Turkish Journal of Science & Technology
Volume 13(1), 65-68, 2018

# Multy Variable Grey Method For Multy Point Deformation Analysis

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**(Received: 15.02.2017, Accepted: 01.04.2017)**

**Abstract**

Grey theory is one of the methods used to study uncertainty. The uncertain systems characterized by small sample and poor information are the study object of grey system theory. Multivariable grey prediction models are part of grey forecasting system. They are presented if there are mutual relations among the factors in the system. They believe that all the influencing factors are not independent of each other and should be regarded as a whole. In multivariable grey forecasting models, the future value of a variable is tried to be forecasted considering the other influential factors in the system. In this study, deformation consisting on the crest of a Dam is aimed to determine by using multivariable grey prediction models.

**Keywords**: Deformation, Grey method, Deformation predict

## Çok Noktalı Deformasyon Analizi İçin Çok değişkenli Gri Sistem

**Özet**

Gri teori, belirsizliği incelemek için kullanılan yöntemlerden biridir. Sınırlı bilgi ve küçük örnekler ile karakterize edilmiş belirsiz sistemler, gri sistem teorisinin inceleme konusudur. Çok değişkenli gri tahmin modelleri gri tahmin sisteminin parçasıdır. Sistemdeki faktörler arasında karşılıklı ilişkiler varsa onlar temsil edilirler. Etkileyen faktörler birbirinden bağımsız değildir ve bir bütün olarak kabul edilmektedir. Çok değişkenli gri tahmin modeli, bir değişkenin gelecek değerini sistemdeki diğer etkilenen faktörleri göz önüne alarak tahmin etmeye çalışmaktadır. Bu çalışmada, çok değişkenli gri tahmin modeli kullanarak bir barajın kreti üzerinde oluşan deformasyonların tahmin edilmesi amaçlanmıştır.

**Anahtar Kelimeler:** Deformasyon, Gri yöntem, Deformasyon tahmini

## 1. Introduction

The evaluation of measurements and geodetic deformation measurements covers a significant portion of the engineering measurements. The monitoring of the movements of big engineering structures begins during the construction of the building and continues throughout life. The evaluation of the data and the interpretation phase of the results is the last and most important part of the deformation study. There are damages that can't be compensated by a wrong decision. As a result, it is necessary to be very careful and the results must be absolutely reliable. A large number of measures are needed to be able to give decisions about the behavior of constructions and to interpret the results. But, being able to make more accurate decisions with less data is very important in every circumstance.

The main strength of grey prediction is that it only requires short-term, current and limited data. Grey systems theory was first proposed by Deng [1]. Grey theory is one of the methods used to study uncertainty. The uncertain system characterized by small sample and poor information are the study object of grey system theories. The grey system puts each stochastic variable as a grey quantity that changes within a given range. It does differ from statistical analysis method to deal with the grey quantity. It deals directly with the original data and searches the intrinsic regularity of the data [2].

## 2. Materials and Methods

### 2.1. Multy Variable Grey Forecast Model

Multivariable grey prediction models are part of grey forecasting system. They are presented if there are mutual relations among the factors in the system. They believe that all the influencing factors are not independent of each other and should be regarded as a whole. In multivariable grey forecasting models, the future value of a variable is tried to be forecasted considering the other influential factors in the system. Because of high accuracy and easy application procedures, multivariable grey models have been applied to many areas in the literature [3-9].

MGM (1, N) model is set up by using Accumulating Generation Operator (AGO). The primitive data are subjected to the AGO to smooth the randomness of the data and to weaken the tendency of variation [10]. Assume that $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(m)\}$ is an original, non-negative data series taken in consecutive order and at equal time intervals, then $X^{(1)}=\{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(m)\}$ is called 1-AGO where $x^{(1)}(k) = \sum_{j=1}^{k} x^{(0)}(j)$, $k = 1, 2, \ldots m$. Suppose that the number of variable is represented by $n$ and the number of observation period by $m$, MGM(1,N) procedure is described in the following parts [10]. The first-order ordinary differential equation for the MGM (1, N) can be written as follows:

$$\begin{aligned}
\frac{dx_1^{(1)}}{dt} &= a_{11}x_1^{(1)} + a_{12}x_2^{(1)} + \ldots + a_{1m}x_m^{(1)} + b_1 \\
\frac{dx_2^{(1)}}{dt} &= a_{21}x_1^{(1)} + a_{22}x_2^{(1)} + \ldots + a_{2m}x_m^{(1)} + b_2 \\
&\ldots\ldots\ldots\ldots\ldots\ldots \\
\frac{dx_n^{(1)}}{dt} &= a_{n1}x_1^{(1)} + a_{n2}x_2^{(1)} + \ldots + a_{nm}x_m^{(1)} + b_n
\end{aligned} \tag{1}$$

