

## New Iterative method to Calculate Base Stress of Footings under Biaxial Bending

**Ibrahim Aydogdu**

Department of Civil Engineering, Akdeniz University  
E-mail address: [aydogdu@akdeniz.edu.tr](mailto:aydogdu@akdeniz.edu.tr)

Received date: November 2016

Accepted date: December 2016

### Abstract

Base stresses of the footings should be calculated under biaxial bending especially for structures in seismic zones. However, calculation of the base stresses is not easy for the footings with large eccentricity due to the fact that interaction of the stresses in both directions should be considered and redistribution of stresses is required in zero stress zone. In this study new iterative method is presented to calculate the base stress of rectangular footings under biaxial bending. A computer program is developed according to this method. A design example (single footing having rectangular shape) is analyzed using the developed program and computed base stresses are compared to previous results in order to test accuracy of the method. The design example is also analyzed using different eccentricities in order to reveal efficiency of the redistribution under biaxial bending.

**Keywords:** Biaxial bending, single footings, base pressures, numerical model

### 1. Introduction

Single footings are mostly preferred in soil having high enough bearing capacity and for structures having less axial load. Several criteria should be considered in design of the footings such as punching, base shear stress and bending. Bearing stress capacity of the soil is one of the most important criteria in order to determine cross sectional dimensions of the footings which depend on the base stresses. Under biaxial bending, the interaction of the two orthogonal directions should be considered in calculation of the base stresses. Determining the interaction is not difficult for single footings having small eccentricity since unstressed zone does not occur and the redistribution of the stresses is not required. However, the same thing cannot be said for the footings with large eccentricity. In that case, unstressed zone occurs and the base stresses in the compression zone should be redistributed. The redistribution is not easy process due to the fact that equilibrium of the footing should be maintained after the redistribution.

In the literature, many studies have been found about footings having large eccentricity. Prakash and Saran developed analytical formulation to calculate bearing capacity of eccentrically loaded footings [1]. Gesund investigated flexural collapse loads for eccentrically loaded single footings [2]. Saran and Agarwal determined bearing capacity of an eccentrically obliquely loaded footing using limit equilibrium analysis [3]. Mahiyar et al. analyzed angle shaped footings under eccentric loadings [4]. Wieneke et al. investigated punching shear behavior of eccentrically loaded footings [5]. Gurfinkel developed a numerical procedure to calculate the maximum stress on the soil [6]. Köseoğlu presented

extensive explanations calculation of base stresses under biaxial bending [7]. Csenki presented an analytic solution to estimate maximum foundation pressure beneath a rectangular footing [8]. Özmen developed iterative method to determine base stresses in rectangular footings under biaxial bending [9]. Gutierrez and Ochoa investigated simplified analytical method that determines the axial load and biaxial moment capacities of a rigid spread footing [10]. Sarkar analyzed isolated footing under axial load and high biaxial moments, and developed numerical approach to calculate stresses [11]. Guan and Zhang developed a computational method for ultimate strength analysis of arbitrary reinforced concrete and composite sections subjected to axial force and biaxial bending [12]. Pardo et al. made an analytical and experimental research about biaxial capacity of rigid footings [13].

The study presents an alternative simplified method to determine the base stresses of rectangular single footings under the biaxial bending. The computer program is developed which is based on this method. The developed program is tested using a design example which is used in previous studies. Obtained results of the design example are compared to the results in the literature. The design example is also solved without the redistribution of the stresses with different eccentricity values. Obtained results are compared in order to investigate efficiency of the redistribution on maximum value of the base stresses.

The remainder of the study is organized as follows; Section 2 describes analysis of the rectangular footings. In Section 3, mathematical modelling of the developed method is depicted. In Section 4, the efficiency and accuracy of the developed method are tested with the design example. Finally, concluding remarks and future research suggestions are provided in Section 5.

## 2. Analysis of the rectangular footings

Owing to ease of calculations, the rectangular footings are generally modelled as 2-D and under uniaxial bending, in case that applied moment of the weak axis is considerably less than a moment of the strong axis and axial force. The basic model of the single footing under uniaxial bending is shown in Fig.1.

