



A GAME-THEORETICAL INTEGRATED APPROACH FOR SUSTAINABLE PORTFOLIO SELECTION: AN APPLICATION ON BIST PARTICIPATION SUSTAINABILITY INDEX STOCKS

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Abstract: Sustainable investment is a hot topic of portfolio selection. This study aims to examine sustainable portfolio selection for conservative investors using the ESG criteria. Thus, we propose a two-stage integrated approach based on two-player zero-sum games. In the first stage, we use a fuzzy multi-criteria decision making (MCDM) approach to calculate the sustainability scores of the stocks based on expert knowledge. In the second stage, we form and solve a linear optimization problem by only adding a sustainability constraint to Young's minimax portfolio selection model. We illustrate the integrated approach using the weekly simple returns of eight stocks. We also compare our results with the results of Young's minimax portfolio selection model. We find that sustainable investment does not necessarily lead to performance loss. Furthermore, it may increase performance in some cases. To the best of our knowledge, this is the first paper on sustainable portfolio selection that depends only on two-player zero-sum games, including the stage of finding sustainability scores.

Keywords: Fuzzy set, Game theory, Multi-criteria decision making, Participation index, Portfolio selection, Sustainability

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1. Introduction

Sustainability is an integrated approach that includes ecological, economic, and social elements (Ok, 2022). It focuses on justice between current and future generations in the consumption of resources. It is defined as the capacity to meet today's needs without compromising the standards of future generations (Mckeown et al., 2002). Sustainable investment is "an investment discipline that considers environmental, social, and governance (ESG) criteria" (Qi and Li, 2020). Markowitz (1952) changes portfolio theory almost entirely by introducing the mean-variance (MV) portfolio selection model. On the other hand, it is not attractive for practitioners due to estimation errors (Goldfarb and Iyengar, 2003). Robust MV models based on the worst-case analysis can be used to overcome this issue (Garlappi et al., 2006; Tütüncü and Koenig, 2004). The models based on expert knowledge can also be used. We give the Bayesian approach introduced by Jorion (1986), the possibilistic MV model introduced by Carlsson et al. (2002), and the minimax model introduced by Ding (2006) as significant examples. The minimax model introduced by Young (1998) is another alternative to Markowitz's MV model. It is based on game theory and only uses historical data.

The models mentioned above are quantitative methods and ignore other criteria such as fundamental analysis,

sustainable investment, etc. On the other hand, Utz et al. (2014), Utz et al. (2015), Gasser et al. (2017), Hilario-Caballero et al. (2020), and Steuer and Utz (2023) modify Markowitz's MV model by adding a third criterion for sustainable investment. Pedersen et al. (2021) analyze the ESG - Sharpe ratio frontier. Ballesterio et al. (2012) use utility theory under uncertainty to integrate sustainable investment into portfolio selection. Qi and Li (2020) modify Markowitz's MV model by adding three additional constraints on the ESG criteria. There are also many approaches, such as a fuzzy multi-criteria model proposed by Calvo et al. (2016), an extended goal programming model proposed by Bilbao-Terol et al. (2018), an intuitionistic fuzzy MCDM approach proposed by Yadav et al. (2023), an intuitionistic fuzzy multi-objective optimization approach proposed by Hanine et al. (2021), and a multi-objective minimax-based portfolio optimization model proposed by Xidonas and Essner (2022).

The ESG criteria are considered as important indicators in measuring and reporting the sustainability performance of businesses (Şişman and Çankaya, 2021). Their emergence is based on socially responsible investors (Staub-Bisnang, 2012). Clearly, this study is intended for socially responsible investors, and there may be no reason for other investors to consider ESG criteria in portfolio selection. Kalayci et al. (2019) say



that Young's minimax model is one of the most important deterministic models, which overcomes the shortcomings of Markowitz's MV model. Thus, we determine it as a basis for this study, which focuses on sustainable investment.

