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Compensation of Errors in Multi-Channel Analog-to-Digital Convertors at Signal Processing of Multidimensional Arrays

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Özet. Çıktı sinyalinin çok boyutlu dizin veya bir çok tek boyutlu dizinler şeklinde olduğu, çok kanallı analog sayısal çevirgeç (ADC)'lerde hataların kaynakları ve telafi yöntemleri dikkate alınmıştır. Analog sayısal çevirgeçlerde yaygın olan bir çok kanal için örnekle ve tut devresinde hata kaynağı analiz edilmiştir. Sinyal kaynağı'nın geometrik koordinatları algoritması zaman ve frekans bölgesinde hataların telafi yöntemleri analiz edilmiştir.

Anahtar Kelimeler. Hata analizi, örnekle ve tut devresi, ADC, veri dizini analizi.

Abstract. Sources of the errors and compensating methods at multi-channel analog-to-digital convertor (ADC), in which an output signal is a many-dimensional array or set of the one-dimensional arrays, are considered. The source of the error in the common sample-and-hold (SH) circuit of the modern ADC for several channels is analyzed. Methods of errors compensation in time and frequency domains for the algorithm of geometrical coordinates of a signal source determining are analyzed.

Keywords. Error analysis, sample-and hold circuit, ADC, data array analysis.

1. Introduction

In many practical applications the analog signals, which are collected from an array of sensors, are converted to a digital format and further considered as a many-dimensional array, or set of the one-dimensional arrays. Typically, the multi-channel ADC, which is one of the embedded units of the modern signal microprocessor, is used to convert the signal from an analog to a digital format. Here is analyzed inaccuracy of multichannel ADC with the common SH circuit for all analog inputs [1]. Sampling of the signal in each channel is performed with a time shift on value of s seconds, defined by the ADC parameters. The signal processing algorithm is based on the measurement of phase differences between pairs of channels to determine the geometrical coordinates of a signal source [2]. In this case, a sample delay in time

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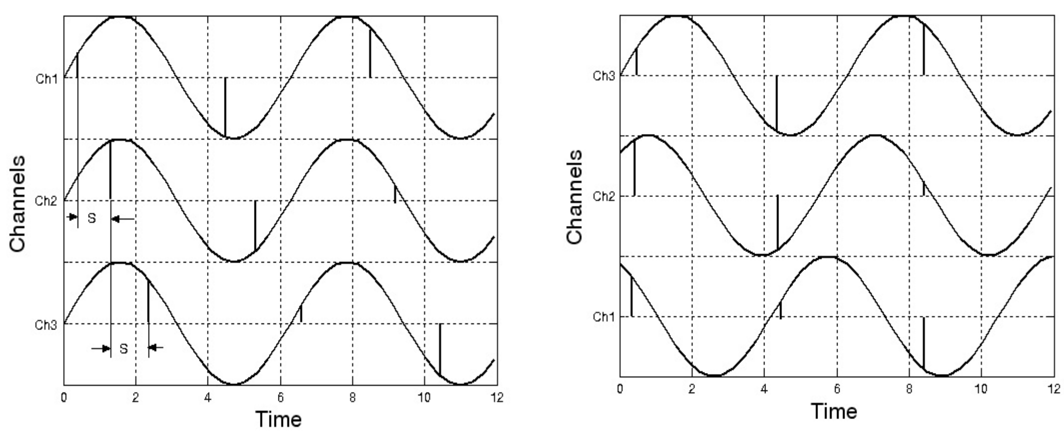
for channels connected to the same SH circuit leads to the presence of systematic errors, and these errors depend on signal frequency. To illustrate, the nature of these errors let us apply the same sine wave to all inputs of an ADC with common SH circuit, where the moment of sampling is shifted in time (Figure 1a) on value of s seconds. Having applied identical signals on ADC inputs in continuous time, we process data arrays that are shifted by phase with respect to each other in discrete time (Figure 1b).



(a) Time shift between channels (b) Processed signals

FIGURE 1. An illustration of a signal phase shift between adjacent channels.