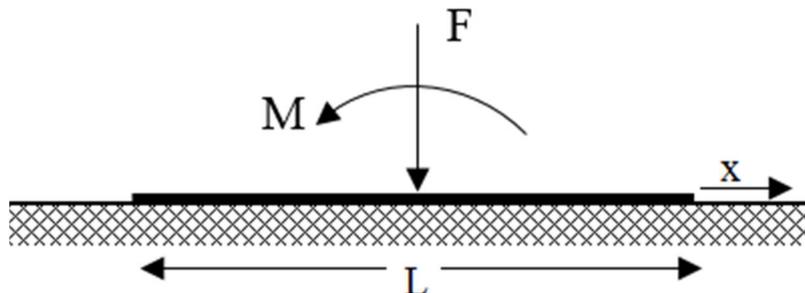


Fig. 1. Basic model of single footing under uniaxial moment, where  $M$ : Moment,  $F$ : Axial force,  $L$ : length of footing

If the footing is considered as rigid, the base stresses can be calculated by using basic stress formula which is described as follows:

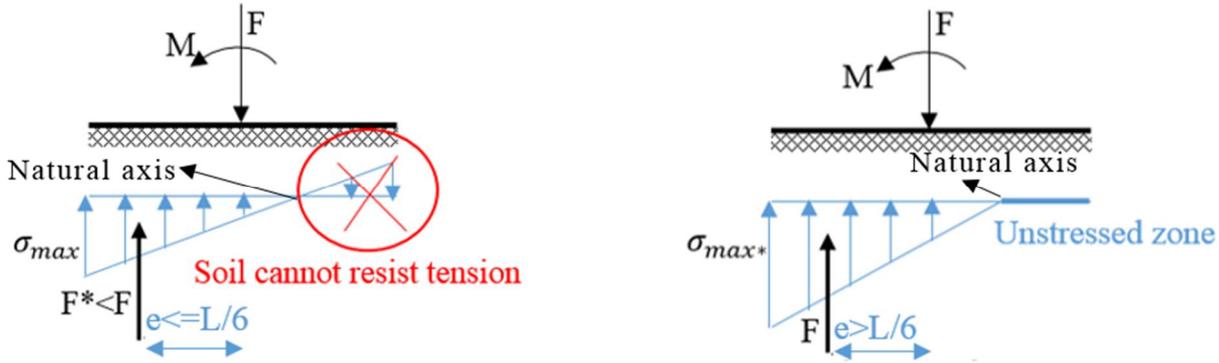
$$\sigma(x) = \frac{F}{B \cdot L} + \frac{12 \cdot M}{B \cdot L^3} x \quad (1)$$

where  $B$  is width of the footing and  $x$  is distance of the base stress to the natural axis. In the model, eccentricity of the footing ( $e$ ) is calculated by using following formula.

$$e = \frac{M}{F} \quad (2)$$

If an absolute value of the eccentricity is greater than  $L/6$ , large eccentricity occurred in the footing. In that case, negative (tension) stress values are computed from Eq. (1). Essentially, tension stresses are not occurred in the base of the footing as the soil cannot resist tension stresses. Therefore, zone where the tension stresses are computed, should be considered as unstressed zone and the base stresses in compression zone are required to be redistributed in such a way that the equilibrium of footing is maintained (see Fig.2). After the redistribution, the maximum compressive base stress is calculated as follows:

$$\sigma_{max} = \frac{4 \cdot F}{3 \cdot B(L - 2 \cdot e)} \quad (3)$$



a) Before redistribution of compressive stress b) After redistribution of compressive stress

Fig. 2. Base stress distribution of footing with large eccentricity

Although, 2-D modeling of the footings brings ease of calculation, the biaxial bending should be taken into account when moments, occurred in the both axis, have considerable value. Turkish Earthquake Code [14] also points out that the interaction of the two orthogonal excitation directions is required in the analysis of footings. Under biaxial bending, the base stresses depend on  $x$  and  $y$  distances of the footing to natural axis which is described as follows:

$$\sigma(x, y) = \frac{F}{B \cdot L} + \frac{12 \cdot M_y}{B \cdot L^3} x + \frac{12 \cdot M_x}{L \cdot B^3} y \quad (4)$$

where  $M_y$  and  $M_x$  are bending moments for Y and X directions respectively (see Fig. 3). The maximum base stress value of the footing with small eccentricity can be calculated using Eq. (4). However, the maximum base stress value cannot be obtained using Eq. (4) for the footing with large

eccentricity. In that case, the interaction of the bending stresses for X and Y directions and redistribution the base stresses are required (See Fig. 4).

