JOURNAL OF SCIENTIFIC PERSPECTIVES

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ABOUT THE JOURNAL

Journal of Scientific Perspectives (JSP) is a **scholarly** and **international peer-reviewed journal**. It is published quarterly in *January, April, July* and *October,* in the fields of **basic sciences, engineering, natural sciences** and **health sciences**. All articles submitted for publication are evaluated by the editor-in-chief, field editor, editorial board and referees. The original research papers, technical notes, letter to the editor, debates, case presentations and reviews, only in *English,* are published in the journal. Thus, it aims to bring together the views and studies of academicians, researchers and professionals working in the fields mentioned above.

Journal of Scientific Perspectives is an *open access electronic journal*. All articles published in the journal, are assigned the *DOI number*. Article submission and follow-up process are carried out by the manuscript submission system.

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EVANS, W.A., 1994, Approaches to Intelligent Information Retrieval, *Information Processing and Management*, 7 (2), 147-168.

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SILVER, K., 1991, Electronic Mail: The New Way to Communicate, *9th International Online Information Meeting*, 3-5 December 1990, London, Oxford: Learned Information, 323-330.

Thesis:

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ICMJE

Idealonline

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Research Article

HYDROGEN GENERATION OF Al-NaCl POWDERS IN DIFFERENT REACTION MEDIUMS

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ABSTRACT

This work is a study of hydrogen generation of Al-NaCl powders in different reaction mediums. Water, KOH and NaOH were used as reaction mediums for the comparison of the hydrogen generation rate. The effects of the milling time, NaCl wt%, reaction temperature and reaction mediums were investigated. Al was milled with NaCl in order to increase the hydrogen generation rate. Increasing NaCl wt% and reaction temperature also increased the hydrogen generation rate. Al reactivity can be enhanced by increasing milling time, NaCl wt%, reaction temperature and by using alkaline solutions.

Key words: *Al-NaCl powders, ball milling, hydrogen generation, alkaline solutions.*

1. INTRODUCTION

Climate changes due to the emergence of CO_2 as a result of the use of fossil fuels have led to the search for alternative fuels. Bio-mass, wind, sun, hydrogen, nuclear etc. alternative energies can be given as examples (Ho, 2017). With its high chemical energy and environmentally friendly properties, hydrogen is one of the important new and clean energy sources (Yang et al., 2018).

Hydrogen can be produced by different methods, steam reforming, other production methods from fossil fuels (partial oxidation, plasma reforming etc.), from water (electrolysis, thermolysis, etc.), etc. (wikipedia, 2020). Hydrogen generation by Al water reaction is preferred because it reduces CO_2 production and has no harmful effects on the environment. Water is a cheap and clean source, Al is an abundant and economical metal. Al has high energy density of 29 MJ/kg, and theoretically it produces 1360 ml/g H_2. Al water reaction is given in equation (1). (Ho, 2017).

$$2Al + 6H_2O \rightarrow 2Al(OH)_3 + 3H_2 \quad \textbf{(1)}$$

Reaction byproducts are non-corrosive, stable, and recyclable (Liu et al., 2012). Al can be remanufactured from $Al(OH)_3$ by-product (Liu et al., 2015; Ho, 2017; Dupiano et al., 2011).

Difficulties in Al water reaction are coating of Al metal surface with protective oxide layer and formation of dense hydroxide as by-product. To overcome these difficulties, various studies have been applied to activate aluminum and develop its hydrolysis properties (Liu et al., 2015). Addition of low melting point metals and forming alloys (Al–Ga, Al–Ga–In, Al–Li, Al–Mg–Fe, etc.) (Yang et al., 2018), use of water-soluble salts while reducing particle size with ball milling (e.g., NaCl, KCl, $CoCl_2$, $NiCl_2$, Na_2CO_3 etc.) (Yang et al., 2018; Rong et al., 2017; Czech and Troczynski, 2010; Razavi-Tousi and Szpunar, 2016; Yolcular and Karaoglu, 2017), use of alkaline solutions (NaOH, KOH, etc) (Pyun and Moon, 2000; Porciúncula et al., 2012). These methods are very effective in the removing of protective oxide layer on the Al particles and keeping the activation of Al powders.

$$2Al + 3H_2O \rightarrow Al_2O_3 + 3H_2 \quad \textbf{(2)}$$

Equation (1) gives the Al water reaction at low temperatures. The by product is $Al(OH)_3$ at low temperatures. Equation (2) gives the Al water reaction at high temperatures. At high temperatures the by product is Al_2O_3 (Rong et al., 2017).

When Al reacts with water firstly $Al(OH)_3$ forms. Then, it further reacts to form Al_2O_3. $Al(OH)_3$ and Al_2O_3 are amphoteric which means that they have both an acid and a base character. Al_2O_3 is not soluble in water. This prevents the Al corrosion in water. However, Al_2O_3 is soluble in alkaline solutions (Mcarthur and Spalding, 2004). Ball milling, increasing reaction temperature or use of alkaline solutions could be applied for the reaction to continue by dissolving the passive oxide films.

The equations of the reaction for NaOH to remove the passive oxide film from the Al particles with equations (3) and (4);

$$Al_2O_3 + 2OH^- + 3H_2O = 2Al(OH)_4^- \quad \textbf{(3)}$$

$$OH^- + Al(OH)_3 \leftrightarrow Al(OH)_4^- \quad \textbf{(4)}$$

The reaction of NaOH with by products Al_2O_3 or $Al(OH)_3$ forms $Al(OH)_4^-$. $Al(OH)_4^-$ dissolves in NaOH solution and satisfies the continuation of the reaction by supplying fresh Al surfaces for the Al water reaction (Jia et al., 2014).

