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A Novel Numerical Solution Method for Semi-explicit Differential-Algebraic Equations

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Abstract. Generally, DAEs do not have a closed form solution, so these equations have to be solved numerically. In this work, an approximate analytic series solution of the semi-explicit DAEs is obtained by using Laplace Adomian Decomposition Method (LADM). Before directly solving the high-index semi-explicit DAEs, we apply the index reduction method to high-index semi-explicit DAEs since solving high-index semi-explicit DAEs is difficult. Then, we use the LADM obtaining the numerical solution. To show computational capability and efficiency of the LADM for the solution of semi-explicit DAEs, a couple of numerical examples are given. It has been shown that the intoduced algorithm has a very good accuricy compared with exact solution for the semi-explicit DAEs. So it can be applied to other DAEs.

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Keywords: Differential algebraic equations, index reduction, Laplace transform, Adomian decomposition method, approximation solution.

1. Introduction

Differential algebraic equations (DAEs) are often obtained during modeling of problems in science and engineering. There are lots of different application areas for DAE systems such as computer-aided design and multi-body systems [23], circuit simulation [5], chemical process modelling, applying the extended Kalman filter and in many other applications [10, 21]. Sometimes studying on the approximate solution of DAEs may be more difficult then the approximate solution of ordinary differential equations (ODEs) [22]. In recent years, many studies have been carried out with DAEs. Zolfaghari at al described a method for analyzing the structure of a system of nonlinear integro-differential–algebraic equations (IDAEs) that generalizes the Σ method for the structural analysis of differential–algebraic equations [34]. Pulch at al, have considered linear dynamical systems composed of differential–algebraic equations (DAEs). They have did sensitivity analysis of random linear differential–algebraic equations using system norms [24]. Hanke and März approach a direct numerical treatment of nonlinear higher-index differential–algebraic equations by means of overdetermined polynomial least-squares collocation [18]. Yan at al, have focused on the numerical solutions of nonlinear delay-differential–algebraic equations with proportional delay, which are transformed into

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nonlinear delay- differential–algebraic equation with constant delay through exponential transformation. Block boundary value methods are extended to solve this type of equation [33]. DAEs is generally seen the following structure in the literature

$$F(\dot{y}(x), y(x), x) = 0, \tag{1.1}$$

where y and F are vector-valued functions and $\dot{y}(x)$ is the derivative of $y(x)$ with respect to x. If the Jacobian $\frac{\partial F}{\partial \dot{y}}$ is nonsingular, then the system (1.1) are implicit ODEs. Therefore, we can get $\dot{y}$ from the system (1.1) by applying the implicit function theorem for obtaining the explicit ordinary differential equations given below

$$\dot{y}(x) = G(y(x), x). \tag{1.2}$$

Here G is

$$F(G(y(x), x), y(x), x) = 0$$

a suitably defined functional. Therefore, by using this approach we may find the solution of implicid ODEs by solving ecplicit ODEs.

For the system (1.1) define differential-algebraic equations, it must be Jacobian $\frac{\partial F}{\partial \dot{y}}$ is singular, as now the function $\dot{y}(x)$ cannot be written in the explicit form (1.2) [9].

The index is a very important concept for the analysis of DAEs and defined as the number of differentiations needed to convert DAEs into a system of ODEs. Different index definitions are available such as the perturbation index [12], differentiation index [11] and the tractability index [26]. All these indices are equal for constant coefficients linear DAEs [27]. It is well known that, it is possible to say that the index is very important for the understant the difficulty level for the numerical solution of DAEs.If DAEs have an index greater than 1 then solving them can be difficult. [6, 29]. So there is need to index reduction before the numerical solution of DAEs [22, 29]. For this reason, to solve semi-explicit DAEs having high-index with Laplace Adomian Decomposition Method (LADM) is difficult and inefficient. We first applied the index reduction method to solve such equations. Then, we used LADM to find the numerical solution.

