over the variance-covariance data corresponds to the suitability value of a solution. Starting from λ = 0 with a chart corresponding to risk aversion and risk-bearing behavior for a certain level of return, an efficient frontier is created with 51 different weight values until λ = 1 with an increase of 0.02 each time.

```
E       is the number of λ values
N       is number of assets
S       is the current solution
C       is the neighbour of S
S_best  is the best solution
α       is the constant cooling factor
Begin
H=∅                                                          /*pareto optimal solution*/
for e=1 to E do
    λ = (e−1)/(E−1)                        /* E=51 and λ values equally spaced in [0-1] */
    select initial solution: S0 e S        /* random initial solution has exactly K assets*/
    evaluate (S0, λ , f(S0), H);
    S ← S0 ; f(S) ← f(S0)
    Sbest ← S0 ; f(Sbest) ← f(S0)                 /* save initial solution as best solution */
    T = |f(S0)/10|                                          /*İnitialise SA paremeters */
    α = 0.95
    for t=1 to iteration number do
        for k=1 to neighbour number do
            C=S                                                   /*neighbour solution* /
            while i ∈ S and j ∉ S
             select a random assets i and j                                      /move*/
              C = S ∪ [j] − [i]                                         /* C has K assets*/
             end while
            evaluate (C, λ , f(C), H)
            if f(C)<f(Sbest) then                        /* C better than current solution S*/
                  Sbest=C
            else
            u: = random number drawn from [0,1]
            delta= (f(C)-f(Sbest))
                if u<exp[-delta/T]   then           /* criteria for accepting worse solution*/
                  Sbest=C                                                  /* update Sbest*/
                end if
            end if
        end for
      T = α*T                                                    /* reduce temperature*/
    end for
end for
end
```

*Figure 1.* Pseudo-Code for Simulated Annealing Algorithm

[1] For details; https://www.mathworks.com/help/optim/ug/quadprog.htm

In order to evaluate the performance of the heuristic algorithm developed, Chang et al. (2000), by taking into account 2000 different values for the return values R, obtained 2000 points on the efficient frontier in the unconstrained portfolio optimization model. Chang et al. (2000), Woodside-Oriakhi, et al. (2011), by calculating this distance between this frontier, which is called standard efficient frontier (SEF) and solved with standard quadratic programming models, and the constrained efficient frontier (CEF) obtained by the heuristic algorithm, different heuristic algorithms, they evaluated the comparison of efficient frontier calculated with different constraint values by performance measures such as mean percentage error, median percentage error, minimum percentage error and maximum percentage error. Fernandez and Gomez (2007) measured this with the variance error and mean return error of the return, and Cura (2009), Sadigh et al. (2012) with the mean Euclidean distance, the variance of return error, and the mean return error. Their formulas are given in Equations (14), (15) and (16). Here, $v_i^s$ , $r_i^s$ (i=1,……..2000) represents the risk and return values of the i point on the standard efficient frontier and $v_j^h$ , $r_j^h$ (j = 1,… ..ξ) the j point on the heuristic efficient frontier. Of the points on the standard efficient frontier, the points closest to the points on the heuristic efficient frontier are represented by $v_{i_j}^s$ , $r_{i_j}^s$. The closest point on the standard efficient frontier represented by $i_j$ is obtained from Equation (13).

$$i_j = \underset{i=1,\ldots.2000}{\arg min}(\sqrt{(v_i^s - v_j^h)^2 + (r_i^s - r_j^h)^2} \quad j = 1,\ldots,\xi \tag{13}$$

Mean Euclidean Distance: $$(\sum\sqrt{(v_{i_j}^s - v_j^h)^2 + (r_{i_j}^s - r_j^h)^2})*\frac{1}{\xi} \tag{14}$$

Variance of Return Error: $$(\sum_{j=1}^{\xi}100\left|v_{i_j}^s - v_j^h\right| / v_j^h)*\frac{1}{\xi} \tag{15}$$

Mean Return Error: $$(\sum_{j=1}^{\xi}100\left|r_{i_j}^s - r_j^h\right| / r_j^h)*\frac{1}{\xi} \tag{16}$$

## 4. Findings

The performance of the developed simulated annealing algorithm was first applied on the Hang Seng data set, which is frequently used in the literature, and the results were compared with previous studies. At this stage, after the algorithm performance was deemed sufficient, it was applied to the BIST30 index, which is the main data set of the study[2]. The daily average returns of the companies included in the BIST30 index were analyzed between 01/09/2020 and 11/12/2020. The companies included in the BIST30 index are given in Appendix A and the income distribution of the companies is given in Figure 2. Data on the weight and closing prices of companies in the BIST30 index can be accessed on the Borsa Istanbul Data Store.