The goal of this paper is to measure the time shifts between channels, analyze phase errors of conversion and then compensate them in determining the coordinates of the signal sources.

2. Effect of the Time Shift in SH Circuit on a Phase Shift of Analyzed Signal

The parameter s is set programmatically and is selected by the system designer within specified bounds. Typically, this value is defined by ADC clocking frequency of the microprocessor and can range from 1 to M periods. Naturally, the phase shift significantly changes and depends on the frequency of the signal to be converted and on the frequency of synchronizing sequence.

The same harmonic signal is applied to inputs of two adjacent channels of an ADC

$$f(t) = A\sin(\omega t + \varphi). \tag{1}$$

If a sample of the input signal of the first channel is taken in an instant $t = t_1$, the measured amplitude of the signal has the value

$$u_1 = A\sin(\omega t_1 + \varphi). \tag{2}$$

Sample of the same signal on the second channel is taken in an instant $t_2 = t_1 - s$. In this equation s is equal to nT, where T - period of clocking frequency of the microprocessor ($1 \leq n \leq M$). Accordingly, the magnitude of the sample in the second channel is

$$u_2 = A\sin(\omega t_2 + \varphi) = A\sin[\omega(t_1 - nT) + \varphi]. \tag{3}$$

It is obvious that the measured magnitudes of the samples on the outputs are different. The phase shift between these channels has the value

$$\Delta\psi = (\omega t_1 + \varphi) - [\omega(t_1 - nT) + \varphi] = \omega nT. \tag{4}$$

The value of phase shift in (4) depends only on signal frequency, as the parameter nT for all samples of an array is the same. Dependence of T on other reasons (temperature, oscillator crystal stability etc.) is not analyzed here.

The diagram of a Figure 2 demonstrates the normalized phase shift $\Delta\psi/T = \omega n$ between two adjacent channels for $1 \leq n \leq 4$, at $T = 10^{-6}$ sec.

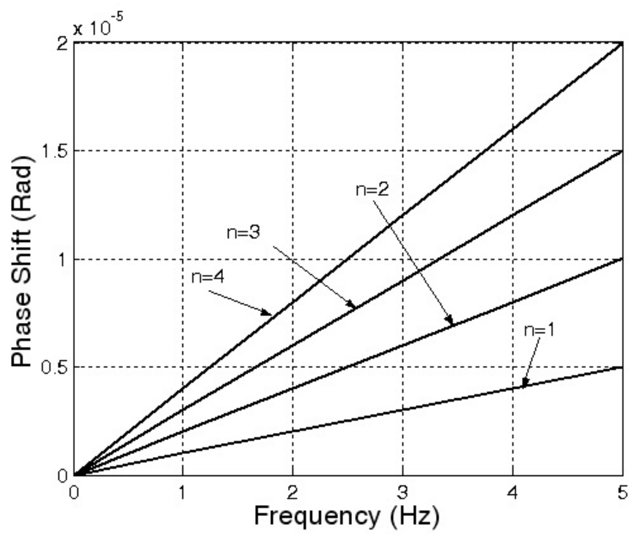


Figure 2. The normalized phase shift between adjacent channels for harmonic signal for the range of frequencies from 0 to 5 Hz.

Such an error in the analysis of low-frequency or narrowband signals, within certain limits, is negligibly small. However, the introduced phase shift can make a significant impact on the inaccuracy of calculated results for signals in the wide frequency range. It is obvious that the phase shift between the first and the last channels connected to the same SH is k times more, where k is the number of the channels connected to SH unit.

