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Effect of Estimation on Simple Linear Profile Monitoring under Non-normality

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Abstract: In recent years, many control charts have been proposed to monitor profiles where the quality of a process/product is expressed as function of response and explanatory variable(s). The methods mostly assume that the in control parameter values are known in Phase II analysis and innovations are normally distributed. However, in practice, the parameters are estimated in Phase I analysis and innovations may be non-normal. In this study, the performance of T^2, EWMA-R and EWMA-3 methods for monitoring simple linear profiles is examined via simulation where the parameters are estimated and innovations have Student's t-distribution. As a performance measure, both the average and standard deviation of the run length is considered. Finally, some recommendations for practitioners are summarized in a table.

Normal Olmayan Dağılımlar Altında Tahminin Basit Doğrusal Profil İzleme Üzerine Etkisi

Anahtar Kelimeler
Kontrol şeması,
Koşu uzunluğu,
Üstel ağırlıklı hareketli ortalama,
Profil izleme,
İstatistiksel süreç kontrol

Özet: Son yıllarda, bir ürün veya sürecin kalitesinin tepki ve açıklayıcı değişken(ler) arasındaki ilişkinin fonksiyonu ile ifade edildiği profillerin izlenmesi için pek çok kalite şeması önerilmiştir. Bu yöntemlerin çoğu Faz II analizlerinde kontrol parametre değerlerinin bilindiğini ve artıkların normal dağıldığını varsaymaktadır. Oysaki uygulamada parametreler Faz I analizlerinde tahmin edilir ve artıklar normal olmayabilir. Bu çalışmada simülasyon ile artıkların t dağıldığı ve parametrelerin tahmin edildiği durumlarda basit doğrusal profillerin izlenmesi için önerilen T^2, EWMA-R ve EWMA-3 yöntemlerinin performansları değerlendirilmiştir. Performans ölçüsü olarak hem ortalama koşu uzunluğu hem de koşu uzunluğu standart sapması dikkate alınmıştır. En sonunda uygulayıcılar için bazı öneriler tablo halinde özetlenmiştir.

1. Introduction

In recent years, there has been a tendency to use control charts to monitor the quality of a process or product in terms of the relation between a response variable and explanatory variable(s), i.e., a "profile". It is of interest to monitor the changes in a profile over time where a profile can be modeled via many models like simple/multiple regression, linear/nonlinear regression, nonparametric regression, mixed models, or wavelet models. For a review of profile monitoring one can refer to Woodall et al. [1] and more detailed discussions are provided by Noorossana, Saghaei, and Amiri [2]. Most of these methods assume that the in-control parameter values are known in Phase II analysis and properties of the introduced methods are also discussed under this assumption. However, in practice, they are unknown and induce an estimation error in Phase II analysis which should be investigated, as mentioned by Woodall and Montgomery [3, 4].

In simple linear profile monitoring, the estimation effect is investigated by Mahmoud [5] and Aly, Mahmoud, and Woodall [6]. Mahmoud [5] investigates it in Phase II analysis for the methods introduced by Kang and Albin [7], Kim, Mahmoud, and Woodall [8] and Mahmoud, Morgan, and Woodall [9] in terms of the average run length (ARL) and standard deviation of run length (SDRL) measures. It is shown by simulation that their performance is severely affected when the in-control profile

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parameters are estimated from a small number of Phase I samples, *m*. In his study, although the method of Kang and Albin is seen to be the least affected method by estimation when the in-control ARL is considered, it is the worst one when the out-of-control performance in detecting slope and standard deviation shifts are considered.

Aly, Mahmoud, and Woodall [6] extend the study of Mahmoud [5] by comparing the same methods for estimation effect in terms of standard deviation of the average run length (SDARL) metric. This study supports the result of Mahmoud [5] by concluding that as the number of samples used in Phase I, *m*, increases, the estimation error decreases and the average in-control ARL values approaches to the desired value. Besides, it is concluded that the method of Kim, Mahmoud, and Woodall [8] has the best performance in terms of both in-control and out-of-control ARL values as well as SDARL values. Therefore, both studies recommend the use of this method for monitoring simple linear profiles when the parameters are estimated from the Phase I samples.

In both studies, it is assumed that the error terms are normally distributed. However, this assumption can be violated in certain situations, yielding misleading results. Mahmoud and Woodall [10] discussed the effect of non-normality for Phase I analysis and recommended to check normality prior to profile analysis since it has a critical effect. Noorossana, Vaghefi, and Dorri [11] discussed the effect of non-normality for Phase II analysis of simple linear profile monitoring when the parameters are known. They compared the charts recommended by Kang and Albin [7] and Kim, Mahmoud, and Woodall [8] when the error terms have Student's t or gamma distribution and found that non-normality could degrade the performance of these charts when the process is in control. However ARL is less affected for the out-of-control case. Moreover, they found that the method introduced by Kim, Mahmoud, and Woodall [8] is more robust to deviations from normality.

Noorossana, Saghaei, and Dorri. [12] consider the case where the error terms are non-normal and autocorrelated and found that both in-control and out-of-control ARL are affected, but the method of Kim, Mahmoud, and Woodall [8] is less sensitive. Williams et al. [13] and Vaghefi, Tajbakhsh, and Noorossana [14] discussed the effect of non-normality for Phase II analysis of nonlinear profiles and mentioned that it could be a problem for small sample sizes. However, there is no study that discusses the estimation effect in Phase II under non-normal innovations so far. Therefore, the aim of this study is to investigate the performance of the mentioned methods for Phase II analysis when the error term is non-normal; specifically has a Student's t distribution, and the parameters are unknown.

One can find the discussion of the methods in Section 2. The in control and out-of-control performance of the charts is discussed in Sections 3 and 4; respectively and finally conclusions are given in Section 5.

2. Phase II methods for Monitoring the Simple Linear Profiles

A sample taken at j^{th} time has n pairs of observations (X: predictor, Y: response) in simple linear profiles while the relation is best represented by

$$Y_{ij} = A + BX_i + \varepsilon_{ij} \ , \ i = 1, 2, \ldots, n \ \text{and} \ j = 1, 2, \ldots \quad (1)$$

where A and B are the regression parameters and ε_{ij} are independent and normally distributed random variables with mean 0 and constant variance, σ^2. It is assumed for simplicity that the X-values are fixed. Moreover, the regression parameters are assumed to be known in Phase II analysis. This is, in fact, unrealistic in applications where parameters are estimated in Phase I analysis from *m* in-control profile samples as

$$\bar{a} = \sum_{j=1}^{m} a_j/m \, , \qquad \bar{b} = \sum_{j=1}^{m} b_j/m \, , \qquad (2)$$

where a_j and b_j are the least squares estimates of the profile parameters for sample j; i.e. $a_j = \bar{Y}_j - b_j\bar{X}$ and $b_j = s_{XY}^{\{j\}}/s_{XX}$; $\bar{Y}_j = \sum_{i=1}^{n} Y_{ij}/n$, $\bar{X} = \sum_{i=1}^{n} X_i/n$, $S_{XY}^{\{j\}} = \sum_{i=1}^{n}(X_i - \bar{X})Y_{ij}$, $S_{XX} = \sum_{i=1}^{n}(X_i - \bar{X})^2$. Similarly, the variance σ^2 is usually estimated by the average of the mean square errors of the m profiles as

$$MSE = {\sum_{j=1}^{m} MSE_j}/{m} \qquad (3)$$

where $MSE_j = SSE_j/(n-2)$, $SSE_j = \sum_{i=1}^{n} e_{ij}^2$, and $e_{ij} = Y_{ij} - a_j - b_jX_i$, $i = 1, 2, \ldots, n$.

