

*Research Article*

Seismic performance of steel moment frames with variable friction pendulum systems under real ground motions

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ABSTRACT

Many researchers have already acknowledged that the base isolation system as the most feasible and economical method for civil engineering structures exposed to the seismic excitation. The Friction Pendulum Systems (FPS) have steel concave surface connected with articulated friction slider and utilized the concept of pendulum for lengthening the period of the superstructure so as to dissipate the seismic energy. The present study investigates on various design approaches for the evaluation of the seismic response of steel frames equipped with FPS. The response of isolated frames is simply adjusted by several parameters such as the friction coefficient (μ), the radius of curvature (R), the isolation period (T) and the axial load and so 2D, three bay 3 and 7-storey steel moment resisting frames (SMRF) are designated as isolated frames in order to examine the effect of variation of the R and the friction coefficient on the seismic response of the isolated frames. The R and μ are predefined as 1, 1.55, 2.25 and 0.025, 0.05, 0.1, respectively. The seismic response of the modelled isolation systems has been evaluated through nonlinear time history analyses, a set of ground motions using SAP2000 software. The local and global deformations are employed to compare the seismic performance of different isolation frames through nonlinear analysis. The results showed that the isolated frames having greatest radius of curvature with lowest friction coefficient exhibited better seismic performance than other models in terms of the local and global deformations.

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1. Introduction

In last decades, numerous researchers have developed several methods as to enhance the energy dissipation capacity of civil engineering structures. Among these methods, base isolation system (BIS) can be accepted the most popular seismic protective system that is extensively used [1-5]. BIS has been utilized for mitigating the destructive effects of the earthquakes on the structure that initially considered as one of the most effective approaches both in the design of the new buildings and in seismic retrofit of the existing buildings. The concept and theory of the base isolation is depended on decoupling the building from the ground and inserted the BIS in order to mitigate the catastrophic effect of the earthquakes. Commonly two type of base isolation systems; such as sliding systems and elastomeric bearings are available. The sliding systems are designed to dissipate the seismic

forces by providing frictional sliding and limiting the transfer of shear while the elastomeric bearings are designed to eliminate horizontal earthquake forces by providing a layer with low horizontal stiffness [6]. Recently, many analytical and experimental studies have been presented with the aim of finding out the seismic response of the FPS. For example, in the study of Landi et al. [7], several time history analyses were performed with near field records for different models of FPS, constructed by the predefined friction coefficient, sliding velocity and vertical force. The analysis results enabled to compare the models considering the influence of the earthquake component with the constant friction coefficient. Similarly, another numerical study was performed to examine the seismic response of elastically isolated frames with FPS designed by three different soil conditions corresponded to various frequency content under a set of 100 artificial earthquakes. It was obtained that greater friction coefficient was supposed in

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case of soft soil with lower isolation period [8]. In the study of Castaldo et al. [9], the seismic reliability of a 3D 4-storey reinforced concrete frame equipped with FPS was evaluated and designed in compliance with the FEMA 274 [10] code. The seismic performance of the frame with FPS satisfied the requirements of corresponding seismic codes by modified lower friction coefficient adjusted with lower uncertainty. In the analytical study of the Jangid [11] multi-storey frames isolated with FPS and analyzed under near fault records in order to find out the optimal design parameters of the FPS. The analysis results showed that the optimum friction coefficient of FPS has less acceleration demand with respect to the non-isolated frames. The optimal friction coefficient changed with the values of the bearing acceleration that is ranged from 5 to 15%.

The main aims of the study are to (i) examine the seismic performance of the steel frames isolated by FPS, (ii) compare the effectiveness of the base isolation systems against seismic excitation, (iii) show the variation of the isolation parameters of the FPS for 3 and 7-storey SMRFs with different isolation systems, and (iv) offer the optimum parameters of the FPS for minimum seismic response of the isolated system under earthquake records. In the light of the previous researchers, 3 and 7-storey SMRF equipped with FPS having wide range of the radius of curvatures (from 1 m to 2.25 m), isolation period (2 s to 3 s) and friction coefficients (from 0.025 to 0.10), these isolated frames were analyzed using three natural accelerograms compatible with seismic hazard levels of 2% probability of exceedance in 50 years. The organization of the present study is as follows: the analytical modeling details of the frames including the original frames and base-isolated frames (namely FPS) are given in Section 2. Additionally, the design parameters of the base isolation systems were given. The information on the earthquake records that are representatives of 2% probability of exceedance in 50 years was described. Then, the results of the nonlinear analysis were comprehensively discussed in Section 3. Finally, the results inferred from the analysis results are presented in a comparative manner in Section 4.