We aim to examine sustainable portfolio selection for conservative investors using the ESG criteria. Our motivation is to achieve this aim with a tractable integrated approach. Thus, we first use a fuzzy MCDM approach, called as Game Theoretical Fuzzy Evaluation System (G-FES), to calculate the sustainability scores of the stocks. G-FES is related to a two-player zero-sum game (Göktaş and Gökerik, 2024). Then, we form and solve a linear optimization problem by only adding a sustainability constraint to Young's minimax portfolio selection model. Both stages of the integrated approach are worst-case-oriented and suitable for conservative investors. To the best of our knowledge, this is the first paper on sustainable portfolio selection that depends only on two-player zero-sum games, including the stage of finding sustainability scores.

The rest of the paper is organized as follows. Section 2 gives the theories of G-FES and Young's minimax portfolio selection model. Section 3 illustrates the integrated approach using the weekly simple returns of eight stocks included in the BIST services and BIST participation sustainability indexes. Section 4 concludes the paper.

2. Materials and Methods

2.1. G-FES

If fuzzy numbers form the decision matrix, fuzzy MCDM methods are used (Chu and Lin, 2009). The membership function of the triangular fuzzy number (c, d, e) is as in Equation 1.

$$\mu(t) = \begin{cases} 1 - \frac{d-t}{d-c}, & c \leq t < d \\ 1, & t = d \\ 1 - \frac{t-d}{e-d}, & d < t \leq e \\ 0, & \text{else} \end{cases} \quad (1)$$

The different views of multiple experts are brought together with G-FES. Linguistic variables and their crisp equivalents are shown in Table 1 (Göktaş and Gökerik, 2024).

The steps of G-FES are as follows (Göktaş and Gökerik, 2024).

Step 1: Using the linguistic variables in Table 1, expert views are taken for each alternative-criterion pair. (There are n alternatives and m criteria.)

Step 2: For the i^{th} alternative and j^{th} criterion pair, their minimum rating is assigned as c_{ij} , their median rating is assigned as d_{ij} , their maximum rating is assigned as e_{ij} , and the fuzzy utility is determined as the triangular fuzzy number (c_{ij}, d_{ij}, e_{ij}) . Then, a fuzzy decision matrix $A_{n \times m}$ is formed.

For the i^{th} alternative and j^{th} criterion pair, the utility's fuzzy mean (m_{ij}) is as in Equation 2, where $E_F()$ is the fuzzy mean operator (Carlsson et al., 2002).

Table 1. Linguistic variables

Linguistic variables	Corresponding crisp number
Extremely good (EG)	1
Very good (VG)	0.75
Good (G)	0.5
A little good (LG)	0.25
Fair (F)	0
A little poor (LP)	-0.25
Poor (P)	-0.5
Very poor (VP)	-0.75
Extremely poor (EP)	-1

$$m_{ij} := E_F \left((c_{ij}, d_{ij}, e_{ij}) \right) = \frac{c_{ij} + 4d_{ij} + e_{ij}}{6} \quad (2)$$

For the i^{th} alternative and j^{th} criterion pair, the utility's fuzzy standard deviation (s_{ij}) is as in Equation 3, where $STD_F()$ is the fuzzy standard deviation operator (Carlsson et al., 2002).

$$s_{ij} := STD_F \left((c_{ij}, d_{ij}, 1) \right) = \frac{1 - c_{ij}}{2\sqrt{6}} \quad (3)$$

Step 3: Fuzzy mean matrix $M_{n \times m} = (m_{ij})$ is formed using Equation 2.

Step 4: Fuzzy standard deviation matrix $S_{n \times m} = (s_{ij})$ is formed using Equation 3.

Let the decision maker's two linear objectives maximize the utility's fuzzy mean and minimize the utility's fuzzy standard deviation. The payoff matrix (P) is defined as in Equation 4, where the first objective's weight (w) is in [0,1]. If the decision maker is risk-neutral i.e. the fuzzy mean is maximized, w equals 1. If the decision maker's risk aversion degree is at the highest level i.e. the fuzzy standard deviation is minimized, w equals 0. If it is at the medium level, w equals 0.5.

$$P := wM - (1-w)S \quad (4)$$

Step 5: Using Equation 4, the payoff matrix (P) is formed for $w \in [0,1]$.