This study is about hydrogen generation of Al-NaCl powders in different reaction mediums. Pure water, NaOH and KOH were used as different reaction mediums in the hydrogen generation experiments. The parameters, reaction temperature, milling time and addition amount of NaCl were changed to investigate their effects in the hydrogen generation. This research examines how milling time, reaction temperature, addition of NaCl and reaction medium affect the Al water reaction.

2. MATERIAL AND METHODS

Al (average diameter 90 μm) and NaCl (Merck, 98%) were mixed and milled in a planetary ball mill (QM-3SP4) with a ball to powder ratio 20:1. NaCl wt% was 0, 10, 20 and 30 and 4, 8 and 16 h milling times were applied. Speed of the rotation was 200 rpm.

Hydrogen generation experiments were done in a 250 ml flask with two openings. When the reaction temperature was reached by placing the flask in the water bath, the ground powder was added to this flask. 35-75 ˚C reaction temperatures were investigated.

Pure water, NaOH (1 M) and KOH (1 M) were used as solution mediums to compare their effects at the hydrogen generation rate. Mixing applied during the reactions.

The hydrogen produced was measured at room temperature with an inverted burette filled with tap water. The amount of hydrogen production was recorded by monitoring the water level changes in the burette at certain intervals.

In our previous study (Yolcular and Karaoglu, 2017), reaction temperature was between 30 - 70 ˚C, NaCl amount was 0, 5, 10 and 20 wt%, milling times were 1, 4 and 12 h. According to our results in our previous study we determined these different temperature, grinding time and NaCl amounts in order to investigate their effects in this study. Unlike our previous study, experiments were carried out in different reaction mediums. The effects of different reaction mediums were investigated with selected reaction temperature, grinding time and NaCl amounts.

3. RESULTS AND DISCUSSION

Ball milling was applied to Al powders in order to increase the surface area by decreasing the particle size. This could be reached with the addition of NaCl during the ball milling. The addition of NaCl prevented cold welding during milling and assisted in the crushing and grinding of the particles. Pure Al and increasing wt% of NaCl additions were investigated in hydrogen production experiments. Figure 1 shows the effect of the amount of NaCl added by the addition of 10, 20 and 30 wt% NaCl. These added NaCl amounts also compared to pure Al powders. As the amount of NaCl increased the particle size decreased and surface area of the particles increased. This Al - water reaction is a surface reaction and the reaction rate is relevant to total surface area of the powders. (Nie et al., 2012) (Bunker and Smith, 2011). The addition of NaCl and milling is effective, but they cannot show the same effect when each is applied individually. So, there is a strong synergy between the grinding process and the presence of a salt (Razavi-Tousi and Szpunar, 2016). From Figure 1, 30 wt% NaCl addition gives the highest hydrogen generation rate.

Figure 1. Hydrogen generation (HG) rate ($ml.min^{-1}.g^{-1}$) vs time (min) with different wt% NaCl additions, in 1 M NaOH solution, with 16 h milling and at 75 ˚C.

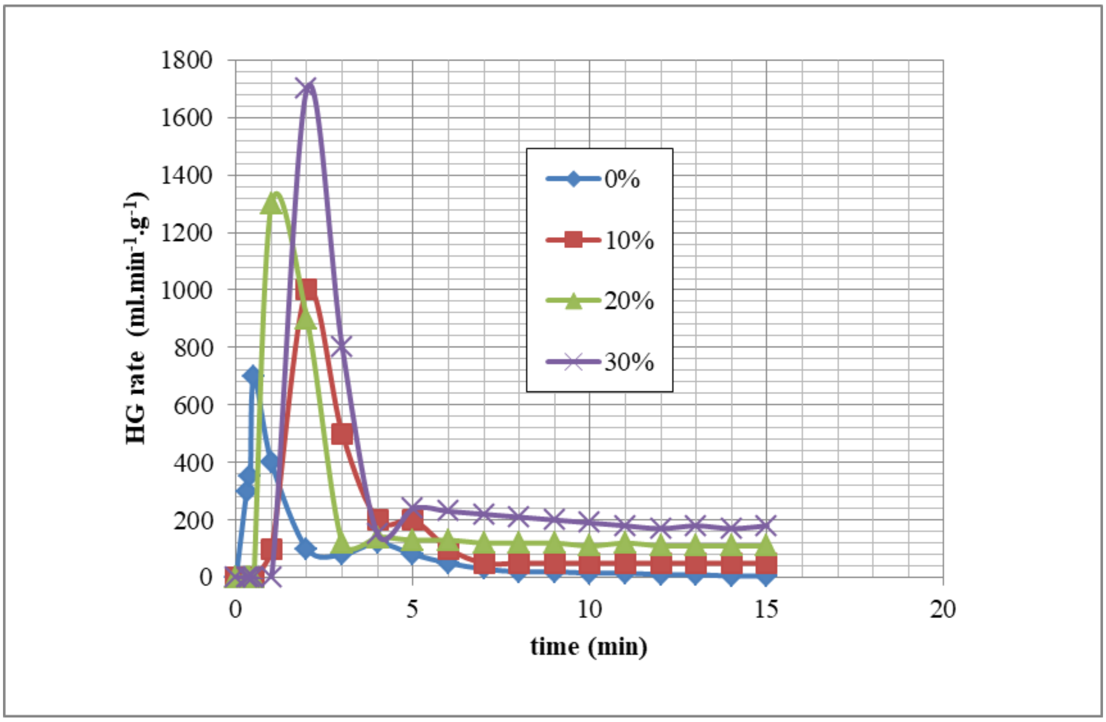


Figure 2 gives the results of increasing milling time. Ball grinding activates the particles so that they can react easily. Hydrogen generation rate increases with increasing milling time. Particle sizes decreased with increasing grinding times, which allowed the particles to have larger surface area. In this study, 16 h milling gave the highest hydrogen generation rate. The milling changes nonporous structure of the particles to porous structure having high surface area. These pores provide a higher surface to the reaction.