The results obtained with the Hang Seng data set were calculated using the values of *K = 10, ε = 0.01 and δ = 0.01 (i = 1,…, N)* and 51 different λ values were set for the efficient frontier. Keeping all the remaining parameters the same, the optimal number of assets to be kept in the portfolio was decided by testing the values 5,10,15 and 20 for *K* in the BIST30 data set.

[2] The Hang Seng data set included in the benchmarking examples can be obtained from http://people.brunel.ac.uk/~mastjjb/jeb/orlib/portinfo.html

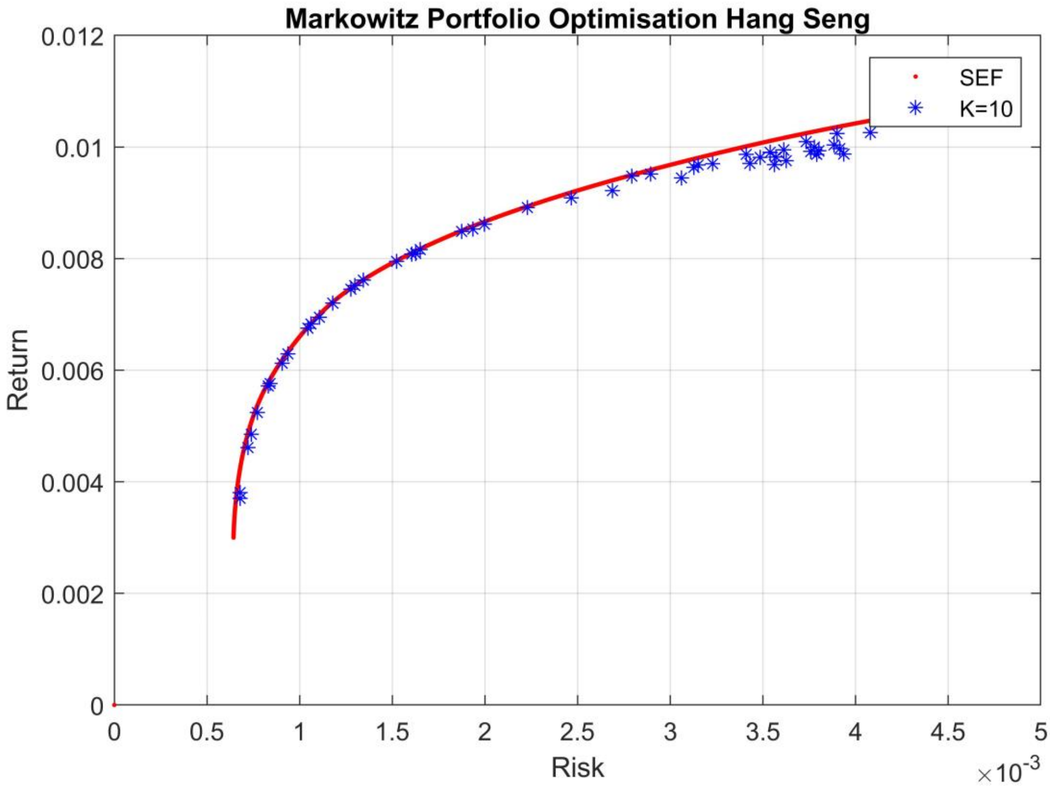


*Figure 2.* Standard and Heuristic Efficient Frontier on The Hang Seng Benchmark Dataset

Unconstrained efficient portfolios, named as standard efficient frontier and obtained with QA solver, are formed in the form of a continuous curve as seen with the "SEF" label in Figure 2. Efficient portfolios with restricted simulated annealing algorithm are formed as dashed lines on the graph with the label *"K = 10"*. It is expected that limited active portfolios will be slightly below the unconstrained efficient frontier. However, it is desired to be as close as possible. How close it is, as mentioned before, is demonstrated by certain performance criteria. Table 1 includes some of the pioneering studies that have achieved highly competitive results in the literature.

