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ON THE TOPOLOGICAL CATEGORY OF NEUTROSOPHIC CRISP SETS

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Abstract. In this work, we explicitly characterize local separation axioms as well as generic separation axioms in the topological category of neutrosophic crisp sets, and examine their mutual relationship. Moreover, we characterize several distinct notions of closedness, compactness and connectedness in **NCSet**, and study their relationship with each other.

1. Introduction

As a generalization of crisp sets, Zadeh [30] introduced fuzzy set theory in 1965. Without a doubt, the fuzzy set theory is effective in dealing with imprecise estimates, yet it was unable to explain the level of dissatisfaction (non-membership). The intuitionistic fuzzy set (IFS) model was established by Atanassov [1] to address these weaknesses of fuzzy sets. This model is more accurate and useful than fuzzy sets since it can manage both membership and nonmembership degrees. The IFSs offer more space in terms of applications for decision-making because they can handle data both in favor (membership value) and against (non-membership value) of the possibilities given.

The concept of a neutrosophic set taking into account the degrees of membership, non-membership, and indeterminacy was first suggested by Smarandache [29] in 1998. Additionally, Salama and Smarandache [28] introduced the idea of a neutrosophic crisp set in a set in 2015. They also provided definitions of neutrosophic crisp empty (resp. whole) set as more than two types, inclusion between two neutrosophic crisp sets, complement of a neutrosophic crisp set and intersection (union) of two neutrosophic crisp sets. In 2017, Hur et al [18] defined several categorical

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properties of neutrosophic crisp set and showed that **NCSet** (the category of neutrosophic crisp spaces and neutrosophic crisp maps) is a cartesian closed topological category.

Categorical topology is that field of mathematics where general topology and category theory overlap, was introduced by Herrlich [17] in 1971, and the purpose was to apply categorical concepts and results to topological settings and to explain not only the original topological phenomena but similar phenomena throughout topology as well as in other fields.

Due to huge importance of neutrosophic crisp sets in decision-making, it motivates us to characterize several fundamental concepts of topology including Hausdorffness, closedness, compactness and connectedness in the topological category of **NCSet**.

The following are the paper's main goals:

(i) to characterize local T_0, T_1, $PreT_2$ objects in the category of neutrosophic crisp sets and to examine how they are related,
(ii) to provide the characterization of generic separation axioms and several distinct version of Hausdorff objects in **NCSet**,
(iii) to give the explicit characterization of several notions of closedness, compactness and connectness in topological category of **NCSet**,
(iv) to compare our results with the ones in some other categories.

2. Preliminaries

All preliminary information and more about neutrosophic crisp spaces can be found in [28].

Definition 1. *[18, 28] Let A be a non-empty set.*

(1) *If $\mathcal{N}$ has the form $\mathcal{N} = (N_1, N_2, N_3)$, where N_1, N_2, and N_3 are subsets of A, then $\mathcal{N}$ is referred to as a neutrosophic crisp set (NCS) on A. The pair $(A, \mathcal{N})$ is called a neutrosophic crisp space (NCSp). The set of all NCSs on A will be represented by $NCS(A)$.*
(2) *The neutrosophic crisp empty set, $\emptyset_{nc}$ is an NCS on A defined by $\emptyset_{nc} = (\emptyset, \emptyset, A)$.*
(3) *The neutrosophic whole set, A_{nc} is an NCS on A defined by $A_{nc} = (A, A, \emptyset)$.*
(4) *Let $\{\mathcal{N}_i\}_{i\in I}$ be a family of NCSs on A, where $\mathcal{N}_i = (N_{i1}, N_{i2}, N_{i3})$. Then*
   (i) *$\bigcap_{i\in I} \mathcal{N}_i$, the intersection of $\{\mathcal{N}_i\}_{i\in I}$, is an NCS on A defined by*
   $$\bigcap \mathcal{N}_i = (\bigcap N_{i1}, \bigcap N_{i2}, \bigcup N_{i3}),$$
   (ii) *$\bigcup_{i\in I} \mathcal{N}_i$, the union of $\{\mathcal{N}_i\}_{i\in I}$, is an NCS on A defined by*
   $$\bigcup \mathcal{N}_i = (\bigcup N_{i1}, \bigcup N_{i2}, \bigcap N_{i3}).$$

Definition 2. *[18] Let $(A, \mathcal{N})$, $(B, \mathcal{M})$ be NCSps and $f\colon A \to B$ be a map. Then $f\colon (A, \mathcal{N}) \to (B, \mathcal{M})$ is called a morphism, if $\mathcal{N} \subset f^{-1}(\mathcal{M})$, equivalently, $N_1 \subset$*

$f^{-1}(M_1)$, $N_2 \subset f^{-1}(M_2)$ and $N_3 \supset f^{-1}(M_3)$, where $\mathcal{N} = (N_1, N_2, N_3)$ and $\mathcal{M} = (M_1, M_2, M_3)$.

Definition 3. *The category of neutrosophic crisp spaces,* **NCSet** *has the pairs $(A, \mathcal{N})$ as objects, where A is any non-empty set and $\mathcal{N}$ is a neutrosophic crisp set on A, and has morphisms. In this case, every morphism in* **NCSet** *is called a* **NCSet***-map.*

Lemma 1. *(cf. [18])*

(1) *Let A be a set, $\{(A_j, \mathcal{N}_j)\}_{j\in J}$ be any families of NCSps and $\{f_j : (A, \mathcal{N}_A) \to (A_j, \mathcal{N}_j)\}_{j\in J}$ be a source. Then,*

$$\mathcal{N}_A = \bigcap_{j\in J} f_j^{-1}(\mathcal{N}_j)$$

is an initial structure on A, where $\mathcal{N}_A = (N_{A1}, N_{A2}, N_{A3})$ and $\mathcal{N}_j = (N_{j1}, N_{j2}, N_{j3})$.