Due to the linearity of the objectives, the weighted objective function equals $x^T P y$, where x and y are the nonnegative weight vectors of the alternatives and criteria, respectively. (The row vector x^T is the transpose of the column vector x.) Then, G-FES is defined with Expression 5, corresponding to a two-player zero-sum game with the payoff matrix $P_{n \times m} = (p_{ij})$ (Göktaş and Gökerik, 2024).

$$\max_x \min_y x^T P y \quad (5)$$

The solution of Expression 5 for the row player (decision-maker) equals the optimal solution of Expression 6, which is a linear optimization problem (Raghavan, 1994; Chen and Larbani, 2006; Sikalo et al.,

2022). Its optimal solution may not be unique, but we assume it is unique in this study.

$$\begin{aligned}
 & \max t \\
 & \text{s.t. } \sum_{i=1}^n p_{ij}x_i \geq t, \text{ for all } j \\
 & \sum_{i=1}^n x_i = 1 \\
 & x_i \geq 0, \text{ for all } i
 \end{aligned} \tag{6}$$

The dual problem of Expression 6 is as in Expression 7. Its optimal solution (y^*) equals the weight vector of the criteria. That is, G-FES objectively determines the criteria weights.

$$\begin{aligned}
 & \min t \\
 & \text{s.t. } \sum_{j=1}^m p_{ij}y_j \leq t, \text{ for all } i \\
 & \sum_{j=1}^m y_j = 1 \\
 & y_j \geq 0, \text{ for all } j
 \end{aligned} \tag{7}$$

Step 6: The alternatives' priority vector (x^*) is found by solving Expression 6. The criteria's weight vector (y^*) is found by solving Expression 7.

Step 7: According to their priority values, alternatives are ranked and/or resources are distributed to the alternatives. In this study, we use them as sustainability scores (ss_i). The stock is more preferable for socially responsible investors if its sustainability score is higher. This study uses the ESG criteria to get the sustainability scores.

2.2. Young's Minimax Portfolio Selection Model and Its Extension

Let $R_{n \times z} = (r_{ik})$ be the simple return matrix for n assets and z periods. Young's minimax portfolio selection model is in Expression 8 (Young, 1998). It gives the solution of a two-player zero-sum game for the row player (investor), where the payoff matrix is $R_{n \times z}$ and the column player is the market (Sikalo et al., 2022). Expression 8 finds the portfolio that maximizes the worst-case return based only on the historical data.

$$\begin{aligned}
 & \max t \\
 & \text{s.t. } \sum_{i=1}^n r_{ik}x_i \geq t, \text{ for all } k \\
 & \sum_{i=1}^n x_i = 1 \\
 & x_i \geq 0, \text{ for all } i
 \end{aligned} \tag{8}$$

In the integrated approach, we use Expression 9 to consider sustainability where $\alpha \in [0,1]$ is a scalar that shows the target level of the portfolio's sustainability score, and ss_i is the sustainability score of the i^{th} stock. Expression 9 finds the portfolio that maximizes the worst-case return based on the historical data and the

sustainability constraint.

$$\begin{aligned}
 & \max t \\
 & \text{s.t. } \sum_{i=1}^n r_{ik}x_i \geq t, \text{ for all } k \\
 & \sum_{i=1}^n x_i = 1 \\
 & \sum_{i=1}^n x_i ss_i \geq \alpha \\
 & x_i \geq 0, \text{ for all } i
 \end{aligned} \tag{9}$$

Remark: Since the sustainability scores depend on the decision maker's (investor's) risk aversion degree (w) due to G-FES, the optimal solution of Expression 9 also depends on it.

3. Results and Discussion

In this section, we illustrate the integrated approach using the weekly simple returns of eight stocks included in the BIST services and BIST participation sustainability indexes. The main limitations of the integrated approach can be listed as follows. The first limitation is that it may not be suitable for non-conservative investors due to the worst-case-orientation. The second limitation is that it cannot be used when short selling is allowed. The third limitation is that historical data may reflect the future poorly. The fourth limitation is that there is no formal procedure to determine the decision-maker's risk aversion degree (w) in G-FES. The fifth limitation is that G-FES can only be used with triangular fuzzy numbers. The sixth limitation is that the experts may poorly evaluate the alternatives' ESG, and the use of expert knowledge may not be practical if the stock number is high.