Figure 2. HG rate ($ml.min^{-1}.g^{-1}$) vs time (min) with different milling times, with 30 wt% NaCl addition, in 1 M NaOH solution and at 75 ˚C.

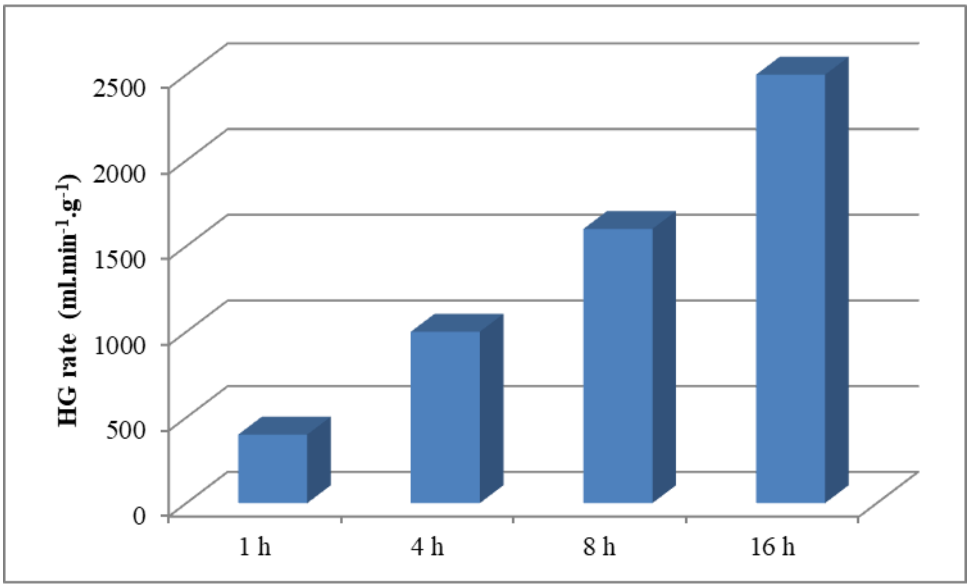


Figure 3 shows the effect of temperature on hydrogen generation with 16 h milling, 30 wt% NaCl addition to Al powders. 35 – 75 ˚C reaction temperatures were investigated for these samples. The increased reaction temperature was resulted in higher reaction rates. As 75 ˚C gives the highest reaction rate, this temperature was used in other experiments. During the reaction the passive oxide film which was formed on the surface of Al powders can be quite permeable at higher temperatures compared to low temperatures. Then the reaction continues for a long time and higher reaction rates also can be reached.

Figure 3. HG rate (ml.min^{-1}.g^{-1}) vs time (min) with different temperatures, with 30 wt% NaCl addition, in 1M NaOH solution and at 16 h milling time.

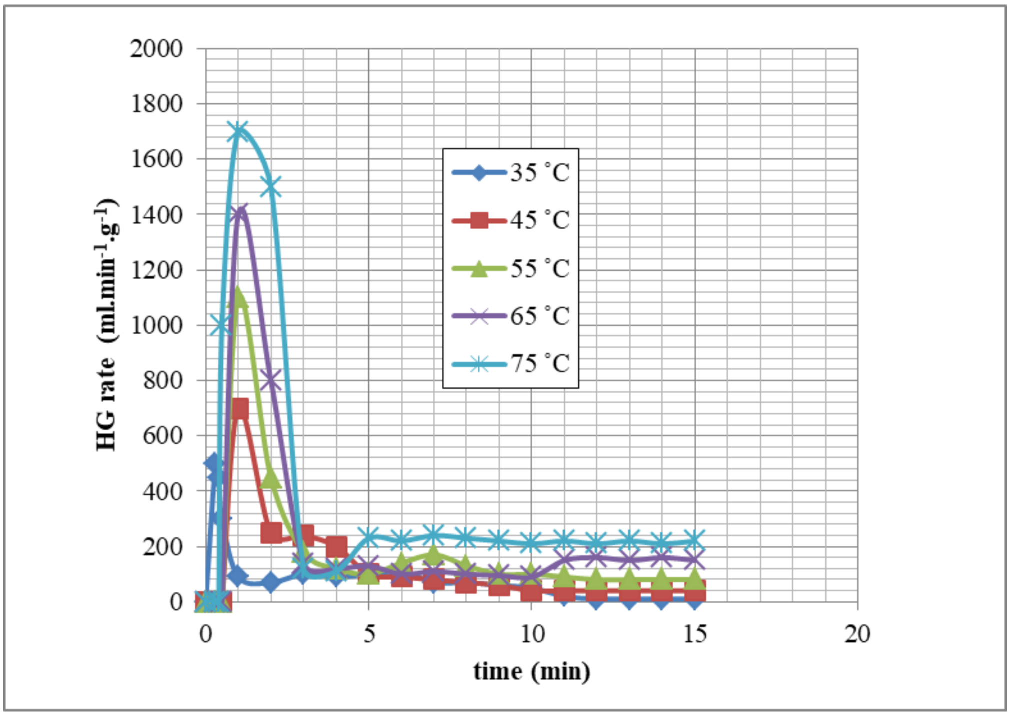
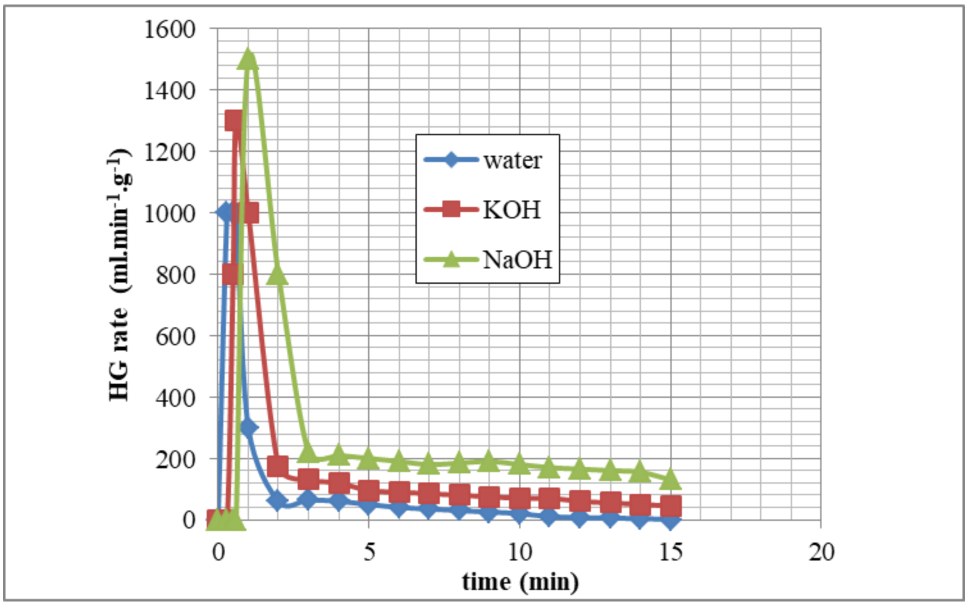


