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Analytical Design of PI Controllers for First Order plus Time Delay Systems

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Abstract

This study proposes an analytical design method of a Proportional Integral controller for the stability and performance of first order plus time delay systems. The method proposed in the study achieves general computation equations for such systems. Inspired from Bode's ideal loop, gain crossover frequency and phase margin specifications are considered for the system. Then, these specifications are used to obtain the parameters of the proportional integral controller. Analytically derived formulas by the proposed method are tested with existing plants in the literature and the results are illustrated graphically. It is shown that the tuning method satisfies desired gain crossover frequency and phase margin specifications.

Keywords: "First Order Plus Time Delay, PI Controller, Bode's ideal loop"

1. Introduction

A large number of industrial processes are approximately modeled by first order plus time delay (FOPTD) transfer functions. FOPTD models are widely used in electronics [1], automation [2], thermal [3] and chemical processes [4] etc. Thus, control of these plants is a challenging area of research. From this idea, researchers are motivated on better design methods or alternative controller ideas [5-7].

Proportional Integral Derivative (PID) controllers are one of the leading industrial controllers for decades. There can be found numerous studies related to control of FOPTD plants by using PID controllers in the literature [8-10]. Formed with the deficiency of the derivative operator, Proportional Integral (PI) controllers are also utilized in so many processes. For instance, Onat et al. presented a PI tuning method for FOPTD plants in [11]. A comparative study of cascaded PI-PD controllers applied on a coupled tank system can be found in [12]. Miao et al. optimized PI parameters for different performance criterias in [13].

This paper proposes a method to tune PI controllers for the stability and performance of FOPTD plants. Phase crossover frequency and phase margin specifications based on Bode's ideal transfer function are determined and parameters of the controller are analytically derived considering these specifications. The method gives generalized parameters of the PI controller for FOPTD plants. Efficiency of the proposed equations are tested with existing plants in the literature and the results are illustratively given.

Organization of this paper is as follows. Section 2 gives remindful information about PI controllers and FOPTD plans. Section 3 presents the computation process of the PI controller. Illustrative examples clarify the process in section 4 and section 5 has the concluding remarks.

2. PI Controllers and FOPTD Plants

This section presents the transfer functions of a PI controller and a FOPTD plant. For instance, general components of a Bode diagram is also briefly reminded. Following equation denotes the general representation of a FOPTD plant.

$$P(s) = \frac{K}{Ts+1} e^{-Ls} \tag{1}$$

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where, K is the gain, T is the time constant and L is the delay. Similarly, transfer function of a PI controller is given as follows.

$$C(s) = k_p + \frac{k_i}{s} = \frac{k_p s + k_i}{s} \tag{2}$$

Thus, the system can be shown with the following equation.

$$G(s) = C(s)P(s) \tag{3}$$

Figure 1 shows the closed loop scheme of the system implemeted in this paper.

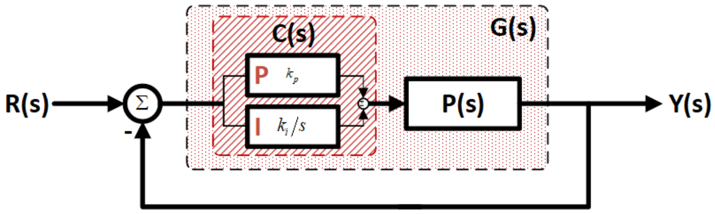

Figure 1. Block diagram of the closed loop system.

where, $R(s)$ is the input signal and $Y(s)$ is the output signal. $P(s)$ is the transfer function of the FOPTD plant in Eq. 1 and $C(s)$ is the PI controller in Eq. 2. Figure 2 shows an instance Bode diagram of an open loop system.

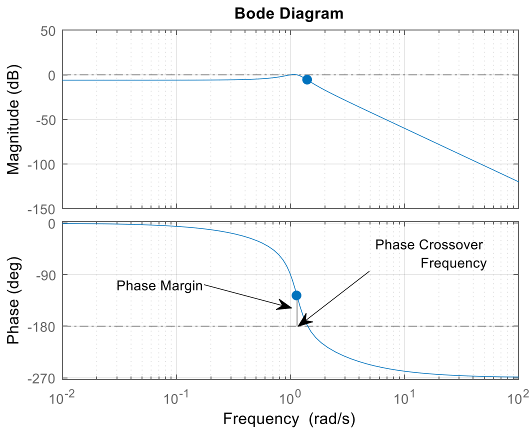

Figure 2. An Instance Bode diagram.

