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Propagation of weakly nonlinear waves in nanorods using nonlocal elasticity theory

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Abstract

The present research examines the propagation of weakly solitary waves in nanorods by employing nonlocal elasticity theory. Many systems in physics, engineering, and natural sciences are nonlinear and modeled with nonlinear equations. Wave propagation, as a branch of nonlinear science, is one of the most widely studied subjects in recent years. Nonlocal elasticity theory represents a technique with increasing popularity for the purpose of conducting the mechanical analysis of microelectromechanical and nanoelectromechanical systems. The nonlinear equation of motion of nanorods is derived by utilizing nonlocal elasticity theory. The reductive perturbation technique is employed for the purpose of examining the propagation of weakly nonlinear waves in the longwave approximation, and the Korteweg-de Vries equation is acquired as the governing equation. The steady-state solitary-wave solution is known to be admitted by the KdV equation. To observe the nonlocal effects on the KdV equation numerically, the existence of solitary wave solution has been investigated using the physical and geometric properties of carbon nanotubes.

Keywords: *Nanorod, nonlocal elasticity theory, Nonlinear waves, reductive perturbation technique.*

Yerel olmayan elastisite teorisi kullanılarak nano ölçekli çubuklarda nonlineer dalga yayılımı

Özet

Bu çalışmada, yerel olmayan elastisite teorisi kullanılarak nano ölçekli çıbuklarda zayıf nonlineer dalga yayılımı incelenmiştir. Mühendislik, fizik ve doğal bilimlerde birçok

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sistem nonlineerdir ve nonlineer denklemlerle modellenir. Lineer olmayan bilimin bir dalı olan dalga yayılımı son yıllarda yaygın olarak çalışılan konulardan biridir. Yerel olmayan elastisite teorisi microelektromekanik ve nanoelektromekanik gibi sistemlerin analizinde gelişen popüler bir tekniktir. Formülasyonlarda Eringen'in yerel olmayan elastisite teorisine dayanan bünye denklemleri kullanılmıştır. Hareket denklemleri malzeme koordinatları cinsinden yazılmış ve nano ölçekli çubuğun doğrusal olmayan hareket denklemleri yerel olmayan elastisite teorisine göre elde edilmiştir. İndirgeyici pertürbasyon metodu kullanılarak zayıf nonlineer dalgaların hareketini yöneten evolüsyon denklemi olarak Korteweg de Vries (KdV) denklemi elde edilmiştir. KdV denkleminde yerel olmayan etkiyi nümerik olarak gözlemleyebilmek için, karbon nanotüplerin fiziksel ve geometrik özellikleri göz önünde bulundurulmuştur.

Anahtar kelimeler: *Nano ölçekli çubuk, Yerel olayan elastisite teorisi, nonlineer dalgalar, indirgeyici pertürbasyon metodu.*

1. Introduction

The accurate characterization of the actual mechanical behavior of nanoscale devices is significant in the design of the devices in question, including carbon nanotubes (CNTs). Carbon nanotubes have high technological potential because of their light weights, having high elasticity module, capable of showing metalic or semi conductivity features and possible modifications of their electronic aspects. However, the implementation of classical continuum theory is controversial while analyzing carbon nanotubes mechanically. Classical continuum theory (classical elasticity theory) is length scale-free. Hence, it cannot accurately account for very small-sized effects. To eliminate the deficiencies of classical continuum theory, continuum theories of higher order, including micro-polar elasticity theory [1-4], nonlocal elasticity theory [5-7], couple stress theory [8] and the modified couple stress approach [9, 10], have received significant attention in the analysis of micro- and nanostructures. Due to the high cost of experiments that operate on the nanoscale, it is of vital importance to introduce suitable physical models for nanobeams (carbon nanotubes) for the establishment of an appropriate theoretical and mathematical framework for nanosized structures [11-13]. Eringen [14] and Eringen and Edelen [15] proposed nonlocal elasticity theory in the 1970s for the purpose of overcoming the deficiencies of classical elasticity models. Eringen obtained equations for nonlocal elasticity theory and made very important contributions to science by achieving results that cannot be achieved by classical means in different studies on one-dimensional elastic wave propagation problems. Furthermore, Eringen and Edelen conducted studies on the derivation of constitutive equations for elastic media in two different ways, including mechanical and variational. Moreover, Demiray [16] obtained constitutive equations for nonlocal dielectric materials. Unlike the conventional theory of elasticity, in the nonlocal theory of elasticity, it is assumed that the strain at a particular point in a continuous domain and the strain at each point in the domain determine the stress at the point in question. Several studies have been performed using this nonlocal model to conduct the analysis of the mechanical behavior of nanosized structures [17-19].

The investigation of vibration and wave propagation in CNTs constitutes the main subject of ongoing studies. Various studies conducted previously have investigated the vibration of CNTs, nanobeams, and rods by employing nonlocal elasticity ([10]-[19]). The free axial vibration of uniform nanorods was examined in the study of Aydogdu

[18] by means of Eringen's nonlocal continuum theory. As a result of the study, it was demonstrated that the nonlocal rod model overestimates the natural frequencies of the nanorod in comparison with the classical model. Furthermore, Narendar and Gopalakrishnan [21] studied the longitudinal vibration of nanorods by utilizing a nonlocal bar model. According to the findings acquired, it was demonstrated that a particular band gap region in the longitudinal wave mode was caused by the small scale parameter of the nonlocal model, which allowed no wave propagation to occur. Moreover, Filiz and Aydogdu [22] investigated the axial vibration of heterojunction CNTs in the context of Eringen's nonlocal continuum theory. Besides, the impact of nonlocality and lengths of CNTs and their segments was studied in a detailed manner. The ultrasonic wave dispersion characteristic of nanorods was examined in the study of Narendar and Gopalakrishnan [23] based on nonlocal strain gradient models. Murmu and Adhikari [24] studied the longitudinal vibration of double-nanorod-system (DNRS) based on nonlocal elasticity theory. The results obtained from the above-mentioned study emphasize the significant impact of the nonlocal impact on the axial vibration of DNRS.