2. Modelling and Analysis of SMRF with FPS

Two different SMRF are designated as bare frames those have firstly designed by Ferraioli et al. [12] according to the Italian Code [13]. These frames have 2D, three bay 3 and 7-storey and the cross sections of the examined frames are given in Table 1. The more information about those fixed base frame can be found concerned paper. In this study, 3 and 7-storey SMRF are modified as base isolated frame with FPB.

Among the BIS, FPS has come into prominence by virtue of its supremacy behavior such as non-ageing, durability, maintenance, thermal condition; thus, it can be accepted as mostly used BIS [14]. It has an articulated and slider surface in contact with the curved frictional area is made of

composite material. The other side of the slider is attached to stainless steel concave, spherical surface and covered with moderate frictional materials [15]. The mechanism of FPS depend on the movement of the bearing along the curved surface and the supported mass is moved upward, thus the movement will provide the restoring force to the system. During a seismic excitation, the bearing goes along with curved surface hereby it moves in small arcs like a pendulum. The bearings decrease the transmission of the seismic forces to the structure by the movement of pendulum and also by frictional pad. The effective stiffness and the isolation period can be easily tuned by the radius of the concave surface.

2.1 Analytical models of frames

Analytical models of the fixed base and isolated frames were provided using nonlinear finite element program Sap2000 that is capable of performing nonlinear static and dynamic analyses [16]. To depict the hysteretic behavior of the isolators of the FPS a nonlinear finite element NLink called as friction pendulum isolator has been selected and the mechanical model of the FPS is rooted in the classic Coulomb theory.

Table 1. The details of 3 and 7-storey frames. [12]

Frames	Storey Level	Beams	Outer Column	Inner Column
3-storey	1	IPE270	HE160B	HE220B
	2	IPE270	HE160B	HE200B
	3	IPE270	HE160B	HE200B
7-storey	1	IPE270	HE180B	HE240B
	2	IPE270	HE180B	HE240B
	3	IPE270	HE180B	HE220B
	4	IPE270	HE160B	HE220B
	5	IPE270	HE160B	HE220B
	6	IPE270	HE160B	HE200B
	7	IPE270	HE160B	HE200B

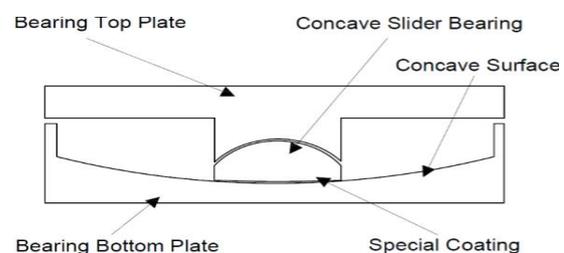


Figure 1. The Cross-section of FPS [14]

The cross-section and hysteretic behavior is plotted in Figures 1 and 2, respectively. For the base isolated models, two different bearings (inner and outer) were designed according to the vertical loads on the column. Designed FPS were replaced under each column of all evaluated models that include rubber isolator as a nonlinear link

element employed by Park et al. [17]. Thus, a total of 9 different cases were considered in this study. The design parameter of the FPS was computed by an iterative solution in compliance with Naeim and Kelly [5]. In the iterative procedure the following equations were employed. The load exerted on the isolator of the FPS is W , the horizontal displacement is D , and the friction coefficient is μ , then the resisting force F is given by

$$F = \frac{W}{R}D + \mu W(\text{sgn}D) \quad (1)$$

where R is the radius of curvature of the dish. The first term is the restoring force due to rise of the mass, providing a horizontal stiffness

$$k_2 = \frac{W}{R} \quad (2)$$

which produces an isolated structure period T given by

$$T = 2\pi\sqrt{\frac{R}{g}} \quad (3)$$

The equivalent stiffness is given by

$$k_{eff} = \frac{W}{R} + \frac{\mu W}{D} \quad (4)$$

effective damping, β_{eff} :

$$\beta_{eff} = \frac{2\mu}{\pi x \left(\frac{D}{R}\right) + \mu} \quad (5)$$

In this study, each isolation systems were labelled based on assumed isolation periods, T , radius of curvature, R and friction coefficient (μ) as to use in figures and throughout the rest of the text. For example, R1.55 μ 5 denotes an isolation system with $R = 1.55$ m, $T = 2.5$ s and $\mu = 0.05$. The isolation system parameters for 7-storey frame with FPS were presented in Table 2, where R ranges from 1 to 2.25 m, T from 2 to 3 s and μ from 2.5 to 10 %. Moreover, the isolation periods of 2, 2.5 and 3 s are corresponded to the radius of curvatures 1, 1.55 and 2.25 m, respectively (see Eqn 3). To determine the seismic performance of the frames with and without FPS, the nonlinear time history analysis was performed using the finite element program of SAP 2000 non-linear version 14 [16] in which FPS was assigned as Nlink (Friction Isolator) components located under the bottom columns of the superstructure, the view of 3-storey SMRF with FPS in SAP2000 was given in Figure 2. The design parameters of FPS calculated using Eqs (1-5) and put into SAP2000 as follows for R1.55 μ 5. Nonlinear Link Type: Friction Isolator, U1 Non/Linear Effective Stiffness: 200000 kN/m, U2 and U3 Linear and Nonlinear Effective Stiffness: 190, 4102 kN/m, U2 and U3 Friction Coefficient: 0,05, U2 and U3 Radius of Sliding Surface: 1.55.