Figure 4 shows that higher reaction rates were achieved with NaOH solution when comparing water, NaOH and KOH solutions. Reaction mechanisms were different for each reaction medium. Activation energies vary according to the reaction medium. Reaction activation energy is higher with water and KOH solution and lower with NaOH solution. This results in higher hydrogen generation rates due to the low activation energy of experiments with NaOH solution. The activation energies were found for this study as; 53.2 kJ.mol^{-1} with water, 49.4 kJ.mol^{-1} with KOH (1 M) solution and 45.7 kJ.mol^{-1} with NaOH (1M) solution by using 30 wt% Al-NaCl powder. According to these results, highest hydrogen generation rate was observed with NaOH solution.

Figure 4. HG rate (ml.min^{-1}.g^{-1}) vs time (min) with water, NaOH and KOH solutions, at 75 ˚C, with 30 wt% NaCl addition and at 16 h milling time.



4. CONCLUSIONS

The addition of NaCl to Al powder affects the hydrogen generation reaction. The structure of the Al particles also affected by both NaCl addition and ball milling. Hydrogen generation rate and produced hydrogen amounts were increased by NaCl addition and ball milling. The salt mixed while grinding is dissolved in water or aqueous solution during the reaction, creating many cavities, voids, tunnels in aluminum particles to make the reaction happen and improves the kinetics of the reaction. The addition of a higher salt ratio allows the particles to have a higher surface area, which increases the reaction kinetics. Then, containing a higher percentage of weight Al powders showed higher hydrogen generation rates. Mixing the reaction solution during the reaction and using higher reaction temperatures prevents the formation of passive oxide film and ensures that the formed film is also permeable. Reaction activation energies change by different reaction mediums. In this study, water, KOH and NaOH solutions were compared and their activation energies also calculated as; 53.2 kJ.mol^{-1} with water, 49.4 kJ.mol^{-1} with KOH (1 M) solution and 45.7 kJ.mol^{-1} with NaOH (1M) solution by using 30 wt% Al-NaCl powder. The results showed that experiments with NaOH solutions has lower activation energies which means that the corrosion reaction continues easily without disruption because of passive oxide film. Highest hydrogen generation rate was observed with NaOH solution as 1500 ml.min^{-1}.g^{-1}. Similar studies on activation energy data are as follows: 46-53 kJ.mol^{-1} with NaOH (between 20-70 ˚C) (Zhuk et al., 2006) 51.5-53.5 kJ.mol^{-1} with 0.1 mol.L^{-1} NaOH (Aleksandrov et al., 2003) 69 kJ.mol^{-1} with addition of lithium as promoting metal without any alkaline solution (Rosenband and Gany, 2010). The activation energy ranges between approximately 40-70 kJ.mol^{-1} and changes according to reaction medium, reaction temperature and Al particle properties. Increasing milling time, NaCl wt%, reaction temperature and by using alkaline solutions enhances Al reactivity. Al can be used effectively to generate hydrogen by Al water reaction.

