Int. J. of Thermodynamics
Vol. 10 (No. 4), pp. 165-176, December 2007 ISSN 1301-9724

Thermodynamic Study of a Low Temperature Difference Stirling Engine at Steady State Operation

Nadia MARTAJ[*], Lavinia GROSU[1*], Pierre ROCHELLE[*,**]

[*] Laboratoire d'Energétique et d'Economie d'Energie
50, rue de sèvres, 92 410 Ville d'Avray
[**] Laboratoire de Mécanique Physique
2, av. de la Gare de Ceinture 78310 Saint Cyr l'Ecole
mgrosu@u-paris10.fr

Abstract

In the current energy economy context, the use of renewable energies and the valuation of lost energies are the subject of many studies. From this point of view, the Stirling engine draws attention of the researchers for its many advantages. This paper presents a thermodynamic analysis of a low temperature Stirling engine at steady state operation; energy, entropy and exergy balances being presented at each main element of the engine. A zero dimensional numerical model describing the variables evolution (pressure, volumes, masses, exchanged energies, irreversibilities...) as function of the crankshaft angle is also presented. The calculated irreversibilities are due to imperfect regeneration and temperature differences between gas and wall in the hot and cold exchangers. A favourable comparison was made with experimental results obtained on an small size engine.

Keywords: Stirling engine, numerical model, thermodynamic analysis, imperfect regeneration.

1. Introduction

In the past few yars the understanding of the Stirling engine has shown considerable growth. Many new applications were developed, one of these applications being the low temperature difference Stirling engine. This new type of Stirling engine is able to operate with very low temperature difference between the source and the sink of the engine. Such an engine can run simply placed on a hot cup of coffee or on the hand. More practical applications (pumps) have also appeared.

This study presents an energetic, entropic and exergetic analysis of a gamma type engine which allowed the development of an equation system describing the processes occurring at every element of the engine. The numerical model allows the evaluation of the processes from the energetic, entropic and exergetic point of view as function of the crankshaft angle (kinematics-thermodynamics coupling).

The hot air engine configuration which one proposes to study is presented on *Figure 1*. The two pistons (working piston and displacer piston) are connected to the same crankshaft with an appropriate out-of-phase angle ; the displacer piston is used jointly as a regenerator.

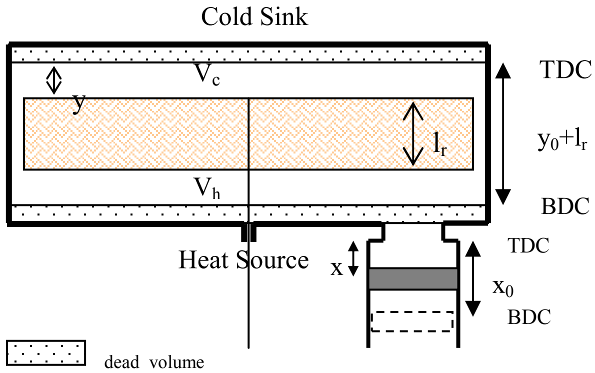

Figure.1. Low temperature difference Stirling engine

2. Model of low temperature difference engine

2.1 Volume and pressure expressions

For this type of engine, compression and expansion spaces are defined by the positions (x and y) of the pistons compared to the corresponding top dead centers. Compression and expansion volumes can be expressed according to the instantaneous pistons positions by using the engine geometry.

[1] Author to whom correspondence should be addressed.

$$V_h = V_{mh} + (y_0 - y)\, A_d + x\, A_p \quad (1)$$

$$V_c = V_{mc} + y\, A_d \quad (2)$$

Pistons positions in cylinders

From a reference state (working piston at BDC), the instantaneous position of the working piston in the small cylinder is written according to the angle of crankshaft like:

$$x = \frac{x_0}{2}(1 + \cos\theta) \quad (3)$$

The position of the displacer piston, which separates compression and expansion spaces in the large cylinder, is given by:

$$y = \frac{y_0}{2}(1 + \cos(\theta - \varphi)) \quad (4)$$

with φ generally approximated by $\frac{\pi}{2}$.

The dead space of the regenerator (annular in this case) is defined by:

$$V_r = \pi.(R_{cyl}^2 - R_d^2).l_r \quad (5)$$

It remains constant during the operation of the engine contrary to hot an cold volumes which change with positions x and y of the pistons.

In the following, the engine is considered as made up of three cells: compression, regeneration and heating cell.

The instantaneous pressure, considered as uniform in the engine and its variation can be expressed by using the mass balance.

The total gas mass m locked up in the engine, which is the sum of the gas masses of the three cells, remains constant during the operation of the engine, therefore, by assuming that the gas is perfect:

$$m_c + m_r + m_h = m = \frac{p}{r}\left(\frac{V_c}{T_c} + \frac{V_r}{T_r} + \frac{V_h}{T_h}\right) \quad (6)$$

$$dm_c + dm_r + dm_h = 0$$

T_r being the mean temperature in the regenerator (see Equation (12)), then :

$$p = \frac{m}{\frac{V_c}{rT_c} + \frac{V_h}{rT_h} + \frac{V_r}{rT_r}} \quad (7)$$

While differentiating (7) and using (6) one obtains dp in the following form, assuming that the temperatures are constant:

$$dp = -\frac{p\left(\frac{dV_c}{T_c} + \frac{dV_h}{T_h}\right)}{\frac{V_c}{T_c} + \frac{V_h}{T_h} + \frac{V_r}{T_r}} = -\frac{p^2}{m}\left(\frac{dV_c}{T_c} + \frac{dV_h}{T_h}\right) \quad (8)$$

2.2 Energy analysis:

The differential form of the energy balance for an open system is written generally by :

$$\delta Q + \delta W + \sum h_i\, dm_i = c_v\, d(mT) \quad (9)$$

2.2.1 Regenerator cell:

The regenerator/displacer reciprocating movement forces the air of the cooling cell towards the heating cell and conversely ; it is also useful to store and release the heat exchanged with the regenerator wall during this transfer. It is supposed that the regeneration is imperfect (*Figure 2*). The fluid temperature at the exit of the regenerator (orifice at abscissa 0) towards the cold cell $T_{c'}$ is higher than T_c and the fluid temperature at the exit of the regenerator (orifice at abscissa *lr*) towards the hot cell $T_{h'}$ is lower than T_h .

