Hacettepe Journal of Mathematics and Statistics
Volume 40 (3) (2011), 359 – 365

ON CERTAIN CLASSES OF MEROMORPHICALLY p-VALENT CONVEX FUNCTIONS

M. K. Aouf*†, A. Shamandy*, A. O. Mostafa* and S. M. Madian*

Received 13 : 10 : 2009 : Accepted 26 : 09 : 2011

Abstract

Making use of a differential operator, which is defined here by means of the Hadamard product (or convolution), we introduce the class $\Sigma_p^n(\alpha_1, \beta_1; \lambda)$ of meromorphically p-valent convex functions. The main object of this paper is to investigate various important properties and characteristics for this class. Further, a property preserving integrals is considered.

Keywords: p-Valent, Hadamard product, Meromorphic, Convex, Jack's lemma.

2000 AMS Classification: 30 C 45.

1. Introduction

Let Σ_p denote the class of functions of the form:

$$f(z) = z^{-p} + \sum_{k=0}^{\infty} a_k z^k \ (p \in \mathbb{N} = \{1, 2, \ldots\}), \tag{1.1}$$

which are analytic and p-valent in the punctured unit disc $U^* = \{z : z \in C \text{ and } 0 < |z| < 1\} = U \backslash \{0\}$. For functions $f(z) \in \Sigma_p$ given by (1.1) and $g(z) \in \Sigma_p$ given by

$$g(z) = z^{-p} + \sum_{k=0}^{\infty} b_k z^k \ (p \in \mathbb{N}), \tag{1.2}$$

we define the Hadamard product (or convolution) of $f(z)$ and $g(z)$ by

$$(f * g)(z) = z^{-p} + \sum_{k=0}^{\infty} a_k b_k z^k = (g * f)(z). \tag{1.3}$$

*Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt. E-mail: (M. K. Aouf) mkaouf127@yahoo.com (A. Shamandy) shamandy16@hotmail.com (A. O. Mostafa) adelaeg254@yahoo.com (S. M. Madian) samar_math@yahoo.com

†Corresponding Author.

For complex parameters $\alpha_1, \alpha_2, \ldots, \alpha_q$ and $\beta_1, \beta_2, \ldots, \beta_s$ $(\beta_j \notin Z_0^- = \{0, -1, -2, \ldots\};$ $j = 1, 2, ..., s)$, we now define the generalized hypergeometric function

$${}_qF_s\ (\alpha_1, \alpha_2, \ldots, \alpha_q;\ \beta_1, \beta_2, \ldots, \beta_s;\ z)$$

by (see, for example, [18, p. 19]),

$$\begin{aligned}&{}_qF_s(\alpha_1, \alpha_2, \ldots, \alpha_q;\ \beta_1, \beta_2, \ldots, \beta_s;\ z) = \sum_{k=0}^{\infty} \frac{(\alpha_1)_k \cdots (\alpha_q)_k}{(\beta_1)_k \cdots (\beta_s)_k\ (1)_k} z^k \\ &(q \le s+1;\ s, q \in \mathbb{N}_0 = \mathbb{N} \cup \{0\};\ z \in U),\end{aligned} \tag{1.4}$$

where $(\theta)_k$ is the Pochhammer symbol, defined in terms of the Gamma function Γ by,

$$(\theta)_v = \frac{\Gamma(\theta + v)}{\Gamma(\theta)} = \begin{cases} 1 & (v = 0;\ \theta \in C^* = C \backslash \{0\}), \\ \theta(\theta+1) \cdots (\theta + v - 1) & (v \in \mathbb{N};\ \theta \in C). \end{cases}$$

Corresponding to the function $h_p(\alpha_1, \alpha_2, \ldots, \alpha_q;\ \beta_1, \beta_2, \ldots, \beta_s;\ z)$, defined by

$$h_p(\alpha_1, \alpha_2, \ldots, \alpha_q;\ \beta_1, \beta_2, \ldots, \beta_s;\ z) = z^{-p}\ {}_qF_s\ (\alpha_1, \alpha_2, \ldots, \alpha_q;\ \beta_1, \beta_2, \ldots, \beta_s;\ z), \tag{1.5}$$