3. Estimation of Error in Defining Signal Source Coordinates Caused by Phase Shifts between Channels

The analysis of the method of measurement of coordinates of a signal source is given in [2] where relationships for determining the position of signal source are based on measurement of the phase difference between pairs of sensors. An array of equally spaced sensors is placed on a two-dimensional plain. We analyze the error estimation of measurement caused by systematic delay in the sampling moment of different channels that use the same SH unit. Actually, if, for example, the two-dimensional array of sensors has dimensionality of 16×16 analog signal receivers (microphones), digital conversion should be fulfilled by the device, having few SH units. Otherwise the phase shift between channels for an array of 256 sensors becomes rather significant. Without considering the circuit diagram of such a specific ADC, we discover the solution for a system of minimal configuration. To estimate the measurement error of the signal source coordinates which is in point D, phase differences between four sensors, placed on a plain, in points with coordinates in three dimensional space $O(0, 0, 0)$, $A(d, 0, 0)$, $B(0, d, 0)$ and a $C(d, d, 0)$ are analyzed (Figure 3). Supposing that the same signal is present in inputs of all ADC channels, as it was earlier, in cases when the phase shifts caused by SH are absent, the coordinates of signal source D are on equidistant spacing interval from all sensors, on line ED (ambiguity of determination of coordinates of this point D on a z-axis is eliminated by adding an additional sensor, or applying a spectral algorithm for an array of sensors). In the presence of SH for all four ADC channels, coordinates of a signal source D are drifted, as is shown in a Figure 3, the time shift for the sensor allocated in a point O, caused by the real SH misses, in a point A - constitute nT, in points B and C - values $2nT$ and $3nT$ accordingly.

The source of a harmonic signal of frequency ω is allocated in the arbitrary point D with coordinates (x_0, y_0, z_0). Then the value of the modulus of a vector OD is

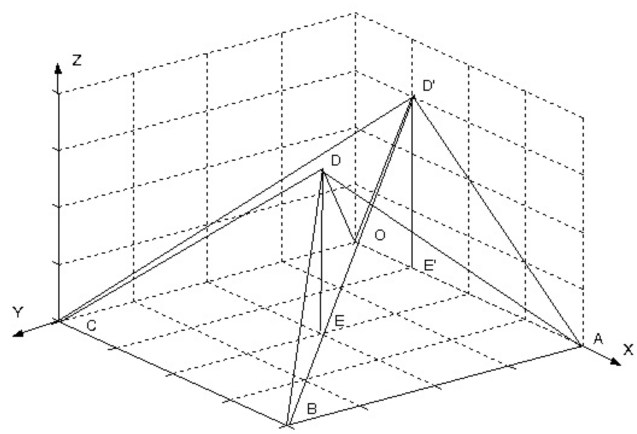


FIGURE 3. Position of the signal source (D - without the multi-channel SH, D' - taking into account influencing of the multi-channel SH).

defined by the formula

$$\rho = \sqrt{x_0^2 + y_0^2 + z_0^2}. \tag{5}$$

Because of a time shift introduced by SH, the signal source is shifted in point D'. By analogy with [2], we introduce the distance differences from D' to the sensor in point A and placement points of other sensors as

$$\begin{aligned} \Delta l_1 &= D'A - D'O = \sqrt{(x_0' - d)^2 + y_0'^2 + z_0'^2} - \rho' \\ \Delta l_2 &= D'B - D'O = \sqrt{x_0'^2 + (y_0' - d)^2 + z_0'^2} - \rho' \\ \Delta l_3 &= D'C - D'O = \sqrt{(x_0' - d)^2 + (y_0' - d)^2 + z_0'^2} - \rho', \end{aligned} \tag{6}$$

where x_0', y_0' and z_0'-coordinates of point D', ρ'-modulus of a vector $D'O$. Having used the solution for coordinates of the signal source obtained in [2], we have

$$\rho' = \frac{1}{2}\frac{(\Delta l_1^2 + \Delta l_2^2 - \Delta l_3^2)}{(\Delta l_3 - \Delta l_1 - \Delta l_2)}, \tag{7}$$

$$x_0' = \frac{d}{2} + \frac{\rho'}{d}\frac{(\Delta l_3 \Delta l_1^2 + \Delta l_1 \Delta l_2^2 - \Delta l_2 \Delta l_1^2 - \Delta l_1 \Delta l_3^2)}{(\Delta l_3^2 - \Delta l_1^2 - \Delta l_2^2)}, \tag{8}$$