Wave propagation is a very effective, nondestructive method used for the characterization of nanostructures. Nanosensor transducers also work on the wave propagation principle. The wave propagation issue has attracted attention around the world [25-31] in different domains of science and engineering because of its importance. In the study carried out by Lim and Yang [25], wave propagation in CNTs was investigated on the basis of nonlocal elastic stress field theory and Timoshenko beam theory, and a novel dispersion and spectrum correlation was acquired. Transverse and torsional waves in single-walled carbon nanotubes (SWCNTs) and double-walled carbon nanotubes (DWCNTs) were investigated on the basis of nonlocal elastic cylindrical shell theory in the study of Hu et al. [26]. The researchers compared the wave dispersion that was estimated by their model with molecular dynamics simulations in the terahertz area and concluded that it was possible to acquire a better prediction of dispersion relations by the nonlocal model. Wu and Dzenis [32] investigated wave propagation in nanofibers. The researchers studied longitudinal and flexural wave propagation in nanofibers by employing local theories in terms of surface impacts. Challamel [33] suggested a dispersive wave equation by utilizing nonlocal elasticity. A mixture theory of a local and nonlocal strain was introduced. The nonlocal scale impacts on the ultrasonic wave feature of nanorods were examined by Narendar and Gopalakrishnan [34] by employing the nonlocal Love rod theory. Narendar [35] used the nonlocal Love-Rayleigh rod theory for the examination of wave propagation in uniform nanorods.

Under natural conditions, the deformation of CNTs is nonlinear [36]. Nevertheless, the research summarized above focuses on linear deformation. The role of nonlinear excitations in physics and engineering is significant, and they are present in different areas including water waves, plasma physics, nonlinear optics, etc. The more precise quantification of the static and dynamic characteristics of CNTs is only possible if nonlinearities in geometry and physics are considered [37-41], which would lead to more extensive areas of application of CNTs. The asymptotic behavior of weakly nonlinear dispersive waves was studied extensively in the past years. One-dimensional propagation of long nonlinear waves in various dispersive systems is defined by the Korteweg-de Vries equation. Different asymptotic methods [42] were employed in the study of Demiray [43] to examine the motion of weakly nonlinear pressure waves in a thin nonlinear elastic tube that was filled with an incompressible fluid. Furthermore, the researcher examined solitary waves in fluid-filled elastic tubes in a weakly dispersive case [44]. The researcher confirmed that the Korteweg-de Vries equation governs the dynamics in case of ignoring the viscosity of blood. The propagation of large amplitude nonlinear waves in a peridynamic solid was studied by Silling [45]. It was shown that peridynamic solitary waves arise from the balance between nonlinearity in the material model and the dispersive characteristic of the model in the natural environment because of nonlocality.

Because of the internal structure of the theory of nonlocal elasticity, there is a possibility to examine dispersive wave propagation in the linear approximation. Different phenomena, for example, solitary waves, may emerge with the condition that nonlinearity is included because of the above-mentioned property of the model, in other words, its dispersive characteristic. As a matter of fact, in case of the separate occurrence of any of the mentioned impacts, dispersion and nonlinearity, no solitary wave solutions present. If there are both impacts, a travelling wave of constant profile and velocity is promoted by the competition between steepening as a consequence of nonlinearity and spreading due to dispersion.

In the present work, nonlinear wave propagation in nanorods based on the nonlocal theory is studied by utilizing the reductive perturbation method. Firstly, a one-dimensional nonlinear field equation is obtained, and the propagation of weakly nonlinear waves in these dispersive media is examined in the long-wave limit by utilizing the reductive perturbation technique. The linear dispersion relation of axial waves is also revealed to see the dispersive character of the environment. It is shown that the Korteweg-de Vries (KdV) equation governs the nonlinear propagation of axial waves in nonlocal elastic media. Moreover, the localized travelling wave solutions for the mentioned evolution equation are presented. To observe the nonlocal effects on the KdV equation numerically, the existence of solitary wave solutions has been investigated using the physical and geometric properties of CNTs.

The organization of the current study is presented below. Section 2 contains information on nonlinear local and nonlinear nonlocal elasticity theory and governing equation of the system. The fundamental principles of the reductive perturbation technique and propagation of nonlinear waves are briefly discussed in Section 3. Section 4 contains the numerical and graphical presentation of the findings. In Section 5, some discussions and conclusions are given.