Liu and Srivastava [16] (see, for details [9] and [10]) introduced a linear operator:

$$H_p(\alpha_1, \alpha_2, \ldots, \alpha_q;\ \beta_1, \beta_2, \ldots, \beta_s) : \Sigma_p \longrightarrow \Sigma_p,$$

which is defined by the following Hadamard product:

$$\begin{aligned}&H_p(\alpha_1, \alpha_2, \ldots, \alpha_q;\ \beta_1, \beta_2, \ldots, \beta_s) f(z) \\ &\qquad = h_p(\alpha_1, \alpha_2, \ldots, \alpha_q;\ \beta_1, \beta_2, \ldots, \beta_s;\ z) * f(z), \\ &(q \le s+1;\ s, q \in \mathbb{N}_0; z \in U).\end{aligned} \tag{1.6}$$

We observe that, for a function $f(z)$ of the form (1.1), we have

$$H_p(\alpha_1, \alpha_2, \ldots, \alpha_q;\ \beta_1, \beta_2, \ldots, \beta_s) f(z) = H_{p,q,s}(\alpha_1) f(z) = z^{-p} + \sum_{k=0}^{\infty} \Gamma_k a_k z^k, \tag{1.7}$$

where

$$\Gamma_k = \frac{(\alpha_1)_{k+p} \cdots (\alpha_q)_{k+p}}{(\beta_1)_{k+p} \cdots (\beta_s)_{k+p} (1)_{k+p}}. \tag{1.8}$$

Then one can easily verify from (1.7) that

$$z(H_{p,q,s}(\alpha_1) f(z))' = \alpha_1 H_{p,q,s}(\alpha_1 + 1) f(z) - (\alpha_1 + p) H_{p,q,s}(\alpha_1) f(z). \tag{1.9}$$

The linear operator $H_{p,q,s}(\alpha_1)$ was investigated recently by Liu and Srivastava [16], Aouf [2], and Aouf and Yassen [5].

We note that:

(i) $H_{p,2,1}(a, 1; c) f(z) = \ell_p(a, c) f(z)$ $(a, c > 0)$ (see Liu and Srivastava [15]);
(ii) $H_{p,2,1}(\nu + p, p; p) f(z) = D^{\nu+p-1} f(z)$ $(\nu > -p;\ p \in \mathbb{N})$ (see[1] and [4]);
(iii) $H_{p,2,1}(\nu, 1; \nu + 1) f(z) = F_{\nu,p}(f)(z)$ $(\nu > 0;\ p \in \mathbb{N})$ (see [1], [22] and [23]).

A function $f(z) \in \Sigma_p$ is said to be in the class $\Sigma_p(\lambda)$ of p-valent meromorphically convex functions of order λ in U^* if and only if (see Kumar and Shukla [14])

$$\Re \left\{ 1 + \frac{z f''(z)}{f'(z)} \right\} < -\lambda,\ (z \in U^*;\ 0 \le \lambda < p). \tag{1.10}$$

In this paper, we introduce the class $J_{p,q,s}(\alpha_1; \lambda)$ of functions $f(z) \in \Sigma_p$ which satisfy the condition:

$$\begin{aligned}&\Re \left\{ \frac{(H_{p,q,s}(\alpha_1 + 1) f(z))'}{(H_{p,q,s}(\alpha_1) f(z))'} - (p+1) \right\} < -\frac{p(\alpha_1 - 1) + \lambda}{\alpha_1} \\ &(z \in U^*;\ \alpha_1 > 0;\ 0 \le \lambda < p).\end{aligned} \tag{1.11}$$

Also we note that:

(i) If $q=2,\ s=1$, the class $J_{p,2,1}(\nu+p,\ p;\ p;\ \lambda)$ $(\nu>-p;\ p\in\mathbb{N})$ reduces to the class $\chi_{\nu+p-1}(\lambda)$ (see [4]);