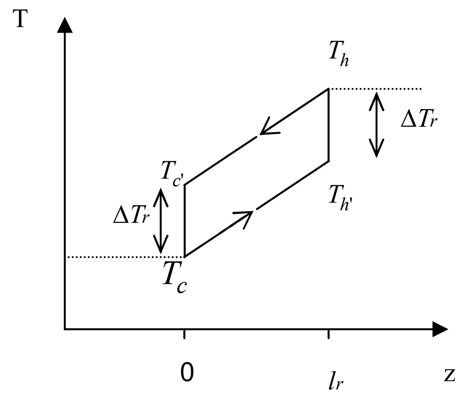

Figure 2. Regenerator T-z diagram

Let define the regenerator efficiency by :

$$\eta_r = \frac{T_h - T_{c'}}{T_h - T_c} = \frac{T_{h'} - T_c}{T_h - T_c} = 1 - \frac{\Delta T_r}{T_h - T_c} \quad (10)$$

where ΔT_r represents the temperature pinch in the regenerator, assumed identical at the two extreme orifices (between C and R and between H and R):

$$\Delta T_r = T_{c'} - T_c = T_h - T_{h'} \quad (11)$$

In this constant-volume cell of heat storage and release, the work exchanged is null and the average temperature is supposed to be constant T_r .

$$T_r = \frac{T_c + T_h}{2} \quad (12)$$

In the regenerator cell, the equation (9) can be written:

$$\delta Q_r = c_p\ T_{rc}\ dm_c + c_p\ T_{rh}\ dm_h + \frac{c_v}{r}\ V_r\ dp \quad (13)$$

the mass dm_i is positive when it enters the volume i.

Two cases arise for each opening:

- flow from (C) towards (R): $dm_c < 0$, then $T_{rc} = T_c$, $T_{rh} = T_{h'}$.

- flow from (H) towards (R): $dm_h < 0$, then $T_{rh} = T_h$, $T_{rc} = T_{c'}$.

To determine the mass in this cell and its differential, the equation of state is written.

$$p\ V_r = m_r\ r\ T_r$$

$$m_r = \frac{p\ V_r}{r\ T_r} \quad (14)$$

and the mass transfer balance gives:

$$dm_r = -dm_c - dm_h = \frac{V_r}{rT_r} dp \quad (15)$$

2.2.2 Cooling cell

For the cooling cell (*Figure 3*), which have a single communication orifice, one can write the equation (9) in the following form:

$$\delta Q_c + \delta W_c + c_p\ T_{rc}\ \mathrm{dm}_c = c_v . d(mT)_c \quad (16)$$

as stated before, if $dm_c > 0$ (elementary mass entering the compression volume) then $T_{rc} = T_{c'}$, else $T_{rc} = T_c$.

$$\delta W_c = -p\ dV_c \quad (17)$$

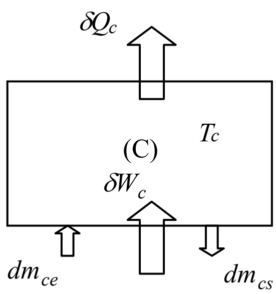

Figure 3. Cooling cell

One determines the air mass and its differential in the compression volume by using the perfect gas state equation:

$$p\ V_c = m_c\ r\ T_c \quad (18)$$

and its differential form :

$$\frac{dp}{p} + \frac{dV_c}{V_c} = \frac{dm_c}{m_c} + \frac{dT_c}{T_c} \quad (19)$$

The cooling cell is supposed to be isothermal. Thus the differential form of the state equation is reduced to:

$$\frac{dp}{p} + \frac{dV_c}{V_c} = \frac{dm_c}{m_c} \quad (20)$$

according to (18):

$$m_c = \frac{p\ V_c}{r\ T_c} \quad (21)$$

and :

$$dm_c = \frac{V_c}{T_c}\ \frac{dp}{r} + \frac{p}{T_c}\ \frac{dV_c}{r} = \frac{1}{r\ T_c}\left(V_c\ dp + p\ dV_c\right) \quad (22)$$

In this way the equation (16) becomes

$$\delta Q_c = -c_p T_{rc} dm_c + \frac{c_v}{r} V_c dp + \frac{c_p}{r} p dV_c$$

$$\delta Q_c = V_c \frac{c_p}{r}\left(\frac{1}{\gamma} - \frac{T_{rc}}{T_c}\right) dp + p \frac{c_p}{r}\left(1 - \frac{T_{rc}}{T_c}\right) dV_c \quad (23)$$

2.2.3 Heating cell

If the equation (9) is applied to this cell, the result is

$$\delta Q_h = -c_p\ T_{rh}\ dm_h + \frac{c_v}{r}\ V_h\ dp + \frac{c_p}{r}\ p\ dV_h \quad (24)$$

If $dm_h > 0$ then $T_{rh} = T_{h'}$, else $T_{rh} = T_h$.

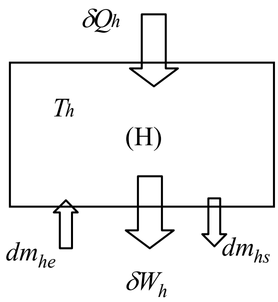

Figure 4. Heating cell

Using the same reasoning as for the previous cells, the air mass and its differential can be obtained:

$$m_h = \frac{p\ V_h}{r\ T_h} \quad (25)$$

$$dm_h = \frac{V_h}{T_h}\frac{dp}{r} + \frac{p}{T_h}\frac{dV_h}{r} = \frac{1}{r\,T_h}\left(V_h\ dp + p\ dV_h\right) \tag{26}$$

In this way the equation (24) becomes

$$\delta Q_h = V_h \frac{c_p}{r}\left(\frac{1}{\gamma} - \frac{T_{rh}}{T_h}\right)dp + p\frac{c_p}{r}\left(1 - \frac{T_{rh}}{T_h}\right)dV_h \tag{26'}$$

2.2.4 Engine balance

During a crankshaft rotation the quantities of heat exchanged in the cooling and heating cells are: $Q_h = \oint \delta Q_h$ and $Q_c = \oint \delta Q_c$ obtained by integration of the equations (23) and (26').