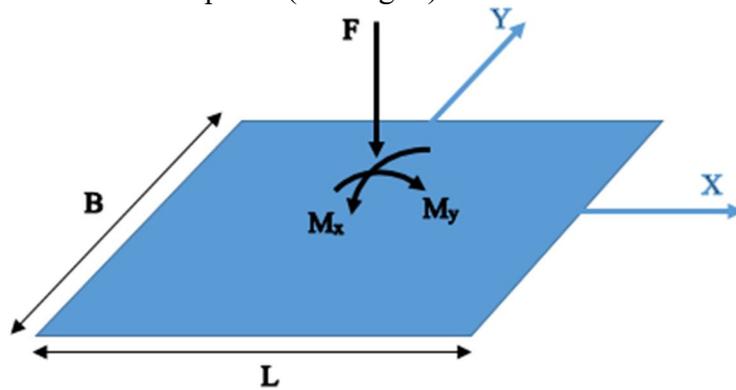


Fig. 3. Single footing under biaxial moment

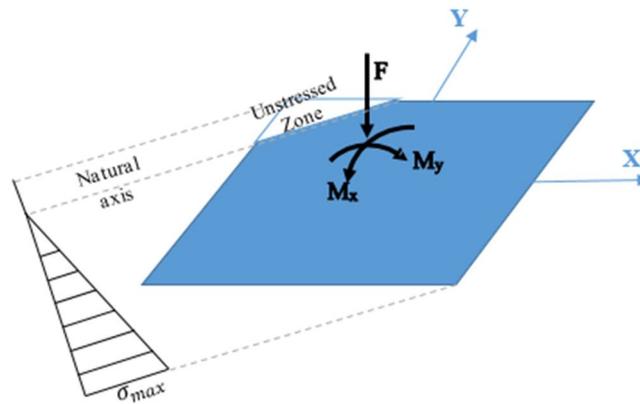


Fig. 4. Base stress distribution of single footing under biaxial moment

### 3. Mathematical modelling of the numerical approach

As mentioned before, calculation of the base stresses is not easy task for the footings with large eccentricity under biaxial bending. Many methods and approaches have been developed in order to calculate the base stresses. However, the approaches are complex and are not easy to convert computer program. The study presents alternative numerical approach to calculate the base stresses under biaxial bending. Theory of the approach is based on the redistribution of the base stresses in the compression zone. According to strength material theory, the base stresses change linearly in case deformation of the footing is neglected. Therefore, general stress formula, described in Eq. (4), can be updated by using constant coefficients after the redistribution of the base stresses. The updated stress formula is defined as follows:

$$\sigma^*(x, y) = a \frac{F}{B \cdot L} + b \frac{12 \cdot M_y}{B \cdot L^3} x + c \frac{12 \cdot M_x}{L \cdot B^3} y \quad x, y \in \sigma^*(x, y) > 0 \quad (5)$$

where  $a$ ,  $b$  and  $c$  are the constant unknown coefficients. In order to compute these coefficients, three equations are required which are described as follows:

$$\iint_{x, y \in \sigma^*(x, y) > 0} \sigma^*(x, y) dA = F \quad (6)$$

$$\iint_{x,y \in \sigma^*(x,y) > 0} \sigma^*(x,y) x dA = M_y \quad (7)$$

$$\iint_{x,y \in \sigma^*(x,y) > 0} \sigma^*(x,y) y dA = M_x \quad (8)$$

where  $x$  and  $y$  are distances of the base stresses to natural axis. The presented approach use simple iterative formula to find these constant coefficients which are described as follows:

$$a^{i+1} = a^i + \frac{(F - F^*)}{F} \quad (9)$$

$$b^{i+1} = b^i + \frac{(M_y - M_y^*)}{M_y} \quad (10)$$

$$c^{i+1} = c^i + \frac{(M_x - M_x^*)}{M_x} \quad (11)$$

where  $F^*$ ,  $M_y^*$  and  $M_x^*$  respectively are axial force, bending moment acting on y direction and bending moment acting on x direction which are calculated using the base stresses that occurs in compression zone. The iterative process is terminated by the time that following criteria is satisfied.

$$\left\| \frac{F^*}{F} - 1 \right\| + \left\| \frac{M_y^*}{M_y} - 1 \right\| + \left\| \frac{M_x^*}{M_x} - 1 \right\| \leq tol \quad (12)$$

where  $tol$  is a predetermined relative tolerance value. The approach can also be described as pseudo code algorithm as follows:

**Start Program**

Read  $B, L, F, M_y, M_x$  and  $tol$

$Rdiff=1$  (should be greater than  $tol$ )

$i=0; a^0=1; b^0=1; c^0=1$

**While**  $rdiff > tol$

    Calculate  $F^*, M_y^*$  and  $M_x^*$  using formulas (5)-(8)