(2) *Let B be a set, $\{(A_j, \mathcal{N}_j)\}_{j\in J}$ be any families of NCSps and $\{g_j : (A_j, \mathcal{N}_j) \to (B, \mathcal{N}_B)\}_{j\in J}$ be a sink. Then,*

$$\mathcal{N}_B = \bigcup_{j\in J} g_j(\mathcal{N}_j)$$

is a final structure on B, where $\mathcal{N}_B = (N_{B1}, N_{B2}, N_{B3})$ and $\mathcal{N}_j = (N_{j1}, N_{j2}, N_{j3})$.

(3) *Let $(A, \mathcal{N})$ be a neutrosophic crisp space (NCSp).*
   (i) *A neutrosophic crisp structure on A is discrete whenever $\mathcal{N} = \emptyset_{nc}$.*
   (ii) *A neutrosophic crisp structure on A is indiscrete whenever $\mathcal{N} = A_{nc}$.*

Remark 1. *The forgetful functor $\mathcal{U}$:* **NCSet** $\to$ **Set** *is topological, i.e., the category* **NCSet** *is topological over* **Set** *[18], but the functor $\mathcal{U}$ is not normalized (i.e., subterminals, have a unique structure) since a singleton set $\{a\}$ has multiple neutrosophic crisp structures on it.*

3. Local Separation Axioms in Neutrosophic Crisp Sets

Let p be a point in a set B and $B \vee_p B$ be the *wedge product* of B at p ([2], p. 334), i.e., two disjoint copies of B identified at p. If a point b in $B \vee_p B$ is in the first component, it is denoted as b_1, and if it is in the second component, it is denoted as b_2.

Definition 4. *[2] Let B^2 denote the cartesian product of B.*

(1) *The map $\mathcal{A}_p : B \vee_p B \to B^2$ is called principal p-axis map iff*

$$\mathcal{A}_p(b_i) = \begin{cases} (b, p), & i = 1 \\ (p, b), & i = 2 \end{cases}$$

(2) *The map $\mathcal{S}_p : B \vee_p B \to B^2$ is called skewed p-axis map iff*

$$\mathcal{S}_p(b_i) = \begin{cases} (b, b), & i = 1 \\ (p, b), & i = 2 \end{cases}$$

(3) *The map $\nabla_p : B \vee_p B \to B$ is called fold map at p provided that $\nabla_p(b_i) = b$ for $i = 1, 2$.*

Definition 5. *[2] Let $U : \mathcal{E} \to$ **Set** be topological, $A \in Ob(\mathcal{E})$ with $U(A) = B$ and $p \in B$.*

(i) *A is $\overline{T_0}$ at p iff the initial lift of the U-source $\{\mathcal{A}_p : B \vee_p B \to U(A^2) = B^2$ and $\nabla_p : B \vee_p B \to UD(B) = B\}$ is discrete, where D is the discrete functor.*

(ii) *A is T_0' at p iff the initial lift of the $\mathcal{U}$-source $\{id : B \vee_p B \to \mathcal{U}(A \vee_p A) = B \vee_p B$ and $\nabla_p : B \vee_p B \to \mathcal{UD}(B) = B\}$ is discrete, where $A \vee_p A$ is the wedge in $\mathcal{E}$, i.e., the final lift of the $\mathcal{U}$-sink $\{i_1, i_2 : \mathcal{U}(A) = B \to B \vee_p B\}$ where i_1, i_2 denote the canonical injections.*

(iii) *A is T_1 at p iff the initial lift of the U-source $\{\mathcal{S}_p : B \vee_p B \to U(A^2) = B^2$ and $\nabla_p : B \vee_p B \to UD(B) = B\}$ is discrete.*

(iv) *A is $Pre\overline{T}_2$ at p iff the initial lift of the $\mathcal{U}$-source $\{\mathcal{A}_p : B \vee_p B \to \mathcal{U}(A^2) = B^2\}$ and the initial lift of the $\mathcal{U}$-source $\{\mathcal{S}_p : B \vee_p B \to \mathcal{U}(A^2) = B^2\}$ agree.*

(v) *A is $PreT_2'$ at p iff the initial lift of the $\mathcal{U}$-source $\{\mathcal{S}_p : B \vee_p B \to \mathcal{U}(A^2) = B^2\}$ and the final lift of the $\mathcal{U}$-sink $\{i_1, i_2 : \mathcal{U}(A) = B \to B \vee_p B\}$ agree.*

(vi) *A is $\overline{T}_2$ at p iff A is $\overline{T}_0$ at p and $Pre\overline{T}_2$ at p.*

(vii) *A is T_2' at p iff A is T_0' at p and $PreT_2'$ at p.*

Remark 2. (1) *Particularly, we have the following for the category of topological spaces, **Top**:*

(a) *$\overline{T}_0$ at p and T_0' at p (resp. T_1 at p) reduce to for each $x \in X$ with $x \neq p$, there exists a neighborhood of x that doesn't contain p or (resp. and) there exists a neighborhood of p that doesn't contain x [5].*

(b) *$Pre\overline{T}_2$ at p and $PreT_2'$ at p are equivalent, and they both reduce to for each point x distinct from p, there exist disjoint neighborhoods of x and p if the set $\{x, p\}$ is not indiscrete [5].*

(c) *$\overline{T}_2$ at p and T_2' at p are equivalent, and they both reduce to for each $x \in X$ with $x \neq p$, there exist disjoint neighborhoods of x and p [5].*

(2) *Local separation axioms are used to introduce the notions of (strong) closedness in set-based topological categories which are defined in [3]. These notions are used in [2,9,10] to generalize each of the notions of Hausdorffness, compactness, perfectness and connectedness to arbitrary set-based topological categories. Additionally, it is shown in [9] that they constitute suitable closure operators in the sense of Dikranjan and Giuli [16] in various well-known topological categories.*

Theorem 1. *Let $(A,\mathcal{N})$, $(B,\mathcal{M})$ be NCSps and $f:(A,\mathcal{N})\to(B,\mathcal{M})$ be a* **NCSet***-map. If $(B,\mathcal{M})$ is discrete, then so is $(A,\mathcal{N})$, i.e., f reflects discreteness.*

