

Adaptive Control Solution for a Class of MIMO Uncertain Underactuated Systems with Saturating Inputs

Ajay Kulkarni^{*1}, Abhay Kumar²

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Abstract: This paper addresses the issue of controller design for a class of multi-input multi-output (MIMO) uncertain underactuated systems with saturating inputs. A systematic controller framework, composed of a hierarchically generated control term, meant to ensure the stabilization of a particular portion of system dynamics and some dedicated control terms designed to solve the tracking problem of the remaining system dynamics is presented. Wavelet neural networks are used as adaptive tuners to approximate the system uncertainties also to reshape the control terms so as to deal with the saturation nonlinearity in an antiwindup paradigm. Gradient based tuning laws are developed for the online tuning of adjustable parameters of the wavelet network. A Lyapunov based stability analysis is carried out to ensure the uniformly ultimately bounded (UUB) stability of the closed loop system. Finally, a simulation is carried out which supports the theoretical development.

Keywords: Underactuated systems, hierarchical control structure, adaptive control, wavelet neural network, actuator saturation.

1. Introduction

Depending on the nature of the complexity encountered, there exist various classes of nonlinear systems with underactuated systems as a typical case amongst them [1]. Underactuated systems are characterized by the facts that they possess lesser number of actuations than the degrees of freedom to be controlled and there exist inherent nonlinear interactions among these degrees of freedom. Thus underactuated systems can be modeled as the group of active and passive subsystems which are nonlinearly coupled [1, 2]. Inadequate actuation simply indicates that dedicated control terms cannot be designed to solve the control problem of individual degrees of freedom and demands a particular actuation to control more than one subsystem output. Most of the controller schemes for underactuated systems are either based on transformation or hierarchical control approaches. First approach deduces a nonlinear coordinate transformation which transforms the original system to some cascade like form suitable for controller design [2]. This approach utilizes the nonlinear subsystem coupling to transfer the control effort to respective subsystem and often results in complicated controller design [2]. Second technique decomposes the overall control term into number of control components which are deduced by applying some hierarchical methodology to the underactuated system under consideration [4-11]. Underactuation has been adopted by several systems like mobile robots, twin rotor system, underwater vehicles, ball-beam etc. Due to the existence of real life systems displaying underactuation, several research findings on controller design for underactuated systems have been cited in the literature [2-12]. Feedback linearization based controller designs have been proved highly effective for the control of nonlinear systems. However, these techniques are model based

and so require completely and accurately modeled system dynamics [13, 14]. This requirement of fully known system dynamics restricts these techniques to a narrow class of nonlinear systems, as in most of the cases complex phenomenon are either inaccurately modeled or left unmodeled. Control laws resulting from such ill-defined system models are rather conservative and can perform well in some local sense only [15,16]. Application of nonparametric function approximation techniques in controller design often relaxes the aforementioned constraint and thereby enhances the application areas of feedback linearization techniques [16].

Due to its ability to approximate any nonlinear function with arbitrary accuracy, wavelet network has emerged as a powerful nonparametric system identification tool. A wavelet network can be considered as the nonlinear regression structure that performs the input- output mapping by using scaled and shifted versions of some mother wavelet function as the regression functions [17-19]. Regression functions used in wavelet network satisfy the norms of multiresolution analysis, possess the property of orthonormality and are localized in space and frequency [18, 19]. Multiresolution analysis provides a systematic framework for the construction of wavelet network. Classical technique is to start with coarser resolution and including the finer resolution according to the tradeoff between accuracy and computational complexity. Orthonormality assures the unique function representation with no redundancy. Orthonormality along with multiresolution allows the explicit representation of the function different resolutions. Localization properties of wavelets allow efficient learning and rapid convergence of training algorithms [19]. Several research findings on the application of wavelet network for system identification and control are cited in the literature [20-24].

One major limitation associated with real time systems is that the actuators cannot reproduce the control effort beyond certain limits and when the control effort tries to exceed these limits

¹Medicaps Institute of Technology and Management, Indore, India

²School of Electronics, DAVV, Indore, India

* Corresponding Author: Email: a.kulkarni17@gmail.com

actuators get saturated. This limited application of control effort deteriorates the system performance, if ignored during the controller design. Research findings on actuator saturation mainly emphasize on dynamic augmentation of baseline controller with additional dynamics to deal with actuator saturation [25-27].

This paper presents a wavelet based control scheme for a class of multi-input multi- output (MIMO) uncertain underactuated systems [12] with actuator saturation. Proposed controller framework is composed of one hierarchically generated control term and some dedicated control terms. Dedicated control terms are designed to solve the tracking control problem of individual subsystems whereas hierarchically generated control term is meant to ensure the stabilization and control of remaining subsystems. To deal with actuator saturation, the original system with saturation dynamics is first transformed to a saturation free system augmented with additional nonlinear dynamics which accounts for actuator saturation. Wavelet networks are not only used as identification tool to mimic the unmodeled dynamics but also to approximate the nonlinear dynamics inserted by actuator saturation thus the wavelet network acts as saturation compensator also. Controller scheme ensures the convergence of system error dynamics and uniform ultimate boundedness of closed loop signals in presence of uncertain dynamics and actuator saturation.

This paper is organized as follows: Preliminaries and system model are given in section 2, whereas section3 describes the designing of wavelet based adaptive controller scheme for underactuated systems with partially known system dynamics and subjected to actuator saturation. Results of the simulation performed for a multi- input multi-output underactuated system are illustrated in Section 4, whereas Section 5 concludes the paper.

2. PROBLEM FORMULATION AND PRELIMINERIES

2.1. Actuator Saturation

Actuators cannot replicate the input applied beyond certain limits and saturates when the input reaches these limits. The input output relation of an actuator can be defined as

$$u = \begin{cases} v; |v| < u_{\max} \\ u_{\max} \operatorname{sgn}(v); |v| \geq u_{\max} \end{cases} \quad (1)$$

where $u_{\max} \in \mathfrak{R}_+$ represents the saturation bound. Whenever the actuator undergoes saturation, a part of control effort $\Delta u = u - v$ is suppressed by the actuator. This suppression often degrades the system performance and its effect can be viewed as an undesired dynamics invoked by actuator saturation [25]. In this work, this additional dynamic is effectively approximated and mitigated by using a wavelet compensator.