The LADM method is achieved by using two methods together to obtaion the numerical solutions of nonlinear equations. These are Laplace Transform Method and Adomian Decomposition Method (ADM). ADM was proposed and established by Adomian [1–4]. With this method, many different types of problems have been solved such that partial differential equations, linear and nonlinear ODEs, integral equations and integro-differential equations. The numerical solution of the second Painleve equation is obtained by using Adomian Decomposition method [13]. Rach has shown in his paper that the Adomian decomposition method cannot be identified with any of the previous methods as it solves a large class of nonlinear and stochastic equations without the often nonphysical assumptions that have become conventional [25]. Wazwaz in his work, compared the performance of the Adomian decomposition method and the Taylor series method applied to the solution of linear and nonlinear ordinary differential equations [30]. The Adomian decomposition method and a modified form of this method were applied to construct the numerical solution of fifth-order boundary value problems with two-point boundary conditions [31]. A modified form of the Adomian decomposition method was applied to construct the numerical solution for sixth-order boundary value problems (BVPs) with two-point boundary conditions [32]. Using Laplace transform gives us important advantage that effectively transform differential equations into algebraic equations. In recent years, the LADM method has been used by many researchers to solve many different problem types. [8, 14–17, 20]. In our researches we have not seen any study used this combined method to DAEs. Therefore, in present work we wish to use LADM to DAEs. By doing this we will manage at first to extent the applications of LADM to DAEs.

This work has been organized and presented as follows. In section 2, we give a few necessary informations about DAEs and Reducing index. In section 3, we demonstrated the LADM practically on some examples for showing the solution prosedure. The last section is reserved for the conclusion.

2. Reducing Index For DAEs

In this part, the index reduction method given by [7, 19, 28] is presented. Let us examine a linear (or linearized) semi-explicit DAEs:

$$X^{(m)} = \sum_{j=1}^{m} F_j X^{(j-1)} + GY + q \tag{2.1}$$
$$0 = HX + r,$$

where $X(\tau) \in R^{n\times 1}, Y(\tau) = R^{k\times n}$ and F_j, G,H are smooth functions of τ , $t_0 \leq \tau \leq \tau_f, F_j(\tau) \in R^{n\times n}, j = 1, \cdots, m, G(\tau) \in R^{n\times k}, H(\tau) \in R^{k\times n}, 1 \leq k \leq n, n \geq 2$ and HG is nonsingular (DAE has index m+1) outside a finite number of isolated points of τ, which in this case, the DAEs (2.1) restriction singularity. The inhomogeneties are $q(\tau) \in R^{n\times 1}$ and $r(\tau) \in R$.

If we take HG as nonsingular, from (2.1), we obtain

$$Y = (HG)^{-1} H\left[X^{(m)} - \sum_{j=1}^{m} F_j X^{(j-1)} - q\right], \tau \in \left[\tau_0, \tau_f\right]. \tag{2.2}$$

If we substitute (2.2) into (2.1), we obtain that

$$\left[I - G(HG)^{-1} H\right]\left[X^{(m)} - \sum_{j=1}^{m} F_j X^{(j-1)} - q\right] = 0.$$

Therefore, Problem (2.1) converts to the following overdeterminant system:

$$\left[I - G(HG)^{-1} H\right]\left[X^{(m)} - \sum_{j=1}^{m} F_j X^{(j-1)} - q\right] = 0 \tag{2.3}$$
$$HX + r = 0,\ \tau \in \left[\tau_0, \tau_f\right].$$

Then, (2.3) systems can be written as a full-rank DAE system having n unknowns and n equations with index m [7], [19].

Let us consider the problem (2.1), for simplicity, for $m = 1$ (index 2 problem occur), k=1,2 and n=2,3. Furthermore, if it is assumed that DAE is not singular, i.e.,

$$H.G(\tau) \neq 0, \tau \in \left[\tau_0, \tau_f\right].$$

Then, Theorem 2.1 and Theorem 2.2 offer us the opportunity to transform the index-2 DAEs into an index-1 DAEs. Thus, we will be able to apply LADM to the obtained index-1 problem.

Theorem 2.1. *For k = 2 and n=2 Problem (2.1) having index-2 is identical to DAE system having index-1 given below.*

$$T_1 X' + T_0 X = \hat{q}.$$

In this system, the following must be provided.

$$T_0 = \begin{bmatrix} b_1a_{21} - b_2a_{21} & b_1a_{22} - b_2a_{12} \\ c_1 & c_2 \end{bmatrix}, T_1 = \begin{bmatrix} b_2 & -b_1 \\ 0 & 0 \end{bmatrix}, \hat{q} = \begin{bmatrix} b_2q_1 - b_1q_2 \\ -r \end{bmatrix}$$
$$Y = (HG)^{-1} H\left[X' - FX - q\right].$$

Proof. Given by [7] □

Now, we give another theorem for index reduction.