(ii) If $q=2,\ s=1$, the class $J_{p,2,1}(a,\ 1;\ c;\ \lambda)$ $(a>0, c>0)$ reduces to the class $J_p(a,c;\lambda)$, where $J_p(a,c;\lambda)$ is defined by

$$\Re\left\{\frac{(\ell_p(a+1,c)f(z))'}{(\ell_p(a,c)f(z))'}-(p+1)\right\}<-\frac{p(a-1)+\lambda}{a}\ (z\in U^*;\ 0\le\lambda<p); \tag{1.12}$$

(iii) If $q=2,\ s=1$, the class $J_{p,2,1}(\nu,1;\nu+1)$ $(\nu>0;\ p\in\mathbb{N})$ reduces to the class $J_p(\nu,\lambda)$, where $J_p(\nu,\lambda)$ is defined by

$$\begin{gathered}\Re\left\{\frac{(F_{\nu+1,p}(f)(z))'}{(F_{\nu,p}(f)(z))'}-(p+1)\right\}<-\frac{p(\nu-1)+\lambda}{\nu},\\ (z\in U^*;\ \nu>0;\ 0\le\lambda<p).\end{gathered} \tag{1.13}$$

The various properties of the class $J_{p,q,s}(\alpha_1;\lambda)$, derived in this paper, would extend the corresponding results obtained earlier by Bajpai [6], Goel and Sohi [11], Uralegaddi *et al.* ([20] and [21]), Aouf and Hossen [3] (see also Ruscheweyh [17], and Srivastava and Owa [19]). Several other subclasses of Σ_p, analogous to the class $J_{p,q,s}(\alpha_1;\ \lambda)$ studied in this paper, were considered (among others) by Cho [7], Aouf [1], and by Kulkarni *et al.* [13].

2. Basic properties of the class $J_{p,q,s}(\alpha_1;\lambda)$

Unless otherwise mentioned, we shall assume in the reminder of this paper that $q\le s+1, s,q\in\mathbb{N}_0,\ 0\le\lambda<p$, $p\in\mathbb{N}$ and $\alpha_1>0$.

We begin by recalling the following result (Jack's lemma), which we shall apply in proving our first inclusion theorems (Theorem 2.2 and Theorem 2.3 below).

2.1. Lemma. [12] *Let the (nonconstant) function $w(z)$ be analytic in U, with $w(0)=0$. If $|w(z)|$ attains its maximum value on the circle $|z|=r<1$ at a point $z_0\in U$, then*

$$z_0w'(z_0)=\xi w(z_0),$$

where ξ is a real number and $\xi\ge 1$. □

2.2. Theorem. *The following inclusion property holds true for the class $J_{p,q,s}(\alpha_1+1;\lambda)$:*

$$J_{p,q,s}(\alpha_1+1;\lambda)\subset J_{p,q,s}(\alpha_1;\lambda).$$

Proof. For $f(z)\in J_{p,q,s}(\alpha_1+1;\lambda)$, we find from (1.11) that

$$\Re\left\{\frac{(H_{p,q,s}(\alpha_1+2)f(z))'}{(H_{p,q,s}(\alpha_1+1)f(z))'}-(p+1)\right\}<-\frac{p\alpha_1+\lambda}{\alpha_1+1},\ (z\in U^*). \tag{2.1}$$

In order to show that (2.1) implies the inequality (1.11), we define $w(z)\in U$ by

$$\frac{(H_{p,q,s}(\alpha_1+2)f(z))'}{(H_{p,q,s}(\alpha_1+1)f(z))'}-(p+1)=-\left\{\frac{p(\alpha_1-1)+\lambda}{\alpha_1}+\frac{p-\lambda}{\alpha_1}\cdot\frac{1-w(z)}{1+w(z)}\right\}. \tag{2.2}$$

