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$(\theta, \mu, \tau)-$NEIGHBORHOOD FOR ANALYTIC FUNCTIONS INVOLVING MODIFIED SIGMOID FUNCTION

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ABSTRACT. In the present investigation we discuss the neighborhoods of analytic functions defined by using modified sigmoid function. Further, we also give some applications of Jack's lemma.

1. INTRODUCTION AND DEFINITIONS

Sigmoid function is once the special functions which is a branch of mathematics which is of the most importance to scientists and engineers who are concerned with actual mathematical calculations such as in physics, engineering, statistics, computer science etc. The theory of special functions was initially out-shined by many other fields like functional analysis, real analysis, topology, differential equations and algebra. There is a collection of three functions known as special functions. They are the ramp function, the threshold function and the sigmoid function. The most popular among them is the sigmoid function of the form $G(s) = \frac{1}{1+e^{-s}}$, $s \in \mathbb{R}$. This function is called as the sigmoidal curve or logistic function and has the following properties:

- It outputs real numbers between 0 and 1.
- It maps a very large input domain to a small range of outputs.
- It never loses information because it is a one-to-one function.
- It increases monotonically.

With all the properties mentioned in [8] sigmoid function is perfectly useful in geometric function theory.

The sigmoid function is defined as

$$G(s) = \frac{1}{1+e^{-s}} = \frac{1}{2} + \frac{s}{4} - \frac{s^3}{48} + \frac{s^5}{480} - \frac{17s^7}{80640} + \cdots .$$

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Let $\tau(s)$ be a modified sigmoid function. That is,

$$\tau(s)=\frac{2}{1+e^{-s}}=1+\frac{s}{2}-\frac{s^3}{24}+\frac{s^5}{240}-\frac{17s^7}{40320}+\cdots.$$

For details see [8], [9].

Let A be the class of functions of the form

$$f(z)=z+\sum_{n=2}^{\infty}a_nz^n, \tag{1}$$

which are analytic in the open unit disk $\mathbb{D}=\{z\in\mathbb{C}:|z|<1\}$.

The function

$$f_\tau(z)=z+\sum_{n=2}^{\infty}\tau(s)a_nz^n,$$

which is analytic and univalent in $\mathbb{D}$ belongs to the class A_τ of the form (1) for $\lim_{s\to\infty}\tau(s)=2$.

Let A_τ be the class of functions $f_\tau(z)=z+\sum\limits_{n=2}^{\infty}\tau(s)a_nz^n$ that are analytic in $\mathbb{D}$. For $f_\tau(z)\in A_\tau$ and $g_\tau(z)\in A_\tau$, $f_\tau(z)$ is said to be $(\theta,\mu,\tau)-$neighborhood for $g_\tau(z)$, if it satisfies $\left|f'_\tau(z)-e^{i\theta}g'_\tau(z)\right|<\mu \quad (z\in\mathbb{D})$ for some $-\pi\le\theta\le\pi$ and $\mu>\sqrt{2(1-\cos\theta)}$.

We denote this neighborhood by $(\theta,\mu,\tau)-N(g_\tau(z))$. Also, we say that

$$f_\tau(z)\in(\theta,\mu,\tau)-M(g_\tau(z)),$$

if it satisfies

$$\left|\frac{f_\tau(z)}{z}-e^{i\theta}\frac{g_\tau(z)}{z}\right|<\mu \quad (z\in\mathbb{D})$$

for some $-\pi\le\theta\le\pi$ and $\mu>\sqrt{2(1-\cos\theta)}$.

Recently, some neighborhoods for analytic functions were considered by Orhan and Kadıoğlu [3], Orhan and Kamali [4], Orhan, Kadıoğlu and Owa [7], Altıntaş, Özkan and Srivastava [1], Orhan, Kamali and Owa [5] and Srivastava and Orhan [6]. Our classes of neighborhoods in the present paper are based on our new considering for the neighborhoods.

2. Some properties

Our first result is contained in

Theorem 1. *If $f_\tau(z)\in A_\tau$ satisfies*

$$\sum_{n=2}^{\infty}n\left|a_n-e^{i\theta}b_n\right|\le\frac{1}{\tau(s)}[\mu-\sqrt{2(1-\cos\theta)}]$$

for some $-\pi\le\theta\le\pi$ and $\mu>\sqrt{2(1-\cos\theta)}$, then $f_\tau(z)\in(\theta,\mu,\tau)-N(g_\tau(z))$.