It would be useful to describe the components of a Bode diagram. The frequency value that the gain curve crosses the $0dB$ line is called as the gain crossover frequency and denoted as ω_c is this paper. Difference of the phase value with the -180 degrees line at the gain crossover frequency is the phase margin and denoted as ϕ_m. Phase margin shows the tolerance of the open loop system phase to ensure the stability of the closed loop system. Now, the desired gain and phase specifications can be given.

3. Design Specifications of PI Controller for FOPTD Plant

In order to analyse the system in the frequency domain, substitution of $s = j\omega$ in Eq. 3 is necessary.

$$G(j\omega) = C(j\omega)P(j\omega) \tag{4}$$

Then, frequency response of the FOPTD plant in Eq.1 can be written as,

$$P(j\omega)=\frac{K}{T\left(j\omega\right)+1}\mathrm{e}^{-L(j\omega)}=\frac{K}{1+jT\omega}\mathrm{e}^{-jL\omega}=\left|P(j\omega)\right|e^{j\angle P(j\omega)}=\sqrt{\frac{K}{1+T^2\omega^2}}e^{-j(\arctan(T\omega)+L\omega)} \quad . \tag{5}$$

Likewise, frequency response of the PI controller is,

$$C(j\omega)=k_p+\frac{k_i}{j\omega}=k_p-\frac{jk_i}{\omega} \tag{6}$$

Magnitude and phase of the FOPTD plant are obtain in the following way.

$$\left|P(j\omega)\right|=\sqrt{\frac{K}{1+T^2\omega^2}}\ , \tag{7}$$

$$\angle P(j\omega)=-\arctan\left(T\omega\right)-L\omega\,. \tag{8}$$

Similarly, magnitude and phase of the PI controller are,

$$\left|C(j\omega)\right|=\sqrt{k_p{}^2+\left(-\frac{k_i}{\omega}\right)^2}=\sqrt{\frac{k_i{}^2+k_p{}^2\omega^2}{\omega^2}}\ , \tag{9}$$

$$\angle C(j\omega)=\arctan\left(\frac{-\frac{k_i}{\omega}}{k_p}\right)=-\arctan\left(\frac{k_i}{k_p\omega}\right). \tag{10}$$

Therefore, magnitude and phase of the system can be written as follows.

$$\left|G(j\omega)\right|=\left|C(j\omega)P(j\omega)\right|=\left|C(j\omega)\right|\left|P(j\omega)\right| \tag{11}$$

$$\angle G(j\omega)=\angle C(j\omega)P(j\omega)=\angle C(j\omega)+\angle P(j\omega) \tag{12}$$

Assuming that the gain crossover frequency is ω_c and the phase margin is ϕ_m, following gain and phase specifications are desired to be satisfied.

$$\left|G(j\omega_c)\right|=1 \tag{13}$$

$$\angle G(j\omega_c)=\phi_m-\pi \tag{14}$$

Considering Eq. 11 and Eq. 13, gain specification of the system can be rewritten as,

$$\left|G(j\omega_c)\right|=\left|C(j\omega_c)\right|\left|P(j\omega_c)\right|=\sqrt{\frac{k_i{}^2+k_p{}^2\omega_c{}^2}{\omega_c{}^2}}\sqrt{\frac{K}{1+T^2\omega_c{}^2}}=1\,. \tag{15}$$

Similarly, considering Eq. 12 and Eq. 14 phase margin specification of the system is,

$$\angle G(j\omega_c)=\angle C(j\omega_c)+\angle P(j\omega_c)=-\arctan\left(\frac{k_i}{k_p\omega_c}\right)-\arctan\left(T\omega_c\right)-L\omega_c=\phi_m-\pi\ . \tag{16}$$

Together solution of Eq. 15 and Eq. 16 leads to the following parameters of the PI controller.

$$k_p=\pm\frac{\sqrt{1+T^2\omega_c{}^2}}{K\sqrt{1+\tan\left(\phi_m+L\omega_c+\arctan\left(T\omega_c\right)\right)^2}} \tag{17}$$

$$k_i=\mp\frac{\omega_c\sqrt{1+T^2\omega_c{}^2}\,\tan\left(\phi_m+L\omega_c+\arctan\left(T\omega_c\right)\right)}{K\sqrt{1+\tan\left(\phi_m+L\omega_c+\arctan\left(T\omega_c\right)\right)^2}} \tag{18}$$

Parameters of the PI controller in Eq. 17 and Eq. 18 can be used to obtain the PI controller to satisfy given gain and phase margin specifications in this paper. It would be clarifying to explain the results on illustrative examples.