REFERENCES

ALEKSANDROV, Y. A., TSYGANOVA, E. I., PISAREV, A. L., 2003. Reaction of aluminum with dilute aqueous NaOH solutions, *Russ. J. Gen. Chem.,* 73 (5), 689694. https://doi.org/10.1023/A:1026114331597

BUNKER, C. E., SMITH, M. J., 2011. Nanoparticles for hydrogen generation, *Journal Materials Chemical*, 21, 12173–12180. https://doi.org/10.1039/C1JM10856E

CZECH, E., TROCZYNSKI, T., 2010. Hydrogen generation through massive corrosion of deformed aluminum in water, *International Journal of Hydrogen Energy* , 35, 1029-1037. https://doi.org/10.1016/j.ijhydene.2009.11.085

DUPIANO, P., STAMATIS, D., DREIZIN, E. L., 2011. Hydrogen production by reacting water with mechanically milled composite aluminum-metal oxide powders, *International Journal of Hydrogen Energy* , 36(8), 4781-4791. https://doi.org/10.1016/j.ijhydene.2011.01.062

HO, C.-Y., 2017, Hydrolytic reaction of waste aluminum foils for high efficiency of hydrogen generation, *International Journal of Hydrogen Energy*, 42 (31), 19622-19628. https://doi.org/10.1016/j.ijhydene.2017.06.104

JIAA Y., SHENA J., MENGA H., DONGA Y., CHAIA Y., WANG N., 2014. Hydrogen generation using a ball-milled Al/Ni/NaCl mixture, *Journal of Alloys and Compounds*, 588, 259–264. https://doi.org/10.1016/j.jallcom.2013.11.058

MCARTHUR, H., SPALDİNG, D., 2004, *Engineering Materials Science: Properties, Uses, Degradation, Remediation*, Elsevier, UK, 1st Edition, Paperback ISBN:9781898563112, eBook ISBN:9781782420491, Imprint: Woodhead Publishing, UK, page count: 608.

LIU S., FAN M.-q., WANG C., HUANG Y.-X., CHEN D., BAI L.-Q., SHU K.-Y., 2012. Hydrogen generation by hydrolysis of Al-Li-Bi-NaCl mixture with pure water, *International Journal Hydrogen Energy,* 37, 1014-1020. https://doi.org/10.1016/j.ijhydene.2011.03.029

LIU, Y., GE, H., WANG, X., LIU, H., DONG, Z., LI, S., YAN, M., 2015. Study on hydrogen generation from the hydrolysis of a ball milled aluminum/calcium hydride composite, *RSC Advances*, 5, 60460-60466. https://doi.org/10.1039/C5RA09200K

NIE, H., SCHOENITZ, M., DREIZIN E. L., 2012. Calorimetric investigation of the aluminum water reaction, *International Journal of Hydrogen Energy,* 37 (15), 11035–11045. https://doi.org/10.1016/j.ijhydene.2012.05.012

PORCIÚNCULA, C. B., MARCILIO, N. R., TESSARO, I. C., GERCHMANN, M., 2012. Production of hydrogen in the reaction between aluminum and water in the presence of NaOH and KOH, *Brazilian Journal of Chemical Engineering*, 29(2), 337-348. https://doi.org/10.1590/S0104-66322012000200014

PYUN, S., MOON, S., 2000. Corrosion mechanism of pure aluminium in aqueous alkaline solution, *J Solid State Electrochem*, 4, 267–272. https://doi.org/10.1007/s100080050203

RAZAVI-TOUSI, S.S., SZPUNAR, J.A., 2016. Effect of addition of water-soluble salts on the hydrogen generation of aluminum in reaction with hot water, *Journal of Alloys and Compounds*, 679, 364-374. https://doi.org/10.1016/j.jallcom.2016.04.038

RONG, S., BAOZHONG, Z., YUNLAN, S., 2017. The effect of $NiCl_2$ and Na_2CO_3 on hydrogen production by Al/H_2O system, *International Journal of Hydrogen Energy*, 42, 3586-3592. https://doi.org/10.1016/j.ijhydene.2016.07.254

ROSENBAND, V., GANY, A., 2010. Application of activated aluminum powder for generation of hydrogen from water, *International Journal Hydrogen Energy*, 35, 10898-10904. https://doi.org/10.1016/j.ijhydene.2010.07.019

WIKIPEDIA CONTRIBUTORS. (2020, June 2). Hydrogen production. In *Wikipedia, The Free Encyclopedia*. Retrieved 13:21, June 26, 2020, from https://en.wikipedia.org/w/index.php?title=Hydrogen_production&oldid=960349940

YANG, B. C., CHAI, Y. J., YANG, F. L., ZHANG, Q., LIU, H., WANG, N., 2018. Hydrogen generation by aluminum-water reaction in acidic and alkaline media and its reaction Dynamics, *International Journal of Energy Research,* 42, 1594-1602. https://doi.org/10.1002/er.3953

YOLCULAR, S., KARAOGLU, S., 2017. Activation of Al powder with NaCl assisted milling for hydrogen generation, *Energy Sources, Part A: Recovery, Utilization, and Environmental Effects*, 39(18), 1919-1927. https://doi.org/10.1080/15567036.2017.1390009

ZHUK, A. Z., SHEINDLIN, B. V., KLEYMOV, V., SHKOLNIKOV, E. I., LOPATIN, M. Y., 2006. Use of low cost Aluminum in electric energy production, *J. Power Sources*, 157, 921-926. https://doi.org/10.1016/j.jpowsour.2005.11.097

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Research Article

A STUDY ON GENERALIZED 5-PRIMES NUMBERS

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Abstract

In this paper, we introduce the generalized 5-primes numbers sequences and we deal with, in detail, three special cases which we call them 5-primes, Lucas 5-primes and modified 5-primes sequences. We present Binet's formulas, generating functions, Simson formulas, and the summation formulas for these sequences. Moreover, we give some identities and matrices related with these sequences.