Proof. Note that

$$\begin{aligned}\left|f_\tau'(z) - e^{i\theta}g_\tau'(z)\right| &= \left|(1-e^{i\theta}) + \sum_{n=2}^{\infty}\tau(s)n(a_n - e^{i\theta}b_n)z^{n-1}\right| \\ &\leq \left|(1-e^{i\theta})\right| + \tau(s)\sum_{n=2}^{\infty} n\left|a_n - e^{i\theta}b_n\right||z|^{n-1} \\ &< \sqrt{2(1-\cos\theta)} + \tau(s)\sum_{n=2}^{\infty} n\left|a_n - e^{i\theta}b_n\right|.\end{aligned}$$

If

$$\sum_{n=2}^{\infty} n\left|a_n - e^{i\theta}b_n\right| \leq \frac{1}{\tau(s)}[\mu - \sqrt{2(1-\cos\theta)}],$$

then we see that $\left|f_\tau'(z) - e^{i\theta}g_\tau'(z)\right| < \mu$ where $z \in \mathbb{D}$.

Thus, $f_\tau(z) \in (\theta,\mu,\tau) - N(g_\tau(z))$. □

Example 2. *For given* $g_\tau(z) = z + \sum_{n=2}^{\infty}\tau(s)b_n z^n \in A_\tau$,*we consider* $f_\tau(z) = z + \sum_{n=2}^{\infty}\tau(s)a_n z^n \in A_\tau$,*with*

$$a_n = \frac{\frac{1}{\tau(s)}[\mu - \sqrt{2(1-\cos\theta)}]}{n^2(n-1)}e^{i\rho} + e^{i\theta}b_n \qquad (-\pi \leq \rho \leq \pi,\ n = 2,3,4,...).$$

Then we get that

$$\begin{aligned}\sum_{n=2}^{\infty} n\left|a_n - e^{i\theta}b_n\right| &= \sum_{n=2}^{\infty} n\left|\frac{\frac{1}{\tau(s)}[\mu - \sqrt{2(1-\cos\theta)}]}{n^2(n-1)}e^{i\rho}\right| \\ &= \frac{1}{\tau(s)}\left(\mu - \sqrt{2(1-\cos\theta)}\right)\left(\sum_{n=2}^{\infty}\frac{1}{n(n-1)}\right) \\ &= \frac{1}{\tau(s)}\left(\mu - \sqrt{2(1-\cos\theta)}\right)\left(\sum_{n=2}^{\infty}\left(\frac{1}{n-1} - \frac{1}{n}\right)\right) \\ &= \frac{1}{\tau(s)}\left(\mu - \sqrt{2(1-\cos\theta)}\right).\end{aligned}$$

Therefore, $f_\tau(z) \in (\theta,\mu,\tau) - N(g_\tau(z))$.

Corollary 3. *If* $f_\tau(z) \in A_\tau$ *satisfies*

$$\sum_{n=2}^{\infty} n\,||a_n| - |b_n|| \leq \frac{1}{\tau(s)}\left(\mu - \sqrt{2(1-\cos\theta)}\right)$$

for some $-\pi \leq \theta \leq \pi$ *and* $\mu > \sqrt{2(1-\cos\theta)}$, *and*

$$\arg a_n - \arg b_n = \theta \ (n = 2,3,4,...),$$

then

$$f_\tau(z) \in (\theta, \mu, \tau) - N(g_\tau(z)).$$

Proof. With Theorem 1, we see that

$$\sum_{n=2}^{\infty} n \left| a_n - e^{i\theta} b_n \right| \leq \frac{1}{\tau(s)} \left(\mu - \sqrt{2(1-\cos\theta)} \right),$$

which implies that

$$f_\tau(z) \in (\theta, \mu, \tau) - N(g_\tau(z)).$$

Since

$$\arg a_n - \arg b_n = \theta,$$

if $\arg a_n = \varphi_n$, then $\arg b_n = \varphi_n - \theta$.Therefore

$$a_n - e^{i\theta} b_n = |a_n| e^{i\varphi_n} - |b_n| e^{i\varphi_n} = (|a_n| - |b_n|) e^{i\varphi_n},$$

which implies

$$\left| a_n - e^{i\theta} b_n \right| = ||a_n| - |b_n|| .$$

This completes the proof. □

Theorem 4. *If $f_\tau(z) \in A_\tau$ satisfies*

$$\sum_{n=2}^{\infty} n \left| a_n - e^{i\theta} b_n \right| \leq \frac{1}{\tau(s)} \left(\mu - \sqrt{2(1-\cos\theta)} \right), (z \in \mathbb{D})$$

for some $-\pi \leq \theta \leq \pi$ and $\mu > \sqrt{2(1-\cos\theta)}$, then $f_\tau(z) \in (\theta, \mu, \tau) - M(g_\tau(z))$.