With the considered assumptions ($T_{c'} - T_c = T_h - T_{h'}$), an additional correcting quantity of heat Q_r, to bring by the hot source, is essential.

This deficit is due to the inequality of the masses transferred at the interfaces cooling cell–regenerator cell and heating cell–regenerator cell (different densities).

$$\oint \delta Q_r = \oint c_p T_{rc} dm_c + \oint c_p T_{rh} dm_h$$

Notice : if η_r=100% then T_{ri} = T_i = cst throughout the cycle and Q_r=0 .

The overall engine balance is written :

$$W + Q_c + Q_h + Q_r = 0$$

$$W = -\left(Q_c + Q_h + Q_r\right) \tag{27}$$

This work is carried out by the working piston during compression and expansion. It can be also calculated by the following expression:

$$dW = dW_c + dW_h$$

dW_c and dW_h having been expressed previously. After integration:

$$W = W_c + W_h \tag{28}$$

This work is also:

$$W = -\oint p dV \tag{29}$$

with p : the internal pressure.

and, dV : the total air volume variation

The thermal engine efficiency can be expressed by:

$$\eta_{th} = \frac{W}{Q_h + Q_r} \tag{30}$$

and the second low efficiency by :

$$\eta_{II} = \frac{\eta_{th}}{\eta_{carnot}} \tag{31}$$

The temperatures of the walls considered as sources are given by the following relations :

$$Q_c = U_c\ A_c\ (T_{wc} - T_c\ )\ \Delta t \tag{32}$$

$$Q_h + Q_r = U_h\ A_h\ \left(T_{wh} - T_h\right)\ \Delta t \tag{33}$$

where Δt is the engine revolution period and U_i is the heat transfer coefficient between the gas and the cell wall *i*.

2.3 Entropy analysis

In the same way as previously, the entropic balance of an open system is written in the form:

$$dS = \frac{\delta Q}{T} + s_e.dm_e - s_s.dm_s + \delta\pi \tag{34}$$

applied to the three cells.

2.3.1 Regenerator cell

The entropy balance of this cell is:

$$dS_r = \frac{\delta Q_r}{T_r} - s_{rc}\ dm_c - s_{rh}\ dm_h + \delta\pi_r \tag{35}$$

s_{rc} and s_{rh} are the specific entropies of the fluid associated with the mass flow through the entries of the cell:

$$s_{rc} = s_0 + c_p.\ln(\frac{T_{rc}}{T_0}) - r.\ln(\frac{p}{p_0}) \tag{36}$$

$$s_{rh} = s_0 + c_p.\ln(\frac{T_{rh}}{T_0}) - r.\ln(\frac{p}{p_0}) \tag{37}$$

s_0, T_0, p_0 are reference parameters.

Two possible cases arise, again:

- first case, flow between (R) and (C) : if $dm_c > 0$ then $s_{rc} = s_{c'}$ (and $T_{rc} = T_{c'}$), else $s_{rc} = s_c$ (and $T_{rc} = T_c$).

- second case : flow between (H) and (R), if $dm_h > 0$ then $s_{rh} = s_{h'}$ (and $T_{rh} = T_{h'}$), else $s_{rh} = s_h$ (and $T_{rh} = T_h$).

2.3.2 Cooling cell

For the cooling cell case, one obtains:

$$dS_c = \frac{\delta Q_c}{T_c} + s_{rc}.dm_c \tag{38}$$

As it was already noted, the specific entropies depend on the direction of the flow:

If $dm_c > 0$ then $s_{rc} = s_{c'}$ (and $T_{rc} = T_{c'}$), else $s_{rc} = s_c$ (and $T_{rc} = T_c$).

The entropy flow follows the heat transfer direction and increases with the temperature difference.

The ***external*** entropy generation $\delta\pi_c$, due to the temperature pinch between the cold source (wall) and the gas (T_{sc} and T_c), is the difference between the entropy received by the cold source and the entropy transferred by the cooling cell (*Figure 5*).

$$\delta\pi_c = dS_{sc} - dS_c \tag{39}$$

$$\delta\pi_c = \left|\delta Q_c\right|\left(\frac{1}{T_{sc}} - \frac{1}{T_c}\right) \tag{40}$$

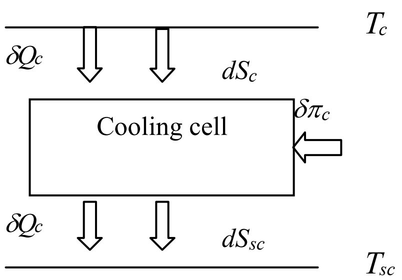

Figure 5. Functional entropy balance diagram of the cooling cell

2.3.3 Heating cell

One writes the entropy balance in the heating cell under the form :

$$dS_h = \frac{\delta Q_h}{T_h} + s_{rh}.dm_h \tag{41}$$

If $dm_h > 0$ then $s_{rh} = s_{h'}$ (and $T_{rh} = T_{h'}$), else $s_{rh} = s_h$ (and $T_{rh} = T_h$).