Theorem 2.2. *For $k = 2$ and $n = 3$ Problem (2.1) index-2 is identical to DAE system having index-1 given below.*

$$\begin{bmatrix} \overline{M} \\ 0 \end{bmatrix} X' + \begin{bmatrix} -\overline{M}F \\ H \end{bmatrix} X = \begin{bmatrix} \overline{M}q \\ -r \end{bmatrix}.$$

In this system the following must be provided.

$$M = \begin{bmatrix} b_{21}b_{32} - b_{22}b_{31} & b_{12}b_{31} - b_{11}b_{32} & b_{11}b_{22} - b_{12}b_{21} \end{bmatrix}_{1\times 3}$$

and

$$Y = (HG)^{-1} H\left[X' - FX - q\right].$$

Proof. Given by [19]. □

3. Laplace Adomian Decomposition Method

In this part, we proposed the LADM with examples to show the capability and performance of this method for semi-indexed DAEs. Using Laplace transform gives us important advantage that effectively transform differential equations into algebraic equations. The LADM method is achieved by using two methods together to obtaion the numerical solutions of nonlinear equations. These are Laplace Transform Method and ADM. In some examples to simplify the computations coefficient functions is expressed in Taylor series. We apply the index reduction method since solving high-indexed semi-explicit DAEs is difficult and inefficient. Then, we solve them by using LADM. The algorithms are implemented by Mathematica 7.0.

Example 3.1. Let us study the following semi-explicit DAEs with the $x_1(0) = 1, x_2(0) = 0$ initial conditions.

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1'(\tau) \\ x_2'(\tau) \end{pmatrix}+\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x_1(\tau) \\ x_2(\tau) \end{pmatrix}=\begin{pmatrix} [\sin\tau + (2.02)]\, e^{(0.02)\tau} \\ [2\sin\tau + 1]\, e^{(0.02)\tau} \end{pmatrix}.$$

The exact solution of the problem is

$$x(\tau) = \begin{pmatrix} e^{(0.02)\tau} \\ \sin\tau e^{(0.02)\tau} \end{pmatrix}.$$

We apply the LADM to this problem to obtain the solution of it. Now, let's apply the Laplace transform

$$\begin{aligned} L\{x_1(\tau)\} &= \tfrac{1}{s} + \tfrac{1}{s} L\left\{[\sin\tau + (2.02)]\, e^{(0.02)\tau}\right\} - \tfrac{2}{s} L\{x_1(\tau)\} - \tfrac{1}{s} L\{x_2(\tau)\}, \\ L\{x_2(\tau)\} &= \tfrac{1}{2} L\left\{[2\sin\tau + 1]\, e^{(0.02)\,\tau}\right\} - \tfrac{1}{2} L\{x_1(\tau)\}. \end{aligned} \tag{3.1}$$

Substituting $x_1 = \sum\limits_{k=0}^{\infty} x_{1k}$, $x_2 = \sum\limits_{k=0}^{\infty} x_{2k}$ into (3.1), it causes

$$\begin{aligned} L\left\{\sum_{k=0}^{\infty} x_{1k}\right\} &= \frac{1}{s} + \frac{1}{s} L\left\{[\sin\tau + (2.02)]\, e^{(0.02)\tau}\right\} - \frac{2}{s} L\left\{\sum_{k=0}^{\infty} x_{1k}\right\} - \frac{1}{s} L\left\{\sum_{k=0}^{\infty} x_{2k}\right\}, \\ L\left\{\sum_{k=0}^{\infty} x_{2k}\right\} &= \frac{1}{2} L\left\{[2\sin\tau + 1]\, e^{(0.02)\tau}\right\} - \frac{1}{2} L\left\{\sum_{k=0}^{\infty} x_{1k}\right\}. \end{aligned}$$

Then, the solution procedure is obtained by using the LADM.

$$\begin{aligned} L\{x_{10}\} &= \frac{1}{s} + \frac{1}{s} L\left\{[\sin\tau + (2.02)]\, e^{(0.02)\tau}\right\},\ L\{x_{1k+1}\} = -\frac{2}{s} L\{x_{1k}\} - \frac{1}{s} L\{x_{2k}\}, \\ L\{x_{20}\} &= \frac{1}{2} L\left\{[2\sin\tau + 1]\, e^{(0.02)\tau}\right\},\ L\{x_{2k+1}\} = -\frac{1}{2} L\{x_{1k}\}. \end{aligned}$$