Table 2. The outer (O) and inner (I) design values of 7-storey isolated frame with FPB

R (m)	T (s)	μ (%)	k_2 (kN/m)	D (m)	k_{eff} (kN/m)	β_{eff}	Side
1.0	2.0	2.5	232.2	0.17	265.1	0.08	O
		5	232.2	0.17	297.8	0.16	
		10	232.2	0.17	363.5	0.30	
	2.0	2.5	504.4	0.22	575.7	0.10	I
		5	504.4	0.22	646.9	0.20	
		10	504.4	0.22	789.5	0.36	
1.55	2.5	2.5	149.8	0.27	176.1	0.12	O
		5	149.8	0.27	202.3	0.27	
		10	149.8	0.27	254.8	0.42	
	2.5	2.5	325.4	0.17	382.4	0.08	I
		5	325.4	0.17	439.5	0.16	
		10	325.4	0.17	553.5	0.30	
2.25	3.0	2.5	103.2	0.22	125.1	0.10	O
		5	103.2	0.22	146.9	0.20	
		10	103.2	0.22	190.7	0.36	
	3.0	2.5	224.2	0.27	271.7	0.12	I
		5	224.2	0.27	319.2	0.23	
		10	224.2	0.27	414.3	0.42	

Furthermore, the frames were exposed to three real ground motion records. These time history records data of the earthquake force obtained from the Pacific Earthquake Engineering Research Center (PEER) [18] used as seismic input, called from text file to carry out the time history analysis. The details of the time history of the earthquake records were presented in Table 3. It should be considered that in the scaling process the mean code spectra or a set of earthquakes should be as close as possible to the mean spectrum. The aim of this is to have the accelerograms of the three earthquakes scaled to approximately the same intensity level such that the responses can be represented. Analyses were carried out using the ground motion records occurring 100% along x direction. The Newmark method with integration parameters $\gamma = 0.5$ and $\beta = 0.25$ and critical damping ratio of 5% were also assumed in the time history analysis.

Table 3. The details of the earthquake records

Earthquake Record	Hills	Cape	Northridge
Year	1978	1992	1994
Magnitude (Mw)	7.35	7.01	6.69
Mechanism	Strike-Slip	Reverse	Reverse
PGA (g)	0.418	0.615	0.795
R_{jb} (km)	0.9	0	0
R_{rup} (km)	0.9	8.2	5.3

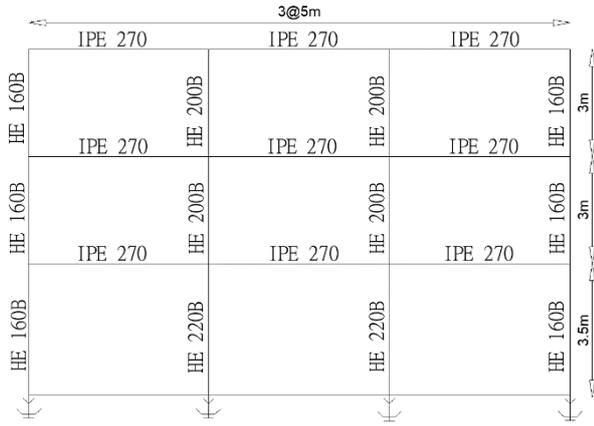


Figure 2. The view of 3-storey SMRF with FPS in SAP2000

3. Results and Discussion

This paper assessed the seismic response of SMRF with different FPS. The inelastic response of the frames with and without FPS are discussed concerning the roof displacement, relative displacement, interstorey drift ratio, absolute acceleration and hysteretic curves by means of the time history analyses. The analysis results showed that the response of frames with FPS changed with the radius of curvature R, friction coefficient μ , number of the storey and also the ground motion records. The analysis results are evaluated by series deformation parameters in Figures 3-17.