$$y_0' = \frac{d}{2} + \frac{\rho'}{d}\frac{(\Delta l_3 \Delta l_2^2 + \Delta l_2 \Delta l_1^2 - \Delta l_1 \Delta l_2^2 - \Delta l_2 \Delta l_3^2)}{(\Delta l_3^2 - \Delta l_1^2 - \Delta l_2^2)}, \tag{9}$$

$$z_0' = \sqrt{\rho'^2 - x_0'^2 - y_0'^2}. \tag{10}$$

The connection of the linear differences with phase shifts is defined by the equation

$$\Delta l_i = \frac{1}{2\pi}\lambda\omega\Delta t_i = \frac{1}{2\pi}\lambda\Delta\psi_i, \tag{11}$$

where Δl_i - the differences introduced in expressions (6), λ - wavelength of frequency ω, Δt_i - a time shift in point of measurement with respect to a point O, and $\Delta\psi_i$-the phase shift at frequency ω.

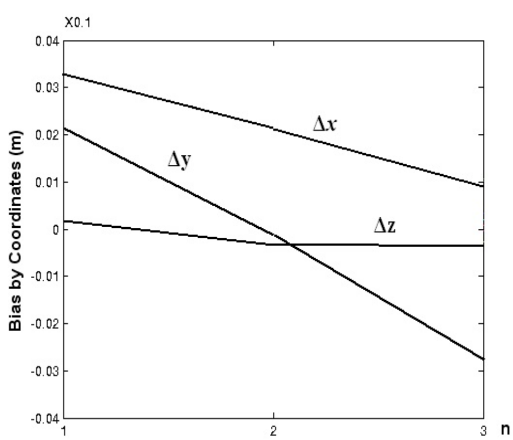
Deflection of the computed coordinates of a signal source from a real signal source can be calculated as

$$\Delta x = x_0' - x_0 = \frac{\rho'}{d}\frac{(\Delta l_3 \Delta l_1^2 + \Delta l_1 \Delta l_2^2 - \Delta l_2 \Delta l_1^2 - \Delta l_1 \Delta l_3^2)}{(\Delta l_3^2 - \Delta l_1^2 - \Delta l_2^2)}, \tag{12}$$

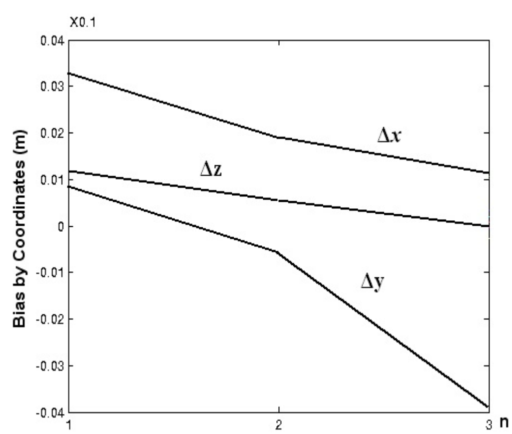
$$\Delta y = y_0' - y_0 = \frac{\rho'}{d}\frac{(\Delta l_3 \Delta l_2^2 + \Delta l_2 \Delta l_1^2 - \Delta l_1 \Delta l_2^2 - \Delta l_2 \Delta l_3^2)}{(\Delta l_3^2 - \Delta l_1^2 - \Delta l_2^2)}, \tag{13}$$

$$\Delta z = z_0' - z_0 = \sqrt{\rho'^2 - x_0'^2 - y_0'^2} - \sqrt{\rho^2 - x_0^2 - y_0^2}. \tag{14}$$

The graphical illustration of errors of coordinates measuring of the signal source for various values n is shown on Figure 4, where $x_0 = y_0 = 1/2$ cm, $z_0 = 10$ cm, $d = 1$ cm and $T = 10^{-6}$ seconds. The inaccuracy of evaluation of coordinates of a signal source at usage of 0, 1, 2 and 3 channels of an ADC (Figure 4a), and 0, 2, 6 and 7 channels (Figure 4b) is shown. It is assumed that all channels work with the same SH circuit, and the harmonic signal of frequency of 10 Hz is applied.