2010 Mathematics Subject Classication. *11B39, 11B83.*

Keywords. *5-primes numbers, Lucas 5-primes numbers, modified 5-primes numbers, generalized Pentanacci numbers.*

1. Introduction

In this paper, we define the generalized 5-primes sequences and we investigate, in detail, three special cases which we call them 5-primes, Lucas-5-primes and modified 5-primes sequences.

The sequence of Fibonacci numbers $\{F_n\}$ and the sequence of Lucas numbers $\{L_n\}$ are defined by

$$F_n = F_{n-1} + F_{n-2}, \quad n \geq 2, \qquad F_0 = 0, \ F_1 = 1,$$

and

$$L_n = L_{n-1} + L_{n-2}, \quad n \geq 2, \qquad L_0 = 2, \ L_1 = 1$$

respectively. The generalizations of Fibonacci and Lucas sequences produce several nice and interesting sequences.

A generalized Pentanacci sequence $\{W_n\}_{n\geq 0} = \{W_n(W_0, W_1, W_2, W_3, W_4; r, s, t, u, v)\}_{n\geq 0}$ is defined by the fifth-order recurrence relations

$$W_n = rW_{n-1} + sW_{n-2} + tW_{n-3} + uW_{n-4} + vW_{n-5}, \qquad W_0 = a, W_1 = b, W_2 = c, W_3 = d, W_4 = e \tag{1.1}$$

where the initial values W_0, W_1, W_2, W_3, W_4 are arbitrary complex (or real) numbers and r, s, t, u, v are real numbers. Pentanacci sequence has been studied by many authors and more detail can be found in the extensive literature dedicated to these sequences, see for example [4], [5], [6]. The sequence $\{W_n\}_{n\geq 0}$ can be extended to negative subscripts by defining

$$W_{-n} = -\frac{u}{v}W_{-(n-1)} - \frac{t}{v}W_{-(n-2)} - \frac{s}{v}W_{-(n-3)} - \frac{r}{v}W_{-(n-4)} + \frac{1}{v}W_{-(n-5)}$$

for $n = 1, 2, 3, ...$ Therefore, recurrence (1.1) holds for all integer n.

As $\{W_n\}$ is a fifth order recurrence sequence (difference equation), it's characteristic equation is

$$x^5 - rx^4 - sx^3 - tx^2 - ux - v = 0 \tag{1.2}$$

whose roots are $\alpha, \beta, \gamma, \delta, \lambda$. Note that we have the following identities:

$$\begin{aligned}
\alpha + \beta + \gamma + \delta + \lambda &= r, \\
\alpha\beta + \alpha\lambda + \alpha\gamma + \beta\lambda + \alpha\delta + \beta\gamma + \lambda\gamma + \beta\delta + \lambda\delta + \gamma\delta &= -s, \\
\alpha\beta\lambda + \alpha\beta\gamma + \alpha\lambda\gamma + \alpha\beta\delta + \alpha\lambda\delta + \beta\lambda\gamma + \alpha\gamma\delta + \beta\lambda\delta + \beta\gamma\delta + \lambda\gamma\delta &= t, \\
\alpha\beta\lambda\gamma + \alpha\beta\lambda\delta + \alpha\beta\gamma\delta + \alpha\lambda\gamma\delta + \beta\lambda\gamma\delta &= -u \\
\alpha\beta\gamma\delta\lambda &= v.
\end{aligned}$$

Generalized Pentanacci numbers can be expressed, for all integers n, using Binet's formula

$$\begin{aligned}
W_n &= \frac{b_1\alpha^n}{(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\alpha-\lambda)} + \frac{b_2\beta^n}{(\beta-\alpha)(\beta-\gamma)(\beta-\delta)(\beta-\lambda)} + \frac{b_3\gamma^n}{(\gamma-\alpha)(\gamma-\beta)(\gamma-\delta)(\gamma-\lambda)} \quad (1.3) \\
&\quad + \frac{b_4\delta^n}{(\delta-\alpha)(\delta-\beta)(\delta-\gamma)(\delta-\lambda)} + \frac{b_5\lambda^n}{(\lambda-\alpha)(\lambda-\beta)(\lambda-\gamma)(\lambda-\delta)},
\end{aligned}$$

where

$$\begin{aligned}
b_1 &= W_4 - (\beta+\gamma+\delta+\lambda)W_3 + (\beta\lambda+\beta\gamma+\lambda\gamma+\beta\delta+\lambda\delta+\gamma\delta)W_2 - (\beta\lambda\gamma+\beta\lambda\delta+\beta\gamma\delta+\lambda\gamma\delta)W_1 + (\beta\lambda\gamma\delta)W_0, \\
b_2 &= W_4 - (\alpha+\gamma+\delta+\lambda)W_3 + (\alpha\lambda+\alpha\gamma+\alpha\delta+\lambda\gamma+\lambda\delta+\gamma\delta)W_2 - (\alpha\lambda\gamma+\alpha\lambda\delta+\alpha\gamma\delta+\lambda\gamma\delta)W_1 + (\alpha\lambda\gamma\delta)W_0, \\
b_3 &= W_4 - (\alpha+\beta+\delta+\lambda)W_3 + (\alpha\beta+\alpha\lambda+\beta\lambda+\alpha\delta+\beta\delta+\lambda\delta)W_2 - (\alpha\beta\lambda+\alpha\beta\delta+\alpha\lambda\delta+\beta\lambda\delta)W_1 + (\alpha\beta\lambda\delta)W_0, \\
b_4 &= W_4 - (\alpha+\beta+\gamma+\lambda)W_3 + (\alpha\beta+\alpha\lambda+\alpha\gamma+\beta\lambda+\beta\gamma+\lambda\gamma)W_2 - (\alpha\beta\lambda+\alpha\beta\gamma+\alpha\lambda\gamma+\beta\lambda\gamma)W_1 + (\alpha\beta\lambda\gamma)W_0, \\
b_5 &= W_4 - (\alpha+\beta+\gamma+\delta)W_3 + (\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta)W_2 - (\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta)W_1 + (\alpha\beta\gamma\delta)W_0.
\end{aligned}$$

Usually, it is customary to choose r, s, t, u, v so that the Equ. (1.2) has at least one real (say α) solutions.

