

# **An Approach to Obtain an Advisable Ratio between Stator and Rotor Tooth Widths in Switched Reluctance Motors for Higher Torque and Smoother Output Power Profile**

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*Received: 31.12.2009 Revised: 06.01.2010 Accepted: 09.02.2010*

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## **ABSTRACT**

This paper is carried out to find out an easy way to determine an advisable and acceptable ratio between stator and rotor tooth width in switched reluctance motors for smoother magnetic field energy, torque and output power profile. As known, different switched reluctance motors (SRMs) have different usage areas because of their different constructions causing different torque, current-voltage and power characteristics. Considering all these effects, eight different SRM designs were held. The designed motors only have different rotor tooth structure. In order to exhibit the most suitable stator/rotor tooth width ratio for an application, software called FEMM 4.2 and LUA scripting language utilizing finite element method (FEM) have been used. The comparison of dynamic output for each motor provides the basis for discussion on the relationship of performance of SRM relating to its physical size. In this study; the differences among the motors designed are explained and the selected and verified simulation results are presented.

**Key Words:** *Switched Reluctance Motor, Tooth Width Ratio, Finite Elements, Static Torque*

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## **1. INTRODUCTION**

The advantages of switched reluctance motors (SRMs) have been discussed in [1-3] and effects of different rotor and stator structures on output characteristics have been shown in [4], [5] and [6]. Performance outputs of SRMs depend on the way that the motors are constructed. The rotor and motor geometry influence the average and maximum torque, current-voltage and output

characteristics. In this study, the following steps have been achieved:

- Based on standard 7.5 kW Oulton SRM [7] (MDL\_6 in Table I), eight different 2D SRMs have been designed using a package program called "Autodesk Inventor" which is a leader software in bringing innovative capabilities to

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the manufacturing market. Each motor has a different rotor/stator tooth width ratio indicated

as  $d_w$ 1...8.


Stator poles	8	
Rotor poles	6	
Stator diameter (mm)	206	
Rotor diameter (mm)	116	
Air gap length (mm)	0.35	
Stack depth (mm)	171	
Stator tooth arc (deg)	20.2	
Rotor tooth arc (deg)	23.5	
Stator tooth width (mm)	20.6	
Rotor tooth width (mm)	23.6	
Number of coil turns	60	
$d_w$ (tooth width ratio)	1.14	
Winding type	series	
Non-oriented steel type	M-19	

Figure 1. 4-phase Oulton SRM [TASC Drives Ltd., UK] and dimensions.

- Afterwards, all motors have been exported to a “\*.dxf” format in order to open them in Finite Element Method Magnetics (FEMM 4.2) by David Meeker [8].
- The electromagnetic problems of models on two-dimensional planar and axisymmetric domains have been solved by defining material properties and boundary conditions.

- And then; a complete, open-source scripting language called LUA which was maintained at PUC-Rio in Brazil has been used to analyze the models that were pre-processed and eventually obtained results have been compared.

In Table 1, the differences in dimensions of designed motors are given. The other dimensions reported in Figure 1 are kept constant.

Table 1. The differences in dimensions of designed motors.

	MDL_1	MDL_2	MDL_3	MDL_4	MDL_5	MDL_6	MDL_7	MDL_8
Rotor tooth arc (deg)	18.43	19.43	20.2	21.44	22.44	23.5	24.00	24.46
Rotor tooth width (mm)	18.6	19.6	20.6	21.6	22.6	23.6	24.1	24.6
$d_w$ (tooth width ratio)	0.9	0.95	1	1.04	1.09	1.14	1.16	1.19

## 2. FINITE ELEMENT MODELS OF THE MOTORS

The finite element method (FEM) (sometimes referred to as finite element analysis) is a numerical technique for finding approximate solutions of partial differential equations (PDE) as well as of integral equations. The solution approach is based either on eliminating the differential equation completely (steady state problems) or rendering the PDE into an approximating system of ordinary differential equations, which are then numerically integrated using standard techniques such as Euler's method, Runge-Kutta, etc. In the analyses, a kind of package program called FEMM 4.2 utilizing finite element method has been used. With the help of FEMM 4.2, different output characteristics (magnetic field energy, inductance variation, Maxwell's static and instantaneous

torque profiles and output power, etc.) have been obtained by using the integral equations (1-2).