The difference between entropy received by air and the entropy transferred by the hot source represents the « external » entropy generation due to the temperature pinch (*Figure 6*).

$$\delta\pi_h = dS_h - dS_{sh} \tag{42}$$

$$\delta\pi_h = \delta Q_h\left(\frac{1}{T_h} - \frac{1}{T_{sh}}\right) \tag{43}$$

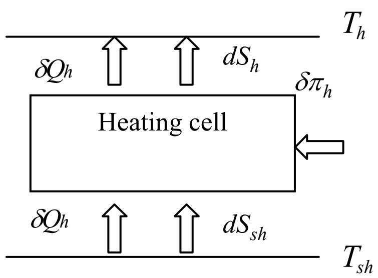

Figure 6. Functional entropy diagram of the heating cell

2.3.4 Engine entropy balance

The engine entropy balance is written excluding external entropy generation below, using (35), (38) and (41) :

$$dS_c + dS_h + dS_r = \frac{\delta Q_c}{T_c} + \frac{\delta Q_h}{T_h} + \frac{\delta Q_r}{T_r} + \delta\pi_r = 0 \tag{44}$$

where $\delta\pi_r$ is the regenerator entropy generation.

$$\delta\pi_r = -\left(\frac{\delta Q_c}{T_c} + \frac{\delta Q_h}{T_h} + \frac{\delta Q_r}{T_r}\right) \tag{45}$$

2.4 Exergy analysis

In order to evaluate the effectiveness of each cell, one now proposes to write the exergetic balance for an open system:

$$dEx = (1 - \frac{T_0}{T})\delta Q + \delta W + p_0.dV + \sum ex_i^f.dm_i - T_0\,\delta\pi_{\text{int}} \tag{46}$$

applied to each one of these cells.

ex_i^f : represents the specific exergy associated with the mass flow dm_i.

$$ex_i^f = \left(h_i - h_0\right) - T_0\left(s_i - s_0\right) \tag{47}$$

In the following, for our particular application, the external exergetic expenditure is from the cold sink ($T_c<T_0$) (we use a cold fluid recuperation as cold sink of the engine, like a ice stones pack); T_0, the ambient temperature (hot source temperature), will be the reference temperature.

2.4.1 Regenerator cell

The regenerator cell volume is constant during the engine operation, $dV_r = 0$ and $\delta W_r = 0$, thus equation (46) becomes :

$$dEx_r = (1 - \frac{T_0}{T_r})\delta Q_r - ex_{rc}^f.dm_c - ex_{rh}^f.dm_h - T_0\,.\delta\pi_r \tag{48}$$

First case : for a flow from (C) to (R)

if $dm_c < 0$, then $ex_{rc}^f = ex_c^f$,

else $ex_{rc}^f = ex_{c'}^f$.

Second case : for a flow from (H) to (R)

if $dm_h < 0$, then $ex_{rh}^f = ex_h^f$,

else $ex_{rh}^f = ex_{h'}^f$.

2.4.2 Cooling cell

The exergy balance in the compression space can be expressed by :

$$dEx_c = (1-\frac{T_0}{T_c})\delta Q_c + \delta W_c + p_0.dV_c + ex^f_{rc}.dm_c \quad (49)$$

$$ex^f_{rc} = \left(h_{rc}-h_0\right)-T_0\left(s_{rc}-s_0\right)$$

$$ex^f_{rc} = c_p\left(T_{rc}-T_0\right)-T_0\left(s_{rc}-s_0\right) \quad (50)$$

where, the interface entropy is expressed by

$$s_{rc} = s_0 + c_p.\ln(\frac{T_{rc}}{T_0}) - r.\ln(\frac{p}{p_0})$$

As in paragraph 2-3, the specific exergies depend on the interface temperatures and the reference state considered :

If $dm_c>0$, then $ex^f_{rc} = ex^f_{c'}$, else $ex^f_{rc} = ex^f_c$.

The functional diagram of the cooling cell (*Figure 7*) reveals the exergies exchanged by the gas with the cold sink.

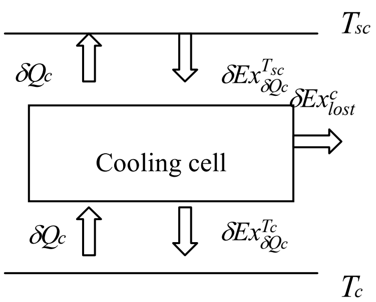

Figure 7. Functional exergetic diagram of the cooling cell

The direction of transfer of exergy is opposed to the direction of transfer of heat since the level of the temperatures is lower than the ambient temperature of reference T_0 . The exergy of a fluid whose temperature is lower than T_0 increases if it is cooled more; it is what occurs in the cooling cell of compression: as $T_c<T_0$, the exergy of the gas increases.

The exergy balance makes it possible to deduce the lost exergy due to the pinch of temperatures between the cooling cell and the cold source.

$$\left|\delta Ex^{T_{sc}}_{\delta Q_C}\right| = \delta Ex^c_{lost} + \left|\delta Ex^{T_c}_{\delta Q_C}\right| \quad (51)$$

where,

$\left|\delta Ex^{T_{sc}}_{\delta Q_C}\right| = \left|(1-\frac{T_0}{T_{sc}})\delta Q_c\right|$ represents the exergy of heat δQ_c at the temperature T_{sc} and,

$\left|\delta Ex^{T_c}_{\delta Q_C}\right| = \left|(1-\frac{T_0}{T_c})\delta Q_c\right|$ is the exergy of heat δQ_c but at the temperature T_c .

Then

$$\delta Ex^c_{lost} = T_0\left(\frac{1}{T_{sc}}-\frac{1}{T_c}\right)\left|\delta Q_c\right| = T_0\,\delta\pi_c \quad (52)$$

2.4.3 Heating cell:

In a similar way to the cell of compression treatment, one makes the exergy balance in the space of expansion and one obtains:

$$dEx_h = (1-\frac{T_0}{T_h})\delta Q_h + \delta W_c + p_0 dV_h + ex^f_{rh}.dm_h \quad (53)$$

$$ex^f_{rh} = c_p\left(T_{rh}-T_0\right)-T_0\left(s_{rh}-s_0\right) \quad (54)$$

where $s_{rh} = s_0 + c_p.\ln(\frac{T_{rh}}{T_0}) - r.\ln(\frac{p}{p_0})$.

If $dm_h > 0$ then $ex^f_{rh} = ex^f_{h'}$ else $ex^f_{rh} = ex^f_h$.