3.1 Displacement

The variation of the storey displacement of 3 and 7-storey frame exposed to three real ground motions are given in Figures 3 and 4, respectively. Further, the frames with FPS accommodated three different radiuses of curvature and friction coefficient are plotted in Figures 5 and 6 under Cape earthquake. It was clearly observed that decrease of the radius of curvature R (corresponding T) and friction coefficient μ are culminated in reduction on the roof displacement for 3 and 7-storey frames with FPS.

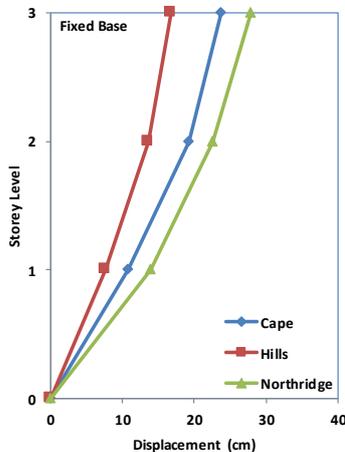


Figure 3. The displacement of 3-storey fixed base frame

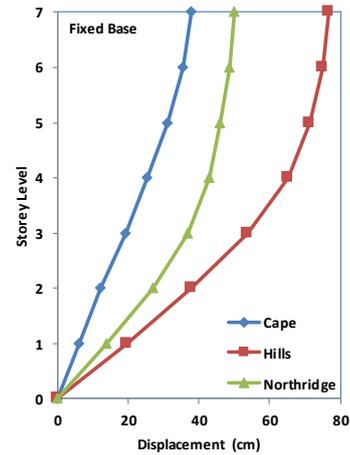


Figure 4. The displacement of 7-storey fixed base frame

The variation of the roof displacement demand for 3 and 7-storey frames was differentiated with the radius of curvature and friction coefficient due to assumption on the design parameters of FPS as shown in Table 2. Among the examined models, two lowest roof displacement of frames with the FPS was experienced in case of $R2.25\mu2.5$ when hit Cape earthquake as shown in Figures 5 and 6. For example, it was observed that the roof displacements of 3 and 7-storey frame with FPS were 23.73 and 37.05 cm, respectively.

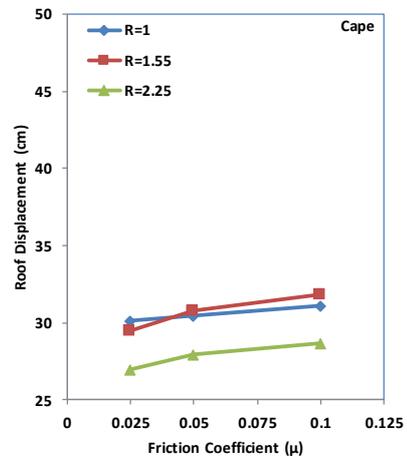


Figure 5. The roof displacement of 3-storey frames with FPS

Further, it was observed that when the friction coefficient was fixed and the radius of curvature was shifted from 1 m to 2.55 m (see Figure 5 and 6) the roof displacement demand of isolated frames was generally decreased parallel to descending trend of the isolators' horizontal and equivalent stiffness (namely, k_2 and k_{eff}) as shown in Table 2. Since increment of the radius of curvature led to enhancement of the isolation period and mitigation of the isolator stiffness, the isolated frames behave more ductile and the demand of the displacement increased as well. Similarly, the roof displacement demand of 3 and 7-storey frame with FPS was generally reduced when the radius of curvature was constant, and the friction coefficient was changed from 0.10 to 0.025 as shown in Figure 5 and 6, respectively.

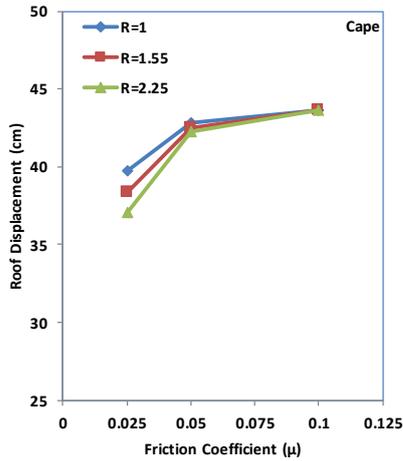


Figure 6. The roof displacement of 7-storey frame with FPS

3.2 Relative Displacement

The relative displacement is computed by subtracting the value of the roof displacement from the base displacement. The relative displacement with storey height of the frames with and without FPS under Northridge earthquake is generated for 9 various models and presented in Figures 7-9. The radius of 2.25 m (correspond to the isolation period of 3 s) with the friction coefficient of 2.5% is produced the lowest relative displacement for 3 and 7-storey isolated frames as shown in Figures 8 and 9. For example, when 3 and 7-storey fixed base frame equipped with $R2.25\mu2.5$ model, the relative displacement reduced from 27.89 cm to 6.83 cm, 50 cm to 19.55 cm respectively.