Kang and Albin [7] proposed two control schemes. The first one is a bivariate T^2 control chart that monitors the regression parameters, A and B jointly. The least-squares estimators a_j and b_j follow a bivariate normal distribution with the mean vector $\mu = (A, B)$ and the variance-covariance matrix $\Sigma = \begin{pmatrix} \sigma_a^2 & \sigma_{ab}^2 \\ \sigma_{ab}^2 & \sigma_b^2 \end{pmatrix}$ where $\sigma_a^2 = \sigma^2(1/n + \bar{X}^2/s_{XX})$, $\sigma_b^2 = \sigma^2/s_{XX}$ and $\sigma_{ab}^2 = -\sigma^2\bar{X}/s_{XX}$. For the j^{th} sample, the bivariate T^2 control chart has the control statistic

$$T_j^2 = (z_j - \mu)^T \Sigma^{-1} (z_j - \mu), \qquad (4)$$

where z_j is the vector of sample least squares estimators. It is well known that when the process is in control, this statistic follows a central chi-square distribution with 2 degrees of freedom. Thus, the control chart has an upper limit of UCL $=\chi_{2,\alpha}^2$ where

$\chi^2_{2,\alpha}$ is the 100(1-α) percentile of the chi-square distribution with 2 degrees of freedom.

The second proposed method which is known as EWMA-R is a combination of an exponentially weighted average (EWMA) control chart used to monitor the average deviation from the in-control profile and range (R) chart to monitor the variation about this profile. The EWMA control chart statistic is

$$EWMA_j = \theta\bar{e}_j + (1-\theta)EWMA_{j-1}, \quad (5)$$

where $0 < \theta \le 1$ is the smoothing parameter determined according to a specified ARL given by Lucas and Saccucci [15], $EWMA_0 = 0$ and $\bar{e}_j = \sum_{i=1}^{n} e_{ij}/n = \sum_{i=1}^{n}(Y_{ij} - A - BX_i)/n$. The control limits are

$$LCL = -L\sigma\sqrt{\frac{\theta}{(2-\theta)n}} \text{ and } UCL = L\sigma\sqrt{\frac{\theta}{(2-\theta)n}}, \quad (6)$$

where L is the multiple of the sample statistic standard deviation that determines the false alarm rate. Typically L = 3. The R control chart statistic is

$$R_j = max_i(e_{ij}) - min_i(e_{ij}). \quad (7)$$

The control limits are

$$LCL = \sigma(d_2 - Ld_3) \text{ and } UCL = \sigma(d_2 + Ld_3) \quad (8)$$

where L > 0, is a constant determined according to a specified in-control ARL and d_2 and d_3 are constants depending on the sample size, n, which are tabulated for a normal population in textbooks. See; for example, Montgomery [16].

Kim, Mahmoud, and Woodall [8] suggested coding the predictor variable to make the average 0 so that the estimators of regression parameters become independent and then constructing separate EWMA charts for each parameter. With the coded X-values; i.e. $X_i' = (X_i - \bar{X})$, the alternative form of model (1) becomes.

$$Y_{ij} = C + DX_i' + \varepsilon_{ij}, \; i = 1, 2, ..., n \text{ and } j = 1, 2, ... \quad (9)$$

where $C = A + B\bar{X}$ and $D = B$. The least squares estimators for the new parameters are $c_j = \bar{y}_j$ and $d_j = b_j$. The EWMA control chart statistic for the new intercept, C, is

$$EWMA_{j,C} = \theta c_j + (1-\theta)EWMA_{j-1,C}, j = 1, 2, ... \quad (10)$$

where $0 < \theta \le 1$ is the smoothing parameter, $EWMA_{0,C} = C$ with the following limits

$$LCL = C - L_C\sigma\sqrt{\frac{\theta}{(2-\theta)n}} \text{ and}$$
$$UCL = C + L_C\sigma\sqrt{\frac{\theta}{(2-\theta)n}} \quad (11)$$

Similarly, the EWMA control chart statistic for the new slope, D, is

$$EWMA_{j,D} = \theta d_j + (1-\theta)EWMA_{j-1,D}, j=1, 2,... \quad (12)$$

where $0 < \theta \le 1$ is the smoothing parameter and $EWMA_{0,D} = D$ with the following limits

$$LCL = D - L_D\sigma\sqrt{\frac{\theta}{(2-\theta)S_{XX}}} \text{ and}$$
$$UCL = D + L_D\sigma\sqrt{\frac{\theta}{(2-\theta)S_{XX}}} . \quad (13)$$

$L_C > 0$ and $L_D > 0$ are chosen to give a specified in-control ARL. Finally, a one-sided EWMA scheme is used to detect increases in the process variability. The chart statistic is as follows:

$$EWMA_{j,E} = \max\{\theta ln(MSE_j) + (1-\theta)EWMA_{j-1,E}, ln(\sigma^2)\}, \; j = 1, 2, ... \quad (14)$$

where $0 < \theta \le 1$ is the smoothing parameter, $EWMA_{0,E} = ln(\sigma^2)$. The upper control limit of the scheme is

$$UCL = L_E\sqrt{\frac{\theta}{(2-\theta)}Var[ln(MSE_j)]} , \quad (15)$$

where $Var[ln(MSE_j)] \approx 2(n-2)^{-1} + 2(n-2)^{-2} + (4/3)(n-2)^{-3} - (16/15)(n-2)^{-5}$ and again $L_E > 0$ is chosen to give a specified in-control ARL. This method will be abbreviated as EWMA-3 for further analysis.

3. In-control Performance Comparisons

In this study, in-control linear profile model, $y_{ij} = 3 + 2x_i + \varepsilon_{ij}$, where the ε_{ij}'s are i.i.d. normal random variables with mean 0 and variance 1, is considered. Furthermore, results are provided where the ε_{ij}'s are distributed as Student's t with ν degrees of freedom. The fixed x_i values of 2, 4, 6, and 8 ($\bar{x} = 5$) are used as in the study of Kang and Albin [7]. For EWMA-3, these values are transformed as $x_i^* = x_i - \bar{x}$ so that the average becomes zero. After transformation, alternative form of the underlying model becomes $y_{ij} = 13 + 2x_i^* + \varepsilon_{ij}$ where the x_i^* values are -3, -1, 1, and 3 with $\bar{x}^* = 0$.

In the simulation study, the effect of the profile number, *m*, and non-normality on the ARL performances of the three methods (T^2, EWMA-R, EWMA-3) are investigated where the profile size, *n*, is taken to be 4. Fortran programming language is used in simulations.