Example 5. *For given*

$$g_\tau(z) = z + \sum_{n=2}^{\infty} \tau(s) b_n z^n \in A_\tau,$$

we define $f_\tau(z)$ by

$$f_\tau(z) = z + \sum_{n=2}^{\infty} \tau(s) a_n z^n \in A_\tau$$

with

$$a_n = \frac{\frac{1}{\tau(s)} \left(\mu - \sqrt{2(1-\cos\theta)} \right)}{n(n-1)} e^{i\rho} + e^{i\theta} b_n \qquad (n = 2, 3, 4, ...).$$

Then, we have

$$\begin{aligned} \sum_{n=2}^{\infty} \left| a_n - e^{i\theta} b_n \right| &= \sum_{n=2}^{\infty} \left| \frac{\frac{1}{\tau(s)} \left(\mu - \sqrt{2(1-\cos\theta)} \right)}{n(n-1)} e^{i\rho} \right| \\ &= \frac{1}{\tau(s)} \left(\mu - \sqrt{2(1-\cos\theta)} \right) \left(\sum_{n=2}^{\infty} \frac{1}{n(n-1)} \right) \end{aligned}$$

$$= \frac{1}{\tau(s)}\left(\mu - \sqrt{2(1-\cos\theta)}\right),$$

which implies that $f_\tau(z) \in (\theta,\mu,\tau) - M(g_\tau(z))$.

Corollary 6. *If* $f_\tau(z) \in A_\tau$ *satisfies*

$$\sum_{n=2}^{\infty} ||a_n| - |b_n|| \leq \frac{1}{\tau(s)}\left(\mu - \sqrt{2(1-\cos\theta)}\right)$$

for some $-\pi \leq \theta \leq \pi$, $\mu > \sqrt{2(1-\cos\theta)}$, *and*

$$\arg a_n - \arg b_n = \theta \ (n = 2, 3, 4, ...),$$

then

$$f_\tau(z) \in (\theta,\mu,\tau) - M(g_\tau(z)).$$

Now, we give the necessary conditions for neighborhoods.

Theorem 7. *If* $f_\tau(z) \in (\theta,\mu,\tau) - N(g_\tau(z))$ *and* $\arg(a_n - e^{i\theta}b_n) = (n-1)\varphi \quad (n = 2, 3, 4, ...)$, *then*

$$\sum_{n=2}^{\infty} n\left|a_n - e^{i\theta}b_n\right| \leq \frac{1}{\tau(s)}[\mu + \cos\theta - 1].$$

Proof. For $f_\tau(z) \in (\theta,\mu,\tau) - N(g_\tau(z))$, we have

$$\begin{aligned}
\left|f'_\tau(z) - e^{i\theta}g'_\tau(z)\right| &= \left|(1-e^{i\theta}) + \tau(s)\sum_{n=2}^{\infty} n(a_n - e^{i\theta}b_n)z^{n-1}\right| \\
&= \left|(1-e^{i\theta}) + \tau(s)\sum_{n=2}^{\infty} n\left|a_n - e^{i\theta}b_n\right| e^{i(n-1)\varphi}z^{n-1}\right| \\
&< \mu
\end{aligned}$$

for all $z \in \mathbb{D}$. Let us consider z such that $\arg z = -\varphi$. Then

$$z^{n-1} = |z|^{n-1} e^{-i(n-1)\varphi}.$$

For such a point $z \in \mathbb{D}$, we see that

$$\begin{aligned}
\left|f'_\tau(z) - e^{i\theta}g'_\tau(z)\right| &= \left|(1-e^{i\theta}) + \tau(s)\sum_{n=2}^{\infty} n\left|a_n - e^{i\theta}b_n\right| |z|^{n-1}\right| \\
&= \left(\left[1 + \tau(s)\sum_{n=2}^{\infty} n\left|a_n - e^{i\theta}b_n\right| |z|^{n-1} - \cos\theta\right]^2 + \sin^2\theta\right)^{\frac{1}{2}} \\
&< \mu
\end{aligned}$$

for $z \in \mathbb{D}$. This implies that

$$(1-\cos\theta)+\tau(s)\sum_{n=2}^{\infty} n\left|a_n - e^{i\theta} b_n\right| |z|^{n-1} < \mu$$

for $z \in \mathbb{D}$. Letting $|z| \to 1^-$, we have that

$$\sum_{n=2}^{\infty} n\left|a_n - e^{i\theta} b_n\right| \leq \frac{1}{\tau(s)}[\mu+\cos\theta-1].$$