**Magnetic field energy (W);** is calculated to denote the energy stored in the magnetic field in the specified region and used as an alternate method of getting inductance for problems that are linear (at least not heavily saturated). In case of nonlinear materials, the energy is computed with equation (1).

$$W = \int \left( \int_0^B H(B') dB' \right) dV \tag{1}$$

where B is the magnitude of flux density (Tesla), H is the magnitude of field intensity (A/m) and V is the volume of

the region obtained from integration over the 2D region of interest multiplied by the stack depth of the machine.

**Torque by Maxwell Stress Tensor ( $T_{st}$ );** is the force applied to one part of the magnetic circuit and that can be obtained by integrating the well-known Maxwell stress tensor along a surface placed in the air-gap. This torque value can be calculated by using equation (2).

$$T_{st} = L \int_S \{ r \times [(1 / \mu_0)(B.n)B - (1 / 2\mu_0)B^2 n] \} dS \quad (2)$$

where  $T_{st}$  is the static torque, L is the length, B is the

induction vector in the elements and r is the lever arm, i.e. the vector which connects the origin to the midpoint of segment dS [9].

The finite-element analysis of each model (MDL\_1.....MDL\_8) has been performed on the SRM geometries which have different rotor/stator tooth width ratio ( $d_w$ ) under the condition that all other remaining dimensions are kept to their original value. As an example, the flux density |B| and flux paths of the SRM model (MDL\_3) of which  $d_{w3}$  value is 1 (stator and rotor tooth widths are equal to each other) are shown in Figure 2 when only phase C is excited with the rated current value of 10 A for Oulton SRM.

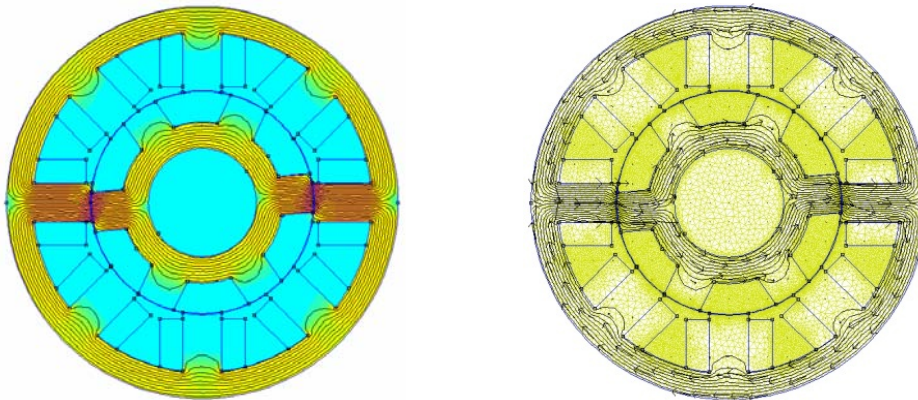


Figure 2. (a) Flux density |B| and (b) mesh generation and flux paths for MDL\_3 ( $d_{w3}=1$ ).

### 3. CURRENT CONTROLLED SYSTEM AND METHOD

SRMs have been analyzed for the output profiles of

inductance variation, magnetic field energy and static/instantaneous torque and output power. In this study, a flow diagram as shown in Figure 3 has been carried out.