For the sake of shortness approximations to the solutions we obtain only following six terms,

$$\begin{aligned} x_1(\tau) &= 1 + 0.02\tau + 0.0002\tau^2 - 0.020832\tau^3 - 0.250104\tau^4 - 0.346834\tau^5 - 0.112309\tau^6, \\ x_2(\tau) &= 1.\tau + 0.02\tau^2 - 0.291467\tau^3 - 0.342499\tau^4 - 0.151411\tau^5 - 0.0115533\tau^6. \end{aligned}$$

Table 1 shows the numerical solutions of Example 3.1. It is also illustrate the absolute errors of exact solution and LADM with 20 terms. Furthermore Figure 1 shows absolute errors graphics between numerical solutions obtained with LADM with 20 terms and exact solutions of Example 3.1. Looking at the Figure 1, it is seen that the difference between the exact solution and the approximate solution is very small. Thus, we can say that we have obtained a good approach with LADM.

Example 3.2. Let us study the following problem. This problem is linear index-2 and semi-explicit DAEs problem and initial values are $x_1(0) = 1, x_2(0) = 0$.

$$\begin{aligned} X' &= FX + GY + q \\ 0 &= HX + r \end{aligned},$$

τ_i	$x_1(\tau_i)$	$err(x_1(\tau_i))$	$x_2(\tau_i)$	$err(x_2(\tau_i))$
0	1.	0.	0.	0.
0.1	1.002	0.	0.100033	2.77556E-17
0.2	1.00401	0.	0.199466	2.498E-16
0.3	1.00602	1.13243E-14	0.297299	4.2466E-14
0.4	1.00803	4.64073E-13	0.392546	1.4827E-12
0.5	1.01005	8.80607E-12	0.484244	2.47669E-11
0.6	1.01207	1.0195E-10	0.571459	2.57993E-10
0.7	1.0141	8.35606E-10	0.6533	1.93189E-9
0.8	1.01613	5.30372E-9	0.728926	1.13295E-8
0.9	1.01816	2.76358E-8	0.797555	5.50185E-8
1	1.0202	1.23063E-7	0.85847	2.29908E-7

TABLE 1. Absolute error between numerical solution obtained with LADM with 20 terms and exact solution of Example 3.1.

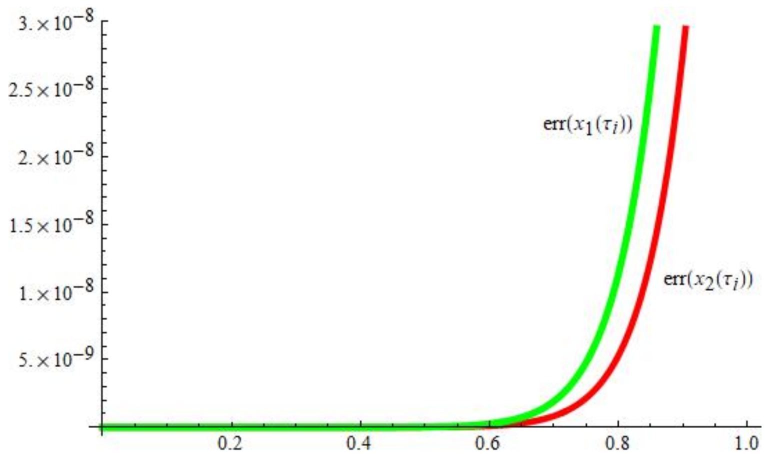

FIGURE 1. Absolute error graphics between numerical solutions obtained with LADM with 20 terms and exact solutions of Example 3.1

where $0 \le \tau \le 1$ and

$$F = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}, G = \begin{bmatrix} 0 \\ 1+2\tau \end{bmatrix}, q = \begin{bmatrix} -\sin(\tau) \\ 0 \end{bmatrix}, H^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, r(\tau) = -\left(e^{-\tau} + \sin(\tau)\right).$$

Here, the exact solutions are

$$x_1(\tau) = e^{-\tau}, x_2(\tau) = \sin(\tau), y(\tau) = \frac{\cos(\tau)}{1+2\tau}.$$

Theorem 2.1 converts the problem having index-2 to the problem having index-1 as follows having the same $x_1(0) = 1$ and $x_2(0) = 0$ initial values:

$$\begin{aligned} x_1 + x_2 &= e^{-\tau} + \sin(\tau) \\ x_1' + x_1 - x_2 + \sin(\tau) &= 0. \end{aligned}$$