Table 1

*Performance of existing studies on Hang Seng data series*

| **Studies and Methods** | **MED** | **VRE** | **MRE** |
|---|---|---|---|
| Cura (2009)-PSO | 0.0049 | 2.2421 | 0.7427 |
| Sadigh (2012)-HNN | 0.0001 | 2.5908 | 0.7335 |
| Baykasoğlu (2015)- GRASP | 0.0001 | 1.6400 | 0.6060 |
| Mozafari et al. (2011)-IPSO-SA | 0.0001 | 1.6388 | 0.6059 |
| Kalaycı et al. (2017b)-ABC | 0.0001 | 1.6432 | 0.6047 |
| Kalaycı et al. (2020)-AC-GA-ABC | 0.0001 | 1.6395 | 0.6085 |
| This study -SA | 0.0001 | 2.2961 | 0.6797 |

PSO: Particle Swarm Optimization, HNN: Hopfield Neural Network, GRASP: Greedy Randomized Adaptive Search Procedure, IPSO: In- Particle Swarm Optimization, SA: Simulated Annealing, GA: Genetic Algorithm, ABC: Artificial Bee Colony, AC: Ant Colony

In Table 1, the performance of the developed SA algorithm has been compared with these studies in the literature. As can be seen in Table 1, the SA algorithm did not pass the best result found so far. However, it can be said to have a comparable performance. Consequently, the experiments on the BIST30 data set were carried out with the conviction that the proposed algorithm can produce certain quality results with an acceptable error.

In Figure 3, the solid straight line standard efficient frontier graph and dashed graphs show the heuristic efficient frontiers calculated with different *K* values. It is observed that better results can be obtained if the number of assets in the portfolio is 10 and 15 in low risk levels and 5 in high risk levels. Considering all the risk levels, it is seen that it would not be appropriate to determine the number of assets as 20 in BIST 30.

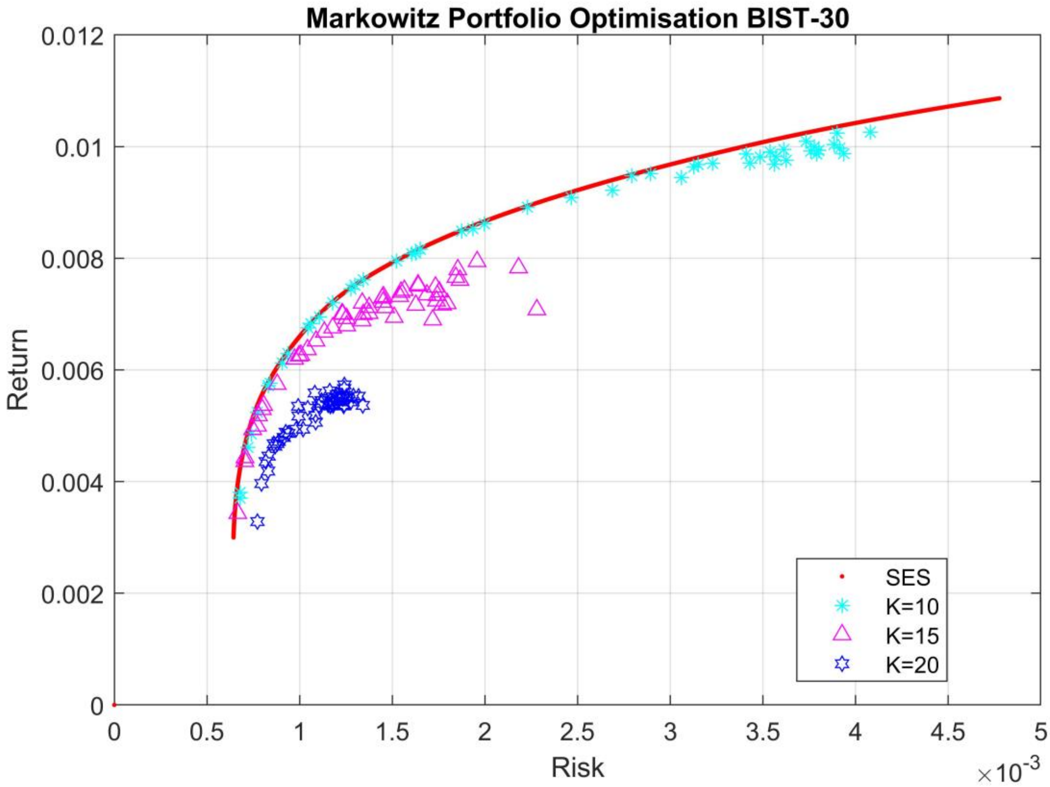


*Figure 3.* Heuristic Efficient Frontier of The BIST30 Data Set Obtained at Standard and Different Constraint Levels

The optimal portfolios, which are considered to be better than their proximity to the efficient frontier, were calculated with the performance criteria mentioned above and the results are reported in Table 2. When the results are examined, it is seen that the optimal number of assets to be kept in the portfolio should be 10 according to each performance criterion, regardless of the risk level. Although the BIST30 data set has been studied over different years and with different heuristic algorithms, Akyer et al. (2018) also obtained better portfolios at all risk levels when the number of assets in the portfolio is 10.