Clearly, $w(z)$ is regular in U and $w(0)=0$. Rewriting (2.2) as

$$\frac{(H_{p,q,s}(\alpha_1+1)f(z))'}{(H_{p,q,s}(\alpha_1)f(z))'}=\frac{\alpha_1+[\alpha_1+2(p-\lambda)]w(z)}{\alpha_1(1+w(z))} \tag{2.3}$$

and differentiating (2.3) logarithmically with respect to z, we obtain

$$\begin{aligned}&\frac{z(H_{p,q,s}(\alpha_1+1)f(z))''}{(H_{p,q,s}(\alpha_1+1)f(z))'}-\frac{z(H_{p,q,s}(\alpha_1)f(z))''}{(H_{p,q,s}(\alpha_1)f(z))'}\\&\qquad=\frac{2(p-\lambda)zw'(z)}{(1+w(z))\left\{\alpha_1+[\alpha_1+2(p-\lambda)]\,w(z)\right\}}.\end{aligned} \tag{2.4}$$

Then one can easily verify from (1.9) that

$$z(H_{p,q,s}(\alpha_1)f(z))'' = \alpha_1 \left(H_{p,q,s}(\alpha_1+1)f(z)\right)' - (\alpha_1+p+1)\left(H_{p,q,s}(\alpha_1)f(z)\right)'. \tag{2.5}$$

Making use of (2.5), (2.4) may be written as

$$\begin{aligned} &\frac{(H_{p,q,s}(\alpha_1+2)f(z))'}{(H_{p,q,s}(\alpha_1+1)f(z))'} - (p+1) + \frac{p\alpha_1+\lambda}{\alpha_1+1} \\ &\qquad = \frac{p-\lambda}{\alpha_1+1}\left\{-\frac{1-w(z)}{1+w(z)} + \frac{2zw'(z)}{(1+w(z))\left\{\alpha_1+[\alpha_1+2(p-\lambda)]\,w(z)\right\}}\right\}. \end{aligned} \tag{2.6}$$

We claim that $|w(z)|<1$ for $z\in U$. Otherwise there exists a point $z_0\in U$ such that $\max\limits_{|z|\le|z_0|}|w(z)| = |w(z_0)| = 1$. Applying Jack's lemma, we have $z_0 w^{'}(z_0) = \xi w(z_0)(\xi\ge 1)$. Writing $w(z_0)=e^{i\theta}(0\le\theta\le 2\pi)$, and putting $z=z_0$ in (2.6), we get

$$\begin{aligned} &\frac{z(H_{p,q,s}(\alpha_1+2)f(z_0))'}{(H_{p,q,s}(\alpha_1+1)f(z_0))'} - (p+1) + \frac{p\alpha_1+\lambda}{\alpha_1+1} \\ &\qquad = \frac{(p-\lambda)}{\alpha_1+1}\left\{-\frac{1-w(z_0)}{1+w(z_0)} + \frac{2\xi w(z_0)}{(1+w(z_0))\left\{\alpha_1+[\alpha_1+2(p-\lambda)]\,w(z_0)\right\}}\right\}. \end{aligned} \tag{2.7}$$

Thus we have

$$\Re\left\{\frac{z(H_{p,q,s}(\alpha_1+2)f(z_0))'}{(H_{p,q,s}(\alpha_1+1)f(z_0))'} - (p+1) + \frac{p\alpha_1+\lambda}{\alpha_1+1}\right\} \ge \frac{p-\lambda}{2\,(\alpha_1+1)\,(\alpha_1+p-\lambda)} > 0, \tag{2.8}$$

which obviously contradicts (2.1). Hence $|w(z)|<1$, and it follows from (2.3) that $f(z)\in J_{p,q,s}(\alpha_1;\lambda)$. This completes the proof of Theorem 2.2. □

2.3. Theorem. *Let $f(z)\in\Sigma_p$ satisfy the condition*

$$\Re\left\{\tfrac{(H_{p,q,s}(\alpha_1+1)f(z))'}{(H_{p,q,s}(\alpha_1)f(z))'} - (p+1)\right\} < \tfrac{(p-\lambda)-2[p(\alpha_1-1)+\lambda](c+1-\lambda)}{2\alpha_1(c+1-\lambda)},\ (c>p-1). \tag{2.9}$$

Then

$$F(z) = \frac{c}{z^{c+p}}\int_0^z t^{c+p-1} f(t)\,dt \tag{2.10}$$

belongs to the class $J_{p,q,s}(\alpha_1;\lambda)$.