Here, $y = (HG)^{-1} H [X - FX - q]$. We apply the LADM to this problem to obtain the solution of it. Now, let's apply the Laplace transform

$$\begin{aligned} L\{x_1(\tau)\} &= \tfrac{1}{s} - \tfrac{1}{s}L\{\sin(\tau)\} + \tfrac{1}{s}L\{x_2(\tau)\} - \tfrac{1}{s}L\{x_1(\tau)\} \\ L\{x_2(\tau)\} &= L\{e^{-\tau}\} + L\{\sin(\tau)\} - L\{x_1(\tau)\}. \end{aligned} \tag{3.2}$$

τ_i	$x_1(\tau_i)$	$err(x_1(\tau_i))$	$x_2(\tau_i)$	$err(x_2(\tau_i))$	$y(\tau_i)$	$err(y(\tau_i))$
0	1.	0.	0.	0.	1.	1.61626E-12
0.1	0.904837	2.49356E-13	0.0998334	8.35498E-13	0.82917	1.16129E-12
0.2	0.818731	6.43818E-13	0.198669	1.38367E-12	0.700048	1.63658E-12
0.3	0.740818	9.1005E-13	0.29552	3.13083E-14	0.597085	3.13616-12
0.4	0.67032	4.23683E-12	0.389418	1.2611E-11	0.511701	1.35715E-10
0.5	0.606531	4.27451E-11	0.479426	1.73343E-10	0.438791	1.4551E-9
0.6	0.548812	2.31555E-10	0.564642	1.33268E-9	0.375153	8.86601E-9
0.7	0.496585	8.07642E-10	0.644218	7.0922E-9	0.318684	3.83198E-8
0.8	0.449329	1.82167E-9	0.717356	2.9105E-8	0.267964	1.30708E-7
0.9	0.40657	1.2984E-9	0.783327	9.82309E-8	0.222003	3.74297E-7
1	0.367879	1.04795E-8	0.841471	2.84294E-7	0.1801	9.36034E-12

Table 2. Absolute error between numerical solution obtained with LADM with 20 terms and exact solution of Example 3.2

Substituting $x_1 = \sum\limits_{k=0}^{\infty} x_{1k}$, $x_2 = \sum\limits_{k=0}^{\infty} x_{2k}$ into (3.2), it causes,

$$\begin{aligned} L\left\{\sum_{k=0}^{\infty} x_{1k}\right\} &= \frac{1}{s} - \frac{1}{s} L\{\sin(\tau)\} + \frac{1}{s} L\left\{\sum_{k=0}^{\infty} x_{2k}\right\} - \frac{1}{s} L\left\{\sum_{k=0}^{\infty} x_{1k}\right\}, \\ L\left\{\sum_{k=0}^{\infty} x_{2k}\right\} &= L\{e^{-\tau}\} + L\{\sin(\tau)\} - L\left\{\sum_{k=0}^{\infty} x_{1k}\right\}. \end{aligned}$$

Then, the solution procedure is obtained by using the LADM.

$$\begin{aligned} L\{x_{10}\} &= \frac{1}{s} - \frac{1}{s} L\{\sin(\tau)\},\ L\{x_{1k+1}\} = \frac{1}{s} L\{x_{2k}\} - \frac{1}{s} L\{x_{1k}\}, \\ L\{x_{20}\} &= L\{e^{-\tau}\} + L\{\sin(\tau)\}, L\{x_{2k+1}\} = -L\{x_{1k}\}. \end{aligned}$$

For the sake of shortness approximations to the solutions we obtain only following six terms:

$$\begin{aligned} x_1(\tau) &= 9 - 12e^{-\tau} - 15\tau + \frac{17\tau^2}{2} - \frac{5\tau^3}{3} + \frac{\tau^4}{12} + 4\cos(\tau) + 2\sin(\tau), \\ x_2(\tau) &= -8 + 8e^{-\tau} + 11\tau - 4\tau^2 + \frac{\tau^3}{3} - 2\sin(\tau), \\ y(\tau) &= \frac{156 - 192e^{-\tau} - 204\tau + 102\tau^2 - 20\tau^3 + \tau^4 + 48\cos(\tau) + 12\sin(\tau)}{12 + 24\tau}. \end{aligned}$$

Table 2 shows the numerical solutions of Example 3.2. It is also illustrate the absolute errors of exact solution and LADM with 20 terms. Furthermore, Figure 2 shows absolute errors graphics between numerical solutions obtained with LADM with 20 terms and exact solutions of Example 3.2. Looking at the Figure 2, it is seen that the difference between the exact solution and the approximate solution is very small. Thus, we can say that we have obtained a good approach with LADM.

