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Thermal Analysis of Hypersonic Inlet Flow with Exergy-Based Design Methods

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Abstract

This paper presents results of work that has been done in developing use of the Second Law of Thermodynamics and methods such as exergy and thermoeconomics into a system-level analysis and design methodology. The application of these methods to the design of a complete flight vehicle is illustrated by considering an integrated airframe/propulsion system as a device to do work. This shows how system-level consideration of exergy applies to all vehicle systems in consistent terms. For the hypersonic inlet flow problem, it is shown that a thermal energy exchange with the inlet flow could be used to position the inlet shock in the optimum shock-on-lip position for off-nominal flight conditions. The thermal heat exchange analysis has been done for a full range of Mach numbers both higher and lower than nominal. It is shown that there is a potential benefit in terms of reduced exergy destroyed using thermal energy addition than by the shock at higher Mach numbers. The paper then discusses how a device to accomplish this result would have to be integrated into a complete vehicle design.

Key words: exergy analysis, thermoeconomics, 2nd Law of Thermodynamics, hypersonic aircraft/vehicles, shock-on-lip positioning, MHD shock control

1. Introduction

The objective of this paper is two-fold and is based on developing a long-term strategy for system-level analysis and optimization, with a special consideration for hypersonic vehicles for which the existing database of experience is very small. The first part illustrates the 'top-down' development of a systems-level approach for vehicle design in exergy terms (Moorhouse 2003). It shows that mission requirements can be stated in terms of work to be accomplished. Then each system of a vehicle can be analyzed as a component in finding the most efficient way to accomplish that work. Ideally, a minimum exergy solution should be the same as a more traditional minimum weight solution, provided that the true global optimum is achieved. The difference is in the manner of trading the advantages and disadvantages of any component or device being considered for incorporation into a system design. Also, we consider it a design requirement to optimize over a range of flight conditions rather than just a single point design.

The second part of the paper deals with the application of these methods to the design of the inlet of a hypersonic vehicle. We consider the addition of a 'device' to keep the shock on the inlet lip for different Mach numbers. This avoids problems of spillage or shock ingestion. The system or device to accomplish this inlet tailoring has to be compared with a mechanical moving inlet system. We have started this

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analysis with a device that uses the concept of adding or extracting thermal energy to the inlet flow in order to tailor the Mach number and shock angle. This shows thermodynamic limits for the design of such a device. Also, changes in the thermal parameters of the inlet flow need to be considered in terms of the combustor. These effects and requirements for an integrated system analysis are then discussed.

There is a considerable body of published work on the Second Law of Thermodynamics for design (e.g., Clarke and Horlock 1975, Moran 1982, Bejan 2000). We believe that this paper is unique in extending previous work to the system-level metric of minimizing total exergy destruction by a flight vehicle.

2. System-Level Requirements

Mission requirements for any vehicle can be stated in terms of the work to be done. Also, in order to illustrate the design problem we make a 'business case' that this work to be done is composed of work the customer wants to purchase together with overhead work (Moorhouse 2003). The customer work consists of giving a specified payload the required altitude (potential energy), velocity (kinetic energy), and power supply to accomplish what is required. This consideration of 'payload' can be interpreted in very general terms. It could be a radar antenna, it could be weapons for delivery, or it could be the second stage of a two–stage–to–orbit system. Thus, there can be a drag associated with the payload.

For the energy part, it is convenient to use a weight specific energy defined as the total of kinetic plus potential energies per unit of weight, i.e. $E_w = h + U^2/2g$. In general, the rate of change of this quantity will be used and is only a function of the rate of climb and the acceleration. For the simplest case of just climbing to a cruise altitude and speed, E_w is obtained very easily. For more complicated missions, the equations must be integrated over all the segments. The rate of doing customer work, i.e. that required to generate the specific energy of some payload of weight, W_p, plus overcome the drag chargeable to the payload, D_P, and supply the required power, P_p, can be written as

$$\frac{dw_c}{dt} = W_p \frac{dE_w}{dt} + P_p + D_p U \qquad (1)$$

The integral of this equation over a total mission represents the quantity of work to be considered as a fixed requirement. This distinction may be considered as artificial, but it is a convenient way to track those items that are not subject to trade-off, i.e. hard constraints on the design process.