2. Theoretical formulations

2.1. Nonlinear local equation of motion of nanorods

The equations of motion of nanorods in local elastic media were derived. It is possible to write the deformation gradient tensor as follows [46]:

$$\mathbf{F} = \vec{\boldsymbol{\nabla}}\mathrm{U} + \mathbf{I}\,. \tag{1}$$

Here, U refers to the displacement component of the motion, while **I** is the unit matrix. Neglecting the body forces on the element in a medium exposed to a finite extension, in terms of material coordinates, the equation of motion can be written as shown below:

$$\nabla.[\mathrm{SF^T}] = \rho_0 \frac{\partial^2 U}{\partial t^2}. \tag{2}$$

Here, ρ_0 is the non-deformed density of the medium, whereas S is the second Piola-Kirchoff stress tensor. It is possible to write Hooke's law as presented below:

$$\mathrm{S} = \mathrm{cE}. \tag{3}$$

Here, c refers to the fourth-order tensor that represents the elastic behavior of the material, while E refers to the Green strain tensor written in the following form:

$$\mathrm{E} = \frac{1}{2}[\mathrm{F^T F} - 1]. \tag{4}$$

By limiting the boundary conditions of the rod and assuming that only axial deformation U(x,t) takes place in the medium, the gradient deformation tensor in the Cartesian coordinates truns into a diagonal matrix:

$$\mathrm{F_{xx}} = 1 + \frac{\partial \mathrm{U}}{\partial \mathrm{x}} \;, \tag{5}$$
$$\mathrm{F_{zz}} = 1, \tag{6}$$
$$\mathrm{F_{yy}} = 1. \tag{7}$$

Referring only to the non-zero element in the Green strain tensor, the following equation can be obtained:

$$\mathrm{E_{xx}} = \left(1 + \frac{1}{2}\frac{\partial \mathrm{U}}{\partial \mathrm{x}}\right)\frac{\partial \mathrm{U}}{\partial \mathrm{x}}. \tag{8}$$

The stress-strain relationships of isotropic materials with the Poisson's ratio ν and modulus of elasticity $\mathrm{E_E}$ are then as follows:

$$\mathrm{S_{ij}} = \frac{E_E}{(1+\nu)}\left[\mathrm{E_{ij}} + \frac{\nu}{1-2\nu}\mathrm{E_{kk}}\delta_{\mathrm{ij}}\right]. \tag{9}$$

δ_{ij} denotes the Kronecker delta. If we insert Eq. (8) into Eq. (9), it is obtained that shear stresses are eliminated, and normal stress elements are shown as below:

$$\mathrm{S_{xx}} = \frac{E_E\,(1-\nu)}{(1+\nu)(1-2\nu)}\left(1 + \frac{1}{2}\frac{\partial \mathrm{U}}{\partial \mathrm{x}}\right)\frac{\partial \mathrm{U}}{\partial \mathrm{x}}, \tag{10}$$

$$\mathrm{S_{yy}} = \mathrm{S_{zz}} = \frac{E_E\,\nu}{(1+\nu)(1-2\nu)}\left(1 + \frac{1}{2}\frac{\partial \mathrm{U}}{\partial \mathrm{x}}\right)\frac{\partial \mathrm{U}}{\partial \mathrm{x}}. \tag{11}$$

By rearranging Eqs. (10) and (11) using Eqs. (5), (6), and (7), Eq. (12) can be acquired as below:

$$\left[\left(\frac{\partial \mathrm{U}}{\partial \mathrm{x}}\right)^2 + 2\frac{\partial \mathrm{U}}{\partial \mathrm{x}} + \frac{2}{3}\right]\frac{\partial^2 \mathrm{U}}{\partial \mathrm{x}^2} = \frac{2\rho_0(1+\nu)(1-2\nu)}{3\,E_E\,(1-\nu)}\frac{\partial^2 \mathrm{U}}{\partial \mathrm{t}^2}\,. \tag{12}$$

For the infinite deformation of the environment, the nonlinear terms in Eq. (12) become not important, and it is reduced to the following equation:

$$\frac{\partial^2 U}{\partial x^2} = \frac{\rho_0 (1+\nu)(1-2\nu)}{E_E\,(1-\nu)} \frac{\partial^2 U}{\partial t^2}. \tag{13}$$

To make the equation non-dimensional, the non-dimensional variables presented below should be introduced:

$$\Psi = \frac{U}{r_0}, \tag{14}$$

$$\zeta = \frac{x}{r_0} \;. \tag{15}$$

Here, r_0 defines the radius of the nano rod. If Eqs. (12) and (13) are reorganized by utilizing Eqs. (14) and (15), it is possible to derive the non-dimensional equation below, as has been obtained by Mousavi and Fariborz [47]:

$$\left[\left(\frac{\partial \psi}{\partial \zeta}\right)^2 + 2\left(\frac{\partial \psi}{\partial \zeta}\right) + \frac{2}{3}\right] \frac{\partial^2 \psi}{\partial \zeta^2} = \frac{2}{3} \delta \frac{\partial^2 \psi}{\partial t^2} \tag{16}$$

and linear equation of motion is

$$\frac{\partial^2 \psi}{\partial \zeta^2} = \delta \frac{\partial^2 \psi}{\partial t^2}, \tag{17}$$

where the coefficient δ is

$$\delta = \frac{\rho_0\, {r_0}^2 (1+\nu)(1-2\nu)}{E_E\,(1-\nu)}. \tag{18}$$

2.2. Nonlinear nonlocal equation of motion of nanorods

It is possible to write the constitutive equation for a nanorod as shown below by utilizing nonlocal elasticity ([14], [17]):

$$[1 - (e_0 a)^2\, \nabla^2][SF^T] = \lambda_L\, E_{rr} \delta_{kl} + 2\, \mu_L\, E_{kl}\;, \tag{19}$$