Table 1. In-Control ARL and SDRL values for t-distributions with different degrees of freedom, υ (Normal for ∞), when *m* Phase I samples of size *n* = 4 are used to estimate the unknown parameters.

			m							
	υ		10	30	70	120	200	300	500	∞
EWMA-R	3	ARL	114.4	106.6	51.4	51.9	46.3	43.6	41.3	39.3
		SDRL	12839.9	12783.7	904.9	1065.7	452.8	276.1	111.5	38.6
	5	ARL	93.7	58.7	52.9	51.2	50.9	50.4	49.9	50.8
		SDRL	3628.2	302.4	81.8	62.1	55.5	53.4	51.8	49.6
	10	ARL	150.6	97.0	89.8	88.6	87.8	86.4	85.9	86.9
		SDRL	1707.0	172.8	115.2	102.9	95.4	91.6	88.7	85.1
	30	ARL	418.5	179.5	155.9	152.3	148.4	147.3	145.4	146.6
		SDRL	12427.2	424.9	227.5	188.3	167.3	160.1	152.6	144.8
	50	ARL	487.9	206.5	175.9	170.0	168.7	164.9	163.4	164.8
		SDRL	8743.4	573.9	259.2	210.8	194.5	181.2	171.8	163.3
	100	ARL	633.9	230.5	192.8	187.8	184.9	182.2	180.3	179.9
		SDRL	21302.9	723.8	290.2	241.8	214.4	200.9	191.1	177.9
	∞	ARL	757.1	254.6	215.0	207.2	202.6	200.3	203.5	197.9
		SDRL	27616.6	669.7	328.6	269.3	237.3	224.5	217.9	195.2
EWMA-3	3	ARL	135.3	290.2	133.4	142.2	136.7	124.8	112.9	103.4
		SDRL	2729.8	32295.7	1447.2	2674.4	2412.1	1034.7	468.3	100.9
	5	ARL	362.6	166.5	131.3	125.2	125.0	122.5	121.9	122.5
		SDRL	26430.2	2756.7	566.5	211.5	191.2	138.8	129.6	118.3
	10	ARL	322.1	164.0	154.9	155.7	157.4	156.7	157.2	159.0
		SDRL	13514.5	940.1	230.4	196.2	179.4	168.3	163.5	154.2
	30	ARL	260.9	176.1	175.1	178.1	181.5	181.9	184.0	185.5
		SDRL	4399.5	510.4	270.5	221.9	205.1	197.4	192.9	181.8
	50	ARL	239.7	178.1	177.5	183.2	187.4	185.7	187.8	190.6
		SDRL	2231.7	408.8	261.3	232.4	213.9	202.3	196.1	185.7
	100	ARL	239.8	179.7	180.1	185.8	189.2	190.6	192.0	195.0
		SDRL	2534.3	423.9	272.9	237.4	214.5	207.7	199.7	191.4
	∞	ARL	249.5	181.2	183.5	188.0	190.4	190.4	197.4	199.2
		SDRL	3833.6	425.2	267.6	237.9	218.1	208.7	206.5	194.8
T^2	3	ARL	69.7	60.9	58.7	57.5	53.8	49.8	47.9	45.7
		SDRL	2078.2	1630.6	1102.9	1060.9	717.2	293.2	257.2	44.8
	5	ARL	96.4	67.2	63.2	61.6	60.5	59.3	60.2	60.4
		SDRL	2693.1	248.9	94.3	70.8	76.0	62.6	61.9	59.8
	10	ARL	161.0	114.4	104.7	102.7	101.5	99.8	99.0	99.8
		SDRL	1007.9	224.3	129.9	118.3	108.9	104.2	101.0	99.1
	30	ARL	502.7	202.1	170.3	165.9	161.7	160.6	158.2	155.9
		SDRL	21077.6	492.0	233.8	200.1	181.8	174.5	165.6	154.4
	50	ARL	526.2	225.8	189.5	184.3	179.0	176.2	173.7	171.7
		SDRL	5641.1	484.6	266.6	228.5	202.6	191.4	181.2	171.0
	100	ARL	716.1	250.4	208.9	201.3	194.8	191.9	188.5	185.9
		SDRL	11675.2	621.1	301.9	250.2	220.3	207.5	198.5	184.7
	∞	ARL	690.6	280.4	226.8	213.9	206.5	202.6	203.9	199.3
		SDRL	6802.3	699.8	331.4	269.7	234.2	221.1	213.5	197.9

The procedure for the simulation of ARL and SDRL of the competing methods is given as follows:

1. A total of 50,000 simulation runs are conducted and in each run 'm' profiles with size 'n = 4' are generated. The control limits are estimated for each method. In EWMA-R and EWMA-3, the smoothing parameter θ is taken to be 0.2. For EWMA-R, the L constants in the control limits of EWMA and R charts are chosen as 3.1151 and for EWMA-3, the constants L_C, L_D and L_E are chosen as 3.0156, 3.0109 and 1.3723; respectively in order to achieve an overall in control ARL of roughly 200 under normality. (Kim, Mahmoud, and Woodall [8])
2. After completing the estimation of control limits (phase I), an additional random profile of size n = 4 is generated to represent the new phase II process information.
3. For each method, the chart statistics are calculated based on the estimated parameters $\bar{a}$, $\bar{b}$ and MSE and they are compared with the corresponding control limits in phase I.

4. Steps 2 and 3 are repeated until the chart gives a signal. When the signal is given, the run length is recorded.
5. Steps 1-4 are repeated 50,000 times to estimate the ARL and the SDRL values.

The in control simulation results for m = 10, 30, 70, 120, 200, 300, 500, ∞ (representing known parameters case) are given in Table 1 where $m = \infty$ values are simulated by the use of true parameter values rather than their estimates. It can be seen from the known parameter case ($m = \infty$) that as the underlying distribution deviates from the normality, ARL values are decreasing for all methods. For example, for EWMA-3, the in control ARL under normality is estimated as 199.2 while it is 103.4 for the t distribution with 3 degrees of freedom. As the distribution becomes platycurtic, ARL values are smaller for all methods, as expected. It can also be mentioned that EWMA-3 is more robust to non-normality than the other methods though its SDRL values are higher than the others. When the estimation effect is considered under normality, it can be observed that the methods T^2 and EWMA-R overshoot the ARL with known parameters indicating fewer false alarm rates. However this is not the case for EWMA-3 unless m=10. The EWMA-3 method, has lower ARL values with known parameters than when m > 10, but it should be noted that its SDRL is much less than the other methods. Therefore, when both ARL and SDRL values are considered, it can be said that EWMA-3 performs better than the rest. For other cases; i.e. estimation effect under non-normality, it can be seen that ARL is higher for small numbers of profiles and decreases as the number of profiles increases. SDRL values are decreasing as the number of profiles is increasing as expected. For small numbers, very large SDRL values are observed for all methods under each distribution. In fact, one reason for these very large deviations is a small number of extremely large run length values, meaning that for a specific sample it is possible not to observe a signal for a long time. It is more probable to observe these extreme run length values when the estimation is done with small number of profiles (m = 10 and m = 30) and the distribution is t distribution with small degrees of freedom (especially when υ=3 and/or υ=5). For example, when the EWMA-3 method is