Proof. From (2.10), we have

$$z(H_{p,q,s}(\alpha_1)F(z))' = (c-p+1)H_{p,q,s}(\alpha_1)f(z) - (c+1)H_{p,q,s}(\alpha_1)F(z). \tag{2.11}$$

Using (2.11) and (1.9), the condition (2.9) may be written as

$$\begin{aligned} &\Re\left\{\frac{\frac{(H_{p,q,s}(\alpha_1+2)F(z))'}{(H_{p,q,s}(\alpha_1+1)F(z))'} - (\alpha_1+p-c)}{\alpha_1-(\alpha_1+p-c-1)\frac{(H_{p,q,s}(\alpha_1)F(z))'}{(H_{p,q,s}(\alpha_1+1)F(z))'}} - (p+1)\right\} \\ &\qquad\qquad < \frac{(p-\lambda)-2[p(\alpha_1-1)+\lambda](c+1-\lambda)}{2\alpha_1(c+1-\lambda)}. \end{aligned} \tag{2.12}$$

In order to prove that (2.12) implies the inequality:

$$\Re\left\{\frac{(H_{p,q,s}(\alpha_1+1)F(z))'}{(H_{p,q,s}(\alpha_1)F(z))'} - (p+1)\right\} < -\frac{p(\alpha_1-1)+\lambda}{\alpha_1},\ (z\in U^*). \tag{2.13}$$

we now define $w(z)$ in U by

$$\frac{(H_{p,q,s}(\alpha_1+1)F(z))'}{(H_{p,q,s}(\alpha_1)F(z))'} - (p+1) = -\left\{\frac{p(\alpha_1-1)+\lambda}{\alpha_1} + \frac{p-\lambda}{\alpha_1}\cdot\frac{1-w(z)}{1+w(z)}\right\}. \tag{2.14}$$

Clearly, $w(z)$ is regular in U and $w(0) = 0$. Rewriting (2.14) as

$$\frac{(H_{p,q,s}(\alpha_1+1)F(z))'}{(H_{p,q,s}(\alpha_1)F(z))'} = \frac{\alpha_1 + [\alpha_1 + 2(p-\lambda)]w(z)}{\alpha_1(1+w(z))} \tag{2.15}$$

and differentiating (2.15) logarithmically with respect to z, we get

$$\begin{aligned} &\frac{(\alpha_1+1)\frac{(H_{p,q,s}(\alpha_1+2)F(z))'}{(H_{p,q,s}(\alpha_1+1)F(z))'} - (\alpha_1+p-c)}{\alpha_1 - (\alpha_1+p-c-1)\frac{(H_{p,q,s}(\alpha_1)F(z))'}{(H_{p,q,s}(\alpha_1+1)F(z))'}} - (p+1) \\ &\qquad = -\left\{\tfrac{p(\alpha_1-1)+\lambda}{\alpha_1} + \tfrac{p-\lambda}{\alpha_1}\cdot\tfrac{1-w(z)}{1+w(z)}\right\} \\ &\qquad\qquad + \frac{2(p-\lambda)zw'(z)}{\alpha_1(1+w(z))\left[(c+1-p)+(p-2\lambda+c+1)w(z)\right]}, \end{aligned} \tag{2.16}$$

where we have also made use (2.5). The remaining part of the proof of Theorem 2.3 is similar to that of Theorem 2.2. □

2.4. Remark. (i) For $q = s+1,\ \alpha_j = 1,\ (j = 1,\dots,s+1);\ \beta_i = 1,\ (i = 1,\dots,s),\ p = 1$ and $\lambda = 0$, we note that Theorem 2.3 extends a result of Goel and Sohi [11]; also Cho and Owa [8];

(ii) For $q = s+1,\ \alpha_j = 1,\ (j = 1,\dots,s+1),\ \beta_i = 1,\ (i = 1,...,s),\ p = c = 1$ and $\lambda = 0$, we note that Theorem —reft2 extends a result of Bajpai [6].