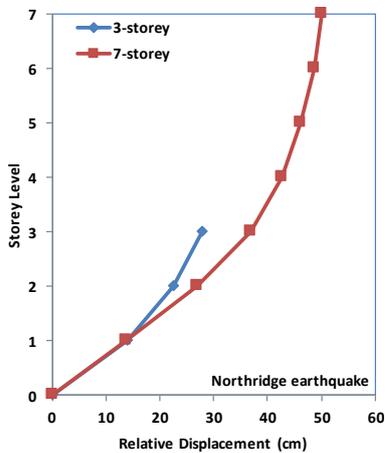


Figure 7. The relative displacement of 3 and 7-storey fixed base frames

The results proved that the use of FPS is very effective in reduction of the relative displacement. Moreover, $R2.25\mu2.5$ is significantly exhibited the most uniform distribution as shown in Figures 6 and 7. Similar to the observed trend on the variation of the displacement, greater radius of curvature R (corresponding T) and lower friction coefficient μ are caused to decrease the relative displacement for 3 and 7-storey frames with FPS under Northridge earthquake.

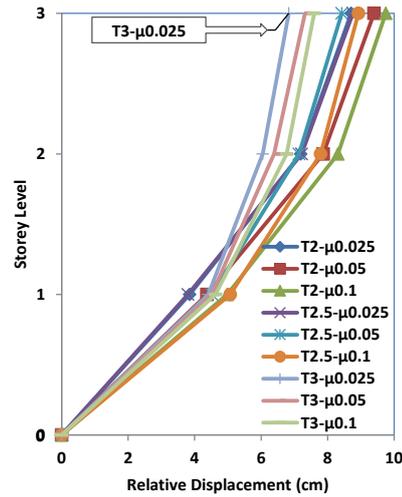


Figure 8. The relative displacement of 3-storey isolated frames against storey height under Northridge earthquakes

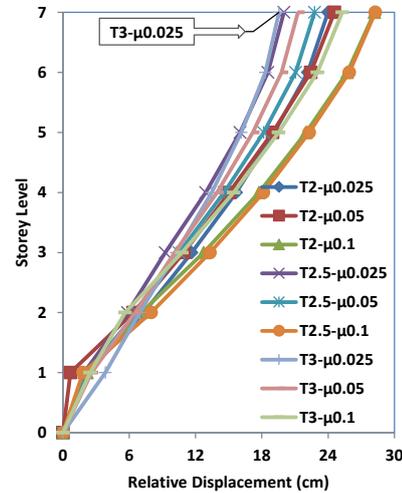


Figure 9. The relative displacement of 7-storey isolated frames against storey height under Northridge earthquakes

3.3 Interstorey drift ratio

The interstorey drift ratio can be admitted as significant benchmark for the seismic performance evaluation and also it can be used for the predicting the structural damage level. The variation of the interstorey drift ratio against storey height of the frames with and without under Hills earthquake is depicted in Figures 10 and 11. The interstorey drift ratio of the fixed base frames is regulated by means of the utilization of the FPS especially in case of $R2.25\mu2.5$. It tended to uniform distribution against other models and fixed base frames. Further, it was observed that the isolation system of $R2.25\mu2.5$ reduced the interstorey drift ratio from 2.2 to 1.4 % and from 6.1 to 1.8 % for 3 and 7-storey fixed base frames, respectively (see Figures 10(a) and 11(a)).

The maximum interstorey drift ratio of the frames equipped with FPS subjected to Cape earthquake obtained for 9 different models and plotted in Figures 12 and 13.

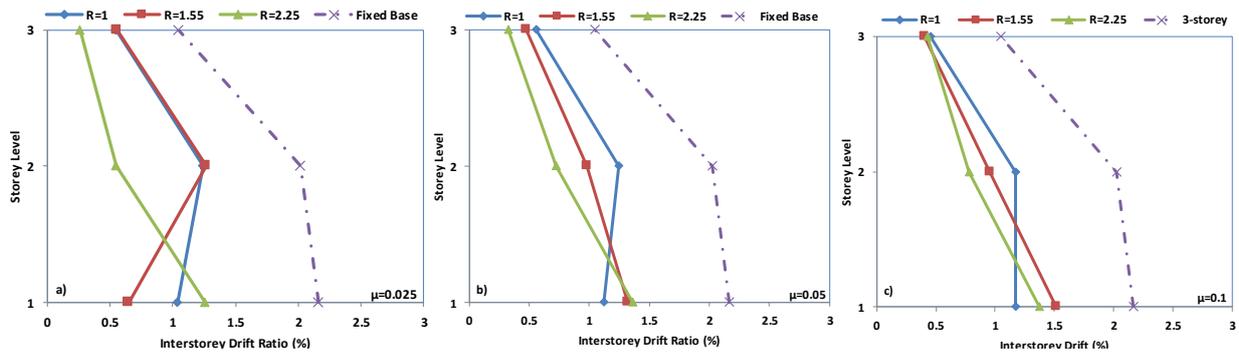


Figure 10. The variation of the interstorey drift ratio of 3-storey against storey height under Hills earthquakes