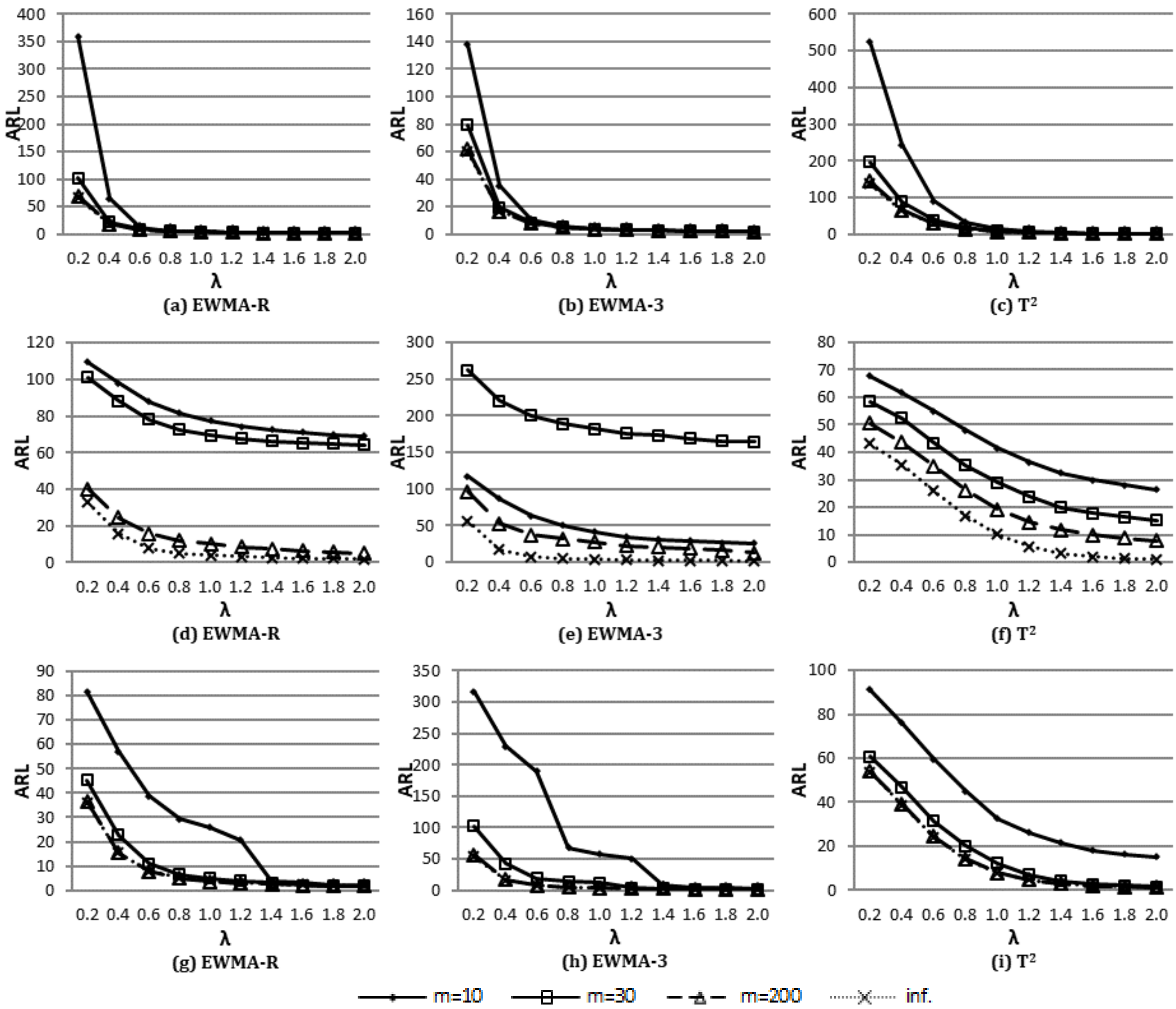

Figure 1. Out of Control ARL performance with intercept shift from A_0 to $A_0 + \lambda\sigma$ under Normal distribution for (a) EWMA-R Control Chart (b) EWMA 3 Control Chart (c) T^2 Control Chart, under t(3) distribution for (d) EWMA-R Control Chart (e) EWMA 3 Control Chart (f) T^2 Control Chart, under t(5) distribution for (g) EWMA-R Control Chart (h) EWMA 3 Control Chart (i) T^2 Control Chart

considered, the SDRL under t distribution with 3 degrees of freedom when 30 profiles are used in estimation is 32295.7 which is an unacceptably large value. It must be noted that t distribution with 3 degrees of freedom has an undefined skewness and infinite kurtosis. Therefore, very high SDRL values are not so unexpected for this distribution. Overall, it can be said that ARL values for EWMA-3 are more close to the theoretical value, 200 and have similar fluctuations in SDRL with other methods.

4. Out-of-control Performance Comparisons

The effect of estimation under non-normality on the out-of-control performance was also investigated. For this purpose, shifts are given to the intercept, slope and variance separately. The simulated out-of-control ARL values are given in Figure 1 for various numbers of profiles with size n = 4 under normal and the t distribution with 3 and 5 degrees of freedom when a shift to the intercept is given. Their corresponding SDRL values can be found in Table 2 where the values that exceed 10000 are reported as >10k.

It can be seen from Figure 1(a)-(c) that when the shift size in the intercept is large; i.e. $\lambda \geq 0.8$, all methods yield similar ARL values with the known parameter ($m = \infty$) case regardless of the number of profiles in phase I used in estimation except T^2 where it is true for $\lambda \geq 1.0$. This means that under normality, estimation effect is negligible for large shifts and

Table 2. SDRL values under Normal, t distribution having 3 and 5 degrees of freedom with intercept shift from A_0 to $A_0 + \lambda\sigma$ when m Phase I samples of size n=4 are used to estimate the unknown parameters.