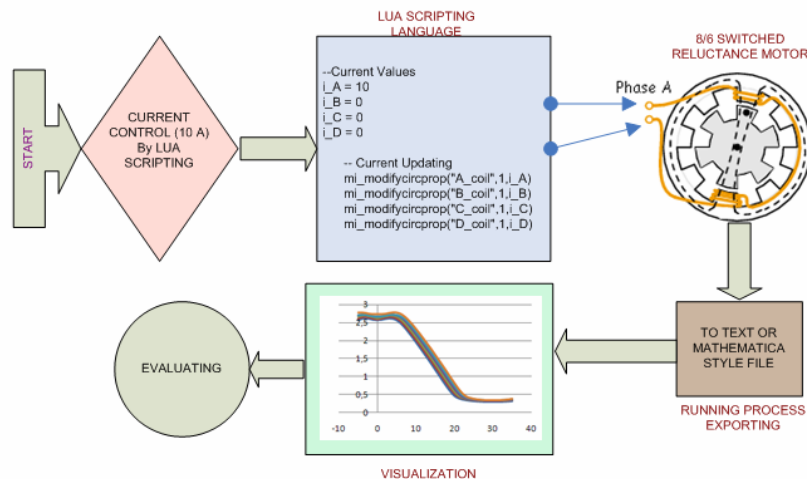


Figure 3. Flow diagram of the analysis.

By keeping the stator tooth width constant to its original value, rotor tooth widths are changed from 18.6 mm to 24.6 mm step by step, and over a rotation cycle between 0° and 30°; consequently magnetic field energy and static

torque by Maxwell stress tensor profiles are obtained for rated phase current of 10 A and for the currents of 7A and 4A. Moreover, the designed motors have been driven using a trapezoidal current waveform script by LUA, and

in this way; output profiles of instantaneous torque of the models have been obtained in Nm for all phases for an operation cycle between 0° and 40°. By using the

**4. SIMULATION RESULTS AND DISCUSSION**

Figure 4 shows static torque profiles of the designed motors only for phase C. The literature is pointing out that the average torque value of the static torque profile over a half cycle (30°) is used for comparison of these profiles to decide the optimum pole size of a motor [10], [11]. One phase is excited for 15° only in theory. Therefore, the next phase is going to be switched on after 15° rotation (SWITCH ON/OFF ANGLE) of the motor. Given a choice, the authors would like to select that 15° where the torque is nearly constant [5]. This choice will

instantaneous torque values, output power values of the motor can be calculated.

ensure maximum average torque with minimum ripple. Within this idea, the torque ripples have been calculated for the given maximum, minimum and average torque values by using the following expression:

$$\%Ripple = \left[ \frac{(T_{max} - T_{min})}{T_{avg}} \right] .100 \tag{3}$$

where  $T_{max}$  is maximum torque in Nm,  $T_{min}$  is the minimum torque in Nm and  $T_{avg}$  is the average torque in Nm for the given rotation angle value.

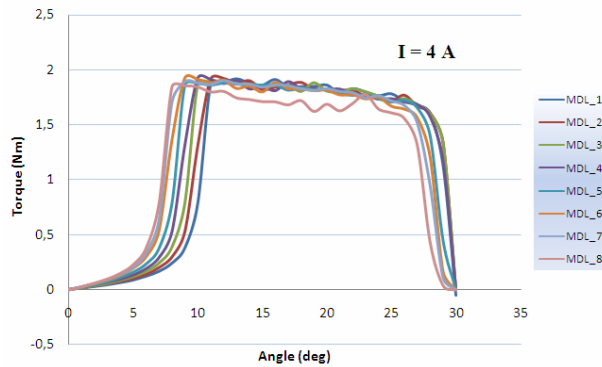


Figure 4. Static torque variation over a rotation between 0° and 30° (only for phase C).

Table 2. Average and maximum torque and torque ripples for various rotor/stator tooth width ratio values.