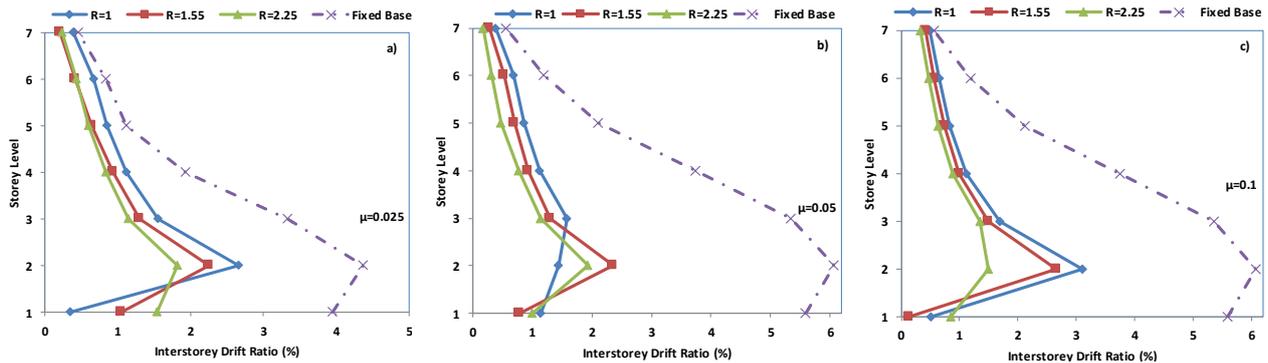


Figure 11. The variation of the interstorey drift ratio of 7-storey against storey height under Hills earthquakes

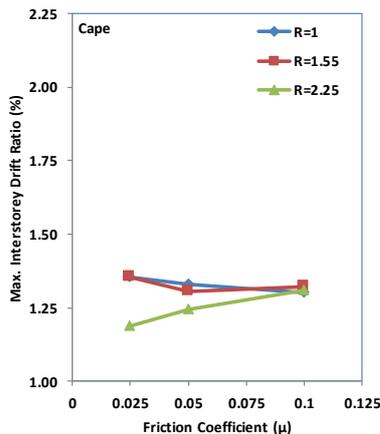


Figure 12. The maximum interstorey drift ratio of 3-storey frame

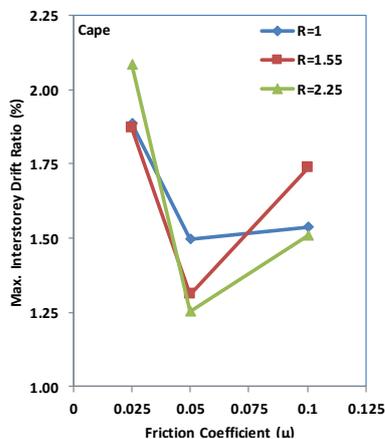


Figure 13. The maximum interstorey drift ratio of 7-storey frame

The the lowest maximum interstorey drift ratio of 1.18 % is experienced in case of R2.25 μ 5 for 3-storey frame with FPS as shown in Figure 12. Further the variation of the radius of curvature, friction coefficient and number of storey visibly fluctuated the drift ratios. However, an amplification trend is observed on the interstorey of 7-storey isolated frames when the friction coefficient varied from 2.5 to 5%. The utilization of optimum FPS in case of lower friction coefficient with greater radius of curvature testified the lower interstorey drift demand than other models as shown in Figures 12 and 13.

3.4 Absolute Acceleration

The variation of the absolute accelerations of the frames with and without FPS against storey height under Northridge earthquake is given in Figures 14 and 15. It was clearly observed that the increase of the isolation period, radius of curvature and friction coefficient led to great reduction on the corresponding storeys' absolute acceleration of 3 and 7-storey frame with FPS. The acceleration initially mitigated towards mid-height and then suddenly fluctuated towards the top storey. It is clearly observed that R2.25 μ 5 presented lowest absolute acceleration for 3 and 7-storey frame. For example, in case of R2.25 μ 2.5 that is caused to mitigate the absolute acceleration of 3 and 7-storey frame with FPS from 12 to 5.7 m/s² and 11.7 to 4.2 m/s², respectively. Further it assisted to tend more uniform distribution than other cases height of the storey especially for 7-storey frame with FPS.