			λ									
		m	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
EWMA-R	Normal	1	5568.2	1594.9	60.8	6.3	2.2	1.4	1.0	0.8	0.7	0.6
		3	282.6	33.9	7.4	2.9	1.7	1.2	0.9	0.8	0.7	0.6
		2	75.9	14.5	4.9	2.5	1.6	1.2	0.9	0.8	0.7	0.6
		∞	61.3	12.7	4.5	2.3	1.4	0.9	0.8	0.6	0.4	0.4
	t (3)	1	>10k	>10k	>10k	>10k	>10k	>10k	>10k	>10k	>10k	>10k
		3	>10k	>10k	>10k	>10k	>10k	>10k	>10k	>10k	>10k	>10k
		2	452.3	451.3	450.3	448.8	447.1	438.3	434.4	425.4	405.8	395.
		∞	31.2	11.7	4.2	2.0	1.2	0.9	0.7	0.5	0.4	0.3
	t (5)	1	3627.2	3561.7	3527.	3511.	3509.	3436.	139.3	138.5	50.5	49.6
		3	296.3	288.8	235.7	222.8	221.9	210.6	158.2	2.9	0.9	0.7
		2	39.5	13.5	4.9	2.5	1.6	1.2	0.9	0.8	0.7	0.6
		∞	33.4	11.3	4.3	2.2	1.4	0.9	0.7	0.6	0.4	0.4
EWMA-3	Normal	1	1325.2	405.3	38.7	5.8	2.0	1.3	0.9	0.7	0.6	0.6
		3	214.4	27.7	6.5	2.7	1.6	1.1	0.8	0.6	0.5	0.5
		2	64.8	12.8	4.4	2.2	1.4	0.9	0.7	0.6	0.5	0.4
		∞	53.8	11.4	4.2	2.2	1.4	0.9	0.7	0.6	0.4	0.4
	t (3)	1	2693.8	2680.8	2653.	2615.	2593.	2579.	2574.	2569.	2565.	2564
		3	>10k	>10k	>10k	>10k	>10k	>10k	>10k	>10k	>10k	>10k
		2	2409.1	2403.3	2394.	2387.	2356.	2288.	2279.	2271.	2228.	2101
		∞	52.6	12.3	4.0	1.9	1.2	0.9	0.7	0.5	0.4	0.3
	t (5)	1	>10k	>10k	>10k	7737.	7611.	7552.	1302.	493.5	297.2	109.
		3	2394.0	2155.9	1802.	1793.	1790.	158.5	158.2	2.9	0.6	0.6
		2	79.5	14.2	4.6	2.2	1.4	0.9	0.7	0.6	0.5	0.4
		∞	48.3	11.6	4.1	2.1	1.3	0.9	0.7	0.5	0.4	0.4
T^2	Normal	1	5769.5	1996.9	963.8	237.6	64.3	24.9	7.3	3.4	1.7	0.9
		3	437.9	182.4	69.1	27.1	11.6	5.5	2.9	1.6	0.9	0.6
		2	164.2	74.1	31.1	14.3	6.9	3.7	2.1	1.3	0.8	0.6
		∞	136.5	62.9	27.3	12.7	6.4	3.5	2.1	1.3	0.9	0.6
	t (3)	1	2081.6	2070.8	2070.	2070.	2067.	2066.	2065.	2065.	2065.	2064
		3	1629.8	1641.6	1614.	1613.	1613.	1612.	1607.	1606.	1606.	1605
		2	720.7	718.4	719.1	715.9	713.2	712.6	710.3	708.7	705.7	699.
		∞	42.5	34.7	25.0	15.9	9.2	4.9	2.6	1.4	0.8	0.4
	t (5)	1	2699.9	2660.7	2654.	2651.	2620.	2620.	2618.	2615.	2611.	2610
		3	243.9	244.5	233.6	244.6	227.9	106.7	35.1	33.1	24.5	19.1
		2	70.5	58.2	27.2	15.7	9.0	5.2	2.7	1.6	0.9	0.6
		∞	53.2	37.6	23.4	12.9	7.1	3.9	2.2	1.3	0.8	0.5

small number of profiles such as 10 can be used in estimation. For small shift sizes, ARL values are naturally higher. Moreover the number of phase I profiles used in estimation becomes crucial and should be at least 200 to eliminate the estimation effect. According to Figure 1(d)-(f), all methods' ARL values are highly affected by estimation under the t distribution having 3 degrees of freedom even for large shifts in the intercept. For example, the ARL value for a shift of size 2.0 is 1.9 for EWMA-R when the parameters are known. It becomes 69.1, 64.0 and 5.0 when m is 10, 30 and 200; respectively. Similar results can be observed for t distribution having 5 degrees of freedom in Figure 1(g)-(i). However, when compared with t distribution having 3 degrees of freedom, this effect is less. For example, the performance of the known parameter case can now be achieved by using 200 profiles (and even with fewer profiles for large shifts) in estimation.

It can be seen from Table 2 that under normality, SDRL values of EWMA-R and EWMA-3 are close to each other as well as to the theoretical value ($m = \infty$) for large shifts ($\lambda \geq 0.8$) except the case m = 10 with $\lambda = 0.8$. However, it must be noted that since T^2 has large SDRL values even for large shifts, other methods can be preferred to it. For small shift sizes, SDRL values increase like the ARL values. When the methods are compared under normality, it can be observed that EWMA-3 is less affected by estimation. The worst performer among these methods is the T^2 chart which requires more profiles in estimation even to detect large shifts.

Although the ARL values are close to the parameters known case as the number of profiles used in estimation increases, SDRL values are far away from being at acceptable levels when the t distribution having 3 degrees of freedom is considered. Therefore it can be concluded that estimation of the parameters highly degrades the performance of the chart under the t distribution with 3 degrees of freedom even when the number of phase I profiles used in estimation is as high as 200. This result dampens when the degrees of freedom increases to 5. The SDRL values become reasonable for quite large shifts say $\lambda \geq 1.6$ when $m \geq 30$ except for the T^2 chart which requires m to be much larger.

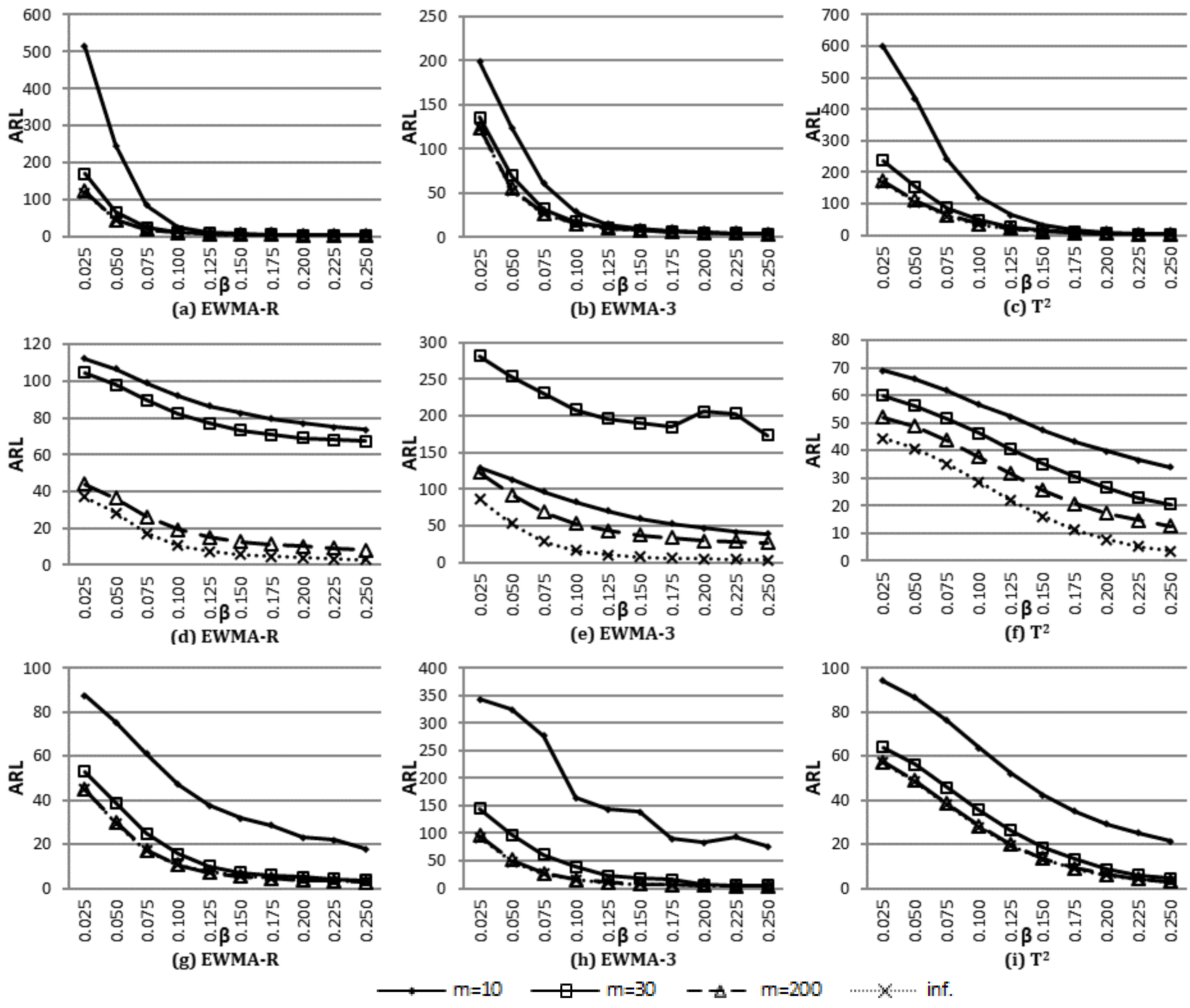

Figure 2. Out of Control ARL performance with slope shift from A_1 to $A_1 + \beta\sigma$ under Normal distribution for (a) EWMA-R Control Chart (b) EWMA 3 Control Chart (c) T^2 Control Chart, under t(3) distribution for (d) EWMA-R Control Chart (e) EWMA 3 Control Chart (f) T^2 Control Chart, under t(5) distribution for (g) EWMA-R Control Chart (h) EWMA 3 Control Chart (i) T^2 Control Chart