□

Theorem 8. *We also have*

$$f_\tau(z) \in (\theta,\mu,\tau)-N(g_\tau(z))$$

and

$$\arg(a_n - e^{i\theta} b_n) = (n-1)\varphi \ (n=2,3,4,...),$$

then

$$\sum_{n=2}^{\infty} \left|a_n - e^{i\theta} b_n\right| \leq \frac{1}{\tau(s)}[\mu+\cos\theta-1].$$

3. Applications of Jack's Lemma

Lemma 9. [2] *Let the function $w(z)$ be analytic in $\mathbb{D}$ with $w(0)=0$. If there exists a point $z_0 \in \mathbb{D}$ such that*

$$\max_{|z|\leq|z_0|} |w(z)| = |w(z_0)|,$$

then

$$z_0 w'(z_0) = k w(z_0),$$

where k is real and $k \geq 1$.

Theorem 10. *If $f_\tau(z) \in A_\tau$ satisfies*

$$\left|f_\tau'(z) - e^{i\theta} g_\tau'(z)\right| < 2\mu - \frac{1}{\tau(s)}\sqrt{2(1-\cos\theta)} \quad (z \in \mathbb{D})$$

for some $-\pi \leq \theta \leq \pi$ and $\mu > \dfrac{\sqrt{2(1-\cos\theta)}}{2\tau(s)}$.

Then

$$\left|\frac{f_\tau(z)}{z} - e^{i\theta}\frac{g_\tau(z)}{z}\right| < \mu + \frac{1}{\tau(s)}\sqrt{2(1-\cos\theta)} \quad (z \in \mathbb{D}).$$

Proof. Let $w(z)$ define by

$$\frac{f_\tau(z)}{z} - e^{i\theta}\frac{g_\tau(z)}{z} - \frac{1}{\tau(s)}(1-e^{i\theta}) = \mu w(z).$$

Then $w(z)$ is analytic in $\mathbb{D}$ and $w(0)=0$. It follows that

$$\left|f_\tau'(z)-e^{i\theta}g_\tau'(z)\right| = \left|\frac{1}{\tau(s)}(1-e^{i\theta})+\mu w(z)\left(1+\frac{zw'(z)}{w(z)}\right)\right|$$

there exists a point

$$< 2\mu-\frac{1}{\tau(s)}\sqrt{2(1-\cos\theta)}.$$

Suppose that $z_0\in\mathbb{D}$ such that

$$\max_{|z|\le|z_0|}|w(z)|=|w(z_0)|=1.$$

by Lemma 9,

Then $w(z_0)=e^{i\theta}$ and $\dfrac{z_0w'(z_0)}{w(z_0)}=k\ge 1$. Therefore, we obtain that

$$\begin{aligned}\left|f_\tau'(z_0)-e^{i\theta}g_\tau'(z_0)\right| &= \left|\frac{1}{\tau(s)}(1-e^{i\theta})+\mu e^{i\theta}(1+k)\right|\\ &\ge \mu(1+k)-\frac{1}{\tau(s)}\left|1-e^{i\theta}\right|\\ &\ge 2\mu-\frac{1}{\tau(s)}\sqrt{2(1-\cos\theta)}.\end{aligned}$$

This contradicts our condition in Theorem 10.

Therefore, there isn't $z_0\in\mathbb{D}$ such that $|w(z_0)|=1$. This implies that $|w(z)|<1$ for all $z\in\mathbb{D}$. Thus we have that

$$\begin{aligned}\left|\frac{f_\tau(z)}{z}-e^{i\theta}\frac{g_\tau(z)}{z}\right| &= \left|\frac{1}{\tau(s)}(1-e^{i\theta})+\mu w(z)\right|\\ &\le \frac{1}{\tau(s)}\left|1-e^{i\theta}\right|+\mu\left|w(z)\right|\\ &< \mu+\frac{1}{\tau(s)}\sqrt{2(1-\cos\theta)}.\end{aligned}$$

Letting $\theta=\dfrac{\pi}{2}$ in Theorem 10 we can obtain the following corollary. □

Corollary 11. *If $f_\tau(z)\in A_\tau$ satisfies*

$$|f_\tau'(z)-ig_\tau'(z)|<2\mu-\frac{\sqrt{2}}{\tau(s)}\qquad(z\in\mathbb{D})$$

for some $\mu>\dfrac{1}{\sqrt{2}\tau(s)}$, then

$$\left|\frac{f_\tau(z)}{z}-i\frac{g_\tau(z)}{z}\right|<\mu+\frac{\sqrt{2}}{\tau(s)}\quad(z\in\mathbb{D}).$$

Similarly, we can prove the following theorem.