Note that the Binet form of a sequence satisfying (1.2) for non-negative integers is valid for all integers n, for a proof of this result see [1]. This result of Howard and Saidak [1] is even true in the case of higher-order recurrence relations.

In this paper we consider the case $r = 2$, $s = 3, t = 5, u = 7, v = 11$ and in this case we write $V_n = W_n$. A generalized 5-primes sequence $\{V_n\}_{n\geq 0} = \{V_n(V_0, V_1, V_2, V_3, V_4)\}_{n\geq 0}$ is defined by the fifth-order recurrence relations

$$V_n = 2V_{n-1} + 3V_{n-2} + 5V_{n-3} + 7V_{n-4} + 11V_{n-5} \tag{1.4}$$

with the initial values $V_0 = c_0, V_1 = c_1, V_2 = c_2, V_3 = c_3, V_4 = c_4$ not all being zero.

The sequence $\{V_n\}_{n\geq 0}$ can be extended to negative subscripts by defining

$$V_{-n} = -\frac{7}{11}V_{-(n-1)} - \frac{5}{11}V_{-(n-2)} - \frac{3}{11}V_{-(n-3)} - \frac{2}{11}V_{-(n-4)} + \frac{1}{11}V_{-(n-5)}$$

for $n = 1, 2, 3, ...$Therefore, recurrence (1.4) holds for all integer n.

(1.3) can be used to obtain Binet formula of generalized 5-primes numbers. Binet formula of generalized 5-primes numbers can be given as

$$\begin{aligned} V_n &= \frac{b_1\alpha^n}{(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\alpha-\lambda)} + \frac{b_2\beta^n}{(\beta-\alpha)(\beta-\gamma)(\beta-\delta)(\beta-\lambda)} + \frac{b_3\gamma^n}{(\gamma-\alpha)(\gamma-\beta)(\gamma-\delta)(\gamma-\lambda)} \\ &\quad + \frac{b_4\delta^n}{(\delta-\alpha)(\delta-\beta)(\delta-\gamma)(\delta-\lambda)} + \frac{b_5\lambda^n}{(\lambda-\alpha)(\lambda-\beta)(\lambda-\gamma)(\lambda-\delta)} \end{aligned}$$

where

$$\begin{aligned} b_1 &= V_4-(\beta+\gamma+\delta+\lambda)V_3+(\beta\lambda+\beta\gamma+\lambda\gamma+\beta\delta+\lambda\delta+\gamma\delta)V_2-(\beta\lambda\gamma+\beta\lambda\delta+\beta\gamma\delta+\lambda\gamma\delta)V_1+(\beta\lambda\gamma\delta)V_0 \quad (1.5) \\ b_2 &= V_4-(\alpha+\gamma+\delta+\lambda)V_3+(\alpha\lambda+\alpha\gamma+\alpha\delta+\lambda\gamma+\lambda\delta+\gamma\delta)V_2-(\alpha\lambda\gamma+\alpha\lambda\delta+\alpha\gamma\delta+\lambda\gamma\delta)V_1+(\alpha\lambda\gamma\delta)V_0, \\ b_3 &= V_4-(\alpha+\beta+\delta+\lambda)V_3+(\alpha\beta+\alpha\lambda+\beta\lambda+\alpha\delta+\beta\delta+\lambda\delta)V_2-(\alpha\beta\lambda+\alpha\beta\delta+\alpha\lambda\delta+\beta\lambda\delta)V_1+(\alpha\beta\lambda\delta)V_0, \\ b_4 &= V_4-(\alpha+\beta+\gamma+\lambda)V_3+(\alpha\beta+\alpha\lambda+\alpha\gamma+\beta\lambda+\beta\gamma+\lambda\gamma)V_2-(\alpha\beta\lambda+\alpha\beta\gamma+\alpha\lambda\gamma+\beta\lambda\gamma)V_1+(\alpha\beta\lambda\gamma)V_0, \\ b_5 &= V_4-(\alpha+\beta+\gamma+\delta)V_3+(\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta)V_2-(\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta)V_1+(\alpha\beta\gamma\delta)V_0. \end{aligned}$$

Here, $\alpha,\beta,\gamma,\delta$ and λ are the roots of the equation

$$x^5-2x^4-3x^3-5x^2-7x-11=0. \quad (1.6)$$

Moreover, the approximate value of the roots $\alpha,\beta,\gamma,\delta$ and λ of Equation (1.6) are given by

$$\begin{aligned} \alpha &= 3.501101503801069 \\ \beta &= 0.3060834095195042+1.329047329711188i \\ \gamma &= 0.3060834095195042-1.329047329711188i \\ \delta &= -1.056634161420038+0.7567376493934506i \\ \lambda &= -1.056634161420038-0.7567376493934506i \end{aligned}$$