3. A further inclusion property

3.1. Theorem. *If $f(z) \in J_{p,q,s}(\alpha_1;\lambda)$, then*

$$G(z) = \frac{1}{z^{1+p}}\int_0^z t^p (f*g)(t)\,dt, \tag{3.1}$$

belongs to the class $J_{p,q,s}(\alpha_1+1;\lambda)$.

Proof. From (3.1), we have

$$(c-p+1)H_{p,q,s}(\alpha_1)f(z) = \alpha_1 H_{p,q,s}(\alpha_1+1)F(z) - (\alpha_1+p-c-1)H_{p,q,s}(\alpha_1)F(z) \tag{3.2}$$

and

$$\begin{aligned} &(c-p+1)H_{p,q,s}(\alpha_1+1)f(z) \\ &\qquad = (\alpha_1+1)H_{p,q,s}(\alpha_1+2)F(z) - (\alpha_1+p-c)H_{p,q,s}(\alpha_1+1)F(z), \end{aligned} \tag{3.3}$$

which, for $c = \alpha_1 + p - 1$, yield

$$\frac{(\alpha_1+1)(H_{p,q,s}(\alpha_1+2)G(z))' - (H_{p,q,s}(\alpha_1+1)G(z))'}{\alpha_1(H_{p,q,s}(\alpha_1+1)G(z))'} = \frac{(H_{p,q,s}(\alpha_1+1)f(z))'}{(H_{p,q,s}(\alpha_1)f(z))'},$$

that is,

$$\frac{(\alpha_1+1)(H_{p,q,s}(\alpha_1+2)G(z))'}{\alpha_1(H_{p,q,s}(\alpha_1+1)G(z))'} - \frac{1}{\alpha_1} = \frac{(H_{p,q,s}(\alpha_1+1)f(z))'}{(H_{p,q,s}(\alpha_1)f(z))'}.$$

Thus we have

$$\begin{aligned} &\Re\left\{\frac{(\alpha_1+1)(H_{p,q,s}(\alpha_1+2)G(z))'}{\alpha_1(H_{p,q,s}(\alpha_1+1)G(z))'} - \frac{1}{\alpha_1} - (p+1)\right\} \\ &\qquad = \Re\left\{\frac{(H_{p,q,s}(\alpha_1+1)f(z))'}{(H_{p,q,s}(\alpha_1)f(z))'} - (p+1)\right\} < -\frac{p(\alpha_1-1)+\lambda}{\alpha_1}, \end{aligned}$$

which leads us at once to the desired inequality:

$$\Re\left\{\frac{(H_{p,q,s}(\alpha_1+2)G(z))'}{(H_{p,q,s}(\alpha_1+1)G(z))'}-(p+1)\right\}<-\frac{p(\alpha_1-1)+\lambda}{\alpha_1},\ (z\in U^*). \tag{3.4}$$

This completes the proof of Theorem 3.1. □

3.2. Corollary. *If $f(z)\in J_p(a,c;\lambda)$, where $J_p(a,c;\lambda)$ is defined by (1.12), then $G(z)\in J_p(a,c;\lambda)$, where $G(z)$ is defined by (3.1).* □

3.3. Corollary. *If $f(z)\in J_p(\nu,\lambda)$, where $J_p(\nu,\lambda)$ is defined by (1.13), then $G(z)\in J_p(\nu,\lambda)$, where $G(z)$ is defined by (3.1).* □

3.4. Remark. Putting $q=2,\ s=1,\ \alpha_1=\nu+p,\ (\nu>-p),\ \alpha_2=p$ and $\beta_1=p$ in our results, we obtain the results obtained by Aouf and Srivastava [4].

References

[1] Aouf, M. K. *New certeria for multivalent meromorphic starlike functions of order alpha*, Proc. Japan. Acad. **69**, 66–70, 1993.

[2] Aouf, M. K. *Certain subclasses of meromorphically multivalent functions associated with generalized hypergeometric function*, Comput. Math. Appl. **55**, 494–509, 2008.