In order to accomplish this mission work, a vehicle is required to carry the payload so that there is a "business overhead". This is the work that has to be done carrying the weight of the vehicle plus the required fuel on the mission. We can then express the rate of doing overhead work, w_o, as

$$\frac{dw_o}{dt} = W \frac{dE_w}{dt} + DU \qquad (2)$$

In this equation, W is the sum of all the vehicle components (except for the payload) and could be broken down to support any analysis. It comprises the empty weight plus the weight of the fuel, and also it is a negative function of time as defined by the fuel flow rate. It may be more useful to write

$$\frac{dw_o}{dt} = \sum \left(W_i \frac{dE_w}{dt} + P_i + D_i U \right) \qquad (3)$$

where W_i is the weight of each component of the vehicle – e.g., wing, fuselage, empennage, etc. – or a device which is being considered as an item for trade off. P_i is the power required by a component and only appears in this breakdown of equation (2) as, for example, the hydraulic power for controls that comes from the engine. In addition, the drag force is a function of flight condition and vehicle weight so that it has also been broken down into components. In this paper, we discuss the trade balance of some additional device for positioning the inlet shock.

Equations (1) and (3) represent the power required or work that has to be done when integrated over the mission, which must equal the exergy of the fuel that is used for the mission. At the system level, it is convenient to use the maximum energy content by weight of the fuel, H, together with an overall system propulsive efficiency, η. This overall efficiency term introduces consideration of the Second Law of Thermodynamics as well as that of all components of the complete installation. The conservation of energy in the time interval dt can be shown to be

$$-\eta H dW = dw_c + dw_0 \qquad (4)$$

In this equation, the negative sign appears because we considered dW to be negative. Alternatively, we could use –dW equal to the fuel flow rate. Equation (4) together with (1) and (3) represent the balance between the work achieved by the exergy derived from the fuel with the changes (increases) in payload and aircraft specific energy plus the exergy destroyed to overcome drag, i.e. the mission work. In this formulation, the design problem is to minimize the overhead work and maximize the overall system efficiency. This allows all subsystems to be designed to system-level metrics, using exergy as the common currency

3. Application to Vehicle Design

A conceptual hypersonic vehicle, as discussed in Chase et al. (1999), is illustrated in *Figure 1*. An MHD by-pass system extracts energy from the inlet flow and adds (some of) it into the exhaust. Here we are considering the 'addition' of such a device to a baseline vehicle design and need to establish the benefits at the system level. If we re-write equation (2) in the "old fashioned way", it becomes:

$$\frac{dw_o}{dt} = W\{\frac{dE_w}{dt} + \frac{D}{L}U\} \tag{5}$$

The assumption that lift equals weight is only true in cruise. Therefore, the validity of this depends on requirements for maneuvering. We see that it supports the traditional 'minimize weight' as the optimization metric together with maximizing L/D. Thus, the additional weight of a new system has to "buy its way on", but how? These requirements would have a negative impact on the overhead work of equation (5), i.e. "weight is bad" by traditional metrics.

If we return to equation (4) with (1) and (3), we can trade weight and overall system-level propulsive efficiency. Now, we have a method to show that if this weight causes sufficient improvement in the overall propulsive efficiency, η, of equation (4), then there is a system-level benefit. We can consider that the additional weight and also the power required to operate the device cause an increment in overhead work, i.e.

$$\frac{dw_{oM}}{dt} = W_M\frac{dE_w}{dt} + P_M \tag{6}$$

which is the increase in overhead work chargeable to the device. If the device weighs less than what it replaces, then it is "good by traditional metrics". For discussion purposes, we are assuming that the device weighs more than the movable inlet system it is replacing. If the only change was this net weight increase and added power consumption on the right hand side of equation (4), then it would require a higher fuel flow rate, dW_M, to achieve the required exergy balance at identical flight conditions. We have postulated, however, that the benefit of the new device is an increase in the overall efficiency, η_M, and maybe a positive increment in thrust, T_M. The final result then becomes

$$\begin{aligned} -\eta_M H\frac{dW_M}{dt} &= \frac{dw_c}{dt} + \frac{dw_0}{dt} \\ &\quad + W_M\frac{dE_w}{dt} + P_M - T_M U \end{aligned} \tag{7}$$