4. Illustrative Examples

This section gives two examples to clarify the given procedure.

Example 1: Consider the following FOPTD plant provided from [14].

$$P_1(s) = \frac{1}{0.4s+1}e^{-0.01s} \tag{19}$$

Desired phase crossover frequency is $\omega_c = 10 rad/\sec$ and the phase margin is $\phi_m = 45^o$. Replacing the unknown variables in Eq. 17 and Eq. 18, following PI controller is obtained.

$$C_1(s) = 2.46369 + \frac{33.06090}{s} \tag{20}$$

Bode diagram of the system $G_1(s) = C_1(s)P_1(s)$ is illustrated in Figure 3. It is clearly seen in the figure that the phase crossover frequency is tuned to be $\omega_c = 10 rad/\sec$ and the phase margin is $\phi_m = 45^o$. Thus, the proposed method is successfully implemented.

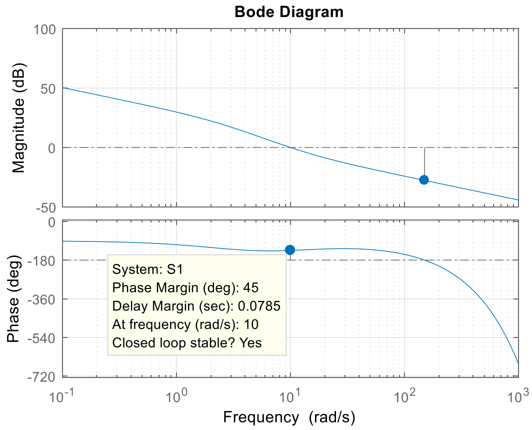

Figure 3. Bode diagram of the system $G_1(s) = C_1(s)P_1(s)$.

We can also check the stability of the system with the step response of the closed loop system given in Figure 4.

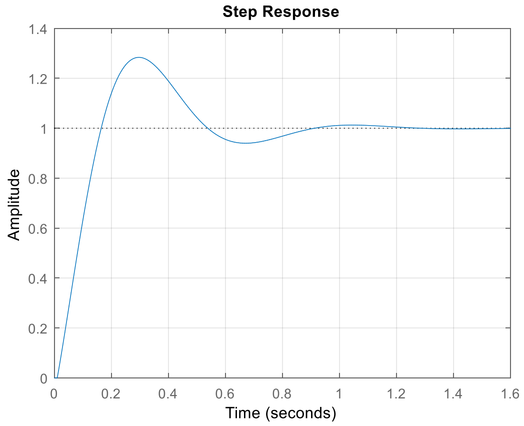

Figure 4. Step response of the closed loop system in Example 1.

The method can also be tested by varying phase margin values. Table 1 lists the parameters of the PI controller found for $\phi_m \in [30^o, 60^o]$ with increment steps of 5^o at $\omega_c = 10 rad/\sec$.

Table 1. Parameters of the PI controller found for $\phi_m \in [30^o, 60^o]$.

ϕ_m	k_p	k_i
30°	1.524059254728363	38.310890603165092
35°	1.852161161722366	36.836800934673008
40°	2.166167003910174	35.082360968399527
45°	2.463687007230828	33.060923051847794
50°	2.742456864811567	30.787871548140370
55°	3.000354969710362	28.280505751726466
60°	3.235418561625795	25.557908230305681

Bode diagrams of the systems with $P_1(s)$ and the 7 controllers listed in Table 1 are given in Figure 5.

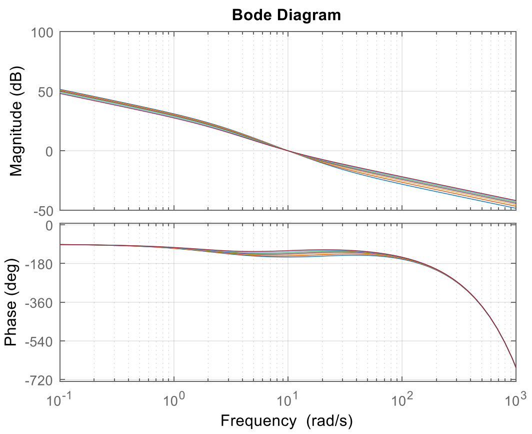

Figure 5. Bode diagrams of the systems with $P_1(s)$ and the 7 controllers listed in Table 1.