3.1. G-FES

In this subsection, we use G-FES to calculate the sustainability scores of the eight stocks based on expert knowledge. (Expert knowledge is used by Yadav et al. (2023) to calculate the sustainability scores. But, their intuitionistic fuzzy MCDM approach is not worst-case oriented, unlike G-FES.) These stocks are AKSEN, BIMAS, DOAS, ENJSA, MAVI, MPARK, PGSUS and THYAO. They are evaluated by five experts using publicly available information before 01.01.2023 (the start of the testing period), where the criteria are environmental issues (C1), social issues (C2), and governance issues (C3). See Refinitiv (2023) for detailed information about them.

Step 1: We take five experts' views for each alternative-criterion pair based on the linguistic variables in Table 1. Table 2 shows the first expert's views. (The experts are either the academicians or sector professionals.) For example, the linguistic ratings of AKSEN are poor (P) for C1, good (G) for C2, and extremely good (EG) for C3.

Step 2: Using the minimum, median, and maximum ratings of expert views for each alternative-criterion pair, we form the fuzzy decision matrix (A) as in Table 3. For

example, the minimum rating for the AKSEN - C3 pair is fair (F), the median rating for the AKSEN - C3 pair is a little good (LG), and the maximum rating for the AKSEN - C3 pair is extremely good (EG).

Table 2. The first expert's views

	C1	C2	C3
AKSEN	P	G	EG
BİMAS	LG	G	LG
DOAS	EG	F	LG
ENJSA	LG	P	G
MAVI	LG	F	G
MPARK	LP	F	LG
PGSUS	LP	G	F
THYAO	LG	LG	G

Table 3. The fuzzy decision matrix (A)

	C1	C2	C3
AKSEN	(-0.75, -0.5, 0)	(-0.25, 0, 0.75)	(0, 0.25, 1)
BİMAS	(0, 0.5, 0.75)	(0.25, 0.25, 0.75)	(0, 0.25, 0.75)
DOAS	(0.5, 0.75, 1)	(-0.25, 0, 0.5)	(0.25, 0.25, 0.5)
ENJSA	(0, 0.25, 0.75)	(-0.5, -0.25, 0.5)	(-0.25, 0.5, 1)
MAVI	(0.25, 0.5, 0.75)	(0, 0.25, 1)	(0, 0.5, 0.75)
MPARK	(-0.5, -0.25, 0.5)	(-0.25, 0.25, 0.5)	(0, 0.5, 0.5)
PGSUS	(-0.25, 0.25, 0.5)	(0, 0.5, 0.75)	(0, 0.25, 0.75)
THYAO	(-0.25, 0, 0.5)	(0, 0.25, 0.75)	(0, 0.25, 0.5)

Step 3: We form the fuzzy mean matrix (M) by using Equation 2. Here, $m_{13} = (0+4 \times 0.25+1)/6$.

Table 4. The fuzzy mean matrix (M)

	C1	C2	C3
AKSEN	-0.4583	0.0833	0.3333
BİMAS	0.4583	0.3333	0.2917
DOAS	0.7500	0.0417	0.2917
ENJSA	0.2917	-0.1667	0.4583
MAVI	0.5000	0.3333	0.4583
MPARK	-0.1667	0.2083	0.4167
PGSUS	0.2083	0.4583	0.2917
THYAO	0.0417	0.2917	0.2500

Step 4: We form the fuzzy standard deviation matrix (S) by using Equation 3. Here, $s_{13} = (1-0)/2\sqrt{6}$.