Proof. Let $(B,\mathcal{M})$ be discrete, i.e., $\mathcal{M}=\emptyset_{nc}$, but $(A,\mathcal{N})$ be not discrete, i.e., $\mathcal{N}\neq\emptyset_{nc}$. Since $f:(A,\mathcal{N})\to(B,\mathcal{M})$ is in **NCSet**, it follows that $\mathcal{N}\subset f^{-1}(\mathcal{M}=\emptyset_{nc})=\emptyset_{nc}$ and consequently $\mathcal{N}=\emptyset_{nc}$, a contradiction. □

Theorem 2. *All objects in* **NCSet** *are $\overline{T_0}$ at p, T_0' at p, and T_1 at p.*

Proof. It is deduced from Definition 5 and Theorem 1. □

Theorem 3. *Let $(A,\mathcal{N})$ be a neutrosophic crisp space and $p\in A$. The following are equivalent.*

(1) *$(A,\mathcal{N})$ is $PreT_2'$ at p.*
(2) *$(A,\mathcal{N})$ is $Pre\overline{T}_2$ at p.*
(3) *$(A,\mathcal{N})$ is $\overline{T_2}$ at p.*
(4) *$(A,\mathcal{N})$ is T_2' at p.*
(5) *$\mathcal{N}=\emptyset_{nc}$ or $p\in\mathcal{N}$.*

Proof. (1) $\Longrightarrow$ (2) : By Theorem 3.1 of [8] we get the result.

(2) $\Longrightarrow$ (3) : It follows from Definition 5 and Theorem 2.

(3) $\Longrightarrow$ (4) : Suppose $(A,\mathcal{N})$ is $\overline{T_2}$ at p. Then by Definition 5, Lemma 1 and Theorem 2, $(\pi_1\mathcal{A}_p)^{-1}\mathcal{N}\cap(\pi_2\mathcal{A}_p)^{-1}\mathcal{N}=(\pi_1\mathcal{S}_p)^{-1}\mathcal{N}\cap(\pi_2\mathcal{S}_p)^{-1}\mathcal{N}$. It follows that $\mathcal{N}=\emptyset_{nc}$ or $p\in\mathcal{N}$. Otherwise the equality does not hold. Because, if $\mathcal{N}\neq\emptyset_{nc}$ and $p\notin\mathcal{N}$, then $(\pi_1\mathcal{A}_p)^{-1}\mathcal{N}\cap(\pi_2\mathcal{A}_p)^{-1}\mathcal{N}=\emptyset_{nc}$ and $(\pi_1\mathcal{S}_p)^{-1}\mathcal{N}\cap(\pi_2\mathcal{S}_p)^{-1}\mathcal{N}=\mathcal{N}\times p\subset A\vee_p A$ by definitions of principal and skewed $p-$axis maps. This is a contradiction.

If $\mathcal{N}=\emptyset_{nc}$, then clearly $(\pi_1\mathcal{S}_p)^{-1}\mathcal{N}\cap(\pi_2\mathcal{S}_p)^{-1}\mathcal{N}=i_1\mathcal{N}\cup i_2\mathcal{N}=\emptyset_{nc}$.

If $p\in\mathcal{N}$, then $(\pi_1\mathcal{S}_p)^{-1}\mathcal{N}\cap(\pi_2\mathcal{S}_p)^{-1}\mathcal{N}=i_1\mathcal{N}\cup i_2\mathcal{N}=\mathcal{N}\vee_p\mathcal{N}$. Hence $(A,\mathcal{N})$ is T_2' at p by Definition 5, Lemma 1 and Theorem 2.

(4) $\Longrightarrow$ (5) : Suppose $(A,\mathcal{N})$ is T_2' at p. Then by Definition 5, Lemma 1 and Theorem 2, $(\pi_1\mathcal{S}_p)^{-1}\mathcal{N}\cap(\pi_2\mathcal{S}_p)^{-1}\mathcal{N}=i_1\mathcal{N}\cup i_2\mathcal{N}$. We must show that $p\in\mathcal{N}$ if $\mathcal{N}\neq\emptyset_{nc}$. Let $\mathcal{N}\neq\emptyset_{nc}$ and $p\notin\mathcal{N}$, then $(\pi_1\mathcal{S}_p)^{-1}\mathcal{N}\cap(\pi_2\mathcal{S}_p)^{-1}\mathcal{N}=(\mathcal{N}\times p)\cap(\mathcal{N}\vee_p\mathcal{N})=\mathcal{N}\times p$ and $i_1\mathcal{N}\cup i_2\mathcal{N}=\mathcal{N}\vee_p\mathcal{N}$ by definitions of skewed $p-$axis map and canonical injections. It follows that $(\pi_1\mathcal{S}_p)^{-1}\mathcal{N}\cap(\pi_2\mathcal{S}_p)^{-1}\mathcal{N}\neq i_1\mathcal{N}\cup i_2\mathcal{N}$ since if $x\in\mathcal{N}$, then $i_2x=(p,x)\in\mathcal{N}\vee_p\mathcal{N}$ but $(p,x)\notin\mathcal{N}\times p$. Consequently, this is a contradiction. Thus $p\in\mathcal{N}$ if $\mathcal{N}\neq\emptyset_{nc}$.