2.2. System Formulation

Consider the following uncertain MIMO underactuated system consisting of interconnected subsystems in Brunovsky canonical form [12]

$$\Sigma_1 = \begin{cases} \dot{x}_{2i-1} = x_{2i} \\ \dot{x}_{2i} = f_i(X) + \sum_{q=1}^p g_{iq}(X)u_q; i = 1, 2, \dots, n \\ y_i = x_{2i-1} \end{cases} \quad (2)$$

where $X = [x_1, x_2, \dots, x_{2n}(t)]^T \in \mathfrak{R}^{2n}$ are state variables of the system, $U = [u_1, u_2, \dots, u_p]^T \in \mathfrak{R}^p$ is the control vector

produced by the actuators of the system with $n > p$, $y_i(t) \in \mathfrak{R}$ is the output of i^{th} subsystem, $f_i(X): \mathfrak{R}^{2n} \rightarrow \mathfrak{R}$ and $g_{iq}(X): \mathfrak{R}^{2n} \rightarrow \mathfrak{R}$ are system nonlinearities abbreviated as f_i and g_{iq} respectively. Nonlinearities f_i are considered as smooth uncertainties while the nonlinear function g_{iq} is bounded away from zero, which means it is either strictly positive or negative and $g_{iq} \in L_\infty$, $\forall x \in S_x, t \geq 0$ here $S_x \in \mathfrak{R}^{2n}$ is some compact set of allowable state trajectories.

The control objective is to track the given desired trajectory in presence of system uncertainties and input nonlinearity. For some desired trajectory vector $y_d \in \mathfrak{R}^n$ selected such that, $y_{id}, \dot{y}_{id}, \ddot{y}_{id} \in L_\infty$, $i = 1, 2, \dots, n$ adaptive controller must ensure the convergence of tracking error $(y_i - y_{id})$ to a small neighborhood of origin and uniform ultimate boundedness of all the closed loop signals. To streamline the controller design, following assumption has been taken regarding the nature of input nonlinearity.

Assumption1: The control input $u_i (i = 1, 2, \dots, p)$ satisfies the saturation nonlinearity described in subsection A. Thus, the input-output relation of actuator i is given as

$$u_i = v_i + \Delta u_i \quad (3)$$

where u_i and v_i are output and input of the actuator respectively whereas Δu_i represents the portion of input concealed by the actuator saturation.

2.3. Wavelet Network Approximation

In system identification, wavelet neural network has been successfully employed as nonparametric regression tool due to their inherent approximation capabilities. Wavelet networks are similar to neural networks but they utilize compactly supported orthonormal wavelet basis functions as activation functions and have a systematic architecture governed by the properties of multiresolution analysis [19]. Wavelet network representation of any square integrable nonlinear function $f(x) \in L^2(\mathfrak{R})$ can be expressed as [19, 20]

$$\begin{aligned} \hat{f}(x) &= \sum_{k \in K_{J_0}} \alpha_{J_0, k} 2^{J_0/2} \phi(2^{J_0} x - k) + \sum_{j=J_0}^J \sum_{k \in K_j} \beta_{j, k} 2^{j/2} \psi(2^j x - k) \\ &= \sum_{k \in K_{J_0}} \alpha_{J_0, k} \phi_{J_0, k}(x) + \sum_{j=J_0}^J \sum_{k \in K_j} \beta_{j, k} \psi_{j, k}(x) \\ &= \alpha^T \phi_{J_0}(x) + \sum_{j=J_0}^J \beta_j^T \psi_j(x) \end{aligned} \quad (4)$$

Where $x \in \Omega_x \in \mathfrak{R}$ is the input argument, $[J_0, J \in \square^2]$ represents the coarsest and finest resolution levels respectively, $K_j \subset \square^{d_j}$ ($j = J_0, J_0 + 1, \dots, J$) represents translates for a particular resolution level, $d_j \in \square$ represents the number of translates used at a particular resolution j . While $\alpha_{J_0} = [\alpha_{J_0, k}]_{k \in K_{J_0}}^T$ and $\beta_j = [\beta_{j, k}]_{k \in K_j}^T$ are the weight vectors and $\phi_{J_0} = [\phi_{J_0, k}]_{k \in K_{J_0}}$ is the vector of translated versions of scaling function at resolution J_0 and $\psi_j = [\psi_{j, k}]_{k \in K_j}^T$ is the vector of translated wavelet functions at resolution j . Any orthogonal wavelet and associated scaling function which satisfy multiresolution analysis can be used for the construction of wavelet network.

It has been proved that, there exist a finite but unknown integer J_N and finite number of translates for each resolution level i.e. K_j such that the unknown nonlinear function $f(x) \in L^2(\mathfrak{R})$, over a compact set $x \in \Omega_x \in \mathfrak{R}$, can be approximated as [19] $f(x) = \alpha_{J_0}^T \phi_{J_0}(x) + \sum_{j=J_0}^{J_N} \beta_j^T \psi_j(x) + \varepsilon(x); \forall x \in \Omega_x \in \mathfrak{R}$ (5)

where $\alpha_{j_0}^*$ and β_j^* represents the optimal weight vectors and $\varepsilon(x)$ is the approximation error and for optimal weight vectors it is assumed to be bounded by $|\varepsilon(x)| \leq \varepsilon^*$.

Optimal weight vectors required for the function approximation are unknown and needs to be estimated. Let $\hat{\alpha}_{j_0}$ and $\hat{\beta}_j$ be the estimates of $\alpha_{j_0}^*$ and β_j^* then the wavelet network estimate of $f(x)$ can be defined as

$$\hat{f}(x) = \hat{\alpha}_{j_0}^T \phi_{j_0}(x) + \sum_{j=J_0}^J \hat{\beta}_j^T \psi_j(x) \quad (6)$$

Defining an estimation error

$$\tilde{f}(x) = f(x) - \hat{f}(x) = \tilde{\alpha}_{j_0}^T \phi_{j_0}(x) + \sum_{j=J_0}^J \tilde{\beta}_j^T \psi_j(x) + \varepsilon(x) \quad (7)$$

where $\tilde{\alpha}_{j_0}$ and $\tilde{\beta}_j$ are weight estimation errors, defined as $\tilde{\alpha}_{j_0} = \alpha_{j_0} - \hat{\alpha}_{j_0}$ and $\tilde{\beta}_j = \beta_j - \hat{\beta}_j$

With appropriate numbers of resolutions and translates at each resolution level, the estimation error $\tilde{f}(x)$ can be made arbitrarily small on the compact set such that the bound $\|\tilde{f}(x)\| \leq \tilde{f}_m$ holds for all $x \in \Omega_x \subset \mathfrak{R}$.