Table 5. The fuzzy standard deviation matrix (S)

	C1	C2	C3
AKSEN	0.3572	0.2552	0.2041
BİMAS	0.2041	0.1531	0.2041
DOAS	0.1021	0.2552	0.1531
ENJSA	0.2041	0.3062	0.2552
MAVI	0.1531	0.2041	0.2041
MPARK	0.3062	0.2552	0.2041
PGSUS	0.2552	0.2041	0.2041
THYAO	0.2552	0.2041	0.2041

Step 5: Using Equation 4, we form the payoff matrix (P) as follows. $P = M$ when $w = 1$, $P = -S$ when $w = 0$, $P = (M-$

$S)/2$ when $w = 0.5$. Clearly, w equals the weight of maximization of fuzzy mean, whereas $1-w$ equals the weight of minimization of fuzzy standard deviation.

Step 6: By solving Expression 6 for different payoff matrices, we uniquely find the priority vectors (x^*) as in Table 6. When $w = 1$, the fuzzy mean (the first moment of the fuzzy utility) is maximized. That is, the focus is to maximize the central tendency. When $w = 0$, the fuzzy standard deviation (the square root of the second moment of the fuzzy utility) is minimized. That is, the focus is to minimize the risk. When $w = 0.5$, there is a balance of these objectives.

Table 6. The priority vectors

	w = 1	w = 0.5	w = 0
AKSEN	0	0	0
BİMAS	0	0.0190	0.6667
DOAS	0	0	0.3333
ENJSA	0	0	0
MAVI	0.6000	0.5666	0
MPARK	0	0	0
PGSUS	0.4000	0.4144	0
THYAO	0	0	0

By solving Expression 7 for different payoff matrices, we uniquely find the criteria's weight vectors (y^*) as in Table 7. y^* leads to the minimum payoff for x^* .

Table 7. The weight vectors of the criteria

	w = 1	w = 0.5	w = 0
C1	0.3000	0.1736	0
C2	0.7000	0.7066	0.3333
C3	0	0.1198	0.6667

Step 7: When the decision maker is risk neutral ($w = 1$), the sustainability scores of MAVI and PGSUS are 0.6000 and 0.4000, respectively. Other scores equal 0. When the decision maker's risk aversion degree is at a medium level ($w = 0.5$), the sustainability scores of MAVI, PGSUS, and BIMAS are 0.5666, 0.4144, and 0.0190, respectively. Other scores equal 0. When the decision maker's risk aversion degree is at the highest level ($w = 0$), the sustainability scores of BIMAS and DOAS are 0.6667 and 0.3333, respectively. Other scores equal 0.

3.2. Young's Minimax Portfolio Selection Model and Its Extension

Table 8 shows the summary statistics for the weekly simple returns of the eight stocks in the training period (2022). We use this data set and the information provided by Section 3.1 in step 7 to derive optimal portfolios. We take the first three quarters of 2023 as the testing period.

Table 9 shows the results when $w = 1$. The weight of MAVI (AKSEN) increases (decreases) with the increase in the target sustainability level (α) of Expression 9. Young's (1998) optimal portfolio has a sustainability score of 0.0358 in this case. When α increases, the relative weight

of the ESG criteria increases with respect to the weight of the absolute value of worst-case return.

Table 10 shows the results when $w = 0.5$. The results are similar to the results in Table 9. The optimal portfolios consist of only MAVI and AKSEN when α equals or

exceeds 0.2. MAVI has the maximum sustainability score when $w = 0.5$ or $w = 1$. 71.41% of Young's (1998) optimal portfolio consists of AKSEN. This portfolio has a sustainability score of 0.0374 in this case.