(5) $\Longrightarrow$ (1) : Assume that $\mathcal{N}=\emptyset_{nc}$ or $p\in\mathcal{N}$. If $\mathcal{N}=\emptyset_{nc}$, then clearly $(\pi_1\mathcal{S}_p)^{-1}\mathcal{N}\cap(\pi_2\mathcal{S}_p)^{-1}\mathcal{N}=i_1\mathcal{N}\cup i_2\mathcal{N}=\emptyset_{nc}$. If $\mathcal{N}\neq\emptyset_{nc}$, then $p\in\mathcal{N}$ by assumption. It follows that $(\pi_1\mathcal{S}_p)^{-1}\mathcal{N}\cap(\pi_2\mathcal{S}_p)^{-1}\mathcal{N}=(\mathcal{N}\vee_p\mathcal{N})\cap(\mathcal{N}\vee_p\mathcal{N})=\mathcal{N}\vee_p\mathcal{N}$, $i_1\mathcal{N}\cup i_2\mathcal{N}=\mathcal{N}\vee_p\mathcal{N}$, and consequently, $(\pi_1\mathcal{S}_p)^{-1}\mathcal{N}\cap(\pi_2\mathcal{S}_p)^{-1}\mathcal{N}=i_1\mathcal{N}\cup i_2\mathcal{N}$. Hence, $(A,\mathcal{N})$ is $PreT_2'$ at p by Definition 5. □

4. Generic Separation Axioms in Neutrosophic Crisp Space

Let B be a non-empty set, B^2 be cartesian product of B with itself and $B^2 \vee_\Delta B^2$ be two distinct copies of B^2 identified along the diagonal. If a point (a,b) in $B^2 \vee_\Delta B^2$ is in the first (resp. second) component, it is denoted as $(a,b)_1$ $(resp.(a,b)_2)$ Clearly, $(a,b)_1 = (a,b)_2$ iff $a = b$ [2].

Definition 6. *[2]*

(1) *The map $\mathcal{A} : B^2 \vee_\Delta B^2 \to B^3$ is called principal axis map iff*

$$\mathcal{A}(a,b)_i = \begin{cases} (a,b,a), & i = 1 \\ (a,a,b), & i = 2 \end{cases}$$

(2) *The map $\mathcal{S} : B^2 \vee_\Delta B^2 \to B^3$ is called skewed axis map iff*

$$\mathcal{S}(a,b)_i = \begin{cases} (a,b,b), & i = 1 \\ (a,a,b), & i = 2 \end{cases}$$

(3) *The map $\nabla : B^2 \vee_\Delta B^2 \to B^2$ is called fold map iff $\nabla(a,b)_i = (a,b)$ for $i = 1,2$.*

Definition 7. *(cf. [2, 6]) Let $\mathcal{U} : \mathcal{E} \to \textbf{Set}$ be a topological functor, A an object in $\mathcal{E}$ with $\mathcal{U}(A) = B$.*

(1) *A is $\overline{T}_0$ iff the initial lift of the $\mathcal{U}-$source $\{\mathcal{A} : B^2 \vee_\Delta B^2 \to \mathcal{U}(A^3) = B^3$ and $\nabla : B^2 \vee_\Delta B^2 \to \mathcal{U}\mathcal{D}(B^2) = B^2\}$ is discrete, where $\mathcal{D}$ is the discrete functor that is a left adjoint to $\mathcal{U}$ [2].*
(2) *A is T_0' iff the initial lift of the $\mathcal{U}-$source $\{id : B^2 \vee_\Delta B^2 \to \mathcal{U}(B^2 \vee_\Delta B^2)' = B^2 \vee_\Delta B^2$ and $\nabla : B^2 \vee_\Delta B^2 \to \mathcal{U}\mathcal{D}(B^2) = B^2\}$ is discrete, where $(B^2 \vee_\Delta B^2)'$ is the final lift of the $\mathcal{U}-$sink $\{i_1, i_2 : \mathcal{U}(A^2) = B^2 \to B^2 \vee_\Delta B^2\}$, i_1 and i_2 are the canonical injections, and $\mathcal{D}(B^2)$ is the discrete structure on B^2 [2].*
(3) *A is T_0 iff A doesn't contain an indiscrete subspace with at least two points [23].*
(4) *A is T_1 iff the initial lift of the $\mathcal{U}-$source $\{\mathcal{S} : B^2 \vee_\Delta B^2 \to \mathcal{U}(A^3) = B^3$ and $\nabla : B^2 \vee_\Delta B^2 \to \mathcal{U}\mathcal{D}(B^2) = B^2\}$ is discrete [2].*
(5) *A is $Pre\overline{T}_2$ iff the initial lift of the $\mathcal{U}$-sources $\{\mathcal{A} : B^2 \vee_\Delta B^2 \to \mathcal{U}(A^3) = B^3\}$ and $\{\mathcal{S} : B^2 \vee_\Delta B^2 \to \mathcal{U}(A^3) = B^3\}$ agree.*
(6) *A is $PreT_2'$ iff the initial lift of the $\mathcal{U}$-source $\{\mathcal{S} : B^2 \vee_\Delta B^2 \to \mathcal{U}(A^3) = B^3\}$ and the final lift of the $\mathcal{U}$-sink $\{i_1, i_2 : \mathcal{U}(A^2) = B^2 \to B^2 \vee_\Delta B^2\}$ agree.*
(7) *A is $\overline{T}_2$ iff A is $Pre\overline{T}_2$ and $\overline{T}_0$.*
(8) *A is T_2' iff A is $PreT_2'$ and T_0'.*
(9) *A is KT_2 iff A is $Pre\overline{T}_2$ and T_0'.*
(10) *A is LT_2 iff A is $PreT_2'$ and $\overline{T}_0$.*
(11) *A is MT_2 iff A is $PreT_2'$ and T_0.*
(12) *A is NT_2 iff A is $Pre\overline{T}_2$ and T_0.*

Remark 3. *Note that for **Top**, all of T_0's or T_1 or $Pre\overline{T}_2$, $PreT_2'$ or all of T_2's reduce to usual T_0 or T_1 or $PreT_2$ (for each distinct pair x, y, there exist disjoint neighborhoods of x and y if the set $\{x, y\}$ is not indiscrete) or Hausdorff separation axioms, respectively [2, 23].*

Theorem 4. *Let $(A, \mathcal{N})$ be an object in* **NCSet**.

(1) $(A, \mathcal{N})$ *is* $\overline{T_0}$.
(2) $(A, \mathcal{N})$ *is* T_0'.
(3) $(A, \mathcal{N})$ *is* T_1.
(4) $(A, \mathcal{N})$ *is* $Pre\overline{T}_2$.
(5) $(A, \mathcal{N})$ *is* $PreT_2'$.
(6) $(A, \mathcal{N})$ *is* $\overline{T_2}$.
(7) $(A, \mathcal{N})$ *is* T_2'.
(8) $(A, \mathcal{N})$ *is* KT_2.
(9) $(A, \mathcal{N})$ *is* LT_2.