Subtracting equations (4) and (7), yields

$$\begin{aligned} \eta_M H\frac{dW_M}{dt} &= \eta H\frac{dW}{dt} \\ &\quad - \left[W_M\frac{dE_w}{dt} + P_M - T_M U\right] \end{aligned} \tag{8}$$

In this equation, the first term on the right hand side is the exergy use rate for a base vehicle without the device. All the other terms are for the effects of the device on that vehicle. The term inside the bracket on the right hand side may be positive (detrimental) or negative (beneficial), depending on the increment in thrust due to effects of the new device. Note that with the definition of exergy as the available work from an energy source (i.e. the fuel), we can now use this equation to analyze the system-level benefits of any component in terms of entropy generated, which is proportional to exergy destroyed. Equation (8) would have to be integrated over a mission to account for effects at different flight conditions, because we have assumed that the device to position the shock on the lip would avoid detrimental effects at off-design conditions.

4. Shock Inlet Tailoring

The example system in *Figure 1* showed a device that extracts 'excess energy' from the inlet flow. In this section, however, we illustrate the exergy process with a device added just to maintain the bow shock on the inlet lip of a hypersonic vehicle with addition or extraction of thermal energy. This is done for fixed inlet geometry to prevent problems of spillage or shock ingestion with the additional device replacing the weight and complexity of a movable inlet system. A given inlet design is characterized by geometry with a known ramp angle, θ, the "shock-on-lip" angle β and a nominal operating Mach number for that fixed geometry, all related by

$$\tan\theta = 2\cot\beta\left(\frac{M_2^2\sin^2\beta - 1}{M_2^2(\gamma + \cos 2\beta) + 2}\right) \tag{9}$$

Given an incoming flow at a Mach number, M_1, the heat input q required to achieve the nominal Mach number is found using the Rayleigh line analysis, namely,

$$q = c_p(T_{o2} - T_{o1}) \tag{10}$$

with the required temperature obtained from

$$\frac{T_{o2}}{T_{o1}} = \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^2 \left(\frac{M_2}{M_1}\right)^2 \left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right) \quad (11)$$

It is assumed that we keep the combustion process at its maximum efficiency over a range of conditions using this process without the complexity of a moving inlet system.

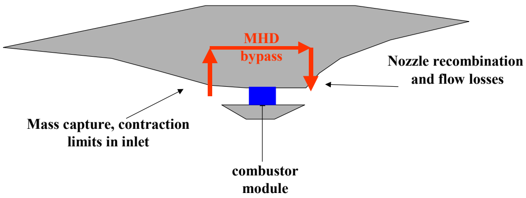


Figure 1. A conceptual hypersonic vehicle with a MHD system

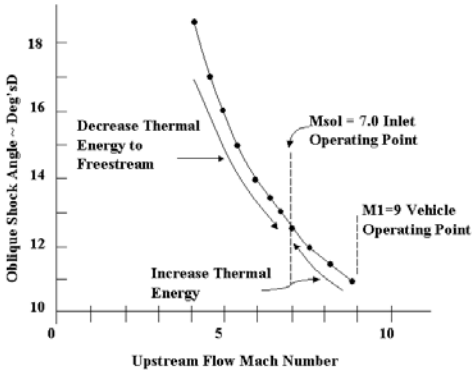
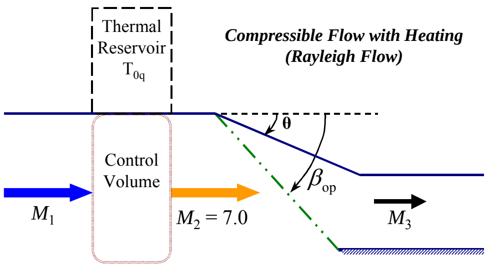


Figure 2. Oblique shock angle versus Mach number



Thermal reservoir is "hot" if M_1>M_2 and "cold" if M_1<M_2

Figure 3. Schematic of tailoring inlet operating shock position using thermal energy.