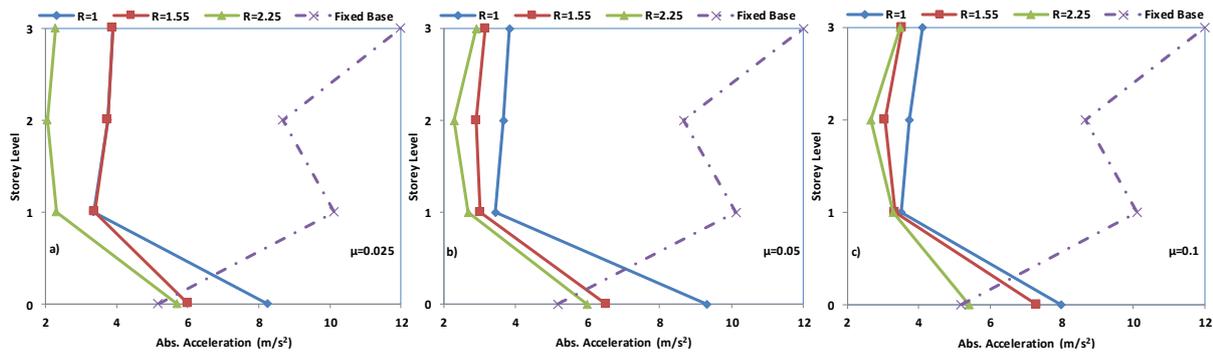


Figure 14. The variation of the absolute acceleration of 3-storey against storey height under Northridge earthquakes

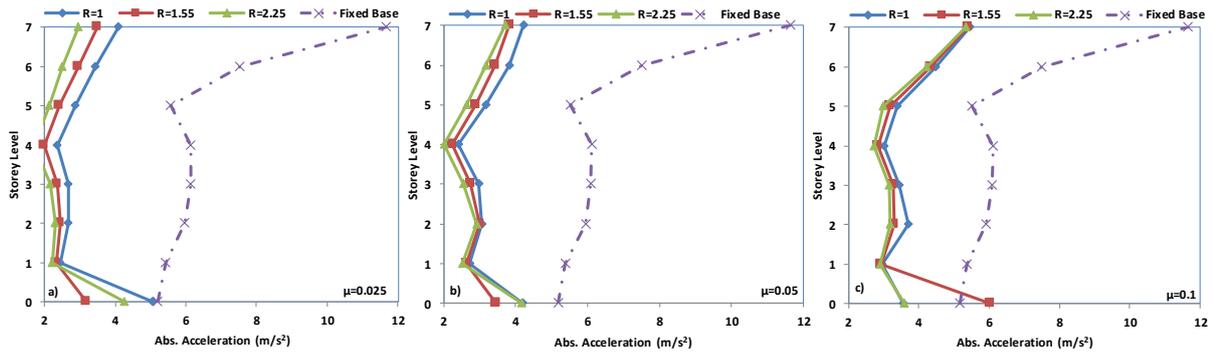


Figure 15. The variation of the absolute acceleration of 7-storey against storey height under Northridge earthquakes

3.5 Base Shear

The base shear demands of the fixed base 3 and 7-storey frames subjected to Cape earthquake are 461.7 and 506.2 kN, respectively. When each of the isolated frames subjected to the ground motions their responses are obtained. For example, the examined isolated frames subjected to Cape earthquake that is depicted in Figures 16 and 17. The use of FPS remarkably reduced the base shear especially 3-storey isolated frames. Besides the variation of the friction coefficient from 0.01 to 0.025, shifting the isolation period from 2.0 s to 3 s (corresponding the radius of curvature is 1m to 2.25 m) also caused steady reduction on the base shear. Additionally, almost similar responses are valid for 7-storey isolated frames as well (see Figure 17). When R2.25μ2.5 was implemented for 3 and 7-storey frames as isolation system that ensured the lowest base shear demand with respect to the other isolation systems as depicted in Figures 16 and 17.

The most remarkable behavior is observed under Cape earthquake, 3 and 7-storey frame with the isolation system of R2.25μ2.5 introduced the lowest base shear of 82.3kN and 164 kN, respectively. It is also noted that the aforementioned nonlinear analysis results on the base shear demand of the examined frames did not differentiate with the earthquake characteristics, similar trend is observed as well.