Table 8. The summary statistics

	Minimum	Maximum	Median	Mean	Standard Dev.
AKSEN	-0.0610	0.2595	0.0170	0.0321	0.0635
BIMAS	-0.0754	0.1860	0.0122	0.0170	0.0550
DOAS	-0.1058	0.2099	0.0239	0.0316	0.0705
ENJSA	-0.1064	0.1419	0.0179	0.0215	0.0562
MAVI	-0.1024	0.1810	0.0105	0.0277	0.0600
MPARK	-0.0947	0.1802	0.0179	0.0230	0.0559
PGSUS	-0.1164	0.1957	0.0419	0.0365	0.0747
THYAO	-0.0733	0.2458	0.0317	0.0402	0.0653

Table 9. The optimal portfolios for risk-neutral investors

	Young (1998)	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$
AKSEN	0.7141	0.7173	0.6667	0.5000	0.3333	0.1667	0
BIMAS	0.0156	0	0	0	0	0	0
DOAS	0	0	0	0	0	0	0
ENJSA	0.0446	0	0	0	0	0	0
MAVI	0	0.1571	0.3333	0.5000	0.6667	0.8333	1.0000
MPARK	0.1362	0.1112	0	0	0	0	0
PGSUS	0.0895	0.0144	0	0	0	0	0
THYAO	0	0	0	0	0	0	0

Table 10. The optimal portfolios when the risk aversion degree is at a medium level

	Young (1998)	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$
AKSEN	0.7141	0.7168	0.6470	0.4705	0.2941	0.1176	
BIMAS	0.0156	0	0	0	0	0	
DOAS	0	0	0	0	0	0	
ENJSA	0.0446	0	0	0	0	0	No feasible solution
MAVI	0	0.1737	0.3530	0.5295	0.7059	0.8824	solution
MPARK	0.1362	0.1057	0	0	0	0	
PGSUS	0.0895	0.0038	0	0	0	0	
THYAO	0	0	0	0	0	0	

Table 11. The optimal portfolios when the risk aversion degree is at the highest level

	Young (1998)	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$
AKSEN	0.7141	0.6448	0.5339	0.3920	0.2419	0.0898	0
BIMAS	0.0156	0.1096	0.2369	0.3847	0.5380	0.6928	0.8742
DOAS	0	0.0808	0.1262	0.1305	0.1239	0.1145	0.0517
ENJSA	0.0446	0	0	0	0	0	0
MAVI	0	0.0176	0.0233	0.0146	0.0020	0	0
MPARK	0.1362	0.0720	0.0125	0	0	0	0
PGSUS	0.0895	0.0752	0.0672	0.0781	0.0941	0.1030	0.0742
THYAO	0	0	0	0	0	0	0

Table 11 shows the results when $w = 0$. The weight of BIMAS (AKSEN) increases (decreases) with the increase in α . Young's (1998) optimal portfolio has a sustainability score of 0.0104 in this case. BIMAS has the maximum sustainability score when $w = 0$. In this case, we derive more diversified optimal portfolios due to fuzzy standard

deviation orientation.

Table 12 shows the results for training and testing periods respectively, when $w = 1$. WCR is the worst-case return, and AR is the average return. Young's portfolio selection model in Expression 8 and its extension in Expression 9 minimize the risk (the absolute value of

WCR) for the training period. AR is the natural return measure. If portfolio A has lower risk and higher return than portfolio B, then we say that A is superior to B, or equivalently B is inferior to A. Table 12 also shows the inferior or superior portfolios to Young's (1998) optimal portfolio when $w = 1$. This portfolio is generally superior to others in the training period. On the other hand, this information is not valid for the testing period.

Table 13 shows the inferior or superior portfolios to

Young's (1998) optimal portfolio when $w = 0.5$. This portfolio is generally superior to others in the training period. On the other hand, this information is not valid for the testing period.

Table 14 shows the inferior or superior portfolios to Young's (1998) optimal portfolio when $w = 0$. This portfolio is generally superior to others in the training period. The opposite is true for the testing period.