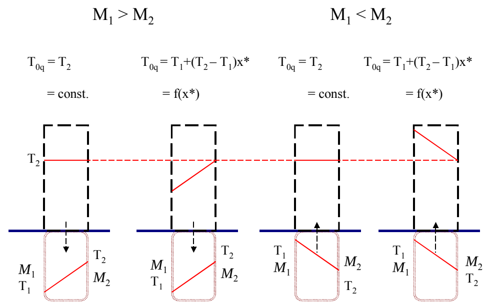


Figure 4. Options for thermal energy exchange.

Figure 2 shows the angle of the shock from an inlet ramp as a function of Mach number. It also shows that thermal energy must be added when M_1 is higher than the inlet design value, although this condition may be considered to have 'excess energy' already. Previously, one example of tailoring Mach 9 flow to a Mach 7 inlet was illustrated as proof of concept (Moorhouse and Suchomel 2001). Here, a more detailed thermodynamic analysis is accomplished to examine the effect over a wider range of operating conditions.

For illustration, we are considering the problem as defined in *Figure 3* for an arbitrary Mach number (behind the bow shock) approaching the inlet. It shows a control volume where thermal energy is exchanged with a notional thermal reservoir in order to tailor this M in an ideal manner. As drawn, for $M_1 > M_2$, thermal energy must be added to the flow from a hot reservoir. Conversely, if $M_1 < M_2$, then thermal energy must be extracted from the flow into a cold reservoir. We could idealize the hot reservoir being supplied from structural heat at these Mach numbers, but creation of a cold reservoir is hard to imagine!

First, we consider the thermodynamic requirements on the thermal energy exchange process. The entropy generation rate produced by the heating process is expressed as

$$\dot{S}_{gen} = \rho_1 V_1 (s_2 - s_1) - \frac{q}{\hat{T}} \quad (12)$$

where

$$s_2 - s_1 = R\left(\frac{1}{\gamma-1}\ln\left(\frac{T_2}{T_1}\right) - \ln\left(\frac{\rho_2}{\rho_1}\right)\right) = -R\ln\left(\frac{P_{o2}}{P_{o1}}\right) \quad (13)$$

Now, the appropriate temperature of the reservoir, $\hat{T}$, must be established for use in equation (12). *Figure 4* presents four options where it is considered that there is a vanishingly small temperature difference to allow ideal heat transfer, i.e. the figure shows limiting conditions. For the cases with constant reservoir temperature, then T_2 is used in equation (12). Then, for the assumed linear profiles, the change in entropy due to heating is

$$\Delta s_q = \Delta q \int_0^1 \frac{1}{T} dx \qquad (14)$$

where x is non-dimensional distance. Now, with $T = T_1\left\{1 + \left(\frac{T_2 - T_1}{T_1}\right)x\right\}$, equation (14) integrates to

$$\Delta s_q = \frac{\Delta q}{T_2 - T_1} \ln \frac{T_2}{T_1} \qquad (15)$$

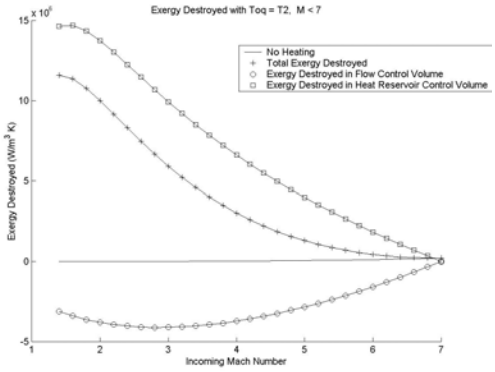


Figure 5. Exergy balance for the case of a constant temperature thermal energy exchange from the reservoir, M < 7.