$a^{i+1} = a^i + (F - F^*)/F$

$b^{i+1} = b^i + (M_y - M_y^*)/M_y$

$c^{i+1} = c^i + (M_x - M_x^*)/M_x$

$Rdiff = \|F^*/F - 1\| + \|M_y^*/M_y - 1\| + \|M_x^*/M_x - 1\|$

$i=i+1$

**End While**

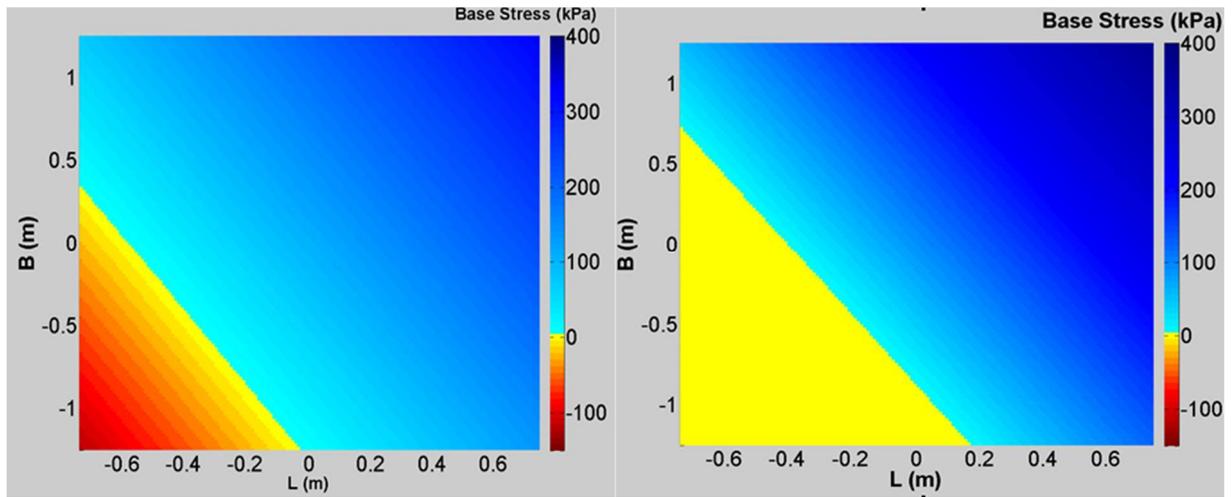
**End Program**

#### 4. Design Example

The presented approach is tested by the single footing which is previously used by Köseoğlu and Özmen [7, 9]. The footing has rectangular shape whose dimensions are:  $L=1.5$  m,  $B=2.5$  m. The

footing is exposed by the axial force and bending moments. Values of the force and moments are  $F=400$  kN,  $M_x=150$  kN.m and  $M_y= 120$  kN.m. The footing is analyzed under two cases. Case 1: without considering the redistribution of the base stresses in compression zone and Case 2: considering the redistribution of the base stresses in compression zone. The relative tolerance ( $tol$ ) value is taken as 0.001 in the algorithm. The algorithm finds updated stress formula in 14 iterations which is described in Eq. (13). Distribution of the base stresses for both cases are displayed as color map in Fig. 5. For the case 2, computed base stresses occurred at corners of the footing are illustrated in Table 2. These stresses are compared with the results of Özmen’s and Köseoğlu’s studies [7, 9]. Convergence history of the constant coefficients ( $a$ ,  $b$  and  $c$  coefficients) are shown in Table 1.

According to Table 2, the maximum difference of computed base stresses between Özmen’s study and the presented studies is only 0.1%. In addition, the maximum base stress obtained from the current study is exactly same as Köseoğlu’s result [7].



(a) Case 1: without considering distribution of the base stresses in compression zone (b) Case 2: considering distribution of the base stresses in compression zone

Fig. 5. Distribution of the base stresses

$$\sigma^*(x, y) = 0.865 \frac{F}{B \cdot L} + 1.236 \frac{12 \cdot M_y}{B \cdot L^3} x + 1.276 \frac{12 \cdot M_x}{L \cdot B^3} y \quad (13)$$

$$x, y \in \sigma^*(x, y) > 0$$

Table 1. Computed base stresses occurred at corners of the footing for the case 2

Corner Poi	Özmen[9]	Köseoğlu[7]	This Study
1	373.3	373.1	373.1
2	56.6	N.A.	56.6
3	0.0	N.A.	0.0
4	128.0	N.A.	128.0