Where SF^T refers to the nonlocal stress tensor, E_{kl} refers to the strain tensor, λ_L and μ_L refer to Lamé constants, a refers to the internal characteristic length, and e_0 represents a constant. For the nonlocal parameter, $(e_0 a)^2$, to select the e_0 parameter is very important to ensure the accuracy of nonlocal models. Eringen [14] suggested e_0= 0.39 by matching lattice dynamic longitudinal wave frequency results at the end of the first Broullin zone ($k = \pi/a$), where k denotes the wave length and the parameter a is chosen as a typical characteristic length extending over the full range of micro-, meso-, and macroscales. For Rayleigh surface waves, e_0= 0.31 has ben proposed by Eringen [7]. Aydogdu [30] obtained that the nonlocal parameter e_0 depends on the material and geometrical properties of CNTs.

Solving Eq. (19) for SF^T and writing the gradient of both sides, the following equation can be obtained:

$$\nabla . [SF^T] = (e_0 a)^2\, \nabla^2 \nabla [SF^T] + \nabla(\lambda_L\, \varepsilon_{rr} \delta_{kl} + 2\, \mu_L\, \varepsilon_{kl}). \tag{20}$$

By writing the gradient of Eq.(2) and multiplying by the nonlocal parameter, the equation presented below is acquired:

$$(e_0 a)^2 \nabla^2 [SF^T] = (e_0 a)^2 \nabla \rho_0 \frac{\partial^2 u}{\partial t^2} . \tag{21}$$

Inserting Eq.(21) into Eq.(20), Eq. (20) becomes as follows:

$$\nabla . [SF^T] = (e_0 a)^2 \nabla^2 \rho_0 \frac{\partial^2 u}{\partial t^2} + \nabla(\lambda_L \, \varepsilon_{rr} \delta_{kl} + 2 \, \mu_L \, \varepsilon_{kl}). \tag{22}$$

By using Eq.(2) and Eq.(22) together, the following differential equation is obtained:

$$\rho_0 \frac{\partial^2 u}{\partial t^2} = (e_0 a)^2 \nabla^2 \rho_0 \frac{\partial^2 u}{\partial t^2} + \nabla(\lambda_L \, \varepsilon_{rr} \delta_{kl} + 2 \, \mu_L \, \varepsilon_{kl}) \, . \tag{23}$$

For a one-dimensional case, using nonzero displacement u gives the following nonlinear non-dimensional equation of motion in the framework of nonlocal elasticity:

$$\left[\left(\frac{\partial \psi}{\partial \zeta}\right)^2 + 2\left(\frac{\partial \psi}{\partial \zeta}\right) + \frac{2}{3}\right] \frac{\partial^2 \psi}{\partial \zeta^2} = \frac{2}{3} \delta \frac{\partial^2 \psi}{\partial t^2} - \frac{2}{3} \delta \, \mu \, \frac{\partial^4 \psi}{\partial \zeta^2 \, \partial t^2} \, , \tag{24}$$

where $\mu = (e_0 a / r_0)^2$ and is called the dimensionless nonlocal parameter. Setting $\mu = 0$ causes the nonlinear equation of motion for the classical elasticity theory.

3. Long-wave approximation in nanorods

Finding exact solutions for nonlinear problems is usually hard. Nonetheless, handling nonlinear problems in case of sufficiently weak nonlinearity is relatively straightforward. In this case, evolution equations originating from the equilibrium between dispersion and nonlinearity can be obtained using the dispersive nature of the medium. The propagation of small-but-finite amplitude waves in nanorods of which dimensionless governing equation is given by Eq. (24) will be investigated in this part. Therefore, the long-wave approximation will be adopted, and the reductive perturbation technique proposed by Jeffrey and Kawahara [42] will be used. To that end, the dispersive characteristic of our model equation is desired to be observed. The dispersion relation presented below is acquired as a result of linearizing the field Eq. (24) and looking for a harmonic wave type of solution to this equation:

$$\delta \omega^2 (1 + \mu \, k^2) - k^2 = 0 \, , \tag{25}$$

where ω refers to the angular frequency and k refers to the wave number. Under the assumption that the wavelength is big in comparison with the radius of the rod, the dispersion relation, $\omega(k)$, can be expanded into a power series of k around $k = 0$ and the following is acquired:

$$\omega(k) = \frac{k}{\sqrt{\delta}} (1 - \mu \, k^2 + 9\mu^2 \, k^4 - \cdots). \tag{26}$$

The introduction of the coordinate stretching below is suggested by the dispersion relation's form (25):

$$\xi = \varepsilon(\zeta - c \, t) \quad , \; \tau = \varepsilon^3 t \, , \tag{27}$$

where ε refers to a small parameter that measures the weakness of dispersion and/or nonlinearity, while c refers to a constant that is demonstrated to be the phase velocity. By introducing the parameters a, b in Eq. (24)

$$\frac{2\delta}{3} = a \;,\; \frac{2\delta}{3}\mu = b\,,$$

the equation presented below is obtained:

$$\frac{\partial}{\partial\zeta}\left[\frac{1}{3}\left(\frac{\partial\psi}{\partial\zeta}\right)^3 + \left(\frac{\partial\psi}{\partial\zeta}\right)^2\right] + \frac{2}{3}\frac{\partial^2\psi}{\partial\zeta^2} - a\frac{\partial^2\psi}{\partial t^2} + b\frac{\partial^4\psi}{\partial\zeta^2\,\partial t^2} = 0. \tag{28}$$