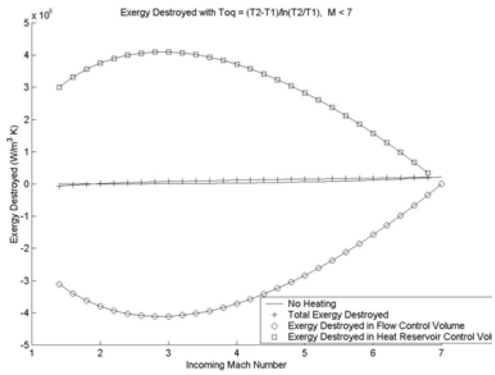


Figure 6. Exergy balance for the case of a linear temperature profile thermal energy exchange from the reservoir, M < 7

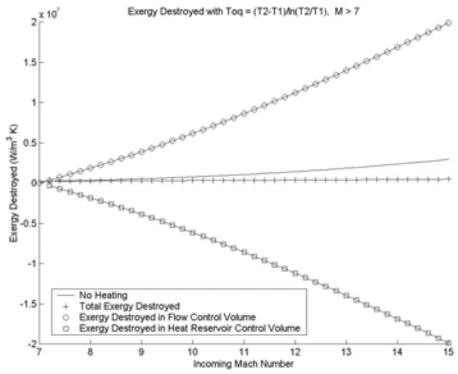


Figure 7. Exergy balance for the case of a constant temperature thermal energy exchange from the reservoir, M > 7.

Equation (15), thus, gives the reservoir temperature to use in equation (12) for the cases with a linear temperature profile. In addition, the entropy created by the jump across the shock (s_3-s_2) can also be calculated using equation (13), and the results for both the heating and shock are combined to find the exergy balance for the entire process, i.e.

$$\dot{X}_{balance} = T_1\dot{S}_{gen} + T_1\rho_1 V_1 (s_3 - s_2) \qquad (16)$$

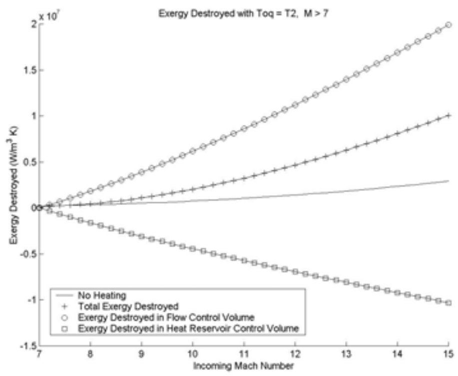


Figure 8. Exergy balance for the case of a linear temperature profile thermal energy exchange from the reservoir, M > 7.

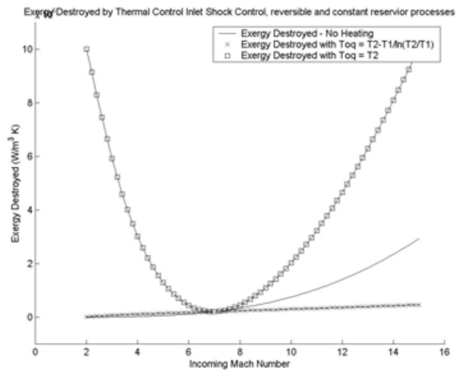


Figure 9. Exergy destroyed with and without heating.

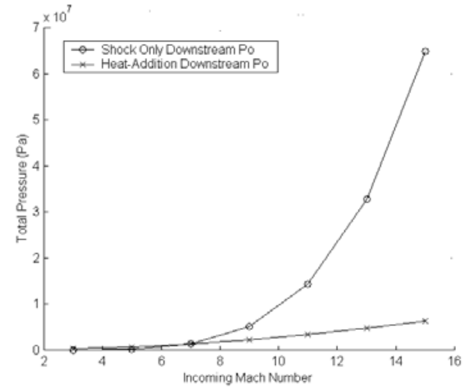


Figure 10. Inlet total pressure with thermal shock tailoring.

Using this method, the exergy balance associated with a design inlet Mach number of 7 over a range of actual operating points was calculated. For illustration purposes, we consider the process moves entropy so that it decreases in one volume and increases in the other (without violating the 2nd Law of Thermodynamics). First consider Mach numbers less than the inlet design value. *Figures 5* and *6* show the exergy (entropy) balances in both the flow control volume and the reservoir that must be colder than the flow. In *Figure 5*, for the case of a constant temperature

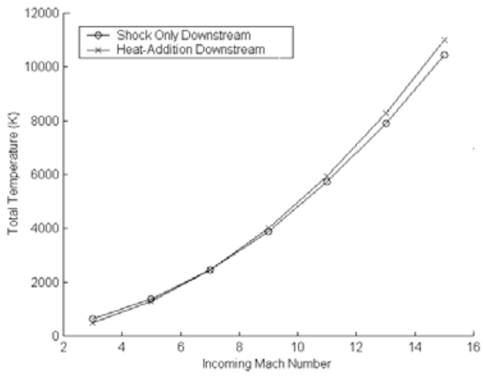


Figure 11. Inlet total temperature with shock inlet tailoring.