For multidimensional functions of the form $f(x): \mathfrak{R}^n \rightarrow \mathfrak{R}$, wavelet network model can be extended to multidimensional wavelet network by tensor product of single dimensional wavelet bases. According to multidimensional multiresolution analysis, for n dimensional case, there exist one scaling function $\phi_{j_0, K}$ which is obtained by the tensor product of single dimensional scaling functions $\phi_{j_0, k_i}(x_i); (i=1, 2, \dots, n)$ and $2^n - 1$ wavelet functions $\psi_{j, K}^q (q=1, 2, \dots, 2^n - 1)$ which are obtained by mixing the single dimensional wavelet and scaling functions in different dimensions [19].

$$\phi_{j_0, K} = \prod_{i=1}^n \phi_{j_0, k_i}(x_i) \quad (8)$$

$$\psi_{j, K}^q = \prod_{i=1}^n \varphi_{j, k_i}(x_i); q=1, 2, \dots, 2^n - 1$$

with

$$\psi_{j, K}^1 = \left(\prod_{i=1}^{n-1} \varphi_{j, k_i}(x_i) \right) \psi_{j, k_n}(x_n)$$

$$\psi_{j, K}^{2^n - 1} = \prod_{i=1}^n \psi_{j, k_i}(x_i)$$

where $K \in \square^n$ and $\varphi_{j, k_i}(x_i)$ is either $\phi_{j, k_i}(x_i)$ or $\psi_{j, k_i}(x_i)$.

3. CONTROLLER DESIGN

This section describes the development of adaptive control scheme for system (2) to achieve desired performance. Control scheme developed in this work can be segregated in two parts, first part details the development of control input u_1 using hierarchical scheme to ensure the stabilization of first $n-p+1$ subsystems. Second part describes the formulation of remaining $p-1$ control efforts which acts as the dedicated controllers for remaining $p-1$ subsystems. To facilitate the controller design, original system model is first transformed to a saturation free model which is used for the subsequent analysis.

Step 1: Considering the first $n-p+1$ subsystems of system (2)

$$\Sigma_2 = \begin{cases} \dot{x}_{2i-1} = x_{2i} \\ \dot{x}_{2i} = f_i(X) + g_{i1}(X)u_1 + \sum_{q=2}^p g_{iq}(X)u_q \\ y_i = x_{2i-1}; i=1, 2, \dots, n-p+1 \end{cases} \quad (9)$$

Substitution of the saturation model governed by (3) into (9) leads to following transformation

$$\Sigma_3 = \begin{cases} \dot{x}_{2i-1} = x_{2i} \\ \dot{x}_{2i} = f_i(X) + \sum_{q=1}^p g_{iq}(X)\Delta u_q + g_{i1}(X)v_1 + \sum_{q=2}^p g_{iq}(X)v_q \\ y_i = x_{2i-1}; i=1, 2, \dots, n-p+1 \end{cases} \quad (10)$$

System Σ_3 (10) can be viewed as a saturation free system with nonlinearity $(f_i(X) + \sum_{q=1}^p g_{iq}(X)\Delta u_q)$. The term $g_{iq}(X)\Delta u_q$ accounts for the nonlinear dynamics due to actuator saturation and is invoked whenever the actuator undergoes saturation. In this work, controller designing is carried out using the transformed system model Σ_3 (10). Following mathematical development leads to the formulation of control component v_{a1} of control term v_1 .

Defining tracking error for i^{th} subsystem as

$$e_{2i-1} = x_{2i-1} - y_{id}; i=1, 2, \dots, n-p+1 \quad (11)$$

Its differentiation leads to

$$\dot{e}_{2i-1} = \dot{x}_{2i} - \dot{y}_{id} \quad (12)$$

Defining a pseudo term as

$$x_{2id} = -k_i e_{2i-1} + \dot{y}_{id} \quad (13)$$

where k_i is a positive constant.

Substitution of (13) in (12) results in

$$\begin{aligned} \dot{e}_{2i-1} &= x_{2i} - x_{2id} - k_i e_{2i-1} \\ &= e_{2i} - k_i e_{2i-1} \end{aligned} \quad (14)$$

Defining filtered tracking error for i^{th} subsystem as

$$s_{i1} = a_i e_{2i-1} + e_{2i} \quad (15)$$

where a_i is a positive constant.

Differentiation of filtered tracking error (15) leads to

$$\begin{aligned} \dot{s}_{i1} &= a_i (e_{2i} - k_i e_{2i-1}) + (f_i(X) + \sum_{q=1}^p g_{iq}(X)\Delta u_q + g_{i1}(X)v_1 + \\ &\quad \sum_{q=2}^p g_{iq}(X)v_q - \dot{x}_{2id}) \end{aligned} \quad (16)$$

Defining an integral error term of the form

$$S_1 = \rho_1 s_{11} + \rho_2 s_{21} + \dots + \rho_{(n-p+1)} s_{(n-p+1)1} \quad (17)$$

where $\rho_i, i=1, 2, \dots, n-p+1$ are coupling parameters.

According to hierarchical methodology for controller design, error term S_1 (17) can be viewed as second level error surface which is obtained by suitably aggregating the subsystem filtered tracking errors $s_{i1} (i=1, 2, \dots, n-p+1)$ (15) which can be considered as the first level error terms [3, 4]. Convergence of the integral error term S_1 (17) ensures the boundedness of first level error terms and hence stabilization of individual subsystems.

Differentiation of error dynamics (17) results in

$$\dot{S}_1 = \sum_{i=1}^{n-p+1} \rho_i \dot{s}_{i1} \quad (18)$$

From (16) we have

$$\dot{S}_1 = \sum_{i=1}^{n-p+1} \left(\begin{array}{l} \rho_i (a_i (e_{2i} - k_i e_{2i-1}) + (f_i(X) + \sum_{q=1}^p g_{iq}(X) \Delta u_q - \dot{x}_{2id})) + \\ \rho_i g_{i1}(X) v_{i1} + \rho_i \sum_{q=2}^p g_{iq}(X) v_{iq} \end{array} \right) \quad (19)$$

Above equation, allows the following formulation of v_{a1}

$$v_{a1} = \sum_{i=1}^{n-p+1} \rho_i v_{i1} + v_{1c} \quad (20)$$

Control term v_{a1} (19) is assumed to be composed of $n-p+1$ control terms v_{i1} ($i=1,2,\dots,n-p+1$) which are defined to stabilize the subsystem error dynamics in (19) while compensating control term v_{1c} is defined to attenuate the approximation error of wavelet network and to improve the convergence of error surface S_1 (17) under the constraints of uncertainties and saturation.

Defining the subsystem control term as

$$v_{i1} = (-a_i (e_{2i} - k_i e_{2i-1}) - Q_i + \dot{x}_{2id} - c s_{i1}); \quad (21)$$

$$i = 1, 2, \dots, n-p+1$$

where $Q_i = f_i(X) + \sum_{q=1}^p g_{iq}(X) \Delta u_q$ and c is a positive constant.