In Eq. (28), the following substitution is permissible for derivation;

$$\frac{\partial}{\partial t} \to \varepsilon\left(\varepsilon^2\frac{\partial}{\partial\tau} - c\frac{\partial}{\partial\xi}\right) \;,\; \frac{\partial}{\partial\zeta} \to \varepsilon\frac{\partial}{\partial\xi}\,,$$
$$\frac{\partial^2}{\partial t^2} \to \varepsilon^2\left(\varepsilon^4\frac{\partial^2}{\partial\tau^2} - 2\varepsilon^2 c\frac{\partial^2}{\partial\tau\,\partial\xi} + c^2\frac{\partial^2}{\partial\xi^2}\right) \quad,\quad \frac{\partial^2}{\partial\zeta^2} \to \varepsilon^2\frac{\partial^2}{\partial\xi^2}. \tag{29}$$

By substituting the derivation expansion (29) into Eq. (28), the field equation presented below is obtained:

$$\varepsilon\frac{\partial}{\partial\xi}\left[\frac{\varepsilon}{3}\left(\frac{\partial\psi}{\partial\xi}\right)^3 + \left(\frac{\partial\psi}{\partial\xi}\right)^2\right] + \frac{2}{3}\frac{\partial^2\psi}{\partial\xi^2} - a\left(\varepsilon^4\frac{\partial^2\psi}{\partial\tau^2} - 2\varepsilon^2 c\frac{\partial^2\psi}{\partial\tau\,\partial\xi} + c^2\frac{\partial^2\psi}{\partial\xi^2}\right) +$$
$$+b\varepsilon^2\left(\varepsilon^4\frac{\partial^4\psi}{\partial\tau^2\,\partial\xi^2} - 2\varepsilon^2 c\frac{\partial^4\psi}{\partial\tau\,\partial\xi^3} + c^2\frac{\partial^2\psi}{\partial\xi^2}\right) = 0. \tag{30}$$

It is further assumed that it is possible to express the field variable as asymptotic series in ε as shown below:

$$\psi = \varepsilon\psi_1(\xi,\tau) + \varepsilon^2\psi_2(\xi,\tau) + \cdots\,. \tag{31}$$

By introducing expression (31) into field equation (30), the differential equation below is obtained:

$$\varepsilon\frac{\partial}{\partial\xi}\left[\frac{\varepsilon}{3}\left(\varepsilon\frac{\partial\psi_1}{\partial\xi} + \varepsilon^2\frac{\partial\psi_2}{\partial\xi} + \cdots\right)^3 + \left(\varepsilon\frac{\partial\psi_1}{\partial\xi} + \varepsilon^2\frac{\partial\psi_2}{\partial\xi} + \cdots\right)^2\right] + \frac{2}{3}\left(\varepsilon\frac{\partial^2\psi_1}{\partial\xi^2} + \varepsilon^2\frac{\partial^2\psi_2}{\partial\xi^2} +\right.$$
$$\left.\cdots\right) - a\left\{\varepsilon^4\left(\varepsilon\frac{\partial^2\psi_1}{\partial\tau^2} + \varepsilon^2\frac{\partial^2\psi_2}{\partial\tau^2} + \cdots\right) - 2\varepsilon^2 c\left(\varepsilon\frac{\partial^2\psi_1}{\partial\tau\,\partial\xi} + \varepsilon^2\frac{\partial^2\psi_2}{\partial\tau\,\partial\xi} + \cdots\right) +\right.$$
$$\left.c^2\left(\varepsilon\frac{\partial^2\psi_1}{\partial\xi^2} + \varepsilon^2\frac{\partial^2\psi_2}{\partial\xi^2} + \cdots\right)\right\} + b\varepsilon^2\left\{\varepsilon^4\left(\varepsilon\frac{\partial^4\psi_1}{\partial\tau^2\,\partial\xi^2} + \varepsilon^2\frac{\partial^4\psi_2}{\partial\tau^2\,\partial\xi^2} + \cdots\right) -\right.$$
$$\left.2\varepsilon^2 c\left(\varepsilon\frac{\partial^4\psi_1}{\partial\tau\,\partial\xi^3} + \varepsilon^2\frac{\partial^4\psi_2}{\partial\tau\,\partial\xi^3}\right) + c^2\left(\varepsilon\frac{\partial^4\psi_1}{\partial\xi^4} + \varepsilon^2\frac{\partial^4\psi_2}{\partial\xi^4} + \cdots\right)\right\} = 0\,. \tag{32}$$

The set of differential equations below is acquired as a result of setting the coefficients of like powers of ε equal to zero;

The first-order, O(ε), equation:

$$\left(-\frac{2}{3} + ac^2\right)\frac{\partial^2\psi_1}{\partial\xi^2} = 0\,, \tag{33}$$

The second-order, $O(\varepsilon^2)$, equation:

$$\left(-\frac{2}{3}+\mathrm{ac}^2\right)\frac{\partial^2\psi_2}{\partial\xi^2}=0\,, \tag{34}$$

The third-order, $O(\varepsilon^3)$, equation:

$$\frac{\partial}{\partial\xi}\left[\left(\frac{\partial\psi_1}{\partial\xi}\right)^2\right]+\left(\frac{2}{3}-\mathrm{ac}^2\right)\frac{\partial^2\psi_3}{\partial\xi^2}+2\mathrm{ac}\frac{\partial^2\psi_1}{\partial\tau\,\partial\xi}+\mathrm{bc}^2\frac{\partial^4\psi_1}{\partial\xi^4}=0\,. \tag{35}$$