The functional diagram of exergy of this cell is presented in *Figure 8*.

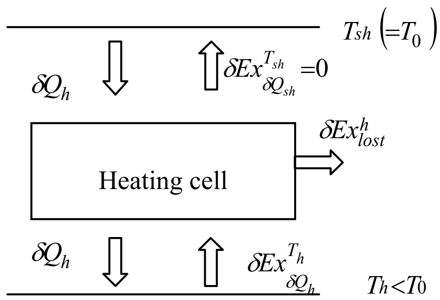

Figure 8. Functional exergy diagram of the heating cell

This time, the temperature T_h is lower than the ambient temperature $T_0 = T_{sh}$, the flow of exergie does not follow the heat flow.

The lost exergy due to the pinch of temperatures between the hot source and the heating cell is written:

$$\delta Ex^h_{lost} = \left|\delta Ex^{T_h}_{\delta Q_h}\right| \quad (55)$$

where $\left|\delta Ex_{\delta Q_h}^{T_{sh}}\right| = (1 - \frac{T_0}{T_{sh}})\delta Q_h = 0$ represents the exergy of heat δQ_h at the temperature $T_{sh} = T_0$ and

$\left|\delta Ex_{\delta Q_h}^{T_h}\right| = \left|(1 - \frac{T_0}{T_h})\right|\delta Q_h$ represents the exergy of heat δQ_h at the temperature T_h .

Then

$$\delta Ex_{lost}^{h} = T_0 \left(\frac{1}{T_h} - \frac{1}{T_{sh}}\right)\delta Q_h = T_0\, \delta\pi_h \qquad (56)$$

2.4.4 Exergetic characteristics of the engine:

The exergetic efficiency is defined generally by the ratio:

Useful effect/Exergetic expenditure, it is written:

$$\eta_{ex} = \frac{|W|}{Ex_{Q_h+Q_r}^{T_{sh}}} = \frac{|W|}{\left(1 - \frac{T_0}{T_{sh}}\right)\left(Q_h + Q_r\right)} \qquad (57)$$

if $T_{sh} > T_0$ and $T_{sc} \approx T_0$.

Or $$\eta_{ex} = \frac{|W|}{Ex_{Q_c}^{T_{sc}}} = \frac{|W|}{\left(1 - \frac{T_0}{T_{sc}}\right)Q_c} \qquad (58)$$

if $T_{sh} \approx T_0$ and $T_{sc} < T_0$ (our case: exergetic expenditure at the cold source).

For this second case of figure, which relates to the numerical application presented below, the maximum exergetic efficiency is the ratio:

$$\eta_{ex\max} = \frac{W_{\max}}{Ex_{Q_c}^{T_{sc}}} \qquad (59)$$

with, $W_{max} = \eta_{Carnot}\ (Q_h + Q_r)$ and, $\eta_{Carnot} = 1 - \frac{T_c}{T_h}$

3. Results and analyses:

The model described previously is used to represent the operation of the engine during one rotation of crankshaft.

The initial data are listed in the following table:

TABLE I. INITIAL DATA

a) Dimensional and mechanical data of the actual engine	
working piston stroke: x_0 (m)	0,007
working piston diameter: D_p (m)	0,0095
working piston swept volume: V_p (m^3)	$4{,}9618.10^{-7}$
displacer piston stroke : y_0 (m)	0,007
displacer piston diameter: D_d (m)	0,077
displacer piston swept volume: V_d (m^3)	$3{,}2596.10^{-5}$
proportion of cold dead volume: V_{mc}/V_d (-)	0,1
proportion of hot dead volume: V_{mh}/V_d (-)	0,1
proportion of regenerator dead volume: V_r/V_d (-)	0,2
out-of-phase angle of the pistons: φ (°)	90
number of revolutions of the engine: N(tr/min)	180
b) Thermodynamic characteristics of the engine (for simulation)	
temperature of gas in cold volume: T_c (K)	290
temperature of gas in hot volume: T_h (K)	292
thermal transfer coefficient (hot side): h_h (W/m^2 K)	10
thermal transfer coefficient (cold side): h_c (W/m^2 K)	10
regenerator efficiency: η_r[%]	50

We defined the data of the state of reference (0) as it follows:

$T_0 = 296{,}3\ K$

$p_0 = 10^5\ Pa$

$s_0 = 6{,}858\ J/kg\ K$

3.1 First simulation results:

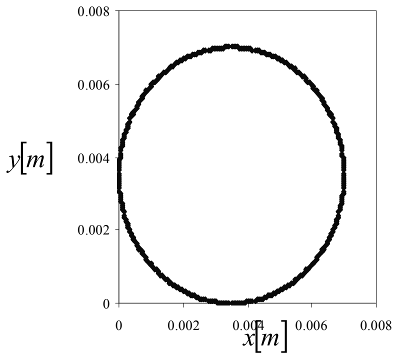

Figure 9. Working piston (x) and displacer piston (y) instantaneous positions

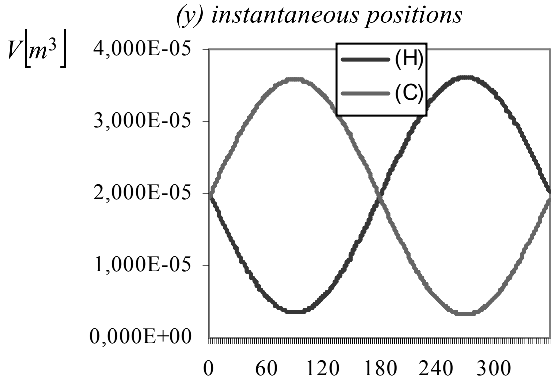

Figure 10. Expansion volume, compression volume and working volume versus crankshaft angle