As the nonlinearity Q_i is uncertain, it is approximated by using a wavelet network and so control term (21) becomes

$$v_{i1} = (-a_i (e_{2i} - k_i e_{2i-1}) - \hat{Q}_i + \dot{x}_{2id} - c s_{i1}); \quad (22)$$

$$i = 1, 2, \dots, n-p+1$$

where \hat{Q}_i is the wavelet approximation of the nonlinear term. Control term (22), so derived is a partial model free version of (21) and also relaxes the requirement of measuring Δu_1 [25]. Compensating control term defined as [21]

$$v_{1c} = -\frac{S_1}{2\kappa^2} \quad (23)$$

here $\left(-\frac{S_1}{2\kappa^2}\right)$ is the robust term included in order to attenuate the approximation error of wavelet network with $0 < \kappa < 1$.

Subsequent portion of this step, details the development of tuning laws for update of wavelet parameters and convergence of the closed loop system. For the development wavelet adaptation laws, consider a cost function of the form [15]

$$J_i = \frac{1}{2} \tilde{Q}_i^2; i = 1, 2, \dots, n-p+1 \quad (24)$$

where \tilde{Q}_i is the wavelet approximation error defined as

$$\tilde{Q}_i = Q_i - \hat{Q}_i \quad (25)$$

Adjustable parameters of the wavelet network are needed to be tuned so as to minimize the cost function J_i . According to the MIT rule for weight adjustment [15], weight updates are

$$\begin{aligned} \dot{\tilde{\alpha}}_{J_{0,i}} &= -\dot{\tilde{\alpha}}_{J_{0,i}} = -\sigma_i \left(\nabla_{\tilde{\alpha}_{J_{0,i}}} J_i \right)^T = -\sigma_i \phi_{J_{0,i}}(X) \tilde{Q}_i \\ \dot{\tilde{\beta}}_{j_i}^q &= -\dot{\tilde{\beta}}_{j_i}^q = -\gamma_i \left(\nabla_{\tilde{\beta}_{j_i}^q} J_i \right)^T = -\gamma_i \psi_{j_i}^q(X) \tilde{Q}_i \end{aligned} \quad (26)$$

where $i = 1, 2, \dots, n-p+1$, $q = 1, 2, \dots, 2^{n-1}$, $\sigma_i > 0$ and $\gamma_i > 0$.

Effectiveness of the tuning laws so developed is reflected by the convergence of wavelet network estimation error to some small residual set. In order to prove the convergence of estimation error and boundedness of weight estimation errors, consider a Lyapunov function of the form [28]

$$V_{wi} = \frac{1}{2\kappa_i} \tilde{\alpha}_{J_{0,i}}^T \tilde{\alpha}_{J_{0,i}} + \frac{1}{2\gamma_i} \sum_{j=J_0}^J \sum_{q=1}^{2^{n-1}} \tilde{\beta}_{j_i}^{qT} \tilde{\beta}_{j_i}^q; i = 1, 2, \dots, n-p+1 \quad (27)$$

Assumption2: To facilitate the subsequent mathematical development it is assumed that over a compact set $x \in \Omega_x \in \mathfrak{R}^n$ scaling, wavelet functions and approximation error terms are bounded i.e. $\phi_{J_{0,i,k}}(x) \in L_\infty$, $\psi_{j_i,k}^q(x) \in L_\infty$ and $\varepsilon_i(x) \in L_\infty$. Differentiation of Lyapunov function V_{wi} (27) and substitution of adaptation laws (26) results in

$$\begin{aligned} \dot{V}_{wi} &= \frac{1}{2\sigma_i} \tilde{\alpha}_{J_{0,i}}^T \dot{\tilde{\alpha}}_{J_{0,i}} + \frac{1}{2\gamma_i} \sum_{j=J_0}^J \sum_{q=1}^{2^{n-1}} \tilde{\beta}_{j_i}^{qT} \dot{\tilde{\beta}}_{j_i}^q \\ &= -\tilde{\alpha}_{J_{0,i}}^T \phi(X) \tilde{Q}_i - \sum_{j=J_0}^J \sum_{q=1}^{2^{n-1}} \tilde{\beta}_{j_i}^{qT} \psi^q(X) \tilde{Q}_i \\ &= -(\tilde{Q}_i - \varepsilon_i) \tilde{Q}_i \\ &= -\tilde{Q}_i^2 + \tilde{Q}_i \varepsilon_i \\ &= -\tilde{Q}_i^2 + \frac{\tilde{Q}_i^2}{2} + \frac{\varepsilon_i^2}{2} - \frac{(\tilde{Q}_i - \varepsilon_i)^2}{2} \\ &\leq -\frac{\tilde{Q}_i^2}{2} + \frac{\varepsilon_i^2}{2} \end{aligned} \quad (28)$$

Under the condition of bounded approximation error, we have

$$\max |\varepsilon_i| = \varepsilon_{i\max} \quad (29)$$

On substituting (29) in (28), we obtain

$$\dot{V}_{wi} = -\frac{\tilde{Q}_i^2}{2} + \frac{\varepsilon_{i\max}^2}{2} \quad (30)$$

It implies that $\dot{V}_{wi} \leq 0$ as long as $|\tilde{Q}_i| \geq \varepsilon_{i\max}$ and therefore indicates the boundedness of weight estimate errors $\tilde{\alpha}_{J_{0,i,k}} \in L_\infty$ and $\tilde{\beta}_{j_i,k}^q \in L_\infty$.

From (7) we have the following inequality

$$|\tilde{Q}_i(x)| \leq \|\tilde{\alpha}_{J_{0,i}}\| \|\phi_{J_{0,i}}(x)\| + \sum_{j=J_0}^J \sum_{q=1}^{2^{n-1}} \|\tilde{\beta}_{j_i}^q\| \|\psi_{j_i}^q(x)\| + |\varepsilon_i(x)| \quad (31)$$

Therefore, we conclude that approximation error \tilde{Q}_i is bounded and when converges to the residual set

$$\Omega_{\tilde{Q}_i} = \left\{ \|\tilde{Q}_i\| \|\tilde{Q}_i\| \leq \varepsilon_{i \max} \right\} \quad (32)$$

$$\|\tilde{\alpha}_{J_{0i}}\| \rightarrow 0 \text{ and } \|\tilde{\beta}_{J_i}^q\| \rightarrow 0.$$

Next part of the section illustrates the development of remaining control terms and details the associated issues of stability.