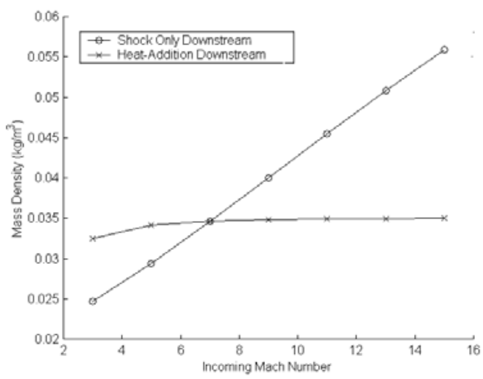


Figure 12. Inlet density with shock inlet tailoring.

thermal energy exchange from the reservoir with T_2 as the limit,we see the "exergy gained/entropy reduced" in the flow control volume by moving it to the reservoir. It is also shown that the global balance, i.e. the sum of the two volumes plus the entropy jump across the Mach 7 shock, shows the amount of exergy destroyed. *Figure 6* shows the result for a notional reservoir with the linear temperature profile, i.e. a perfect heat exchanger. In this case, the total exergy destroyed is very small. *Figures 7* and *8* show the results for higher Mach numbers, i.e. for a hot reservoir adding thermal energy to the flow. A reservoir with constant temperature thermal energy exchange shows a total exergy destroyed worse than that for the shock wave without thermal heating (*Figure 7*). For a linear temperature profile thermal energy exchange from the reservoir, *Figure 8* shows that the total exergy destroyed is less than the amount without the thermal addition. The preceding results are summarized in *Figure 9*, recognizing that the curves for the thermal energy tailoring are ideal boundary conditions. Even so, a simple heat exchanger with constant temperature thermal energy exchange from the reservoir shows significantly more exergy destroyed than the shock without thermal tailoring. For Mach numbers higher than the inlet design value, there is an exergy benefit if we can design a heat exchanger to create the linear temperature profile with close to ideal heat exchange. The purpose of moving the shock to the lip is to maximize engine efficiency, but interestingly this analysis shows that there are benefits associated with reducing the incoming Mach number through heating irrespective of the propulsion benefits of having the shock placed on the inlet lip.

Any high-speed vehicle has to consider the heating that may cause structural problems, so that thermal energy is "available". Could a subsystem be designed using structural heat from the reservoir supplying thermal energy to the inlet flow? The results presented here show the thermodynamic limits for such a subsystem, but even an optimized heat exchanger design based on minimum exergy destruction would not meet these limits. The weight consideration for such a system has been discussed previously. There would also be a volume requirement that might cause an increment in airframe weight that would be chargeable to the new device. All these factors would have to be analyzed to supply the parameters for equation (8) to answer the question at the system level together with other considerations discussed next.

5. Other Considerations

We have shown that thermal energy addition/extraction can produce 'an ideal inlet shock on lip condition' for a fixed inlet. In isolation, this indicates a design space for Mach numbers above the inlet nominal operating point. As discussed earlier, the effects of this on other systems must be included in the analysis. First, the changes of the inlet flow parameters are presented in TABLE I and we see that there are large effects. We consider only the higher Mach numbers where there is an exergy benefit of thermal shock tailoring and illustrate the differences in *Figures 10* to *12*. *Figure 10* shows the decrease in total pressure, so that thermal heating increases the reduction from the upstream value (from 85% to 96% reduction at Mach 11). *Figure 11* shows that total temperatures are increased by 4% at Mach 11. Finally, the change in flow density caused by the shock is almost eliminated by the thermal tailoring (*Figure 12*). We started with a presumption that keeping the inlet shock on the lip, i.e. keeping inlet Mach number constant, was beneficial to the combustor for off-design conditions. We see large changes in the thermodynamic properties of the inlet flow. The effects of these changes on the combustion performance are expected to be significant and would need to be calculated for the system analysis discussed above. Alternatively, can the combustor geometry, etc. be optimized to take advantage of the effects? The optimum answer would only come from an integrated design rather than simply the addition of a discrete device, but exergy considerations would guide such a design.