The simulated out-of-control ARL values are given in Figure 2 for different number of profiles with size n = 4 under normality and the t distribution with 3 and 5 degrees of freedom when a shift to the slope is given. Their corresponding SDRL values can be found in Table 3 where the values that exceed 10000 are reported as >10k.

Under normality, similar behaviour to Figure 1 can be observed from Figure 2 when $\beta \geq 0.175$; that is, ARL values are very close to the values for known parameter case regardless of the number of phase I profiles used in the estimation. However, as seen from Table 3, since SDRL values of T^2 method for $\beta \geq 0.175$ are larger than those of the other methods, it is better to use at least 30 profiles with this method for estimation. Again, under normality the EWMA-3 chart is seen to be less affected by estimation when there is a slope shift. Under non-normality, estimation affects ARL and SDRL values more as the distribution deviates more from normality. As in the case of a shift in intercept, performance of the chart highly degrades under the t distribution with 3 degrees of freedom even when the number of profiles used in estimation is 200. However, using 200 profiles in estimation becomes sufficient when the distribution has 5 degrees of freedom.

The simulated out-of-control ARL values are given in Figure 3 for different number of profiles with size n = 4 under normality and the t distribution with 3 and 5 degrees of freedom when a shift to the variance is

Table 3. SDRL values under Normal, t distribution having 3 and 5 degrees of freedom with slope shift from A_1 to $A_1 + \beta\sigma$ when m Phase I samples of size n=4 are used to estimate the unknown parameters

			β									
		m	0.025	0.050	0.075	0.100	0.125	0.150	0.17	0.20	0.22	0.25
EWMA-R	Normal	10	6768.8	5161.4	1849.8	413.3	51.6	16.1	3.8	2.2	1.7	1.3
		30	475.7	173.4	43.4	14.0	6.4	3.5	2.3	1.7	1.4	1.1
		200	140.7	47.2	16.9	7.8	4.4	2.9	2.1	1.6	1.3	1.1
		∞	113.9	38.8	14.8	7.1	4.0	2.6	1.9	1.4	1.1	0.9
	t (3)	10	>10k	>10k	>10k	>10k	>10k	>10k	>10k	>10k	>10k	>10k
		30	>10k	>10k	>10k	>10k	>10k	>10k	>10k	>10k	>10k	>10k
		200	452.1	451.9	452.1	452.2	451.4	450.9	450.	449.	441.	437.
		∞	36.1	25.6	13.3	6.6	3.8	2.4	1.7	1.3	0.9	0.8
	t (5)	10	3627.4	3626.5	3604.4	3569.	3548.	3542.	3538	2750	2748	2656
		30	299.5	293.4	289.3	283.4	228.7	223.3	221.	221.	210.	158.
		200	48.9	32.6	15.6	7.8	4.4	2.9	2.1	1.6	1.3	1.1
		∞	43.05	26.15	12.87	6.66	3.88	2.54	1.80	1.37	1.08	0.90
EWMA-3	Normal	10	1805.8	1173.2	550.4	511.1	49.1	28.3	14.2	4.4	2.6	1.9
		30	300.4	163.9	61.1	21.0	9.9	5.3	3.4	2.4	1.8	1.5
		200	139.6	56.7	23.4	11.2	6.3	4.0	2.8	2.1	1.6	1.3
		∞	118.0	47.0	20.4	10.2	5.9	3.9	2.7	2.0	1.6	1.3
	t (3)	10	2718.6	2693.4	2684.0	2682.	2671.	2666.	2652	2613	2597	2592
		30	>10k	>10k	>10k	>10k	>10k	>10k	>10k	>10k	>10k	>10k
		200	2411.6	2409.2	2452.7	2447.	2433.	2430.	2425	2377	2374	2365
		∞	83.5	48.9	23.7	11.8	6.4	3.9	2.6	1.9	1.5	1.2
	t (5)	10	>10k	>10k	>10k	>10k	>10k	>10k	>10k	>10k	>10k	8650
		30	2764.3	2431.4	2336.1	2153.	1804.	1800.	1794	234.	228.	212.
		200	150.6	60.2	26.5	12.9	6.8	4.3	2.9	2.1	1.6	1.3
		∞	87.7	43.6	20.5	10.6	6.0	3.9	2.7	1.9	1.6	1.3
T^2	Normal	10	4921.6	4171.6	2398.3	1175.	632.6	318.5	82.8	41.8	24.8	12.3
		30	544.5	363.9	181.2	89.5	46.5	24.9	13.9	7.9	5.0	3.1
		200	195.4	125.5	70.4	38.9	22.2	12.9	8.0	5.1	3.4	2.3
		∞	164.5	105.1	60.7	34.2	19.7	11.9	7.4	4.8	3.2	2.2
	t (3)	10	2081.8	2081.5	2081.4	2070.	2071.	2070.	2070	2070	2069	2071
		30	1631.1	1629.6	1629.3	1629.	1628.	1628.	1627	1626	1609	1609
		200	719.8	721.2	720.7	720.6	720.5	718.8	717.	719.	719.	716.
		∞	43.6	40.2	34.4	27.7	21.1	15.2	10.5	6.9	4.5	2.8
	t (5)	10	2699.8	2698.7	2688.3	2657.	2631.	2622.	2622	2619	2618	2612
		30	245.2	241.8	237.6	242.3	227.8	220.7	217.	49.9	40.8	38.5
		200	73.2	66.1	57.9	39.2	22.3	15.1	10.4	6.9	4.7	3.5
		∞	56.6	47.7	37.2	27.1	18.5	12.3	8.1	5.3	3.5	2.4

given. Their corresponding SDRL values can be found in Table 4 where the values that exceed 10000 are reported as >10k.

It is seen that although the behavior in Figure 3 is similar to that in Figures 1 and 2, i.e., 30 profiles are enough even for small shifts to achieve parameters known case performance under normality. For larger shifts ($\gamma > 1.6$) one can even use 10 profiles in order to have the similar performance of the charts with known parameters. Again there is an increasing estimation effect as the distribution deviates more from normality. However, it can be observed that this effect is less when compared to the shifts in slope and intercept. Thus, it can be concluded that when there is a shift in the standard deviation, the charts are less sensitive to estimation than the case where there is a shift in slope or intercept. For example, under the t distribution with 3 degrees of freedom, the ARL values for m = 200 are close to the known parameter case for large shifts and SDRL values are less when compared to the other types of shifts, although they are still high. For the t distribution having 5 degrees of freedom, 30 profiles are enough for decreasing the estimation effect for large shifts while it requires 200 profiles for small shifts.