Step 2: This step describes the reason and associated mathematical developments for designing of remaining $p-1$ control terms. These control terms are assigned as dedicated control terms to each of the remaining $p-1$ subsystems and are designed to solve the control problem of respective subsystems. Consider the remaining portion of system (2)

$$\Sigma_4 = \begin{cases} \dot{x}_{2(n-p+i)-1} = x_{2(n-p+i)} \\ \dot{x}_{2(n-p+i)} = f_{n-p+i}(X) + g_{(n-p+i)i}(X)u_i + \sum_{\substack{q=1 \\ q \neq i}}^p g_{(n-p+i)q}(X)u_q \\ y_{n-p+i} = x_{2(n-p+i)-1}; i=2,3,\dots,p \end{cases} \quad (33)$$

Under the restrictions of actuator saturation, using the saturation dynamics (3), system dynamics (33) is transformed into the following form

$$\Sigma_5 = \begin{cases} \dot{x}_{2(n-p+i)-1} = x_{2(n-p+i)} \\ \dot{x}_{2(n-p+i)} = Q_{n-p+i} + g_{(n-p+i)i}(X)v_i + \sum_{\substack{q=1 \\ q \neq i}}^p g_{(n-p+i)q}(X)v_q \\ y_{n-p+i} = x_{2(n-p+i)-1}; i=2,3,\dots,p \end{cases} \quad (34)$$

$$Q_{n-p+i} = f_{n-p+i}(X) + \sum_{q=1}^p g_{(n-p+i)q}(X)\Delta u_q$$

where

Defining error variable for $(n-p+i)^{th}$, $(i=2,3,\dots,p)$ subsystem as

$$e_{2(n-p+i)-1} = x_{2(n-p+i)-1} - y_{(n-p+i)d} \quad (35)$$

Differentiating $e_{2(n-p+i)-1}$ (35) along the system trajectories yields

$$\dot{e}_{2(n-p+i)-1} = x_{2(n-p+i)} - \dot{y}_{(n-p+i)d} \quad (36)$$

Virtual control term $x_{2(n-p+i)d}$ is designed as

$$x_{2(n-p+i)d} = -k_{(n-p+i)}e_{2(n-p+i)-1} + \dot{y}_{(n-p+i)d} \quad (37)$$

where $k_{n-p+i} > 0$.

Defining filtered tracking error s_{n-p+i} for $(n-p+i)^{th}$ subsystem as

$$s_{n-p+i} = a_{(n-p+i)}e_{2(n-p+i)-1} + e_{2(n-p+i)} \quad (38)$$

where $e_{2(n-p+i)} = x_{2(n-p+i)} - x_{2(n-p+i)d}$ and $a_{n-p+i} > 0$.

Differentiating filtered tracking error (38)

$$\begin{aligned} \dot{s}_{n-p+i} &= a_{(n-p+i)}\dot{e}_{2(n-p+i)-1} + \dot{e}_{2(n-p+i)} \\ &= a_{(n-p+i)}\dot{e}_{2(n-p+i)-1} + \dot{x}_{2(n-p+i)} - \dot{x}_{2(n-p+i)d} \\ &= \left(a_{(n-p+i)}(e_{2(n-p+i)} - k_{(n-p+i)}e_{2(n-p+i)-1}) + Q_{n-p+i} \right) \\ &\quad + g_{(n-p+i)i}(X)v_i + \sum_{\substack{q=1 \\ q \neq i}}^p g_{(n-p+i)q}(X)v_q - \dot{x}_{2(n-p+i)d} \end{aligned} \quad (39)$$

Defining an error vector as

$$S = [s_1, s_{n-p+2}, \dots, s_n]^T \quad (40)$$

With the consideration of dynamics (40), defining the control

$$\text{vector } v = [v_1, v_2, \dots, v_p]^T \text{ as}$$

$$v = G^{-1} [v_{a1}, v_{a2}, \dots, v_{ap}]^T \quad (41)$$

where the control term v_{a1} is already defined (20) while v_{ai} ($i=2,3,\dots,p$) is meant to solve the control problem of $(n-p+i)^{th}$ subsystem and is defined as

$$v_{ai} = v_{ip} + v_{ic} \quad (42)$$

where v_{ip} is the principle control term and is defined as

$$v_{ip} = \left(-a_{(n-p+i)}(e_{2(n-p+i)} - k_{(n-p+i)}e_{2(n-p+i)-1}) - \hat{Q}_{n-p+i} + \dot{x}_{2(n-p+i)d} - cs_{n-p+i} \right) \quad (43)$$

while the compensating control term v_{ic} is defined as

$$v_{ic} = -\frac{S_{n-p+i}}{2\kappa^2} \quad (44)$$

here \hat{Q}_{n-p+i} is the wavelet approximation of Q_{n-p+i} and

$G \in \mathfrak{R}^{pp}$ is defined as

$$G = \begin{bmatrix} \sum_{i=1}^{n-p+1} \rho_i g_{i1} & \sum_{i=1}^{n-p+1} \rho_i g_{i2} & \cdots & \sum_{i=1}^{n-p+1} \rho_i g_{ip} \\ g_{(n-p+2)1} & g_{(n-p+2)2} & \cdots & g_{(n-p+2)p} \\ \vdots & \vdots & \cdots & \vdots \\ g_{n1} & g_{n2} & \cdots & g_{np} \end{bmatrix}$$

Adaptation laws to update the adjustable wavelet parameters are selected as

$$\dot{\hat{\alpha}}_{J_{0,n-p+i}} = -\hat{\alpha}_{J_{0,n-p+i}} = -\sigma_{n-p+i} \phi_{J_{0,n-p+i}}(X) \tilde{Q}_{n-p+i} \quad (45)$$

$$\dot{\hat{\beta}}_{J_{n-p+i}}^q = -\hat{\beta}_{J_{n-p+i}}^q = -\gamma_{n-p+i} \psi_{J_{n-p+i}}^q(X) \tilde{Q}_{n-p+i}$$

where $i=2,3,\dots,p$, $q=1,2,\dots,2^n-1$ and \tilde{Q}_{n-p+i} is the wavelet estimation error (25). Issues related to the boundedness of estimation error have already been discussed in step1 of control design (28, 29 and 30). In order to perform the convergence analysis of the system under consideration, consider a Lyapunov function [13]

$$V = \frac{1}{2} S^T S \quad (46)$$

Differentiating V (46)

$$\begin{aligned} \dot{V} &= S^T \dot{S} \\ &= S^T [\dot{S}_1, \dot{S}_{n-p+2}, \dots, \dot{S}_n]^T \\ &= S^T [E - Q - X_d + Gv] \end{aligned} \quad (47)$$

where

$$E = \begin{bmatrix} \sum_{i=1}^{n-p+1} \rho_i a_i (e_{2i} - k_i e_{2i-1}) \\ a_{(n-p+2)} (e_{2(n-p+2)} - k_{(n-p+2)} e_{2(n-p+2)-1}) \\ \vdots \\ a_n (e_{2n} - k_n e_{2n-1}) \end{bmatrix};$$

$$Q = \begin{bmatrix} \sum_{i=1}^{n-p+1} \rho_i \tilde{Q}_i \\ Q_{n-p+2} \\ \vdots \\ Q_n \end{bmatrix}; X_d = \begin{bmatrix} \sum_{i=1}^{n-p+1} \rho_i \dot{x}_{2id} \\ \dot{x}_{2(n-p+2)d} \\ \vdots \\ \dot{x}_{2nd} \end{bmatrix}$$