marked as:

$$\frac{dX_1^{(1)}}{dt} = AX^{(1)} + B \tag{2}$$

The estimate value is obtained by using the least square method:

$$H=(L^T L)^{-1} L^T Y \tag{3}$$

where

$$L = \begin{bmatrix} \bar{x}_1^{(1)}(2) & \bar{x}_2^{(1)}(2) & \ldots & \bar{x}_n^{(1)}(2) & 1 \\ \bar{x}_1^{(1)}(3) & \bar{x}_2^{(1)}(3) & \ldots & \bar{x}_n^{(1)}(3) & 1 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \bar{x}_1^{(1)}(m) & \bar{x}_2^{(1)}(m) & \ldots & \bar{x}_n^{(1)}(m) & 1 \end{bmatrix}$$

$$Y = \begin{bmatrix} x_1^{(0)}(2) & x_2^{(0)}(2) & \ldots & x_n^{(0)}(2) \\ x_1^{(0)}(3) & x_2^{(0)}(3) & \ldots & x_n^{(0)}(3) \\ \ldots & \ldots & \ldots & \ldots \\ x_1^{(0)}(m) & x_2^{(0)}(m) & \ldots & x_n^{(0)}(m) \end{bmatrix}$$

and $\bar{x}_i^{(1)}(k) = \frac{1}{2}(x_i^{(1)}(k) + x_i^{(1)}(k-1))$. The estimated value Â and $\hat{B}$ can be obtained from H:

$$\hat{A} = \begin{bmatrix} \hat{a}_{11} & \hat{a}_{21} & \cdots & \hat{a}_{n1} \\ \hat{a}_{12} & \hat{a}_{22} & \cdots & \hat{a}_{n2} \\ \ldots & \ldots & \ldots & \\ \hat{a}_{1m} & \hat{a}_{2m} & \cdots & \hat{a}_{nm} \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \ldots \\ \hat{b}_n \end{bmatrix} \tag{4}$$

Then accumulated predicted values can be calculated by using Equation (4).

$$\hat{X}^{(1)}(k) = e^{\hat{A}(k-1)}(\hat{X}^{(1)}(1) + \hat{A}^{(-1)}\hat{B}) - \hat{A}^{(-1)}\hat{B} \tag{5}$$

where

$$e^{\hat{A}(k-1)} = I + \sum_{i=1}^{\infty} \frac{\hat{A}^i}{i!}(k-1)^i \tag{6}$$

After the inverse first-order accumulated generation operation (1-IAGO), we obtain the sequence as $\hat{X}^{(0)}(k) = \hat{X}^{(1)}(k) - \hat{X}^{(1)}(k-1)$. The average fitting precision of the model is

$$\sigma^2 = \frac{\sum_{i=1}^{n} V_i^T V_i}{nm} \tag{7}$$

where

$$v_i k = x_i^{(0)}(k) - \hat{x}_i^{(0)}(k), V_i = (v_i(1), v_i(2), \ldots v_i(m))^T \tag{8}$$

## 3. Application

Multy Variable Grey Prediction Model which is very efficient methodology in the situation of the limited number of the data has been used to predict deformation in a dam. In this study 11 periods' real measurement values has been used to predict deformations depending on the water level for object points 11, 12 and 13 which are located in the middle of the rockfill dam crest. The first 6 periods' measurements have been used to establish multivariable grey prediction model, and the last six periods' measurement are used to test the accuracy of the proposed method. The original and predicted values are shown in Table 1. The outcomes also presented in Figure 1-3.

The 7$^{th}$ column in the Table 1 shows predicted values for object point 11 depending on the water level. 8$^{th}$ and 9$^{th}$ columns show the predicted values when we apply grey prediction procedure for two object points at the same time. The last three columns show the results if we apply prediction procedure for all object points in this study.