$d_w$	$T_{avg}$ -Average over a half cycle of 30° (Nm)	$T_{max}$ -Torque value of 15° (Nm)	Ripple (%)
<b>0.9</b>	1.15	1.86	61.73
<b>0.95</b>	1.18	1.82	54.23
<b>1</b>	1.21	1.86	53.71
<b>1.04</b>	1.23	1.85	50.4
<b>1.09</b>	1.23	1.86	51.21
<b>1.14</b>	1.24	1.80	45.16
<b>1.16</b>	1.24	1.84	48.38
<b>1.19</b>	1.17	1.71	46.15

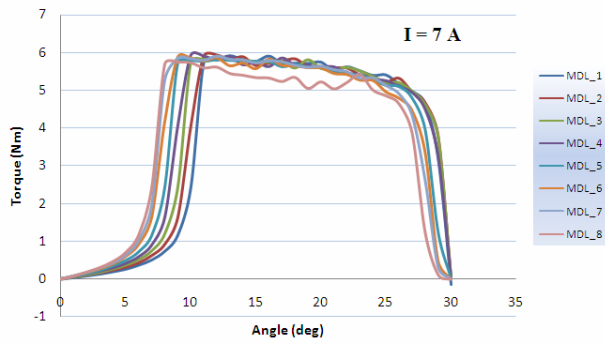


Figure 5. Static torque variation over a rotation between 0° and 30° (only for phase D).

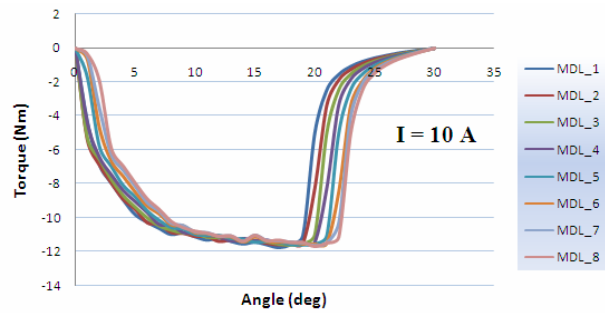


Figure 6. Static torque variation over a rotation between 0° and 30° (only for phase A).

Table 2 gives the entire calculated average torque values over a rotation of 30° and the best torque values at 15° (only C phase is excited at 4A). If the values are compared, it can be seen on the table that the maximum torque value at 15° is 1.86 Nm relating to an average of 1.23 Nm for the model MDL\_5 of which  $d_{w5}$  value is 1.09. And, the torque ripple is % 51.21 that is an acceptable value when compared to other models.

Table 3. Average and maximum torque and torque ripples for various rotor/stator tooth width ratio values.

$d_w$	$T_{avg}$ -Average over a half cycle of 30° (Nm)	$T_{max}$ -Torque value of 15° (Nm)	Ripple (%)
<b>0.9</b>	3.52	5.77	63.92
<b>0.95</b>	3.62	5.67	56.62
<b>1</b>	3.7	5.76	55.67
<b>1.04</b>	3.76	5.75	52.92
<b>1.09</b>	3.75	5.78	53.13
<b>1.14</b>	3.76	5.58	48.4
<b>1.16</b>	3.8	5.71	50.26
<b>1.19</b>	3.59	5.32	48.18

Table 4. Average and maximum torque and torque ripples for various rotor/stator tooth width ratio values

$d_w$	$T_{avg}$ -Average over a half cycle of 30° (Nm)	$T_{max}$ -Torque value of 15° (Nm)	Ripple (%)
<b>0.9</b>	6.65	11.40	76.24
<b>0.95</b>	6.61	11.33	58.57
<b>1</b>	6.96	11.34	55.83
<b>1.04</b>	7.04	11.36	63.8
<b>1.09</b>	7.06	11.41	63.7
<b>1.14</b>	7.09	11.11	62.9
<b>1.16</b>	7.08	11.01	64.12
<b>1.19</b>	7.05	10.99	65.24

Similarly, Table 3 gives the average torque values and the best torque values of 15° for a current at rated 7A for phase D. The maximum torque value at 15° is 5.78 Nm relating to an average of 3.75 Nm for the same model. And, the torque ripple between maximum 15° and average torque is % 53.13, which is acceptable, too.