Proof. For $(1) - (3)$, the proofs are deduced from Definition 7 and Theorem 1.

Let $(A, \mathcal{N})$ be a neutrosophic crisp space and $(A^2, \mathcal{N}^2)$ be the product neutrosophic crisp space. Note that the product neutrosophic crisp structure $\mathcal{N}^2$ is given by $\mathcal{N}^2 = \pi_1^{-1}\mathcal{N} \cap \pi_2^{-1}\mathcal{N}$.

Let $\mathcal{M} = (\pi_1\mathcal{A})^{-1}\mathcal{N} \cap (\pi_2\mathcal{A})^{-1}\mathcal{N} \cap (\pi_3\mathcal{A})^{-1}\mathcal{N}$, $\mathcal{M}' = (\pi_1\mathcal{S})^{-1}\mathcal{N} \cap (\pi_2\mathcal{S})^{-1}\mathcal{N} \cap (\pi_3\mathcal{S})^{-1}\mathcal{N}$, $\mathcal{M}'' = i_1\mathcal{N}^2 \cup i_2\mathcal{N}^2$ and it follows that $\mathcal{M} = \mathcal{M}' = \mathcal{M}'' = \mathcal{N}^2 \vee_\Delta \mathcal{N}^2$. Then by Definition 7 and Lemma 1, $(A, \mathcal{N})$ is $Pre\overline{T}_2$ since $\mathcal{M} = \mathcal{M}'$, and by Definition 7 and Lemma 1, $(A, \mathcal{N})$ is $PreT_2'$ since $\mathcal{M}' = \mathcal{M}''$, and consequently, $(A, \mathcal{N})$ is $\overline{T_2}$, T_2', KT_2 and LT_2 by Definition 7. □

Theorem 5. $(A, \mathcal{N})$ *in* **NCSet** *is T_0 if and only if $cardA \leq 1$.*

Proof. Assume that $(A, \mathcal{N})$ is a T_0 neutrosophic crisp space and $cardA > 1$, i.e., A is not a one-point set. Then there exist distinct points a and b of A. It follows that $(\{a, b\}, \{a, b\}_{nc})$ is the indiscrete subspace of $(A, \mathcal{N})$ contradicting to $(A, \mathcal{N})$ is being T_0. Hence, $cardA \leq 1$.

If $cardA \leq 1$, i.e., $A = \emptyset$ or A is a one-point set, then clearly by Definition 7, $(A, \mathcal{N})$ is a T_0. □

Theorem 6. $(A, \mathcal{N})$ *in* **NCSet** *is MT_2 (resp. NT_2) if and only if $cardA \leq 1$.*

Proof. It is deduced from Definition 7 and Theorems 4, 5. □

5. Closedness, Compactness and Connectedness in NCSet

Let p be a point in a set B and $\vee_p^\infty B$ be the *infinite wedge product* of B at p, that is formed by taking countably separate copies of B and identifying them at p. If a point b in $\vee_p^\infty B$ is in the i-th component, it is denoted as b_i.

Definition 8. *[3] Let $\vee_p^\infty B$ be the infinite wedge product at p and $B^\infty = B \times B \times ...$ be the countable cartesian product of B with itself.*

(i) *The map* $\mathcal{A}_p^\infty : \vee_p^\infty B \to B^\infty$ *is called infinite principle axis map at* p *provided that* $\mathcal{A}_p^\infty(b_i) = (p, p, ..., p, b, p, ...)$.

(ii) *The map* $\nabla_p^\infty : \vee_p^\infty B \to B^\infty$ *is called infinite fold map at* p *provided that* $\nabla_p^\infty(b_i) = b$ *for all* $i \in I$.

Definition 9. *[3] Let* $\mathcal{U} : \mathcal{E} \to$ **Set** *be a topological functor,* $A \in Ob(\mathcal{E})$ *with* $\mathcal{U}(A) = B$ *and* $p \in B$*. Let* C *be a subset of* B*. We denote* A/C *as the final lift of the epi* $\mathcal{U}$*-sink* $q : \mathcal{U}(A) = B \to B/C = (B\backslash C) \cup \{*\}$*, where* q *is the epi map that is the identity on* $B\backslash C$ *and identifying* C *with a point* $\{*\}$*.*

(i) $\{p\}$ *is closed provided that the initial lift of the* $\mathcal{U}$*-source* $\{A_p^\infty : \vee_p^\infty B \to \mathcal{U}(A^\infty) = B^\infty$ *and* $\nabla_p^\infty : \vee_p^\infty B \to \mathcal{U}\mathcal{D}(B^\infty) = B^\infty\}$ *is discrete, where* $\mathcal{D}$ *is the discrete functor.*

(ii) $C \subset A$ *is closed provided that* $\{*\}$*, the image of* C*, is closed in* A/C *or* $C = \emptyset$*.*

(iii) $C \subset A$ *is strongly closed provided that* A/C *is* T_1 *at* $\{*\}$ *or* $C = \emptyset$*.*

(iv) $C \subset A$ *is (strongly) open provided that* C^c*, the complement of* C*, is (strongly) closed in* A*.*

Remark 4. *In* ***Top****,* C *is strongly closed provided that* C *is closed and there exists a neighbourhood of* C *missing* x *for each* $x \notin C$*, and the notion of closedness coincides with the usual one. Moreover, the notions of strong closedness and closedness coincide for* T_1 *topological spaces [3].*

Theorem 7. *Every point is closed in* A *for* $(A, \mathcal{N})$ *in* **NCSet**.