Similarly, stability of the systems with $P_1(s)$ and the 7 controllers can be checked with the step responses in Figure 6.

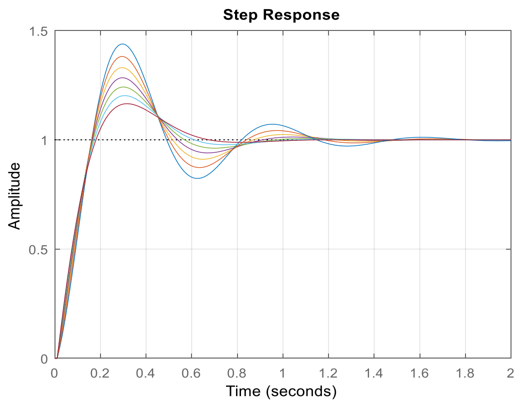

Figure 6. Step responses of the systems with $P_1(s)$ and the 7 controllers.

From this example, efficiency of the proposed method is clearly shown. It would be advantageous to apply the proposed method in another example.

Example 2: Consider the following FOPTD plant [11].

$$P_2(s) = \frac{1}{s+1} e^{-s} \tag{21}$$

Phase crossover frequency for this example is desired to be $\omega_c = 1 rad/\sec$. Phase margin is assumed to change in the interval $\phi_m \in [30^o, 60^o]$ with increment steps of 5^o. Table 2 shows the parameters of the PI controller obtained for this case.

Table 2. Parameters of the PI controller found for $\phi_m \in [30^o, 60^o]$.

ϕ_m	k_p	k_i
30°	0.951706372124047	1.046066432526389
35°	1.039255538940336	0.959139158194384
40°	1.118895343585638	0.864912255724463
45°	1.190019679058772	0.764102848740180
50°	1.252087246220741	0.657478157699069
55°	1.304625673408020	0.545849660881703
60°	1.347235111466145	0.430066918551990

Figure 7 shows the Bode diagrams and Figure 8 gives the step responses of the systems with $P_2(s)$ and the 7 controllers in Table 2 respectively.

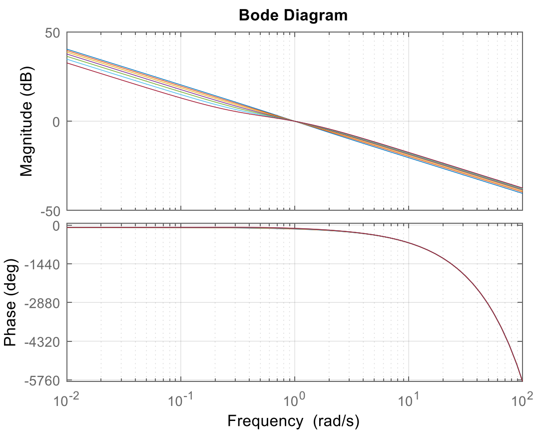

Figure 7. Bode diagrams of the system with $P_2(s)$ and the 7 controllers in Table 2.

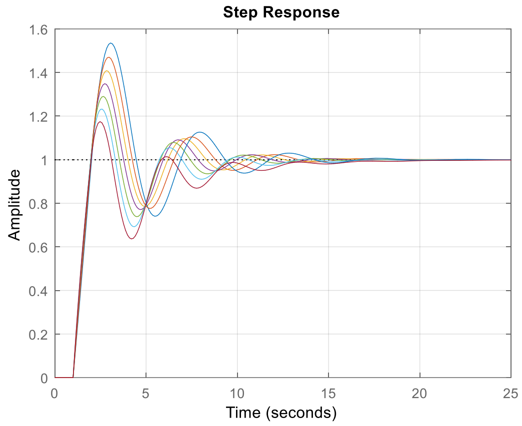

Figure 8. Step responses of the systems with $P_2(s)$ and the 7 controllers in Table 2.

Thus, the method is proved illustratively.

5. Conclusion

This paper proposes a design scheme of proportional integral controllers for performance and stability of first order plus time delay plants. Gain and phase specifications for the system are inspired from Bode's ideal loop. The method analytically obtains general computation equations for the mentioned systems. Illustrative examples are given with two existing plants from the literature and the results are shown graphically. It is investigated that proposed method is successfully implemented.

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