The design example is also analyzed considering different eccentricities for the aforementioned cases in order to reveal the effect of redistribution of the compression stresses. The eccentricities ( $e_x$  and  $e_y$  values) of the footing vary from  $0.05 \cdot L$  to  $0.25 \cdot L$  (increment:  $0.05 \cdot L$ ) for the X axis, from  $0.05 \cdot B$  to  $0.25 \cdot B$  (increment:  $0.05 \cdot B$ ) for the Y axis. The computed maximum base stresses for each eccentricities are plotted to Fig. 6. Differences between the maximum stresses obtained from two

cases are illustrated in Table 3. According to the analysis results, the differences reach 33% which is considerably high.

Table 2. Results of constants coefficients

Iteration	<i>a</i>	<i>b</i>	<i>c</i>
1	0.949	1.099	1.119
2	0.918	1.151	1.179
3	0.899	1.182	1.215
4	0.887	1.201	1.237
5	0.88	1.214	1.251
6	0.875	1.221	1.26
7	0.871	1.227	1.265
8	0.869	1.23	1.269
9	0.868	1.232	1.272
10	0.867	1.234	1.273
11	0.866	1.235	1.275
12	0.866	1.236	1.275
13	0.866	1.236	1.276
14	0.865	1.236	1.276

Table 3. Differences between the maximum stresses obtained for the cases

$e_y/B$ \ $e_x/L$	0.05	0.1	0.15	0.2	0.25
0	0%	0%	0%	3%	3%
0	0%	2%	6%	11%	19%
0	2%	5%	8%	12%	18%
0	3%	6%	11%	18%	26%
0	9%	14%	19%	26%	33%

## 7. Conclusions

In the present study, the new iterative approach is introduced to compute the base stresses of the rectangular shape single footings with large eccentricity under biaxial bending. The computer program is developed based on the approach. The program is tested with the previously used numerical example in order to check accuracy of the approach. According to the test, the approach has given the same results as the literature results. Hence, the approach can be used for rectangular footings under biaxial with larger eccentricity.

In addition, the numerical example is analyzed considering different eccentricities to investigate efficiency of the redistribution of the base stresses on the maximum base stress value. The analysis reveals that considerable amount of the differences are observed when the redistribution of the base stresses is taken into account. Therefore, the approach should be used in the footings with large eccentricity.

In future research, the approach can be applied on raft foundation and the footing deformation can be taken into consideration.