To sum up the simulation results, it is observed that when the number of phase I profiles used in estimation is small under the t distribution with 3

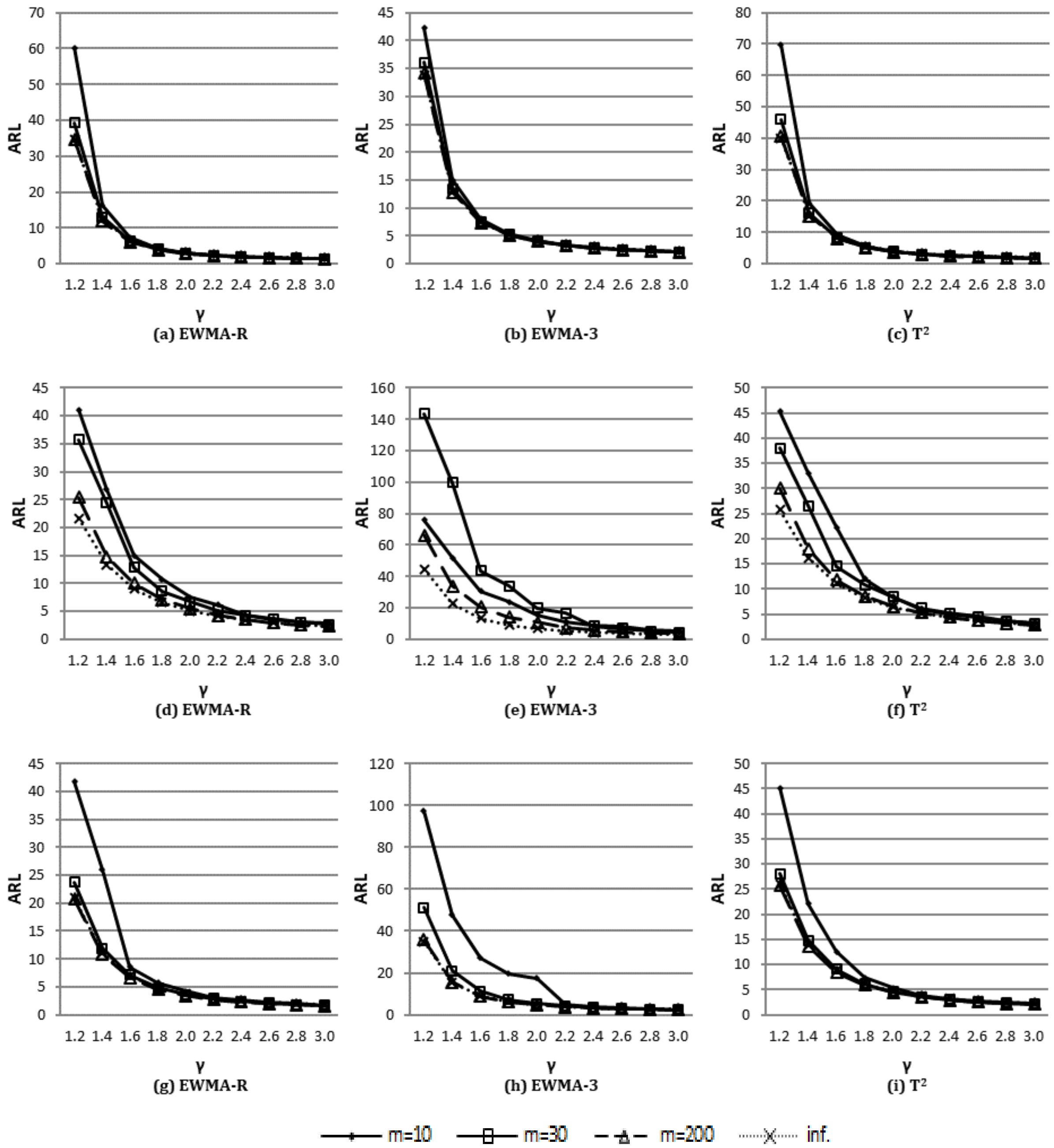

Figure 3. Out of Control ARL performance with standard deviations shift from σ to γσ under Normal distribution for (a) EWMA-R Control Chart (b) EWMA 3 Control Chart (c) T^2 Control Chart, under t(3) distribution for (d) EWMA-R Control Chart (e) EWMA 3 Control Chart (f) T^2 Control Chart, under t(5) distribution for (g) EWMA-R Control Chart (h) EWMA 3 Control Chart (i) T^2 Control Chart

Table 4. SDRL values under Normal, t distribution having 3 and 5 degrees of freedom with standard deviations shift from σ to γσ when *m* Phase I samples of size *n*=4 are used to estimate the unknown parameters

			γ									
		m	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
EWMA-R	Normal	10	457.5	50.9	12.2	5.9	3.3	2.2	1.6	1.3	1.0	0.9
		30	64.5	16.6	6.9	3.9	2.5	1.8	1.4	1.1	0.9	0.8
		200	36.9	11.9	5.7	3.3	2.3	1.7	1.3	1.1	0.9	0.8
		∞	33.0	11.2	5.4	3.3	2.3	1.7	1.3	1.1	0.9	0.8
	t (3)	10	1986.9	1643.5	465.0	400.7	277.9	276.8	69.7	67.4	66.2	66.1
		30	1611.7	1594.1	246.6	108.7	103.7	61.7	47.5	43.2	39.7	35.6
		200	313.6	108.9	102.5	46.4	40.9	28.9	27.3	18.0	16.2	15.4
		∞	20.8	12.5	8.1	5.7	4.2	3.3	2.6	2.2	1.8	1.6
	t (5)	10	2644.9	2608.8	61.4	54.8	51.0	17.9	16.7	10.3	10.2	7.8
		30	119.2	31.2	16.9	9.9	8.7	4.9	4.4	3.4	1.9	1.3
		200	21.8	11.0	6.4	4.1	2.9	2.2	1.7	1.4	1.2	1.0
		∞	19.9	10.0	5.9	4.0	2.9	2.2	1.7	1.4	1.2	1.1
EWMA-3	Normal	10	235.4	29.2	10.9	5.0	3.2	2.2	1.8	1.5	1.2	1.1
		30	54.5	14.5	6.4	3.7	2.5	1.9	1.5	1.3	1.1	1.0
		200	33.6	10.7	5.2	3.2	2.3	1.7	1.4	1.2	1.1	0.9
		∞	30.6	10.1	5.0	3.1	2.3	1.7	1.4	1.2	1.1	0.9
	t (3)	10	2169.6	2130.8	1348.	1339.	747.1	460.2	407.8	406.9	319.9	319.
		30	>10k	>10k	4087.	3582.	1607.	1594.	243.6	239.2	105.5	102.
		200	1184.2	599.5	421.2	318.7	312.1	107.4	103.9	65.3	56.9	41.9
		∞	42.2	20.7	11.4	7.3	5.0	3.7	2.9	2.4	2.0	1.7
	t (5)	10	4219.6	3590.3	2657.	2608.	2608.	85.4	80.7	17.9	17.6	15.3
		30	910.4	320.2	154.9	67.3	54.1	18.9	5.7	4.7	3.7	2.6
		200	40.1	15.3	7.8	4.6	3.1	2.4	1.9	1.6	1.4	1.2
		∞	31.9	12.8	6.8	4.3	3.0	2.3	1.9	1.6	1.3	1.2
T^2	Normal	10	384.9	43.4	14.8	7.3	4.4	3.0	2.3	1.9	1.5	1.3
		30	71.5	20.9	9.2	5.3	3.6	2.6	2.1	1.7	1.4	1.2
		200	42.5	15.2	7.7	4.7	3.3	2.5	1.9	1.6	1.4	1.2
		∞	39.0	14.4	7.4	4.6	3.2	2.4	1.9	1.7	1.4	1.3
	t (3)	10	1993.2	1981.6	1623.	389.2	223.9	71.5	68.6	67.9	66.2	65.9
		30	1604.8	1593.5	290.4	236.5	232.5	97.1	95.9	95.4	35.2	33.2
		200	407.0	200.8	51.9	41.2	27.9	26.1	25.3	22.6	14.4	12.6
		∞	24.9	15.3	10.3	7.3	5.5	4.3	3.5	2.9	2.4	2.1
	t (5)	10	2610.7	1073.8	486.2	95.5	81.2	17.8	17.3	15.7	15.5	13.9
		30	56.8	32.0	18.4	11.9	6.6	5.2	4.4	3.8	2.5	2.1
		200	27.1	14.0	8.3	5.5	4.0	3.1	2.4	2.0	1.7	1.5
		∞	25.1	12.9	7.9	5.3	3.9	3.0	2.4	2.0	1.7	1.5