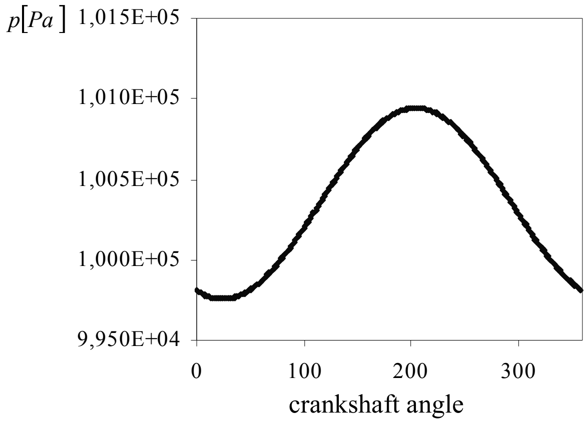

Figure 11. Pressure versus crankshaft angle

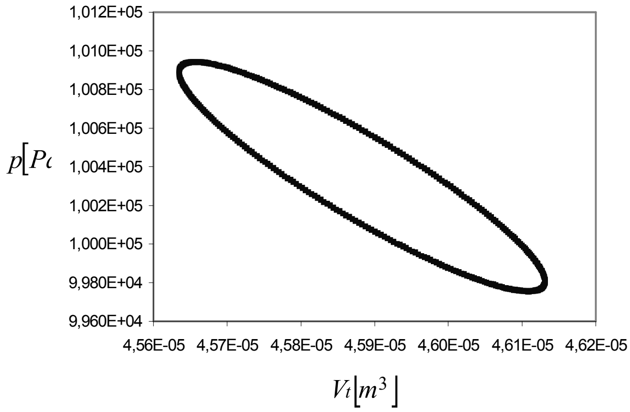

Figure 12. Engine p-V diagram

The out-of-phase movements of the two pistons (engine piston and displacer piston) can be followed on *Figure 9*.

Figure 10 shows the variations of hot and cold volumes according to the crankangle. It illustrates well the low variations of the working volume compared to those of the hot or cold volumes. Variation of the working volume is not shown because it is nearly one percent of the displacer swept volume.

Figures 11 and *12* show the variations of pressure in the cylinder according to the angle of crankshaft or the total volume of the engine (p-V diagram).

The following table provides the results of the simulation:

TABLE II. RESULTS FOR A COMPLETE CYCLE SIMULATION

cycle work: $W_{cycle}[J]$	$-1{,}909.10^{-4}$
cold side heat: $Q_c[J]$	$-6{,}705.10^{-2}$
hot side heat: $Q_h[J]$	$6{,}720.10^{-2}$
regenerator heat: $Q_r[J]$	$3{,}768.10^{-5}$
thermal efficiency: $\eta_{th}[\%]$	0,28
hot side temp.: $T_h[K]$	292
cold side temp.: $T_c[K]$	290
Carnot efficiency.: $\eta_{carnot}[\%]$	0,7
hot wall temp.: $T_{wh}[K]$	296,3
cold wall temp.: $T_{wc}[K]$	285,6
reg. entropy prod.: $\pi_r[J/K]$	$9{,}292.10^{-7}$
cold entropy prod.: $\pi_c[J/K]$	$3{,}495.10^{-6}$
hot entropy prod.: $\pi_h[J/K]$	$3{,}363.10^{-6}$
regen. exergy loss: $Ex^r_{lost}[J]$	$2{,}753.10^{-4}$
cold exergy loss: $Ex^c_{lost}[J]$	$1{,}035.10^{-3}$
hot exergy loss: $Ex^h_{lost}[J]$	$9{,}967.\ 10^{-4}$
exergetic efficiency: $\eta_{ex}[\%]$	7,63
max. exergetic eff.: $\eta_{ex\max}[\%]$	18,7

3.2 Sensitivity study:

The central parameters of this study of sensitivity are:

- The proportion of the regenerator dead volume $V_{mr}/V_d = 0{,}2$
- The temperature of the cold side gas (air) $T_c = 290K$
- The regenerator efficiency $\eta_r = 50\%$.

3.2.1 Regenerator dead volume:

TABLE III. SENSITIVITY OF THE EFFICIENCY OUTPUTS VERSUS PROPORTION OF REGENERATOR DEAD VOLUME

$V_r/V_d\ [-]$	$\eta_{th}\ [\%]$	$\eta_{ex}\ [\%]$
0,1	0,30	7,77
0,2	0,28	7,64
0,3	0,27	7,50

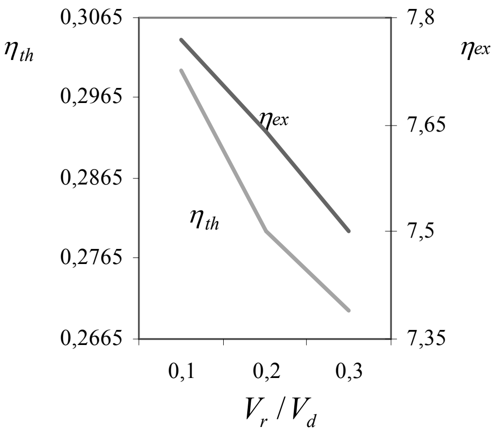

Figure 13. η_{th} and η_{ex} variations versus regenerator dead volume ratio V_{mr}/V_d

Figure 13 presents the influence of regenerator dead volume on the thermal and exergetic efficiencies. It is obviously necessary to limit as far as possible this dead volume since it is a volume of gas which handicaps the engine performance.