4. Conclusion

In this study LADM has been applied to semi-explicit DAEs. To show computational capability and efficiency of the method for the solution of semi-explicit DAEs a couple of numerical examples are given. We apply the index reduction method to high-indexed semi-explicit DAEs since solving high-indexed semi-explicit DAEs is difficult. Then we use the LADM obtaining the numerical solution. By examining Table 1, Figure 1, Table 2 and Figure 2 we can easily see that the absolute errors changes between $10^{-7}$and 10^{-17}. It has been shown that the introduced algorithm has a

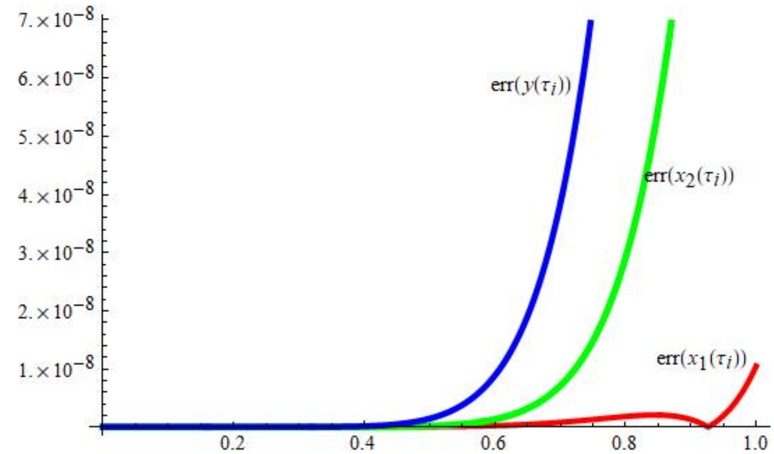

FIGURE 2. Absolute error graphics between numerical solutions obtained with LADM with 20 terms and exact solutions of Example 3.2

very good accuricy compared with exact solution for the semi-explicit DAEs. All the computations are performed by Mathematica 7.0. Therefore, the calculated results are quite reliable. So it can be suitable to other DAEs.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

Authors Contribution Statement

All authors jointly worked on the results and they have read and agreed to the published version of the manuscript.

References

[1] Adomian,G., Nonlinear Stochastic Operator Equations, Academic Press, San Diego, 1986.
[2] Adomian,G., *A review of the decomposition method in applied mathematics*, J. Math. Anal. Appl., **135**(1988), 501–544.
[3] Adomian,G., Rach,R., *Analytic solution of nonlinear boundary-value problems in several dimensions by decomposition*, J. Math. Anal. Appl., **174**(1993) , 118–137.
[4] Adomian,G., Solving Frontier Problems of Physics: The Decomposition Method, Kluwer, Boston, 1994.
[5] Ali, G., Bartel, A., Rotundo, N., *Index-2 elliptic partial differential-algebraic models for circuits and devices*, Journal of Mathemtical Analysis and Applications, **423**(2015), 1348-1369.
[6] Ascher, U.M., Lin, P., *Sequential regularization methods for higher index differential-algebraic equations with constraint singularities: the linear index-2 case*, SIAM J Anal, **33**(1996),1921–1940.
[7] Babolian, E., Hosseini, M.M., *Reducing index, and pseudospectral methods for differential-algebraic equations*, Appl Math Comput, **140**(2003),77–90.
[8] Babolian, E., Biazar,J., Vahidi, A.R., *A new computational method for Laplace transforms by decomposition method*, Applied Mathematics and Computation, **150**(2004), 841–846.
[9] Bai, Z.Z., Yang, X., *On convergence conditions of waveform relaxation methods for linear differential-algebraic equations*, Journal of Computational and Applied Mathematics, **235**(2011), 2790–2804.
[10] Beykal, B., Onel, M., Onel, O., Pistikopoulos, E.N., *A data-driven optimization algorithm for differential algebraic equations with numerical infeasibilities*, AIChE J., **66**(2020), e16657.
[11] Brenan, K.E., Campbell, S.L., Petzold, L.R., Numerical Solution of Initial-Value Problems in Differential-Algebraic Equations, 2nd edn. SIAM, Philadelphia, 1996.
[12] Bujakiewicz, P., Maximum Weighted Matching for High Index Differential Algebraic Equations. Doctor's dissertation, Delft University of Technology, 1994.
[13] Dehghan,M., Shakeri,F., *The numerical solution of the second Painleve equation*, Numer. Methods PDEs, **25**(2009),1238–1259.
[14] Doğan, N., *Solution of the system Of ordinary differential equations by combined Laplace transform-Adomian decomposition method*, Mathematical and Computational Applications An International Journal, **17**(2012), 203-211.
[15] Doğan, N., Akin, Ö., *Series solution of epidemic model*, TWMS Journal of Applied and Engineering Mathematics, **2**(2)(2012), 238-244.
[16] Doğan, N., *Numerical treatment of the model for HIV infection of CD4+T cells by using multi-step Laplace Adomian decomposition method*, Discrete Dynamics in Nature and Society, **2012**(2012), Article ID 976352.