Proof. It is deduced from Definition 9 and Theorem 1. □

Theorem 8. *Let* $(A, \mathcal{N})$ *be in* **NCSet***. Each* $C \subset A$ *is both strongly closed and closed, so it is strongly open and open.*

Proof. It is deduced from Definition 9 and Theorem 1. □

Definition 10. *[7] Let* $\mathcal{E}$ *be a topological category over* **Set**, $A, B \in Ob(\mathcal{E})$*, and* $f : A \to B$ *a morphism.*

(1) f *is (strongly) closed provided that the image of each (strongly) closed subobject of* A *is a (strongly) closed subobject of* B*.*

(2) A *is (strongly) compact provided that for each* $B \in Ob(\mathcal{E})$*, the projection* $\pi_2 : A \times B \to B$ *is (strongly) closed.*

Remark 5. (1) *In* ***Top****, the notions of compactness and closed morphism reduce to the usual ones ([15] p. 97 and 103).*

(2) *The notions of compactness and strong compactness are different for an arbitrary topological category, in general, since the notions of strong closedness and closedness are different, in general ([3] p. 393).*

Theorem 9. *Every neutrosophic crisp space is (strongly) compact.*

Proof. Let $(A, \mathcal{N})$ be a neutrosophic crisp space. By Definition 10, we need to show that $\pi_2 : (A, \mathcal{N}) \times (B, \mathcal{M}) \to (B, \mathcal{M})$ is (strongly) closed for all $(B, \mathcal{M})$ in **NCSet**. Suppose $C \subset A \times B$ is (strongly) closed. By Theorem 8, it follows that $\pi_2(C)$ is (strongly) closed and consequently, $(A, \mathcal{N})$ is (strongly) compact. □

Corollary 1. *Let $(A, \mathcal{N})$ and $(B, \mathcal{M})$ be in* **NCSet** *and $f : (A, \mathcal{N}) \to (B, \mathcal{M})$ be an* **NCSet***-map.*

(1) *Each* **NCSet***-map f is (strongly) closed.*
(2) *If $(A, \mathcal{N})$ is (strongly) compact, then $(f(A), \mathcal{M})$ is (strongly) compact.*

Now, we give the characterizations of the various notions of connected objects in **NCSet**.

Definition 11. *Let $\mathcal{E}$ be a topological category over* **Set** *and $A \in Ob(\mathcal{E})$.*

(i) *A is strongly connected (connected) provided that the only subsets of A both open (strongly open) and closed (strongly closed) are A and $\emptyset$ [10].*
(ii) *A is D-connected provided that any morphism from A to any discrete object is constant [10, 26].*
(iii) *A is (strongly) hereditarily disconnected provided that the only (strongly) connected subspaces of A are singletons and $\emptyset$ [11].*
(iv) *A is said to be (strongly) irreducible if X, Y are (strongly) closed subobjects of A and $A = X \cup Y$, then $X = A$ or $Y = A$ [13].*

Remark 6. *In* **Top***,*

(1) *The notions of D-connectedness and strong connectedness coincide with the usual notion of connectedness. Moreover, if a topological space X is T_1, then the notions of D-connectedness, connectedness and strong connectedness coincide [10].*
(2) *The notion of irreducibility coincides with the usual irreducibility [13]. Note that if a topological space (X, τ) is irreducible, then (X, τ) is connected, and if (X, τ) is T_1, then the notions of of irreducible spaces and strongly irreducible spaces coincide. [13].*

Theorem 10. *Let $(A, \mathcal{N})$ be a neutrosophic crisp space. Then the following are equivalent.*

(1) *$(A, \mathcal{N})$ is (strongly) connected.*
(2) *$(A, \mathcal{N})$ is (strongly) irreducible.*
(3) *$cardA \leq 1$.*

Proof. (1) $\implies$ (2) : Let $(A, \mathcal{N})$ is strongly connected (resp. connected). Then the only subsets of A both open (strongly open) and closed (strongly closed) are A and $\emptyset$. Suppose $(A, \mathcal{N})$ is not (strongly) irreducible. Let B be a subset of A. By Theorem 8, B and B^c are closed (strongly closed). Since $A = B \cup B^c$ and $(A, \mathcal{N})$ is not (strongly) irreducible, then $B \neq A$ and $B^c \neq A$. It follows that $\emptyset \neq B \subset A$ is a both open (strongly open) and closed (strongly closed). Given that $(A, \mathcal{N})$

is strongly connected (resp. connected), this is a contradiction. Hence, $(A, \mathcal{N})$ is (strongly) irreducible.

(2) $\implies$ (3) : Suppose $(A, \mathcal{N})$ is (strongly) irreducible and $cardA > 1$. Then there exist distinct points a and b of A. By Theorem 8, both $\{a\}$ and $\{a\}^c$ are (strongly) closed subsets of A and $A = \{a\} \cup \{a\}^c$ contradicting to $(A, \mathcal{N})$ is being (strongly) irreducible. Hence, $cardA \leq 1$.

(3) $\implies$ (1) : Suppose $cardA \leq 1$. We show that $(A, \mathcal{N})$ is strongly connected (resp. connected). Since $cardA \leq 1$, $A = \emptyset$ or $A = \{a\}$ (one-point set). If $A = \{a\}$, then A and $A^c = \emptyset$ is closed (strongly closed). It follows that $A = \{a\}$ is both closed (strongly closed) and open (strongly open). Similarly, we have $A = \emptyset$ is both closed (strongly closed) and open (strongly open). Hence, $(A, \mathcal{N})$ is strongly connected (resp. connected). □

Theorem 11. *All objects in* **NCSet** *is (strongly) hereditarily disconnected.*

Proof. It is deduced from Definition 11 and Theorem 10. □

Theorem 12. $(A, \mathcal{N})$ *in* **NCSet** *is D-connected provided that $cardA \leq 1$ and $\mathcal{N} = \emptyset_{nc}$.*

Proof. Suppose $(A, \mathcal{N})$ is D-connected. Let $(B, \emptyset_{nc})$ be a discrete neutrosophic crisp space. By the definition of D-connectedness, every **NCSet**-map $f : (A, \mathcal{N}) \to (B, \emptyset_{nc})$ is constant. Since f is an **NCSet**-map, $\mathcal{N} \subset f^{-1}(\emptyset_{nc}) = \emptyset_{nc}$ and we have $\mathcal{N} = \emptyset_{nc}$. We show that $cardA \leq 1$. Suppose $cardA > 1$. Let $B = \{0, 1\}$, E be a non-empty proper subset of A and $f : A \to B$ be map given by

$$f(x) = \begin{cases} 0, & x \in E \\ 1, & x \in E^c \end{cases}$$

The map $f : (A, \emptyset_{nc}) \to (B, \emptyset_{nc})$ is an **NCSet**-map, but it is not constant. Given that $(A, \mathcal{N})$ is D-connected, this is a contradiction. Hence, $cardA \leq 1$.