Substitution of control term (41) in (47) results in

$$\dot{V} = S^T \left[\tilde{Q} - cS + \frac{S}{2\kappa^2} \right] \quad (48)$$

where

$$\tilde{Q} = \begin{bmatrix} \sum_{i=1}^{n-p+1} \rho_i \tilde{Q}_i \\ \tilde{Q}_{n-p+2} \\ \vdots \\ \tilde{Q}_n \end{bmatrix}$$

Under the condition of boundedness of estimation error \tilde{Q}_{n-p+i}

over a compact set $\Omega_{Q_{n-p+i}} \subset \mathfrak{R}$.

Let $\max \left| \sum_{i=1}^{n-p+1} \rho_i \tilde{Q}_i \right| = \chi_1$ and $\max |\tilde{Q}_{n-p+i}| = \chi_{n-p+i}$

$$\dot{V} = S^T \begin{bmatrix} \sum_{i=1}^{n-p+1} \rho_i \tilde{Q}_i \\ \tilde{Q}_{n-p+2} \\ \vdots \\ \tilde{Q}_n \end{bmatrix} - \begin{bmatrix} \frac{S_1}{2\kappa^2} \\ \frac{S_{n-p+2}}{2\kappa^2} \\ \vdots \\ \frac{S_n}{2\kappa^2} \end{bmatrix} - cS^T S$$

$$\leq S^T \begin{bmatrix} \chi_1 \\ \chi_{n-p+2} \\ \vdots \\ \chi_n \end{bmatrix} - \begin{bmatrix} \frac{S_1}{2\kappa^2} \\ \frac{S_{n-p+2}}{2\kappa^2} \\ \vdots \\ \frac{S_n}{2\kappa^2} \end{bmatrix} - cS^T S \quad (49)$$

$$\leq S^T \begin{bmatrix} \chi_1 \\ \chi_{n-p+2} \\ \vdots \\ \chi_n \end{bmatrix} - \begin{bmatrix} \frac{S_1}{2\kappa^2} \\ \frac{S_{n-p+2}}{2\kappa^2} \\ \vdots \\ \frac{S_n}{2\kappa^2} \end{bmatrix} - cS^T S$$

$$\leq \left(-cS_1^2 + \frac{\kappa^2}{2} \chi_1^2 - \frac{1}{2} \left(\kappa \chi_1 + \frac{S_1}{\kappa} \right)^2 + \sum_{i=2}^p \left(-cS_{n-p+i}^2 - \frac{1}{2} \left(\kappa \chi_{n-p+i} + \frac{S_{n-p+i}}{\kappa} \right)^2 + \frac{\kappa^2}{2} \chi_{n-p+i}^2 \right) \right)$$

$$\leq -cS_1^2 + \frac{\kappa^2}{2} \chi_1^2 + \sum_{i=2}^p \left(-cS_{n-p+i}^2 + \frac{\kappa^2}{2} \chi_{n-p+i}^2 \right)$$

Equation (49) indicates that the elements of error vector S are bounded and converges to a compact sets defined as

$$\Omega_{S_1} = \left\{ S_1 \mid |S_1| \leq \left| \frac{\kappa \chi_1}{\sqrt{2c}} \right| \right\}$$

$$\Omega_{S_{n-p+i}} = \left\{ S_{n-p+i} \mid |S_{n-p+i}| \leq \left| \frac{\kappa \chi_{n-p+i}}{\sqrt{2c}} \right| \right\}; i = 2, 3, \dots, p \quad (50)$$

Above equation indicates the uniform ultimate boundedness of error terms and associated closed loop signals.

From (50), we have

$$S_1 \in L_\infty \quad (51)$$

Hence

$$s_{i1} \in L_\infty, i = 1, 2, \dots, n-p+1 \quad (52)$$

Boundedness of S_1 (17) implies the boundedness of associated subsystem error dynamics s_{i1} (15).

Succeeding section illustrates the results of simulation carried out with the adaptive controller strategy developed in this section.

Remark: Update laws (26 and 43) developed for the tuning of wavelet parameters could not be implemented as approximation error \tilde{Q}_i (25) is not available for measurement. To make the implementation feasible, following analysis is carried out.

Consider the following error dynamics from (38)

$$\dot{S} = [\dot{S}_1, \dot{S}_{n-p+2}, \dots, \dot{S}_n]^T = [E - Q - X_d + Gv] \quad (53)$$

Substitution of control law (41) results in

$$\dot{S} = \left[\tilde{Q} - cS + \frac{S}{2\kappa^2} \right] \quad (54)$$

From (52) we have

$$\tilde{Q} = \left[\dot{S} + cS - \frac{S}{2\kappa^2} \right] \quad (55)$$

Above equation results in

$$\tilde{Q}_i = \left[\dot{s}_i + cs_i - \frac{s_i}{2\kappa^2} \right]; i = 1, 2, \dots, n \quad (56)$$

As the elements of \dot{S} cannot be calculated as it contains uncertainties and so is approximated as

$$\dot{s}_{i1} = \frac{s_{i1}(t) - s_{i1}(t - \Delta t)}{\Delta t} \quad (57)$$

where Δt is a small positive constant [28].