3.1 The solution of field equations

In order to have the non-vanishing solution for ψ_1 in Eq. (33), the coefficient of $\frac{\partial^2\psi_1}{\partial\xi^2}$ must vanish; i.e. $-\frac{2}{3}+\mathrm{ac}^2=0$ or $c=\sqrt{\frac{2}{3\mathrm{a}}}$. Here, c represents the phase velocity of the wave. Accordingly, the coefficients of $\frac{\partial^2\psi_2}{\partial\xi^2}$ and $\frac{\partial^2\psi_3}{\partial\xi^2}$ in Eqs. (34) and (35) must vanish. Finally, the form of Eq. (35) may be expressed as follows:

$$\frac{\partial}{\partial\xi}\left[\left(\frac{\partial\psi_1}{\partial\xi}\right)^2\right]+2\mathrm{ac}\frac{\partial^2\psi_1}{\partial\tau\,\partial\xi}+\mathrm{bc}^2\frac{\partial^4\psi}{\partial\xi^4}=0\,. \tag{36}$$

By setting $U=\frac{\partial\psi_1}{\partial\xi}$ in Eq.(36), the Korteweg-de Vries (KdV) equation that is widely known can be obtained:

$$\frac{\partial U}{\partial\tau}+\frac{1}{\mathrm{ac}}U\frac{\partial U}{\partial\xi}+\frac{\mathrm{bc}}{2\mathrm{a}}\frac{\partial^3U}{\partial\xi^3}=0\,, \tag{37}$$

which originates from the equilibrium of non-linearity and dispersion. A steady solution of the form of the KdV equation is presented below:

$$U(\xi,\tau)=U_\infty+\,d\,sech^2\eta\quad,\quad\eta=\left(\frac{d}{6bc^2}\right)^{1/2}\left[\xi-\frac{1}{ac}\left(U_\infty+\frac{d}{3}\right)\tau\right], \tag{38}$$

where $U_\infty>0$ represents the value of U as $\eta=\pm\infty$, whereas d represents the amplitude of the wave in relation to the constant solution U_∞ at infinity.

4. Numerical results and discussion

In this research, the nonlinear wave propagation in nanorods is examined. The nonlinear equation of motion of nanorods is acquired by utilizing nonlocal elasticity theory. The propagation of weakly nonlinear waves in the longwave approximation is examined by using the reductive perturbation technique, and the Korteweg-de Vries equation is acquired as the governing equation. The steady-state solitary-wave solution is known to be admitted by the KdV equation. The calculation of the coefficient that describes the nonlinear character of the governing equation is performed for nonlocal parameter μ, and the results are presented in figures.

The alteration of the solitary wave profile of the KdV equation at some different values of μ at a spatial time is presented in Fig.1. As is seen in the figure in question, the profile of solitary wave becomes steepened with the decreasing values of μ. In accordance with the open literature, Poisson's ratio of nanotubes is not agreed upon. The suggested values range in a wide band of $0.19 \sim 0.34$ [48]. Therefore, in the present study, ν is chosen as 0.3, and nonlocal parameter μ is taken as $0 \sim 4\text{x}10^{-18} nm^2$. Some material features are selected as $\rho_0 = 2300\ kg/m^3$, $r_0 = 10^{-9} m$, $\mathrm{E} = 1\ TPa$.

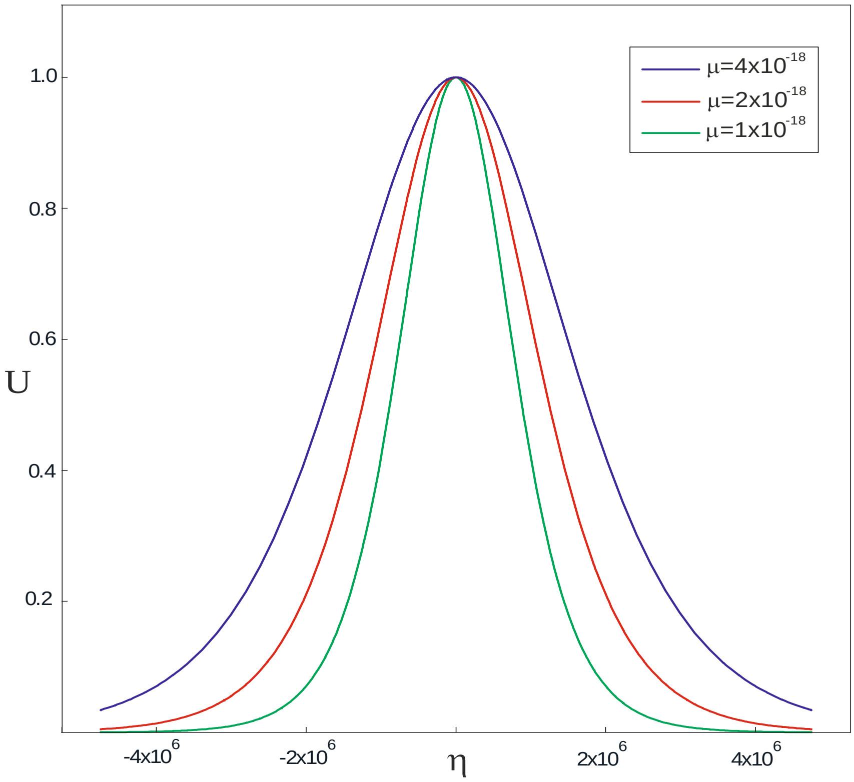

Fig.1. The variation of solitary wave profile for KdV equation some different values of μ.