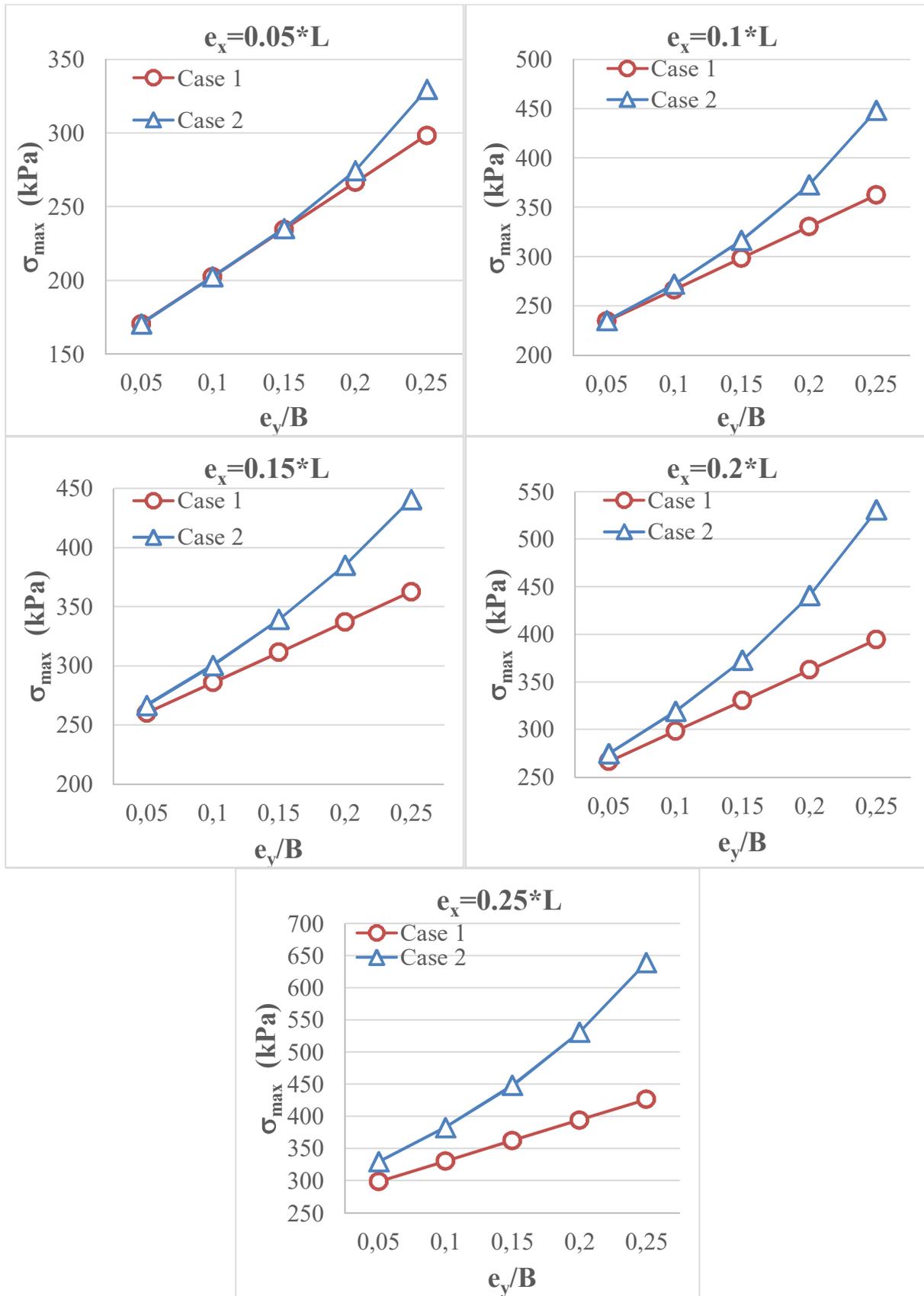


Fig. 6. Maximum base stresses for different eccentricities

## References

- [1] Prakash, S., Saran, S., Bearing capacity of eccentrically loaded footings. *Journal of Soil Mechanics & Foundations Div*, 1971.
- [2] Gesund, H., Flexural limit design of column footings. *Journal of Structural Engineering*, 111(11), 2273-2287, 1985.
- [3] Saran, S., Agarwal, R., Bearing capacity of eccentrically obliquely loaded footing. *Journal of Geotechnical Engineering*, 117(11), 1669-1690, 1991.
- [4] Mahiyar, H., Patel, A., Analysis of angle shaped footing under eccentric loading. *Journal of geotechnical and geoenvironmental engineering*, 126(12), 1151-1156, 2000.
- [5] Wieneke, K., Kueres, D., Siburg, C., Hegger, J., Investigations on the punching shear behaviour of eccentrically loaded footings. *Structural Concrete*, 2016.
- [6] Gurfinkel, G., Analysis of footings subjected to biaxial bending. *Journal of the Structural Division*, 96(6), 1049-1059, 1970.
- [7] Köseoğlu, S., *Temeller: statığı ve konstruksiyonu* 1986; Matbaa Teknisyenleri Basımevi, 1986.
- [8] Csenki, A. Foundation Pressures Beneath Rectengular Footings. *Proceedings of the Institution of Civil Engineers*, 83, 465-473, 1987.
- [9] Özmen, G., Determination of Base Stresses in Rectangular Footings under Biaxial Bending. *Teknik Dergi*, 22(110), 2011.
- [10] Rodriguez-Gutierrez, J., Aristizabal-Ochoa, J.D., Rigid spread footings resting on soil subjected to axial load and biaxial bending. I: Simplified analytical method. *International Journal of Geomechanics*, 13(2), 109-119, 2012.
- [11] Sarkar, B., Analysis of isolated footing subjected to axial load and high biaxial moments and numerical approach for its solution. *The Indian Concrete Journal*, 2014(6), 60-80, 2014.
- [12] Guan, Z., Zhang, J., Li, J., A robust computational method for ultimate strength analysis of arbitrary reinforced concrete and composite sections subjected to axial force and biaxial bending. *Advances in Structural Engineering*, 17(1), 83-96, 2014.
- [13] Smith-Pardo, J.P., Ortiz, A., Blandon, C.A., Biaxial capacity of rigid footings: Simple closed-form equations and experimental results. *Engineering Structures*, 69, 149-157, 2014.
- [14] Code, T.E., Turkish code for buildings in seismic zones, in Turkish Seismic Code, The Ministry of Public Works and Settlement. 2007: Ankara, Turkey.