(a) Channels 0, 1, 2, 3



(b) Channels 0, 2, 6, 7

FIGURE 4. The error of coordinates measuring of a signal source for the different sets of the ADC channels.

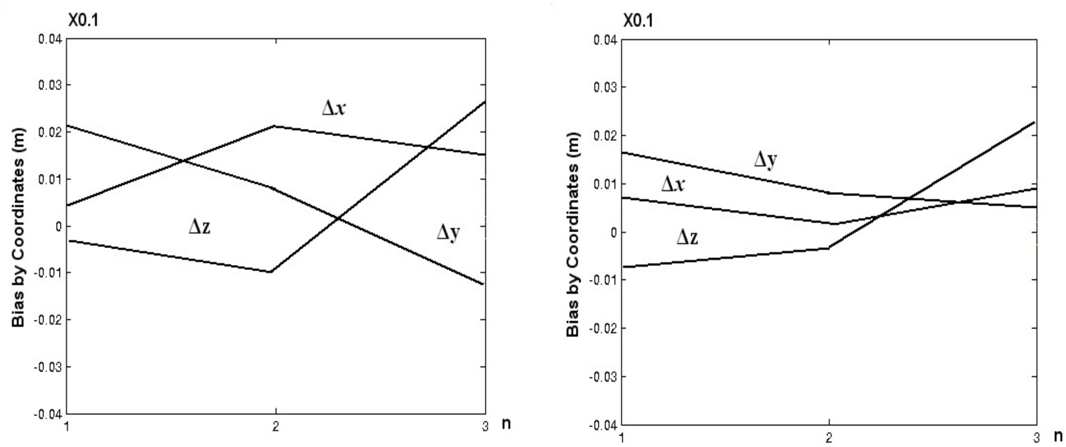
Despite having an approximately equal order of inaccuracy of measurement, their absolute values depend on what ports of an ADC are used in the evaluation of the coordinates of a signal source. Both figures demonstrate inaccuracy of measuring of the same frequency of a signal source. At the same time, this error, as appears from expression (11), depends on the frequency of a signal source, and is enlarged with frequency ascending. Real signals are not monochromatic, and they contain a spectrum of components. The error at evaluation for each component will differ significantly. In order to obtain the true result of computation, these errors should be taken into account and must be corrected. Depending on the applied algorithm at the digital signal processing, the correction method can be realized both in time and frequency domains.

4. A Phase Shift Error Reduction in Time Domain

If in input of j-th port of an ADC acts the signal of arbitrary form $u_j(t)$, the signal samples $u_j[m]$ of an array on output will be taken at instants of time $mT_d - nT$, where T_d - sampling period and m - integer number, that defines index number of array element. We need to compensate the time shift caused by a component nT.

Actually, to realize correction in time-domain, it is necessary to define new values of samples of the array to be processed by using known methods of interpolation. Then, knowing that the time delay caused by a common SH circuit for each of channels is constant, its compensation does not cause problems, and preciseness of evaluations is ensured with a selection of an optimum method of interpolation [3]. If ADC is reprogrammed, i.e., diverse values of n for adjacent channels of ADC are entered, this argument should be automatically transmitted to interpolation routine, to realize an indispensable correction. Increasing the preciseness of interpolation may be achieved by increasing of mathematical manipulations. Loss of additional time, which is not a bottleneck for the modern processors, allows within certain limits, the realization of information processing in real time mode. Experimental research on the effects of compensation has been studied for square and cubic interpolation. The inaccuracy cancelation effect appears to be the best for the "plain" parts of a signal associated with a low-frequency range in the signal spectrum, and is worse for sections with "sharp" magnitude variations where high-frequency components of the signal are observed. If the sampling frequency f_d is selected according to Nyquist criterion $f_d \geq 2f_{\max}$ where $f_{\max}$-maximum frequency in a spectrum of an signal analyzed, there is a larger number of samples on a period for low-frequency components than for high-frequency ones.