Conversely, suppose that $cardA \leq 1$ and $\mathcal{N} = \emptyset_{nc}$. Let $(B, \emptyset_{nc})$ be a discrete neutrosophic crisp space. $A = \emptyset$ or $A = \{a\}$. If $A = \emptyset$, then $f : (\emptyset, \emptyset_{nc}) \to (B, \emptyset_{nc})$ is an **NCSet**-map. If $A = \{a\}$, then $f : (\{a\}, \emptyset_{nc}) \to (B, \emptyset_{nc})$ is an **NCSet**-map and it is constant. It follows that every morphism from A to $(B, \emptyset_{nc})$ is constant. By Definition 11, we have that $(A, \mathcal{N})$ is D-connected. □

6. Comparative Evaluation

In this section, we compare our results with the ones in some other categories.

(1) In **Top**,
   (a) All T_2's are equivalent, i.e., $\overline{T_2} = T_2' = KT_2 = LT_2 = MT_2 = NT_2$. Moreover, $\overline{T_2} \implies T_1 \implies \overline{T_0} = T_0' = T_0$ and $\overline{T_2} \implies Pre\overline{T}_2 = PreT_2'$ [6].
   (b) $\overline{T_2}$ at $p = T_2'$ at $p \implies T_1$ at $p \implies \overline{T_0}$ at $p = T_0'$ at p and $\overline{T_2}$ at $p \implies Pre\overline{T}_2$ at $p = PreT_2'$ at p [5].

(c) If a topological space (X,τ) is $\overline{T_0}$ (resp. T_0', T_1, $Pre\overline{T}_2$, $PreT_2'$, $\overline{T}_2$, or T_2'), then (X,τ) is $\overline{T}_0$ at p (resp. T_0' at p, T_1 at p, $Pre\overline{T}_2$ at p, $PreT_2'$ at p, $\overline{T}_2$ at p, or T_2' at p), since **Top** is a normalized category [5].

(d) Strong closedness implies closedness. In addition, in the realm of T_1 topological spaces, the notions of strong closedness and closedness coincide [3]. Based on this, the notions of strong compactness and compactness are different, in general, and in the realm of T_1 property, these notions coincide [7].

(e) D-connectedness and strong connectedness coincides with the usual connectedness [10], and in the realm of T_1 property, then all the notions of connectedness coincide [10]. Moreover, the notion of strong hereditary disconnectedness coincides with the usual hereditary disconnectedness [10], and if a topological space is T_1, then hereditary disconnectedness and strong hereditary disconnectedness coincide [11].

(f) The notion of irreducibility coincides with the usual irreducibility [13]. In addition, in the realm of T_1 topological spaces, the notions of irreducibility and strong irreducibility coincide. [13].

(2) In **NCSet**, we can infer the following results.

(a) By Theorems 2 and 3, if a neutrosophic crisp space $(A,\mathcal{N})$ is $Pre\overline{T}_2$ at p, $PreT_2'$ at p, $\overline{T_2}$ at p or T_2', then $(A,\mathcal{N})$ is $\overline{T_0}$ at p, T_0' at p or T_1 at p, but the reverse implication is not true, in general.

(b) By Theorems 4, 5, and 6, if a neutrosophic crisp space $(A,\mathcal{N})$ is T_0, NT_2 or MT_2, then $(A,\mathcal{N})$ is $\overline{T_0}$, T_0', T_1, $Pre\overline{T}_2$, $PreT_2'$, $\overline{T_2}$, T_2', KT_2 or LT_2, but the reverse implication is not true, in general.

(c) By Theorems 2 and 4, a neutrosophic crisp space $(A,\mathcal{N})$ is $\overline{T_0}$ (resp. T_0', or T_1) iff $(A,\mathcal{N})$ is $\overline{T_0}$ at p (resp. T_0' at p, or T_1 at p). But, by Theorems 3 and 4, if $(A,\mathcal{N})$ is $Pre\overline{T}_2$ (resp. $PreT_2'$, $\overline{T}_2$, or T_2'), then $(A,\mathcal{N})$ is not necessary to be $Pre\overline{T}_2$ at p (resp. $PreT_2'$ at p, $\overline{T_2}$ at p, or T_2' at p).

(d) By Theorems 8, closedness and strong closedness are equivalent, and all subsets of a neutrosophic crisp space are (strongly) closed.

(e) Let $(A,\mathcal{N})$ be a neutrosophic crisp space. By Theorems 9 and 11,
  (i) $(A,\mathcal{N})$ is (strongly) compact.
  (ii) $(A,\mathcal{N})$ is (strongly) hereditary disconnected.

(f) Let $(A,\mathcal{N})$ be a neutrosophic crisp space. By Theorems 5 and 10, the following are equivalent:
  (i) $A=\emptyset$ or A is a one-point set.
  (ii) $(A,\mathcal{N})$ is T_0.
  (iii) $(A,\mathcal{N})$ is (strongly) connected.
  (iv) $(A,\mathcal{N})$ is (strongly) irreducible.

(g) By Theorems 10 and 12, D-connectedness implies (strong) connectedness or (strong) irreducibility, but in general, the converse of implication does not hold. For instance, if $A = \{a\}$ and $\mathcal{N} = A_{nc}$, then $(A, \mathcal{N})$ is (strongly) connected and (strongly) irreducible, but not D-connected.