4. Simulation Results

To demonstrate the effectiveness of the adaptive controller scheme (39), a simulation study is carried out considering a multi- input multi- output underactuated system with uncertain dynamics and the output of the actuators are assumed to be restricted by saturation nonlinearity. Consider the following multi- input multi -output underactuated system with three interconnected subsystems and two actuations subjected to actuator saturation

$$\Sigma_6 = \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1 + g_{11}u_1 + g_{12}u_2 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2 + g_{21}u_1 + g_{22}u_2 \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = f_3 + g_{31}u_1 + g_{32}u_2 \\ y_1 = x_1 \\ y_2 = x_3 \\ y_3 = x_5 \end{cases} \quad (58)$$

where $y = [y_1, y_2, y_3]^T$ is the output vector and $u = [u_1, u_2]^T$ is the vector of plant inputs and output of actuators subjected to saturation (3) with input vector $v = [v_1, v_2]^T$. As per the controller scheme presented, control term v_1 is meant for effective stabilization of system states x_1 and x_3 whereas v_2 is assigned to solve the control problem of x_5 . Simulation is carried out with following nonlinear dynamics

$$f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} 0.7 \sin x_3 x_1 - x_4^2 \cos x_3 \\ 0.5 x_4^2 \cos^2 x_3 \\ 0.5 \cos x_2 \cos x_4 \end{bmatrix}$$

$$g = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \\ g_{31} & g_{32} \end{bmatrix} = \begin{bmatrix} \frac{-0.5}{.5 \cos x_3 \cos x_5 - 1} & \frac{0.1}{1 + .5 \cos x_3 \cos x_1} \\ \frac{0.5}{1 + .5 \cos x_1 \cos x_3} & \frac{0.2}{1 + x_1^2 x_3^2} \\ \frac{0.5}{1.5 - \cos x_5 \sin x_1} & \frac{0.1}{1.5 - \sin x_5 \sin x_3} \end{bmatrix}$$

Elements of nonlinear vector field f are considered as uncertain system dynamics. According to the actuator saturation dynamics (3), system (58) can be transformed to the following form

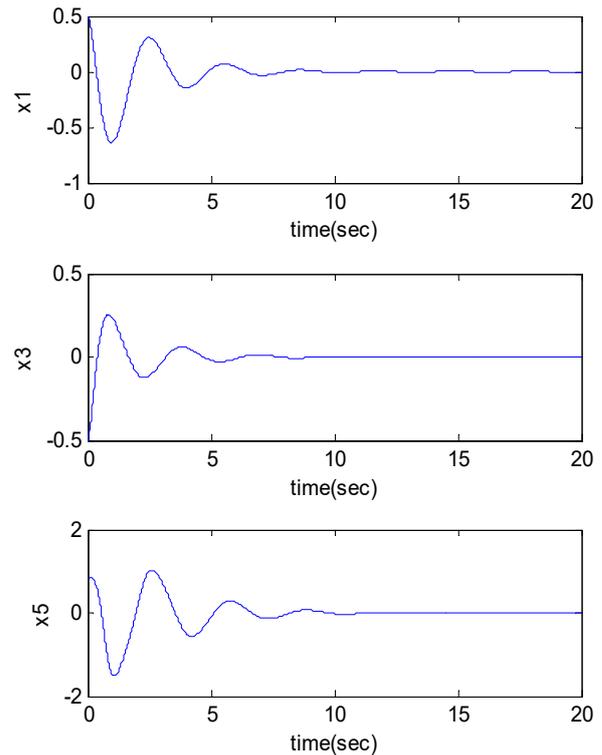


Figure 1. Trajectories of state variables with unconstrained actuators

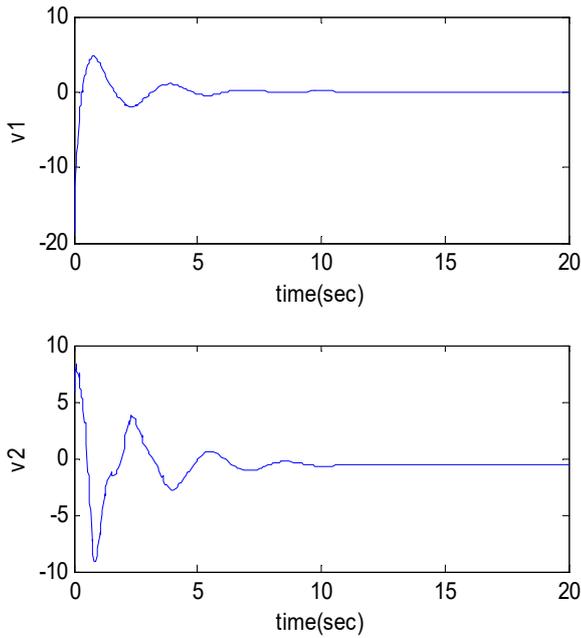


Figure 2. Control efforts with unconstrained actuators

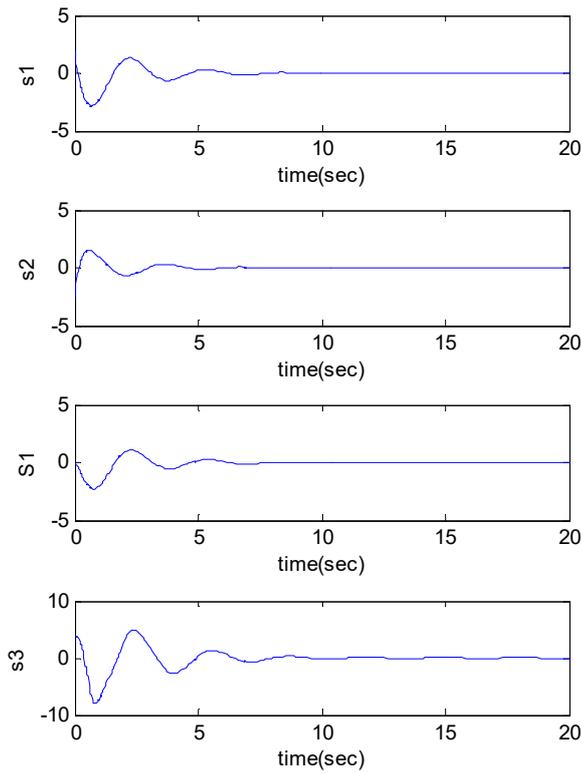


Figure 3. Error signals with unconstrained actuator

$$\Sigma_7 = \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = Q_1 + g_{11}v_1 + g_{12}u_2 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = Q_2 + g_{21}v_1 + g_{22}u_2 \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = Q_3 + g_{31}u_1 + g_{32}v_2 \\ y_1 = x_1 \\ y_2 = x_3 \\ y_3 = x_5 \end{cases} \quad (59)$$

Where

$$Q = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} f_1 + g_{11}\Delta u_1 \\ f_2 + g_{21}\Delta u_1 \\ f_3 + g_{32}\Delta u_2 \end{bmatrix}$$

Here the elements of the vector field Q , represents the system nonlinearity as well as nonlinearity inserted by the saturation dynamics. These elements are approximated by using wavelet neural network.