The alteration of wave frequency with wave number is shown in Fig.2 for different values of μ. As is clearly seen, the profile is decreasing with the increasing nonlocal parameter. When μ=0, the curve is increasing linearly.

The alteration of wave frequency with the nonlocal parameter μ for some different values of wavenumber is given in Fig. 3. As seen in the figure in question, a decrease in frequencies occurs with the increasing nonlocal parameter. The frequency curves become close to each other with the increasing nonlocal parameter.

Fig.4 shows the longitudinal wave dispersion relation for the nanorods of radii of $100, 200, 250\ nm$. It can be found that phase velocity decreases with increasing radius values and wave numbers. At μ=0 (local case), the phase velocity remains constant. The nonlocal phase velocities are lower in comparison with the local case. The reason for this is the scale impact of nanorods [49]. The phase velocity curves are approaching each other, especially when higher wave numbers are considered. As the nanorod radius grows, the phase velocity decreases with the nanorod radius. This result is compatible with [22].

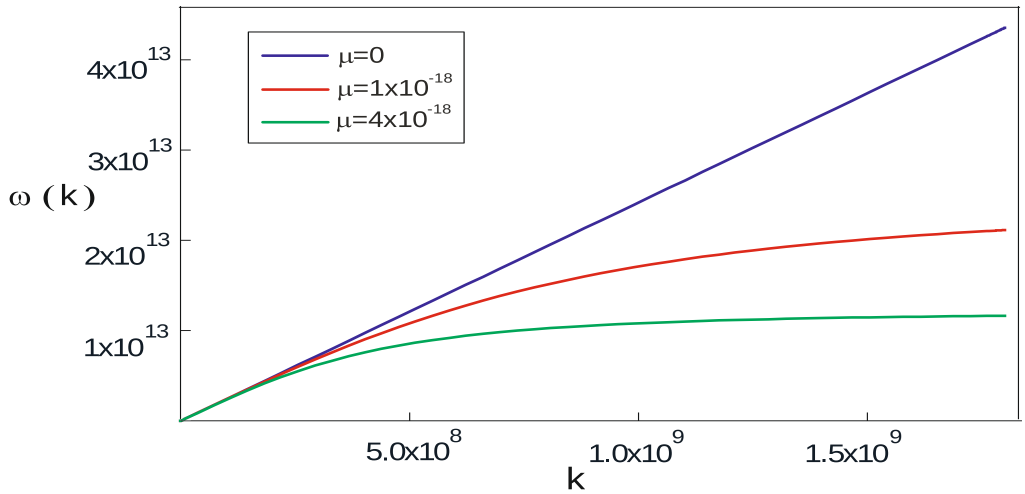

Fig.2. The variation of wave frequency with wave number some different values of μ.

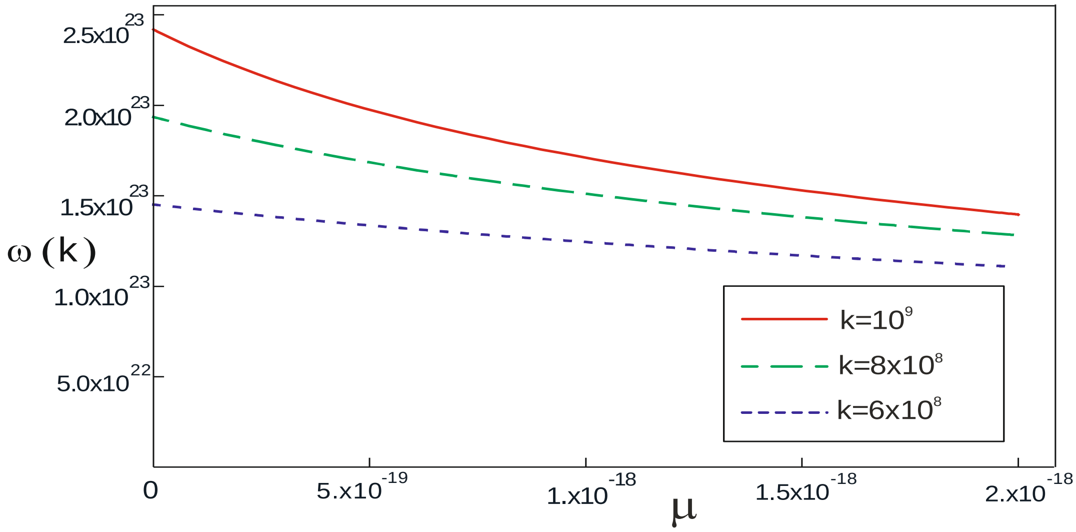

Fig.3. The variation of wave frequency with nonlocal parameter μ for some diferrent values of wave number k.