(h) By Theorems 10 and 11, (strong) connectedness or (strong) irreducibility implies hereditary disconnectedness, the reverse implication is not true, in general. For instance, the indiscrete neutrosophic crisp space $(A, \mathcal{N})$ with $cardA = 2$ is hereditary disconnected, but neither (strongly) connected nor (strongly) irreducible.

(3) In **Prox**, the category of proximity spaces and proximity maps,

(a) $\overline{T_0} = T_1 = PreT_2' = \overline{T_2} = T_2' \implies T_0' = Pre\overline{T}_2$ [20].

(b) $\overline{T_0}$ at $p = T_1$ at $p = PreT_2'$ at $p = \overline{T_2}$ at $p = T_2'$ at $p \implies T_0'$ at $p = Pre\overline{T}_2$ at p [19, 22].

(c) Since **Prox** is a normalized category, if a topological space (X, δ) is $\overline{T_0}$ (resp. T_0', T_1, $Pre\overline{T}_2$, $PreT_2'$, $\overline{T}_2$, or T_2'), then (X, δ) is $\overline{T}_0$ at p (resp. T_0' at p, T_1 at p, $Pre\overline{T}_2$ at p, $PreT_2'$ at p, $\overline{T}_2$ at p, or T_2' at p).

(d) By Remark 4.11 of [19], the notions of closedness and strong closedness coincide. Moreover, by Lemma 4.3 of [21], (strong) closedness implies (strong) compactness since all objects are (strongly) compact.

(e) By Theorem 4.5 of [25], a proximity space (X, δ) is (strongly) connected if and only if (X, δ) is (strongly) irreducible.

(4) In **L-GS**, the category of quantale-valued gauge spaces and $\mathcal{L}$-gauge morphisms,

(a) $\overline{T_2} = T_1 \implies \overline{T_0} \implies T_0$. Moreover, an $\mathcal{L}$-gauge space $(X, \mathcal{G})$ is $\overline{T_2}$, then $(X, \mathcal{G})$ is both NT_2 and $Pre\overline{T_2}$, and in the realm of Pre-Hausdorff quantale-valued gauge spaces, $\overline{T_0}$, T_1 and $\overline{T_2}$ are equivalent [24].

(b) By Theorems 3.6 and 3.9 of [27], T_1 at $p \implies \overline{T_0}$ at p, and if an $\mathcal{L}$-gauge space $(X, \mathcal{G})$ is $\overline{T_0}$ (or T_1), then $(X, \mathcal{G})$ is $\overline{T_0}$ at p (or T_1 at p) [24, 27].

(c) There is no relation between D-connectedness and the notion of closedness or T_1 at p [27].

(5) In **pqsMet**, the category of extended pseudo-quasi-semi metric spaces and contraction maps,

(a) $T_1 = PreT_2' = T_2' = \overline{T_2} \implies \overline{T_0} \implies T_0 \implies T_0'$ and $\overline{T_2} \implies NT_2 \implies Pre\overline{T}_2 = KT_2$ [14].

(b) T_1 at $p = PreT_2'$ at $p = T_2'$ at $p = \overline{T_2}$ at $p \implies \overline{T_0}$ at $p \implies T_0'$ at p and $\overline{T_2}$ at $p \implies Pre\overline{T}_2$ at p [12].

(c) Since **pqsMet** is a normalized category, if an extended pseudo-quasi-semi metric space (X, d) is $\overline{T_0}$ (resp. T_0', T_1, $Pre\overline{T}_2$, $PreT_2'$, $\overline{T}_2$, or T_2'), then (X, d) is $\overline{T}_0$ at p (resp. T_0' at p, T_1 at p, $Pre\overline{T}_2$ at p, $PreT_2'$ at p, $\overline{T}_2$ at p, or T_2' at p).

(d) By Theorem 3.4 of [13], strong closedness implies closedness. By Theorem 3.20 of [14], an extended pseudo-quasi-semi metric space (X,d) is KT_2 or NT_2, then the notions of strong closedness and closedness coincide. Moreover, in the realm of $\overline{T_2}$, T_2' or T_1 property, each subset of X is (strongly) closed.

(e) By Theorem 4.9 of [13], an extended pseudo-quasi-semi metric space (X,d) is strongly connected, then (X,d) is connected. In addition, the notions of connectedness and D-connectedness coincide.

(f) By Theorem 5.4 of [13], irreducibility implies strong irreducibility or strong connectedness. Also, strong irreducibility implies connectedness or D-connectedness.

(6) For any arbitrary topological category,

(a) $\overline{T_0} \implies T_0'$ and there is no relationship between $\overline{T_0}$ or T_0' and T_0 [3]. In addition, it is shown in [6], that $\overline{T_2} \implies NT_2$ and $LT_2 \implies T_2'$, also the notions of $\overline{T_2}$ and NT_2, or T_2' and MT_2 are independent of each other, in general. Moreover, $PreT_2' \implies Pre\overline{T}_2$ [8].

(b) Let $\mathcal{U} : \mathcal{E} \to \boldsymbol{Set}$ be a topological functor, A an object in $\mathcal{E}$ and $p \in \mathcal{U}(A)$ be a retract of A, i.e., the initial lift $h : \overline{1} \to A$ of the $\mathcal{U}$-source $p : 1 \to \mathcal{U}(A)$ is a retract, where 1 is the terminal object in $\boldsymbol{Set}$, or more precisely let $\mathcal{U}$ be normalized. Then if A is $\overline{T}_0$ (resp. T_1, $Pre\overline{T}_2$, or $\overline{T}_2$), then A is $\overline{T}_0$ at p (resp. T_1 at p, $Pre\overline{T}_2$ at p, or $\overline{T}_2$ at p), but the reverse implication is not true, in general ([4], Theorem 2.6 and Corollary 2.7).

(c) The notions of closedness and strong closedness are independent of each other, in general [3]. Even if $A \in \mathcal{E}$ is T_1, where $\mathcal{E}$ is a topological category, then these notions are still independent of each other [3]. Based on this, the notions of compactness and strong compactness are different, in general.

(d) There are no implications between the notions of strong connectedness and connectedness, or hereditary disconnectedness and strong hereditary disconnectedness [11].

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