Table 12. The risk-return analysis for the risk-neutral investors

	Young (1998)	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$
Tra-WCR	-0.0262	-0.0307	-0.0419	-0.0570	-0.0722	-0.0873	-0.1024
Tra-AR	0.0305	0.0304	0.0306	0.0299	0.0291	0.0284	0.0277
Training P.	N/A	Inferior	N/A	Inferior	Inferior	Inferior	Inferior
Test-WCR	-0.1117	-0.1156	-0.1214	-0.1242	-0.1279	-0.1316	-0.1353
Test-AR	0.0034	0.0035	0.0046	0.0074	0.0103	0.0131	0.0159
Testing P.	N/A	N/A	N/A	N/A	N/A	N/A	N/A

Table 13. The risk-return analysis when the risk aversion degree is at a medium level

	Young (1998)	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$
Tra-WCR	-0.0262	-0.0312	-0.0437	-0.0597	-0.0757	-0.0917	
Tra-AR	0.0305	0.0304	0.0305	0.0297	0.0290	0.0282	
Training P.	N/A	Inferior	Inferior	Inferior	Inferior	Inferior	No feasible solution
Test-WCR	-0.1117	-0.1160	-0.1209	-0.1249	-0.1288	-0.1327	
Test-AR	0.0034	0.0035	0.0049	0.0079	0.0109	0.0139	
Testing P.	N/A	N/A	N/A	N/A	N/A	N/A	

Table 14. The risk-return analysis when the risk aversion degree is at the highest level

	Young (1998)	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$
Tra-WCR	-0.0262	-0.0282	-0.0315	-0.0349	-0.0384	-0.0421	-0.0544
Tra-AR	0.0305	0.0300	0.0285	0.0265	0.0243	0.0220	0.0192
Training P.	N/A	Inferior	Inferior	Inferior	Inferior	Inferior	Inferior
Test-WCR	-0.1117	-0.1057	-0.0985	-0.0901	-0.1041	-0.1372	-0.1758
Test-AR	0.0034	0.0049	0.0073	0.0104	0.0138	0.0172	0.0199
Testing P.	N/A	Superior	Superior	Superior	Superior	N/A	N/A

Since the integrated approach adds a sustainability constraint to Young's minimax portfolio selection model, we expect an inevitable increase in the risk (the absolute value of WCR) and a potential decrease in the return (AR) for the training period due to the nature of the optimization. On the other hand, this expectation is not valid for the future (testing period). That is, sustainable investment does not necessarily lead to performance loss. It is compatible with the results given by Hilario-Caballero et al. (2020), Utz et al. (2014), Utz et al. (2015), and Qi and Li (2020). Furthermore, it may increase performance in some cases, as in this study. It is compatible with the results given by Pedersen et al. (2021) and Xidonas and Essner (2022). On the other hand, Ballesteros et al. (2012) find that sustainable investment increases the risk, whereas Gasser et al. (2017) find that it decreases the return. The models in the studies mentioned above are not intended for conservative investors except for the model proposed by Xidonas and Essner (2022). This information may

increase the relative importance of the integrated approach intended for conservative investors.

4. Conclusion

Game theory is a widely used decision-making tool. Since two-player zero-sum games depend on minimax optimization problems, it is convenient for conservative decision-makers. Sustainability is a nonnegligible concept for socially responsible investors. ESG criteria are accepted as important indicators of sustainability. Thus, this study proposes an integrated approach using ESG criteria and two-player zero-sum games for conservative investors' sustainable portfolio selection. We also illustrate the integrated approach with a real-world example. Our findings show that sustainable investment does not necessarily lead to performance loss than conventional investment. Furthermore, it may result in increased performance in some cases. These results are compatible with the many studies in the literature.

Socially responsible investors can prefer the integrated approach to Young's minimax model, which is one of the key models in deterministic portfolio selection. This is because the integrated approach's second stage is formed by only adding a sustainability constraint to Young's minimax portfolio selection model. The fundamental analysis could be combined into the integrated approach in future research.

Author Contributions

The percentage of the author(s) contributions is presented below. The author reviewed and approved the final version of the manuscript.

	F.G.
C	100
D	100
S	100
DCP	100
DAI	100
L	100
W	100
CR	100
SR	100
PM	100
FA	100

C=Concept, D= design, S= supervision, DCP= data collection and/or processing, DAI= data analysis and/or interpretation, L= literature search, W= writing, CR= critical review, SR= submission and revision, PM= project management, FA= funding acquisition.

Conflict of Interest

The author declared that there is no conflict of interest.

Ethical Consideration

Ethics committee approval was not required for this study because of there was no study on animals or humans.

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