Table 2

*Distances between the standard efficient frontier and the constrained efficient frontier obtained on the Bist30 data set*

| K | MED | VRE | MRT |
|---|---|---|---|
| 5 | 0.0875 | 24.5717 | 2.5204 |
| 10 | 0.0323 | **8.0012** | **1.6437** |
| 15 | 0.0914 | 38.2803 | 2.9773 |
| 20 | 0.1444 | 62.3945 | 2.9079 |

## 5. Conclusion

Portfolio optimization is one of the important research topics in finance. In this study, a solution has been made with simulated annealing algorithm, which is a powerful heuristic search algorithm whose success has been proven in many combinatorial optimization problems, over the average variance asset constrained portfolio optimization problem. In the study, firstly, in order to understand whether the performance of the developed algorithm is sufficient or not, it has been solved with a well-known Hang-Seng benchmark data set and a comparison with the past solution results in Cura (2009)-PSO, Sadigh (2012)-HNN, Baykasoğlu (2015)- GRASP, Mozafari et al. (2011)-IPSO-SA, Kalaycı et al. (2017b)-ABC and Kalaycı et al. (2020)-AC-GA-ABC is presented. It seems difficult to compete with hybrid methods, as the algorithms created by hybridizing population-based heuristics and local search-based heuristics use the advantages of both methods simultaneously.

According to these results, it has been observed that the developed algorithm produces comparable reasonable results, even though it is not the best solution presented so far. Then, a special application of the algorithm with the BIST30 data set was demonstrated and portfolios with different numbers of assets were created. These results provide an assessment of what should be the optimal number of assets that investors will keep in their portfolios at different risk levels. Akyer et al. (2018) found that increasing the number of assets does not reduce the portfolio risk. After looking at that BIST-30 study, it was concluded that the optimal number of assets in the portfolio should be 10. The answer to the question of what the optimal number of assets should be was sought with the SA algorithm, which searches on the same index with a different approach, and among the portfolios with 5, 10, 15 and 20 assets, the number of portfolios with the highest average return and the lowest risk was found to be 10. In this study, portfolio optimization with the simulated annealing approach was applied for the BIST30 index and it was concluded that the number of assets at low risk level and the number of assets at higher risk level should be less in order to approach the efficient frontier in the model developed based on the Markowitz mean-variance model. According to the findings, in addition to the risk-return assumptions, the number of assets required to be in the portfolio will be beneficial for the investor in the decision-making process by contributing to the conditions of the Markowitz mean-variance model.

## Author statement

### Research and publication ethics statement

This study has been prepared in accordance with the ethical principles of scientific research and publication.

### Approval of ethics board

Ethics committee approval is not required for this study.

### Author contribution

Seyyide Doğan: Research idea, research design, methodology, data analysis, writing, review and control.

Müge Sağlam Bezgin: Research idea, research design, literature review, data collection, writing, review and control.

Emine Karaçayır: Research idea, research design, literature review, data collection, writing, review and control.

### Conflict of interest

There is no conflict of interest arising from the study for the authors or third parties.

### Declaration of support

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## Appendix 1

A BIST 30 companies

| | |
|---|---|
| Akbank Inc. | Arcelik Inc. |
| Aselsan Electronic Industry and Trade Inc. | Bim United Stores Inc. |
| Emlak Konut Real Estate Investment Trust Inc. | Dogan Group of Companies Holding Inc. |
| Eregli Iron and Steel Factories Inc. | Hacı Omer Sabancı Holding Inc. |
| Kardemir Karabük Iron and Steel Inc. | Fetrilizer Mills Inc. |
| Koc Holding Inc. | Koza Gold Enterprises Inc. |
| Koza Anadolu Metal Mining Enterprises Inc. | Migros Trade Inc. |
| Oyak Cement Factories Inc. | Pegasus Air Transpotation Inc. |
| Petkim Petrochemical Holding Inc. | Tav Airports Holding Inc. |
| Ekfen Holding Inc. | Turkcell Communication Services Inc. |
| Tupras-Turkiye Oil RefineriesInc. | Turkish Airlines Inc. |
| Turk - Telecommunication Inc. | Turkiye Garanti Bank Inc. |
| Turkiye Halk Bank Inc. | Turkiye İs Banı Inc. |
| Turkiye Industrial Development Bank Inc. | Turkiye Sise and Cam Factories Inc. |