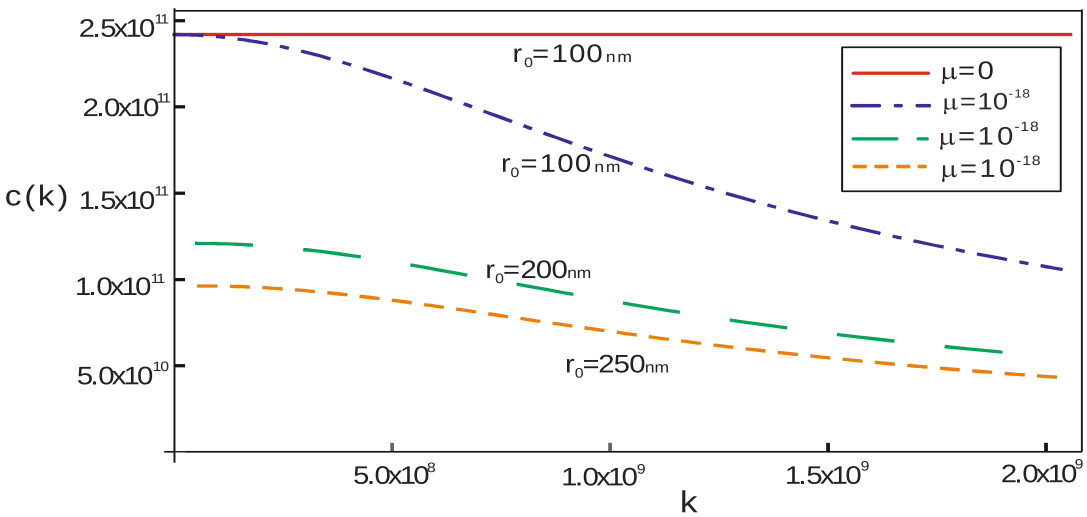

Fig.4. The variation of phase velocity with wave number some different values of μ and radius r_0.

The variation of phase velocity with radius for some different values of wave number is illustrated in Fig.5. As is also observed from the figure in question, with the increasing radius values, the phase velocities decrease rapidly for different wave numbers.

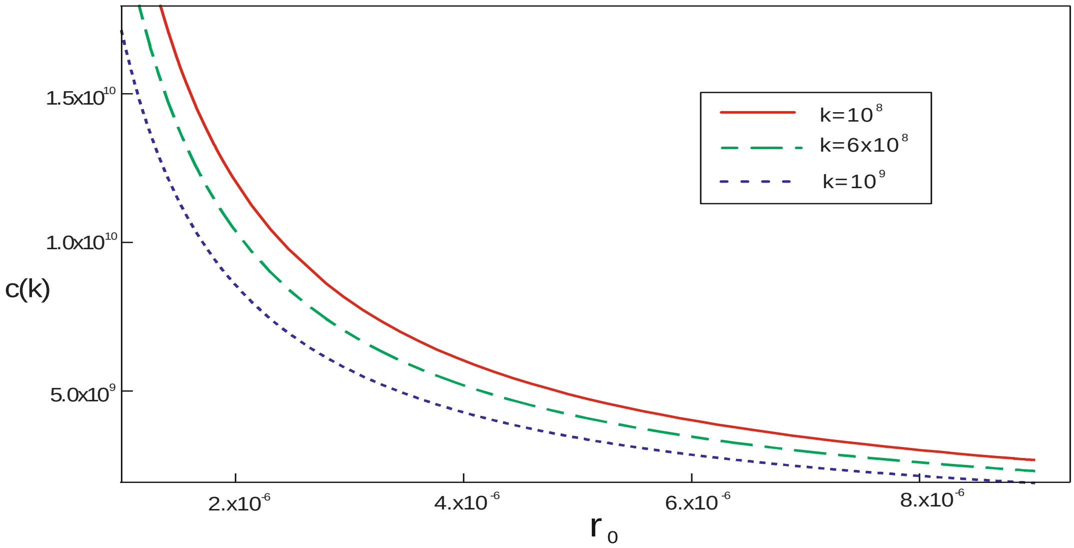

Fig.5. The variation of phase velocity with radius r_0 for some diferrent values of wave number k.

5. Conclusions

A solitary wave represents a wave travelling as a single pulse without dispersion or alteration of shape in time. Differently from a shock that represents another type of wave pulse, a solitary wave leaves the medium through which it passes without alterations. Shock waves are not possible in the absence of dissipative terms in the material model or artificial viscosity. There are significant applications of solitary waves in different fields of science, such as plasma physics, water waves, and solid state physics.

In the present research, due to nonlocality, the elastic medium has a dispersive character. The nonlocal parameter μ demonstrates the dispersive character of the medium. If $\mu = 0$, this leads to the nonlinear motion of classical elasticity. In our wave propagation analysis, both linear local situation and nonlinear local situation represent non-dispersion, while linear nonlocal situation represents dispersion. However, for the nonlinear nonlocal situation, nonlinearity and dispersion balance each other and a solitary wave profile arises. There is no solitary wave for the local nonlinear case.

Wave frequency curves and phase velocity curves are plotted with the wave number and shown that nonlocal frequency and phase velocity curves are lower than local ones in accordance with the literature. To observe the scale effect of nanorods, the variation of wave frequencies was examined, and it was determined that phase velocities changed with the radius of nanorods. Frequencies and phase velocities are shown to decrease with the increasing nanorod radius. The results of various recent experiments have shown that the size effect is important in mechanical properties when the size of the model or the volume of the material investigated is reduced. Classical continuity theories are thought to be unsuccessful when the size of the model is comparable to the internal length dimension of the material. With this study, it was understood that nonlocal effects were much stronger than classical elasticity in understanding the mechanical behaviors of nanostructures. The nonlinear propagation demonstrated here is expected to be beneficial for future studies on nanostructure.

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