TABLE I. INLET FLOW PARAMETERS WITH AND WITHOUT THERMAL TAILORING.

	Upstream Properties		Downstream Properties	
	Incoming Mach Number	Upstream Properties	Shock-Only	Heat adjusted (M=7 shock)
P	3	1,173.3	1,832.3	592.3
ρ		0.018	0.025	0.033
T		227.1	258.6	63.5
Po		43,100	24,237	273,374
To		636	636	510
P	5	1,173.3	2,365.2	1,568.0
ρ		0.018	0.029	0.034
T		227.1	280.2	160.1
Po		620,809	249,589	723,639
To		1,363	1363	1286
P	7	1,173.3	3,031.4	3,031.4
ρ		0.018	0.0346	0.0346
T		227.1	305.2	305.2
Po		4,857,491	1,399,036	1,399,036
To		2,453	2,453	2,453
P	9	1,173.3	3,835.1	4,982.7
ρ		0.018	0.0400	0.0348
T		227.1	333.8	498.9
Po		24,761,586	5,154,304	2,299,564
To		3,906	3,906	4,009
P	11	1,173.3	4,785.1	7,421.8
ρ		0.018	0.045	0.035
T		227.1	366.5	740.9
Po		94,260,729	14,323,222	3,425,226
To		5,724	5,724	5,954
P	13	1,173	5,886	10,348
ρ		0.018	0.0508	0.0350
T		227.1	403.6	1,031.4
Po		291,713,318	32,721,330	4,776,019
To		7,904	7,904	8,288
P	15	1,173	7,145	13,763
ρ		0.018	0.0559	0.035
T		227.1	445.5	1,370.3
Po		774,606,815	64,859,083	6,351,945
To		10,448	10,448	11,012

Other work has considered the possibility of using a non-thermal technique for inlet flow tailoring. Quoting from Macheret et al. (2002), "Calculations show that an MHD system with reasonable parameters could bring shocks back to the cowl lip when flying at Mach numbers higher than those for which the inlet was optimized". The work also shows effects on flow parameters, requiring an integrated analysis. In another context, the MHD device illustrated in *Figure 1* is intended to extract energy from the inlet flow. This would certainly influence the inlet shock position, but another factor that needs to be recognized is that the freestream air is typically at rest, i.e. there is no "free supply" of kinetic energy. It is the vehicle that has the kinetic energy derived from the exergy of the fuel. Again, there is a requirement for an integrated system analysis, and we are suggesting a method to analyze or optimize the efficiency of that total integrated process.

6. Conclusions

The intent of this paper was to show that exergy analysis and the 1st and 2nd Laws of Thermodynamics on which it is based provides a sound basis for designing aerospace vehicles with system-level metrics that form a framework for the design of all components. First, the mission requirements were expressed in terms of work to be done. It was shown how a component could be considered for addition to a vehicle design by assessing the positive and negative influences on the system exergy balance. Then we considered a device to tailor the shock position of the inlet of a hypersonic vehicle, using the addition/extraction of thermal energy. If we consider an ideal heat exchange device to add thermal energy to the inlet flow at Mach numbers higher than the design value, there appears to be a strong beneficial effect of lowering the entropy generated. This result was provided for an inlet in isolation. The parameters of the inlet flow are changed significantly such that there may be negative influences on the combustion process. This leads to the requirement for more efficient overall system design optimization, as opposed to component optimization. This paper is a step towards developing exergy as a common currency for system-level analysis, design, and optimization.

Nomenclature

D	Drag
E_w	(Potential + kinetic energy)/weight
H	Maximum enthalpy per unit weight of fuel
M	Mach number
P	Power required by any component
P	Pressure
Q	Heat flux
R	Gas constant
S	Entropy
T	Temperature
T	Time
U	Velocity
V	Volume
W	Weight
w	Work
γ	Ratio of specific heats
ρ	Mass density
η	Overall system efficiency

Subscripts

c	customer
o	overhead
p	payload
M	parameters of an additional device

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