3.2.2 Cold side gas temperature:

TABLE IV. SENSITIVITY OF THE EFFICIENCY OUTPUTS COMPARED TO THE COLD GAS TEMPERATURE

$T_c[K]$	$T_h - T_c$	$\eta_{th}\ [\%]$	$\eta_{carnot}[\%]$	$\eta_{ex}\ [\%]$	$\eta_{II}[\%]$
291	1	0,20	0,34	8,16	58,62
290	2	0,28	0,68	7,64	41,41
289	3	0,33	1,03	6,59	32,07
288	4	0,36	1,37	5,67	26,14

One notices on *Figure 14* the evolution of the efficiencies according to the temperature of the gas (air) on the cold side, therefore of the difference of

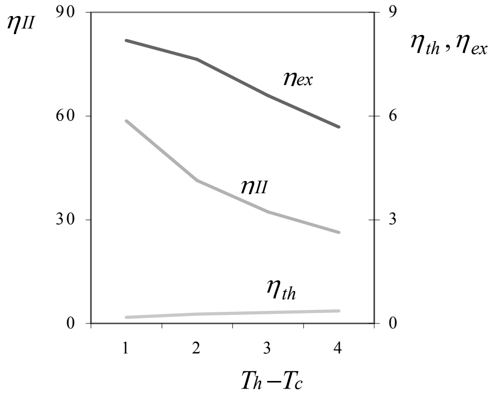

Figure 14. η_{th}, η_{ex} and η_{II} versus temperature difference $T_h - T_c$

temperature $T_h - T_c$. It is obvious that for a more important difference of temperature the engine uses better the heat provided at each cycle what implies a higher thermal efficiency.

On the other hand, since the air will be colder on the cold side, the exergetic expenditure which represents the exergy Q_c at temperature T_{SC} also increases, which makes decrease the exergetic efficiency.

On *Figure 15*, one traces entropy generation according to the difference of the hot and cold temperatures of the gas. One realizes that if the difference in temperature increases, entropy generation in the three cells of the engine increase, which cause a drop in the degree of quality of the engine.

TABLE V. SENSITIVITY OF ENTROPY PRODUCTIONS VERSUS TEMPERATURE OF THE COLD GAS

$T_c[K]$	T_h-T_c	$\pi_c[J/K]$	$\pi_h[J/K]$	$\pi_r[J/K]$
291	1	$1{,}718.10^{-6}$	$1{,}677.10^{-6}$	$2{,}306.10^{-7}$
290	2	$3{,}495.10^{-6}$	$3{,}364.\ 0^{-6}$	$9{,}292.10^{-7}$
289	3	$5{,}949.10^{-6}$	$5{,}638.10^{-6}$	$2{,}106.10^{-6}$
288	4	$9{,}119.10^{-6}$	$8{,}507.10^{-6}$	$3{,}772.10^{-6}$

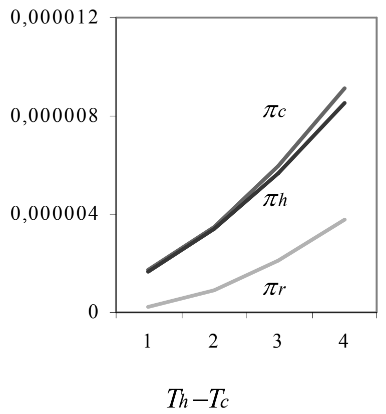

Figure 15. Entropy productions π_c, π_h and π_r versus T_h-T_c

3.2.3 Regenerator efficiency:

TABLE VI. SENSITIVITY OF THE OTHER EFFICIENCY OUTPUTS VERSUS REGENERATOR EFFICIENCY

$\eta_r[\%]$	$\eta_{th}[\%]$	$\eta_{ex}[\%]$	$\eta_{II}[\%]$
0	0,18	3,23	26,15
25	0,22	4,75	32,07
50	0,28	7,64	41,46
75	0,40	14,25	58,62
100	0,68	35,26	100

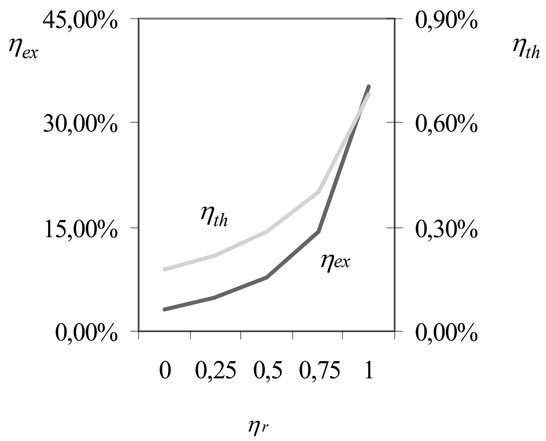

Figure 16. Variations of η_{th} and η_{ex} versus η_r

As one can see on *Figure 16* and TABLE VI, the efficiency of the regenerator has a great influence on the thermal and exergetic efficiencies of the engine. A better regeneration increases the thermal efficiency, since same work will be provided for a lower energy expenditure from the hot source.

The exergetic efficiency increase is due to the reduction of the exergy expenditure at the cold sink. In fact, the temperature of this sink increases, approaching the temperature of the gas (a constant for this study of sensitivity) which decreases its energy quality, as $T_{SC}<T_0$.

TABLE VII. SENSITIVITY OF THE TEMPERATURE OF THE WALLS TO THE EVOLUTION OF THE REGENERATOR EFFICIENCY (with fixed T_c and T_h)

$\eta_r[\%]$	$T_{wh}[K]$	$T_{wc}[K]$	ΔT_w
0	298,867	283,144	15,723
25	297,599	284,412	13,187
50	296,331	285,680	10,651
75	295,063	286,948	8,115
100	293,795	288,216	5,579

On *Figure 17*, one sees that if the regenerator efficiency increases, the difference of temperatures between the hot and cold walls decreases with a

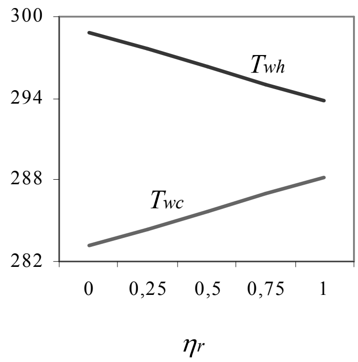

Figure17. Variations of T_{wc} and T_{wh} versus η_r

constant $(T_c - T_h)$. This is due to the reduction of the transferred heat quantities.