[17] Doğan, N., *Numerical solution of chaotic Genesio system with multi-step Laplace Adomian decomposition method*, Kuwait Journal of Science, **40**(1) (2013), 109–121.
[18] Hanke, M., März, R., *Convergence analysis of least-squares collocation methods for nonlinear higher-index differential–algebraic equations*, Journal of Computational and Applied Mathematics, **387**(2021), 112514.
[19] Hosseini, M.M., *An index reduction method for linear Hessenberg systems*, J Appl Math Comput, **171**(2005), 596–603.
[20] Khuri, S.A., *A Laplace decomposition algorithm applied to a class of nonlinear differential equations*, Journal of Applied Mathematics, **1**(4)(2001), 141–155.
[21] Peng, H., Li, F., Liu, J., Ju,Z., *A symplectic instantaneous optimal control for robot trajectory tracking ith differential-algebraic equation models*, in IEEE Transactions on Industrial Electronics, **67**(5)(2020), 3819-3829.
[22] Petzold, L.R., *Differential/algebraic equations are not ODE's*, SIAM Journal of Science and Statistical Computing, **3**(3)(1982), 367-384.
[23] Pöll, C., Hafner, I., *Index reduction and regularisation methods for multibody systems*, IFAC-Papers OnLine, **48**(2015), 306-311.
[24] Pulch, R., Narayan, A., Stykel, T., *Sensitivity analysis of random linear differential–algebraic equations using system norms*, Journal of Computational and Applied Mathematics, **397**(2021), 113666.
[25] Rach, R., *On the Adomian decomposition method and comparisons with Picards method*, J. Math. Anal. Appl., **128**(1987), 480–483.
[26] Schwarz, D.E., Tischendorf, C., *Structural analysis of electric circuits and consequences for MNA*, Int. J. Circ. Theory Appl., **28**(2000), 131–162.
[27] Schulz, S., Four Lectures on Differential-Algebraic Equations. Technical Report 497, The University of Auckland, New Zealand, 2003.
[28] Soltanian, F., Karbassi, S.M., Hosseini, M.M., *Application of He's variational iteration method for solution of differential-algebraic equations*, Chaos, Solitons and Fractals, **41**(2009), 436–445.
[29] Tang, J., Rao, Y., *A new block structural index reduction approach for large-scale differential algebraic equations*, Mathematics, **8**()2020), 2057.
[30] Wazwaz, A.M., *A comparison between Adomian decomposition method and Taylor series method in the series solutions*, Appl. Math. Comput., **79**(1998), 37–44.
[31] Wazwaz, A.M., *The numerical solution of fifth-order boundary value problems by the decomposition method*, J. Comput. Appl. Math., **136**(2001), 259–270.
[32] Wazwaz, A.M., *The numerical solution of sixth-order boundary value problems by the modified decomposition method*, Appl. Math. Comput., **118**(2001), 311–325.
[33] Yan, X., Qian, X., Zhang, H., Song, S., *Numerical approximation to nonlinear delay-differential–algebraic equations with proportional delay using block boundary value methods*, Journal of Computational and Applied Mathematics, **404**(2022), 113867.
[34] Zolfaghari, R., Taylor, J., Spiteri, R. J., *Structural analysis of integro-differential–algebraic equations*, Journal of Computational and Applied Mathematics, **394**(2021), 113568.