[3] Aouf, M. K. and Hossen, H. M. *New certeria for meromorphic p-valent starlike functios*, Tsukuba J. Math. **17** (2) (1993), 481–486, 1993.

[4] Aouf, M. K. and Srivastava, H. M. *A new criterion for meromorphically p-valent convex functions of order alpha*, Math. Sci. Res. Hot-Line **1** (8), 7–12, 1997.

[5] Aouf, M. K. and Yassen, M. F. *On certain classes of meromorphically multivalent functions associated with the generalized hypergeometric function*, Comput. Math. Appl. **58**, 449–463, 2009.

[6] Bajpai, S. K. *A note on a class of meromorphic univalent functions*, Rev. Roum. Math. Pures Appl. **22**, 295–297, 1977.

[7] Cho, N. E. *Some new criteria for meromorphically p-valent starlike functions*, in: Current Topics in Analytic Function Theory (H.M. Srivastava and S. Owa, Editors) (World Scientific publishing Company, Singapore, New Jersey, London, and Hong Kong, 1992), 86–93.

[8] Cho, N. E. and Owa, S. *Certain differential operators for meromorphically p-valent convex functions*, Proc. Japan Acad. Ser. A Math. Sci. **68**, 28–31, 1992.

[9] Dziok, J. and Srivastava, H. M. *Classes of analytic functions associated with the generalized hypergeometric function*, Appl. Math. Comput. **103**, 1–13, 1999.

[10] Dziok, J. and Srivastava, H. M. *Certain subclasses of analytic functions associated with the generalized hypergeometric function*, Integral Transforms Spec.Funct. **14**, 7–18, 2003.

[11] Goel, R. M. and Sohi, N. S. *On a class of meromorphic functions*, Glas. Mat. **17**, 19–28, 1981.

[12] Jack, I. S. *Functions starlike and convex of order α*, J. London Math. Soc. **3** (2), 469–474, 1971.

[13] Kulkarni, S. K., Naik, U. H. and Srivastava, H. M. *A certain class of meromorphically p-valent quasi-convex functions*, PanAmer. Math. J. **8** (1), 57–64, 1998.

[14] Kumar, V. and Shukla, S. L. *Certain class of meromorphically functions*, Bull. Austral. Math. Soc. **25**, 85–97, 1982.

[15] Liu, J. -L. and Srivastava, H. M. *A linear operator and associated with the generalized hypergeometric function*, J. Math. Anal. Appl. **259**, 566–581, 2000.

[16] Liu, J. -L. and Srivastava, H. M. *Classes of meromorphically multivalent functions associated with the generalized hypergeometric function*, Math. Comput. Modelling **39**, 21–34, 2004.

[17] Ruscheweyh, St. *New criteria for univalent functions*, Proc. Amer. Math. Soc. **49**, 109–115, 1975.

[18] Srivastava, H. M. and Karlsson, P. W. *Multiple Gaussian Hypergeometric Series* (Halsted Press, Ellis Horwood Limited, Chichester, 1985) (John Wiley and Sons, New York, Chichester, Brisbane, Toronto).

[19] Srivastava, H. M. and Owa, S. (Editors) *Current Topics in Analytic Function Theory* (World Scientific Publishing Company, Singapore, New Jersey, London and Hong Kong, 1992).

[20] Uralegaddi, B. A. and Ganigi, M. D. *A new criterion for meromorphic convex functions*, Tamakang J. Math. **19** (1), 43–48, 1988.

[21] Uralegaddi, B. A. and Patil, H. S. *A new criterion for p-valent meromorphic convex functions*, Math. Chronicle **18**, 75–77, 1989.

[22] Uralegaddi, B. A and Somanatha, C. *Certain classes of meromorphic multivalent functions*, Tamkang J. Math. **23**, 223–231, 1992.

[23] Yang, D. *On a class of meromorphic starlike multivalent functions*, Bull. Inst. Math. Acad. Sinica **24**, 151–157, 1996.