**Table 1.** Original and predicted values

| Years | Period | Heights of Point AS11 (m.) | Heights of Point AS12 (m.) | Heights of Point AS13 (m.) | Dam Lake Water Level (m.) | Multy Variable | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | MGM(1,2) | MGM(1,3) | | MGM(1,4) | | |
| | | | | | | AS11 | AS11 | AS12 | AS11 | AS12 | AS13 |
| | | | | | | Predicted value | Predicted value | Predicted value | Predicted value | Predicted value | Predicted value |
| 13.01.1975 | 1.Period | 851.659 | 851.727 | 851.796 | 803.960 | 851.660 | 851.659 | 851.727 | 851.659 | 851.727 | 851.796 |
| 21.03.1975 | 2.Period | 851.653 | 851.721 | 851.791 | 805.290 | 851.620 | 851.555 | 851.623 | 851.6049 | 851.673 | 851.7416 |
| 24.06.1976 | 3.Period | 851.540 | 851.605 | 851.662 | 842.090 | 851.564 | 851.391 | 851.456 | 851.475 | 851.539 | 851.597 |
| 01.06.1977 | 4.Period | 851.506 | 851.568 | 851.623 | 836.210 | 851.532 | 851.299 | 851.361 | 851.413 | 851.475 | 851.531 |
| 24.03.1978 | 5.Period | 851.493 | 851.554 | 851.610 | 828.040 | 851.411 | 851.231 | 851.292 | 851.379 | 851.441 | 851.496 |
| 20.11.1979 | 6.Period | 851.502 | 851.563 | 851.618 | 828.500 | 851.501 | 851.173 | 851.234 | 851.356 | 851.417 | 851.472 |
| **30.07.1980** | **Predicted.1** | **851.448** | **851.508** | **851.563** | **841.400** | **851.471** | **851.115** | **851.176** | **851.335** | **851.396** | **851.451** |
| **04.03.1982** | **Predicted.2** | **851.398** | **851.456** | **851.514** | **834.290** | **851.441** | **851.057** | **851.118** | **851.315** | **851.377** | **851.432** |
| **17.05.1982** | **Predicted.3** | **851.402** | **851.496** | **851.514** | **844.350** | **851.411** | **850.995** | **851.056** | **851.295** | **851.357** | **851.412** |
| **15.11.1984** | **Predicted.4** | **851.369** | **851.420** | **851.479** | **831.830** | **851.381** | **850.941** | **851.002** | **851.276** | **851.338** | **851.392** |
| **14.05.1985** | **Predicted.5** | **851.361** | **851.419** | **851.475** | **835.210** | **851.350** | **850.881** | **850.942** | **851.256** | **851.319** | **851.373** |

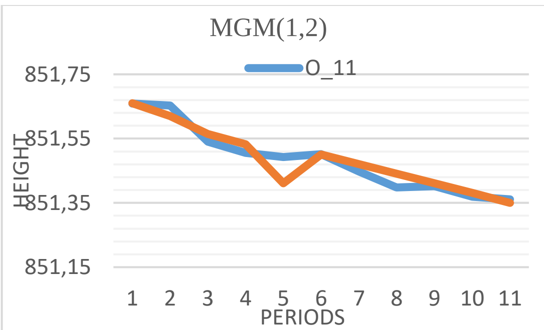


**Figure 1.** Multivariable Grey Model (1, 2)

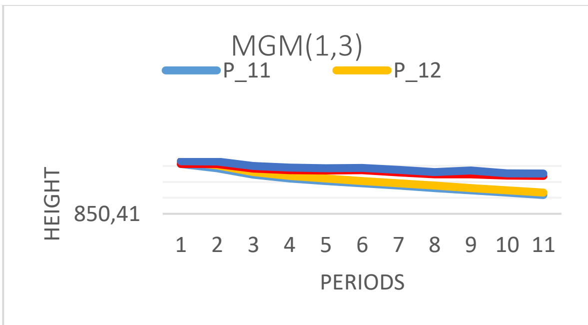


**Figure 2.** Multivariable Grey Model (1, 3)

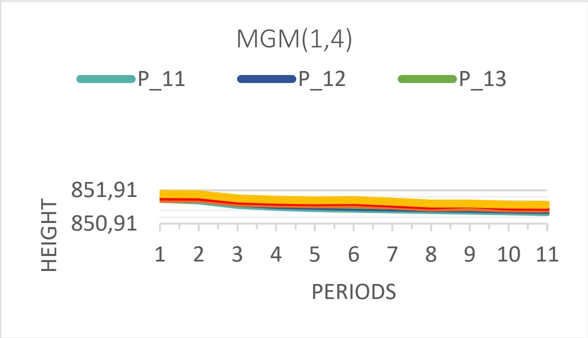


**Figure 3.** Multivariable Grey Model (1, 4)

## 4. Results

The Grey System Theory provides a solution to the problem even in the case of very limited number of data. The Multi Variable Grey Prediction Model can be used to predict deformation not only for one point but also for many points at the same time. In this study deformation level of the three different object points in a dam crest has been tried to be forecasted depending on the water level by multi variable grey model. Results show that there are great consistency between grey prediction values and real values. Based on the outcomes it is possible to conclude that grey prediction model is a very reliable prediction model in limited data circumstances.

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