TABLE VIII. SENSITIVITY OF ENTROPY GENERATIONS TO THE REGENERATOR EFFICIENCY

$\eta_r[\%]$	$\pi_c[J/K]$	$\pi_h[J/K]$	$\pi_r[J/K]$
0	$8{,}883.10^{-6}$	$8{,}381.\ 10^{-6}$	$1{,}858.\ 10^{-6}$
25	$5{,}874.\ 10^{-6}$	$5{,}60.\ 10^{-6}$	$1{,}393.\ 10^{-6}$
50	$3{,}495.\ 10^{-6}$	$3{,}36.\ 10^{-6}$	$9{,}291.\ 10^{-7}$
75	$1{,}736.\ 10^{-6}$	$1{,}690.\ 10^{-6}$	$4{,}645.\ 10^{-7}$
100	$5{,}907.\ 10^{-7}$	$5{,}84.\ 10^{-7}$	0

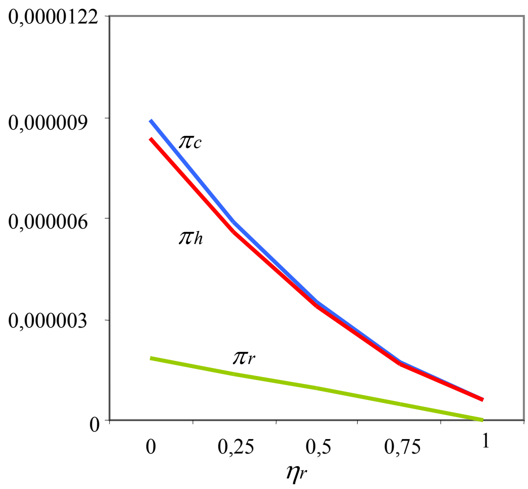

Figure 18. Variations of π_c, π_h and π_r versus η_r

Generation of entropy in the three cells of the engine decrease with the regenerator efficiency increase : higher is this efficiency, less important are the losses due to the differences of temperature and the losses due to imperfect regeneration.

4. Comparison with experimental results:

Tests on a nearly identical engine as ours with low ΔT were carried out by H. Roussel [7]. For two experimental points, he published the following results :

$T_h - T_c$	ΔT_w	$N[tr/\min]$	$W[J]$
3	21	199	$2{,}89.10^{-4}$
2,6	5 ,5	20	$2{,}46.10^{-4}$

The results obtained by our model are located in the same order of magnitude while taking $\eta_r = 0$ and $T_h - T_c = 3°C$. Our calculated work in this case, $W_{cal} = 2{,}91.10^{-4} J$, and that measured by H. Roussel ($2.89.10^{-4}$ J) are very close. We carried out tests on our engine, very similar to those of H. Roussel, which confirm his results.

A test bench is under development ; pressure and piston position pick-ups will allow the direct calculation of the indicated work provided by gas, by the layout of the real cycle on a p-V diagram ; temperature sensors, a controlled electric heating, an effective insulation, various mechanical adjustments (strokes, out-of-phase angle) supplement this equipment and will make it possible to obtain the indicated efficiency and the Carnot efficiency of the engine, as well as a modulation of the kinematics of the engine. This will make it possible to better validate this thermodynamic model.

5. Conclusion:

In this paper, we present a thermodynamic study of one γ type Stirling engine at low temperature difference and steady state operation. Energy, entropy and exergy balances were carried out at each element of the engine, according to the kinematics of the pistons (displacer and working pistons).

A sensitivity study of the engine efficiencies and creations of entropy with respect to various parameters of the model (regenerator efficiency, cold temperature of gas, dead volume of the regenerator) was carried out.

The results of simulation make it possible to locate the optimum conditions for operation of this engine which give the best efficiencies and the smallest productions of entropy, to minimize the operating cost. This model was validated by a comparison with experimental results of actual engines.

Nomenclature

A	Section, [m^2]
c_p	Specific heat at p=cst, [J/kg K]
c_v	Specific heat at V= cst, [J/kg K]
Ex	Exergy, [J]
ex	Specific exergy, [J/kg]
h	Specific enthalpy, [J/kg]
l	Length, [m]
p	Pressure, [Pa]
Q	Heat, [J]
R	Radius, [m]
r	Specific gas constant, [J/kg K]
S	Entropy, [J/K]
s	Specific entropy, [J/kg K]
T	Temperature, [K]
t	Time, [s]
U	Heat exchange coefficient, [W/m^2 K]
u	Velocity, [m/s]
V	Volume, [m^3]
W	Work, [J]
x	Position of the working piston, [m]
y	Position of the displacer, [m]
θ	Angle of crankshaft, [-]
π	Entropy generation, [J/K]
φ	Out-of-phase angle, [-]
η	Efficiency, [-]

Indices:

a	ambiant conditions
c	cold
cyl	cylinder
d	displacer
i	inlet, outlet

g	gas
h	hot
int	internal
m	dead (volume)
p	working piston
r	regenerator
sc	cold source
sh	hot source
th	thermal
w	wall
0	reference state

References

FEIDT M., LESAOS K., COSTEA M., PETRESCU S., "Optimal allocation of HEX inventory associated with fixed power output or fixed heat transfer rate input", Int. J. Applied Thermodtnamics, vol 5, n°1, p25-36, mars 2002

MARTAJ N., GROSU L., ROCHELLE P., 2006 "Exergetical analysis and design optimization of the Stirling engine", Int. J. of Exergy, Vol.3, No. 1, pp.45-67

PRIETO J.I., GARCIA D., 2005,"Comparaison between Kolin's law for power and other criteria for preliminary design of Kinematic Stirling engines" Thermo- and GFD modelling of Stirling machines, pp.389-397.

ROBSON A., 2005, "Development of a computer model to simulate a low temperature differential Ringbom Stirling engine", Thermo- and GFD modelling of Stirling machines, pp.350-357.

ROCHELLE P., 2005, "LDT Stirling engine simulation and optimization using finite dimension thermodynamics", Thermo- and GFD modelling of Stirling machines, pp.358-366

WAGNER A., SYRED N., ELSNER M., 2005, "First order calculation of Stirling engines", Thermo- and GFD modelling of Stirling machines, pp.367-379.

www.ent.ohiou.edu/~urieli/stirling/me422.html

www.photologie.net/indexStirBlue.html