Theorem 12. *If $f_\tau(z) \in A_\tau$ satisfies*

$$\operatorname{Re}\left((f'_\tau(z) - e^{i\theta} g'_\tau(z)\right) > \frac{1}{\tau(s)}(1-\cos\theta) - \frac{3}{4}\mu \qquad (z \in \mathbb{D})$$

for some $-\pi \le \theta \le \pi$ and $\mu > 0$, then

$$\operatorname{Re}\left(\frac{f_\tau(z)}{z} - e^{i\theta}\frac{g_\tau(z)}{z}\right) > \frac{1}{\tau(s)}(1-\cos\theta) - \frac{\mu}{2} \quad (z \in \mathbb{D}).$$

Proof. Let $w(z)$ define by

$$\frac{f_\tau(z)}{z} - e^{i\theta}\frac{g_\tau(z)}{z} - \frac{1}{\tau(s)}(1-e^{i\theta}) = \mu\frac{w(z)}{1-w(z)} \qquad (w(z) \neq 1).$$

Then $w(z)$ is analytic in $\mathbb{D}$ and $w(0) = 0$. Note that

$$f'_\tau(z) - e^{i\theta} g'_\tau(z) = \frac{1}{\tau(s)}(1-e^{i\theta}) + \mu\frac{w(z)}{1-w(z)} + \mu\frac{zw'(z)}{(1-w(z))^2}.$$

We suppose that there exists a point $z_0 \in \mathbb{D}$ such that

$$\max_{|z|\le|z_0|} |w(z)| = |w(z_0)| = 1.$$

Then, by using Lemma 9, we can write that $w(z_0) = e^{i\theta}$ and $z_0 w'(z_0) = ke^{i\theta}$ $(k \ge 1)$.

Therefore, we have that

$$\begin{aligned}
\operatorname{Re}\left((f'_\tau(z_0) - e^{i\theta} g'_\tau(z_0)\right) &= \operatorname{Re}\left(\frac{1}{\tau(s)}(1-e^{i\theta}) + \mu\frac{e^{i\theta}}{1-e^{i\theta}} + \mu\frac{ke^{i\theta}}{(1-e^{i\theta})^2}\right) \\
&= \frac{1}{\tau(s)}(1-\cos\theta) - \frac{\mu}{2} - k\mu\frac{1}{2(1-\cos\theta)} \\
&\le \frac{1}{\tau(s)}(1-\cos\theta) - \frac{\mu}{2} - \frac{\mu}{4} \\
&= \frac{1}{\tau(s)}(1-\cos\theta) - \frac{3}{4}\mu,
\end{aligned}$$

which contradicts our condition of the theorem.

Thus there isn't $z_0 \in \mathbb{D}$ such that $|w(z_0)| = 1$. This implies that $|w(z)| < 1$ for $z \in \mathbb{D}$, that is,

$$\operatorname{Re}\left(\frac{w(z)}{1-w(z)}\right) > -\frac{1}{2} \quad (z \in \mathbb{D}).$$

Finally, we have

$$\operatorname{Re}\left(\frac{f_\tau(z)}{z} - e^{i\theta}\frac{g_\tau(z)}{z}\right) > \frac{1}{\tau(s)}(1-\cos\theta) - \frac{\mu}{2} \quad (z \in \mathbb{D}).$$

□

If we take $\theta = \dfrac{\pi}{2}$ in Theorem 12, then we get the following corollary.

Corollary 13. *If* $f_\tau(z) \in A_\tau$ *satisfies* $\operatorname{Re}\left(f_\tau'(z) - i g_\tau'(z)\right) > \frac{1}{\tau(s)} - \frac{3}{4}\mu \quad (z \in \mathbb{D})$*for some* $\mu > 0$, *then* $\operatorname{Re}\left(\frac{f_\tau(z)}{z} - i\frac{g_\tau(z)}{z}\right) > \frac{1}{\tau(s)} - \frac{\mu}{2} \quad (z \in \mathbb{D})$.*Furthermore, if* $\mu = 2(1-\beta)$ $(0 \leq \beta < 1)$, *then* $\operatorname{Re}\left(f_\tau'(z) - i g_\tau'(z)\right) > \frac{1}{\tau(s)} - \frac{3}{2}(1-\beta) \quad (z \in \mathbb{D})$ *implies that* $\operatorname{Re}\left(\frac{f_\tau(z)}{z} - i\frac{g_\tau(z)}{z}\right) > \frac{1}{\tau(s)} + \beta - 1 \quad (z \in \mathbb{D})$.

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