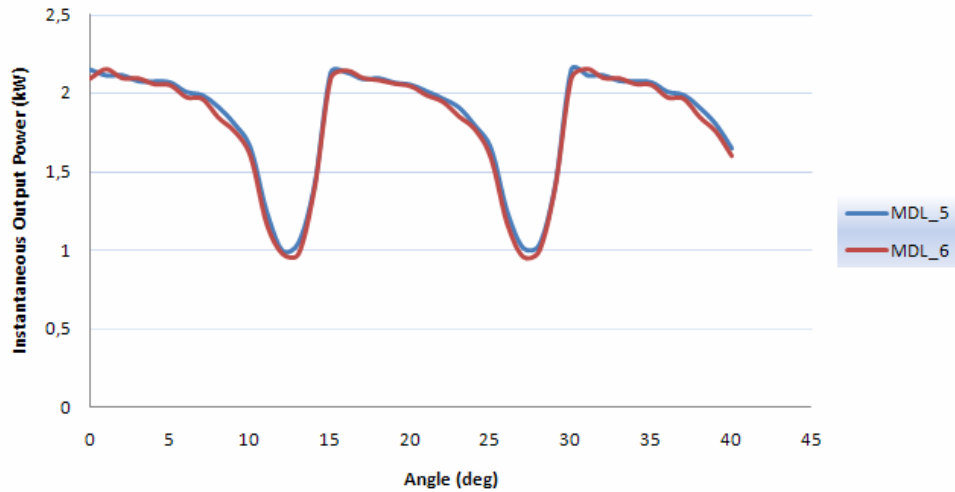


Figure 7. Instantaneous output power variation (in kW) of two motors over a rotation between 0° and 40°.

Table 5. Comparison of output power profiles.

Motor Type	Maximum Output Power (kW)	Minimum Output Power (kW)	Average Output Power (kW)	Output Power Ripple (%)
MDL_5 ( $d_{w5}=1.09$ )	2.14	0.99	1.84	62.5
MDL_6 ( $d_{w6}=1.14$ )	2.15	0.95	1.80	66.67

Figure 7 gives output power profile of two motors over a rotation cycle between 0° and 40°. The reason of taking into consideration only these two models is that these models have given the best results in comparison of static torque values. The better and smoother static torque output means better instantaneous output power. Once the designed motors have been driven using a trapezoidal current waveform script by LUA, and in this way; output profiles of instantaneous torque of the models have been obtained, by using the instantaneous torque values obtained according to the rotation angle; output power of the motor for a given angle can be calculated via:

$$P_{out(\theta)} = \left( \frac{T_{int(\theta)}}{1000} \right) \cdot \frac{2\pi \cdot n_r}{60} \quad (4)$$

where  $P_{out}$  is the output power in kW,  $T_{int}$  is the instantaneous torque in Nm and  $n_r$  is the rotational speed in rpm (this is a constant value, 1800 rpm, which is the rated speed for Oulton SRM).

The concept about output power profile is “smoother instantaneous power profile”. If the curves in Figure 7 and the values in Table 5 are observed carefully, it can be seen that the instantaneous output power variation of “MDL\_5” is smoother than Oulton SRM.

## 5. CONCLUSION

As a result, it is observed that for the MDL\_5 with existing stator tooth width of 20.6 mm and a changed rotor tooth width of 22.6 mm has the best  $d_w$  value (1.09), as it gives smoother output profile and the highest

average torque with an acceptable ripple, which will help in getting more average torque. Generally, it can be pointed out that for an 8/6 SRM of which  $d_{w5}=1.09$  will be the best acceptable and advisable value. By using this ratio, more average torque with an acceptable ripple for both current values of 7A and 4A has been achieved.

## ACKNOWLEDGEMENT

The authors would like to thank Gazi University Scientific Researches Unit (BAP) under project number 07/2008-02 for their financial support.

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