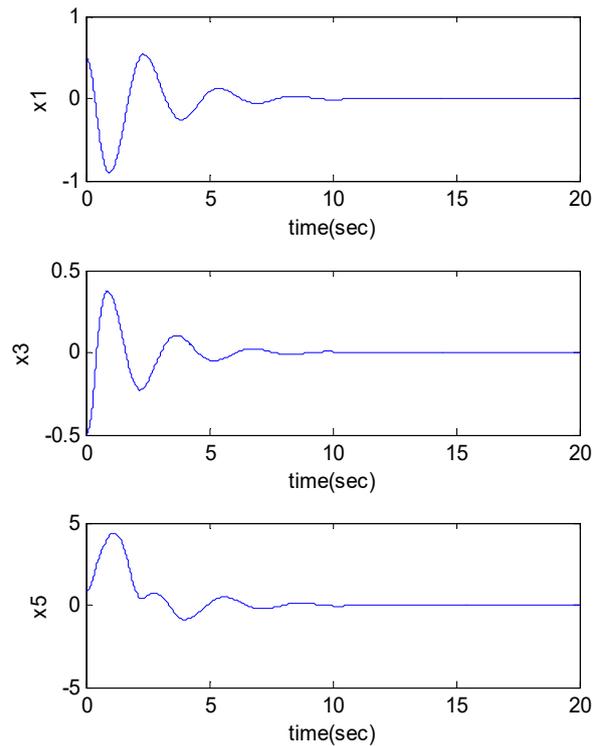


Figure 4. Trajectories of State variables under actuator saturation

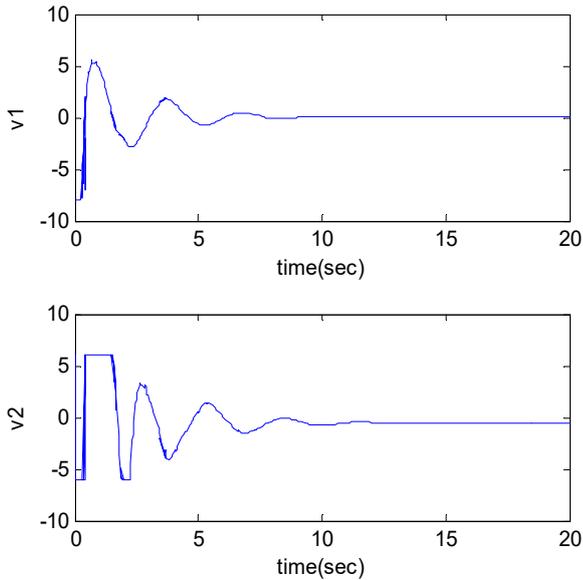


Figure 5. Control efforts under actuator saturation

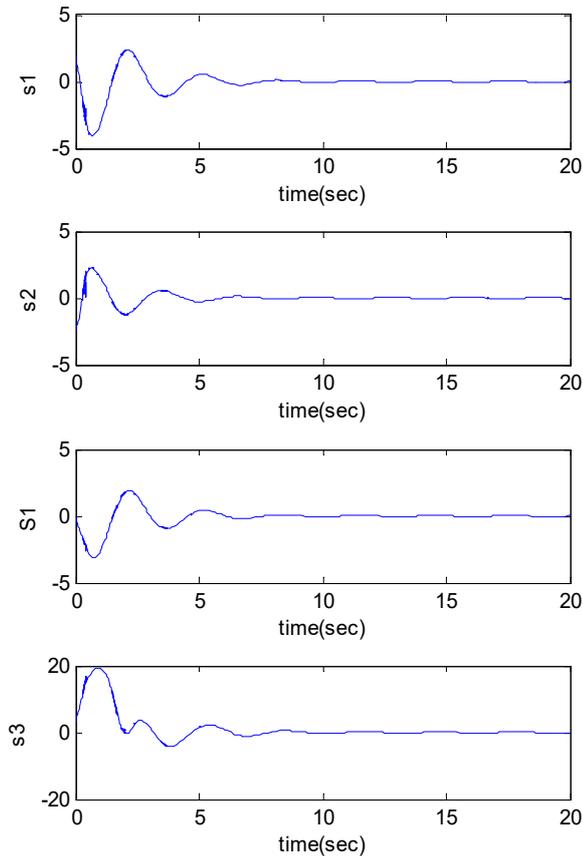


Figure 6. Error signals under actuator saturation

Wavelet approximation of the nonlinear saturation dynamics also relaxes the requirement of measuring the saturation error (3). Wavelet networks are constructed using Daubechies wavelet (db3) with $n=4$, coarsest and finest resolution levels are selected as $J_0=1$ and $J=3$ respectively. Number of translates of single dimensional wavelet basis at coarsest resolution level are taken as $K_1=3$ and are made double when resolution is

increased by 1. Online adaptation laws (26 and 43) are used for adjustment of weight parameters with initial conditions for all the wavelet parameters set to zero. To get an insight of the controller performance under the condition of saturation, the simulation is carried out in two phases. During the first phase, it is assumed that the actuator is saturation free, the simulation is carried out with following initial conditions and controller settings:

$$x(0) = [0.5, 0, -0.5, 0, 0.8, 0]^T$$

$$k = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 1.21 \\ 3.37 \\ 4.10 \end{bmatrix}; a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 1.5 \\ 2.2 \end{bmatrix}; \rho = \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix} = \begin{bmatrix} 1.3 \\ 1 \end{bmatrix}; c = 1;$$

$$\kappa = 0.02; u_{1\max} = 8; u_{2\max} = 6$$

while learning rates for the wavelet networks are taken as $\sigma=0.5$ and $\gamma=0.5$. Simulation results are shown below in figure (1, 2 and 3). Convergence of system states and filtered tracking error to the close neighborhood of origin within a short span of time can be observed. During the second phase of simulation, actuator saturation is considered and the simulation is carried out for same initial conditions and controller parameter settings. Results obtained are shown in figure (4, 5, and 6). As clearly reflected by the figures, system performance is almost similar to that obtained during the first phase. Initially the control efforts undergo saturation thereby causing a slight deterioration of the system performance and introducing rapidly rising nonlinearities. However, due to the promising capability of the wavelet network to approximate such nonlinearities accurately and rapidly, the control efforts reshape themselves and come out of the saturation within a short span of time and thereafter retune the system response rapidly to its original unconstrained form. As the controller scheme tackles the actuator saturation in antiwindup paradigm, the performance deterioration does not disturb the system stability and thus the system stability is preserved.

5. Conclusion

This paper presents an adaptive control scheme to solve the control problem of a class of uncertain multi- input multi- output underactuated systems with actuator saturation. Controller scheme presented is composed of some hierarchically derived control terms which are meant to ensure the stabilization of certain subsystems whereas few controllers are assigned as dedicated controllers to some of the subsystems. Control terms so derived guarantee the uniform ultimate boundedness of all the closed loop signals. Wavelet neural networks are used to approximate the unknown nonlinear system dynamics as well as the nonlinear dynamics invoked by the actuator saturation. Convergence analysis of the error dynamics is carried out in the Lyapunov sense. Effectiveness of the controller scheme is illustrated through the simulation.

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