and 5 degrees of freedom, SDRL values become so high which makes ARL values questionable. Especially in Tables 2, 3 and 4, it is observed that SDRL values for EWMA-3 method when m = 30 are very large so that it seems like they are increasing as the number of profiles m increasing which is rather counter-intuitive. As we discuss before for Table 1, these large SDRL values are because of a small number of extremely large run length values meaning that for a specific sample it is possible not to observe a signal for a long time. It is more probable to observe these extreme run length values when the estimation is done with small number of profiles (m = 10 and m = 30) and the distribution is a t distribution with a small degrees of freedom. When SDRL values are high, using ARL as a performance measure to compare the methods would be unreliable. Therefore, the mentioned methods are also compared according to the deviation of ARL values from the ones obtained under normality with known parameters (named as bias) and the corresponding SDRL values, and the best performer among them are reported in Table 5. However, it must be noted that the best performer does not mean that the chart performance is good and can be used safely. For an indication of that, the best bias (BB) and worst bias (WB) done by the best performer are also reported in Table 5. For example, the best performer under the t distribution with 5 degrees of freedom is EWMA-R when the number of profiles used in estimation, 'm', is less than 200. In this case, the BB is 0.11 which is obtained when λ = 2.0, m =10 and the WB is 39.02 which is obtained

Table 5. Summary of the comparison of the methods under all shifts

m		t(3)	t(5)	Normal	
				For all λ	
< 200	Method	T^2	EWMA-R	EWMA-3	
	BB	-2.04 (λ=0.4, *m*=10)	0.11 (λ=2.0, *m*=10)	-0.05 (λ=1.0, *m*=10)	
	WB	34.28 (λ=0.8, *m*=10)	39.02 (λ=0.4, *m*=10)	78.06 (λ=0.2, *m*=10)	
≥ 200	Method	EWMA-R	EWMA-3	EWMA-3	
	BB	3.0 (λ=2.0, *m*=200)	-0.08 (λ=1.8, *m*=200)	-0.08 (λ=1.6, 1.8, *m*=200)	
	WB	-26.22 (λ=0.2, *m*=200)	-2.93 (λ=0.2, *m*=200)	1.74 (λ=0.2, *m*=200)	
				For all β	
< 200	Method	T^2	EWMA-R	EWMA-3	
	BB	0.81 (β=0.075, *m*=10)	0.52 (β=0.25, *m*=30)	-0.05 (β=0.225, *m*=10; β=0.175, *m*=30) 0.05 (β=0.2, *m*=10)	
	WB	-105.67 (β=0.025, *m*=30)	-65.02 (β=0.025, *m*=30)	75.66 (β=0.025, *m*=10)	
≥ 200	Method	EWMA-R	EWMA-3	EWMA-3	
	BB	4.88 (β=0.25, *m*=200)	-0.06 (β=0.150, *m*=200)	0.03 (β=0.1, *m*=200)	
	WB	73.98 (β=0.025, *m*=200)	-27.77 (β=0.025, *m*=200)	1.72 (β=0.050, *m*=200)	
		For all γ		**γ < 1.8**	**γ ≥ 1.8**
< 200	Method	EWMA-R	EWMA-R / T^2	Similar performance	Similar performance
	BB	0.78 (γ=3.0, *m*=200)	0.04 (T^2,γ=3.0, *m*=200)	-0.05 (T^2, γ=1.4, *m*=200)	-0.13 (EWMA-R, γ=3.0, *m*=200; EWMA-3, γ=1.8, *m*=200)
	WB	-9.13 (γ=1.2, *m*=200)	-14.32 (T^2,γ=1.2, *m*=200)	0.45 (T^2, γ=1.2, *m*=200)	-0.33 (EWMA-R, γ=1.8, *m*=200)
≥ 200	Method	EWMA-R	EWMA-R / T^2	Similar performance	Similar performance
	BB	0.78 (γ=3.0, *m*=200)	0.04 (T^2,γ=3.0, *m*=200)	-0.05 (T^2, γ=1.4, *m*=200)	-0.13 (EWMA-R, γ=3.0, *m*=200; EWMA-3, γ=1.8, *m*=200)
	WB	-9.13 (γ=1.2, *m*=200)	-14.32 (T2,γ=1.2, *m*=200)	0.45 (T^2, γ=1.2, *m*=200)	-0.33 (EWMA-R, γ=1.8, *m*=200)

when λ = 0.4, *m* =10. The practitioners should be aware of the big biases that degrade the chart performance while using them.

5. Conclusion

There is an increasing number of charts suggested to monitor profiles. Most of them are based on the assumption that the model parameters are known in Phase II analysis and the error terms are normally distributed which are both unrealistic in many applications. However, there are only a few studies that examine the estimation effect and violation of normality assumption for simple linear profiles in the literature and no one has investigated both effects at the same time. Therefore, in this study, the effect of both estimation and violation of normality assumption on the most popular T^2, EWMA-R and EWMA-3 charts in terms of ARL and SDRL measures is investigated for simple linear profile monitoring. In this study, the errors are assumed to have Student's t-distribution and it is found that violation of both assumptions highly degrades the charts' performance. The estimation effect increases as the distribution deviates from normality more and it requires more profiles to be used in estimation stage. However, in some situations even 200 profiles are not enough to dampen this effect. Furthermore, it is observed that using only ARL as a performance measure might be misleading. Instead, the run length standard deviations should also be taken into consideration since in most cases they are unacceptably large even though the ARL values are close to their theoretical counterparts. As a result, practitioners should check the normality assumption and should be particularly aware of the estimation effect if this assumption is violated.

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