For these experiments the signal introduced in the previous section has been taken at the same layout of sensors. Measurement of inaccuracies of coordinates of a signal source has been made at usage 0, 1, 2 and 3 channels of an ADC. Figure 5 illustrates an absolute value of deflection of measured geometrical coordinates of a signal source for all axes from its real position, for square (a) and cubic (b) methods of interpolation. The experiment demonstrates how even the elementary methods of interpolation diminish error in coordinate measurement of a signal source by the order, which is sufficient for practical application.



(a) Square interpolation (b) Cubic interpolation

FIGURE 5. The error reduction in measuring of coordinates of a signal source in time domain.

5. A Phase Shift Error Reduction in Frequency Domain

If the algorithm of a signal array analysis supposes the subsequent secondary processing in frequency domain, then the compensation of an error of signal sources coordinates measurement can be made in the same domain. All signal samples will be taken at instants of time $mT_d - nT$, because of multi-channel ADC with common SH unit. From this, in reality, modified signal $u'_j(t) = u_j(t - nT)$ will be processed. Denoting spectrum of this signal as $U'_j(j\omega)$, we can write

$$U'_j(j\omega) \overset{F}{\Leftrightarrow} u'_j(t). \tag{15}$$

Due to property of a time shift [4], spectrum $U'_j(j\omega)$ of the signal is presented by equation

$$U'_j(j\omega) = U_j(j\omega)e^{-j\omega nT} \tag{16}$$

where $U_j(j\omega)$ - spectrum of signal with compensated time shift, and the factor $e^{-j\omega nT}$ indicates a phase shift caused by time delay nT. Multiplying both sides by factor $e^{j\omega nT}$ gives

$$U_j(j\omega) = U'_j(j\omega)e^{j\omega nT}. \tag{17}$$

Replacing complex spectrum $U'_j(j\omega)$ by its magnitude $|U'_j(j\omega)|$ and phase $\varphi(\omega)$ we have

$$U_j(j\omega) = |U'_j(j\omega)|e^{j\varphi(\omega)}e^{j\omega nT} = |U'_j(j\omega)|e^{j[\varphi(\omega)+\omega nT]}. \tag{18}$$

Equation (18) shows the algorithm of error compensation in frequency domain. The phase of each spectrum component must be corrected by adding the value of ωnT. Experimental research on error compensation in frequency domain has been carried out for the signal introduced earlier and is shown in Figure 6. The order of an absolute value of an error is approximately the same as for compensation method in a time domain, which is expected.

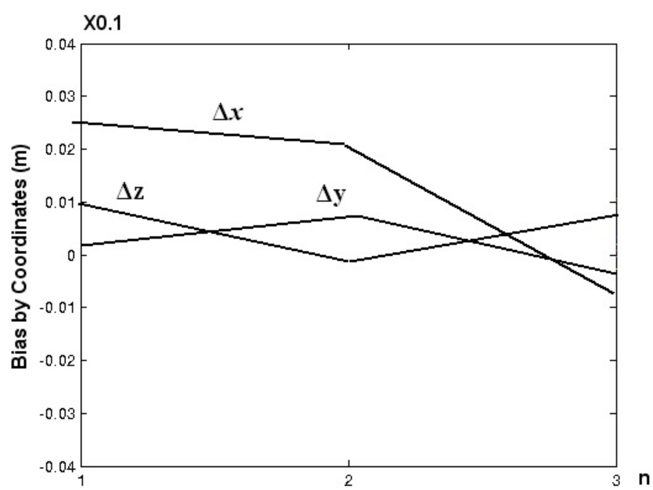


FIGURE 6. The error reduction in measurement of coordinates of a signal source in frequency domain.

6. Summary

The inaccuracies generated by a multi-channel ADC with a common SH unit for several channels have been analyzed. Methods of compensation of these errors in time and frequency domains applied to algorithm of measuring of geometrical coordinates of a signal source location on the basis of measuring of phase shifts have been considered. It is shown that error compensation in frequency domain for this algorithm is more effective when compared to that in time domain. For the algorithms based on signal processing in time domain, without conversion to frequency domain, the method of a signal interpolation is easier and more effective for narrow-band signals for which the elementary methods of interpolation (square, cubic) may be recommended. For wide-band signals, methods of interpolation of higher orders are more effective.

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