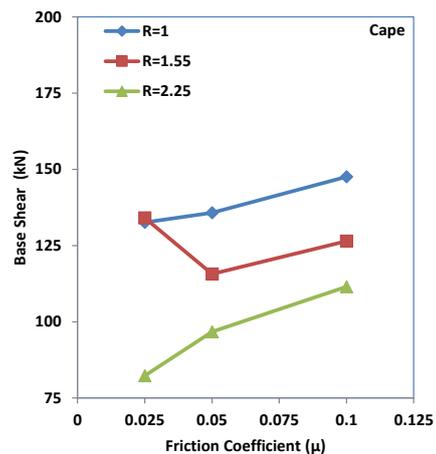


Figure 16. The base shear of 3-storey isolated frame with isolation period and Q/W ratio

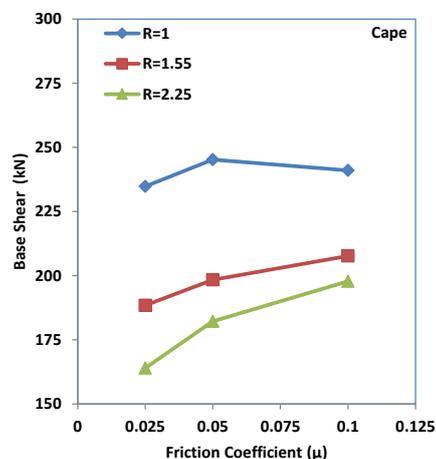


Figure 17. The base shear of 7-storey isolated frame with isolation period and Q/W ratio

3.6 Hysteretic Curve

The hysteretic curves of the 9 isolation models including different radius of curvature and friction coefficient for 3 and 7-storey frames under Hills earthquake is plotted in Figures 18 and 19. These obtained curves were similar to the bi-linear force-deformation as characteristic hysteretic curve of FPS. When the isolation parameters were computed in the light of the Equations (1-6) 9 different isolation systems were acquired and also the positions of the hysteretic curve shifted while it abided by the original force-deformation curve. Because of limited number of pages only the friction coefficient of 0.1 is plotted for 3 and 7-storey frames with FPS. Among the isolation systems, the smallest and largest hysteresis curves was performed for the radius of curvature of 1 m and 2.25 m, respectively and the other model was remained between them. Further, the effectiveness of SMRF with FPS can be very easily tuned by managing the appropriate radius of curvature and friction coefficient.

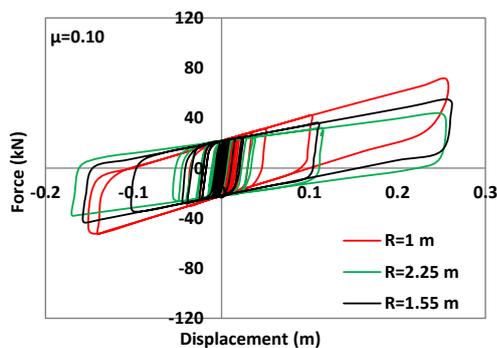


Figure 18. The hysteretic curve of 3-storey frame with FPS under Hills earthquake

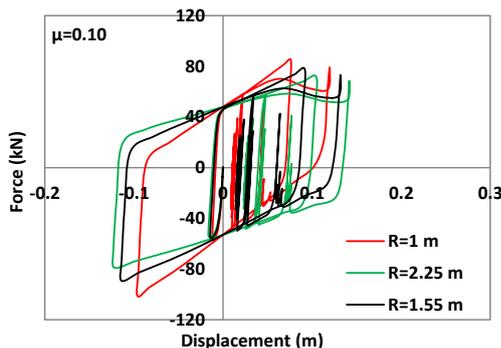


Figure 19. The hysteretic curve of 7-storey frame with FPS under Hills earthquake

4. Conclusions

In this study, it is showed that the radius of curvature, R and friction coefficient, μ on the design of base-isolated frame subjected to three natural accelerograms compatible with seismic hazard levels of 2% probability of exceedance in 50 years were considered. The response of FPS was remarkably changed by the radius of curvature

and friction coefficient. For all models, the amplification of the radius of curvature (the isolation period, T) and reduction of the friction coefficient remarkably led to great reduction in the roof displacement. For the $R2.25\mu2.5$, it was experienced the most uniform relative displacement, interstorey drift ratio and absolute acceleration height of storey with respect to the both fixed base frame. To rehabilitate the 3 and 7-storey frames in terms of the local and global deformations $R2.25\mu2.5$ obviously seemed to be the most favourable model when the seismic performance of the frames with and without FPS compared. According to the result of the analysis, it can be predicted that the larger radius of curvature and lower friction of coefficient for SMRF with FPS provided better seismic performance than other models and fixed base in real life.

Nomenclature

F	: The resisting force
k_2	: The horizontal stiffness
T	: The period of isolated structure
k_{eff}	: Effective stiffness
β_{eff}	: Effective damping
W	: Total weight on the isolator
g	: gravitational force
R	: The radius of curvature
D	: The horizontal displacement
μ	: The friction coefficient

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