Note that

$$\begin{aligned} \alpha+\beta+\gamma+\delta+\lambda &= 2, \\ \alpha\beta+\alpha\lambda+\alpha\gamma+\beta\lambda+\alpha\delta+\beta\gamma+\lambda\gamma+\beta\delta+\lambda\delta+\gamma\delta &= -3, \\ \alpha\beta\lambda+\alpha\beta\gamma+\alpha\lambda\gamma+\alpha\beta\delta+\alpha\lambda\delta+\beta\lambda\gamma+\alpha\gamma\delta+\beta\lambda\delta+\beta\gamma\delta+\lambda\gamma\delta &= 5, \\ \alpha\beta\lambda\gamma+\alpha\beta\lambda\delta+\alpha\beta\gamma\delta+\alpha\lambda\gamma\delta+\beta\lambda\gamma\delta &= -7 \\ \alpha\beta\gamma\delta\lambda &= 11. \end{aligned}$$

The first few generalized 5-primes numbers with positive subscript and negative subscript are given in the following Table 1.

Table 1. A few generalized 5-primes numbers

n	V_n	V_{-n}
0	V_0	
1	V_1	$\frac{1}{11}V_4 - \frac{5}{11}V_1 - \frac{3}{11}V_2 - \frac{2}{11}V_3 - \frac{7}{11}V_0$
2	V_2	$\frac{2}{121}V_1 - \frac{6}{121}V_0 - \frac{1}{121}V_2 + \frac{25}{121}V_3 - \frac{7}{121}V_4$
3	V_3	$\frac{64}{1331}V_0 + \frac{19}{1331}V_1 + \frac{293}{1331}V_2 - \frac{65}{1331}V_3 - \frac{6}{1331}V_4$
4	V_4	$\frac{2903}{14\,641}V_1 - \frac{239}{14\,641}V_0 - \frac{907}{14\,641}V_2 - \frac{194}{14\,641}V_3 + \frac{64}{14\,641}V_4$
5	$11V_0 + 7V_1 + 5V_2 + 3V_3 + 2V_4$	$\frac{33\,606}{161\,051}V_0 - \frac{8782}{161\,051}V_1 - \frac{1417}{161\,051}V_2 + \frac{1182}{161\,051}V_3 - \frac{239}{161\,051}V_4$
6	$22V_0 + 25V_1 + 17V_2 + 11V_3 + 7V_4$	$\frac{33\,606}{1771\,561}V_4 - \frac{183\,617}{1771\,561}V_1 - \frac{87\,816}{1771\,561}V_2 - \frac{69\,841}{1771\,561}V_3 - \frac{331\,844}{1771\,561}V_0$

Now we define three special cases of the sequence $\{V_n\}$. 5-primes sequence $\{G_n\}_{n\geq 0}$, Lucas 5-primes sequence $\{H_n\}_{n\geq 0}$ and modified 5-primes sequence $\{E_n\}_{n\geq 0}$ are defined, respectively, by the third-order recurrence relations

$$G_{n+5} = 2G_{n+4} + 3G_{n+3} + 5G_{n+2} + 7G_{n+1} + 11G_n, \qquad G_0 = 0, G_1 = 0, G_2 = 0, G_3 = 1, G_4 = 2, \tag{1.7}$$

$$H_{n+5} = 2H_{n+4} + 3H_{n+3} + 5H_{n+2} + 7H_{n+1} + 11H_n, \qquad H_0 = 5, H_1 = 2, H_2 = 10, H_3 = 41, H_4 = 150, \tag{1.8}$$

and

$$E_{n+5} = 2E_{n+4} + 3E_{n+3} + 5E_{n+2} + 7E_{n+1} + 11E_n, \qquad E_0 = 0, E_1 = 0, E_2 = 0, E_3 = 1, E_4 = 1, \tag{1.9}$$

The sequences $\{G_n\}_{n\geq 0}$, $\{H_n\}_{n\geq 0}$ and $\{E_n\}_{n\geq 0}$ can be extended to negative subscripts by defining

$$G_{-n} = -\frac{7}{11}G_{-(n-1)} - \frac{5}{11}G_{-(n-2)} - \frac{3}{11}G_{-(n-3)} - \frac{2}{11}G_{-(n-4)} + \frac{1}{11}G_{-(n-5)}, \tag{1.10}$$

$$H_{-n} = -\frac{7}{11}H_{-(n-1)} - \frac{5}{11}H_{-(n-2)} - \frac{3}{11}H_{-(n-3)} - \frac{2}{11}H_{-(n-4)} + \frac{1}{11}H_{-(n-5)} \tag{1.11}$$

and

$$E_{-n} = -\frac{7}{11}E_{-(n-1)} - \frac{5}{11}E_{-(n-2)} - \frac{3}{11}E_{-(n-3)} - \frac{2}{11}E_{-(n-4)} + \frac{1}{11}E_{-(n-5)} \tag{1.12}$$

for $n = 1, 2, 3, ...$ respectively. Therefore, recurrences (1.10), (1.11) and (1.12) hold for all integer n.

Note that the sequences G_n, H_n and E_n are not indexed in [7] yet. Next, we present the first few values of the 5-primes, Lucas 5-primes and modified 5-primes numbers with positive and negative subscripts:

Table 2. The first few values of the special fifth-order numbers with positive and negative subscripts.

n	0	1	2	3	4	5	6	7	8	9
G_n	0	0	0	1	2	7	25	88	311	1082
G_{-n}		0	$\frac{1}{11}$	$-\frac{7}{121}$	$-\frac{6}{1331}$	$\frac{64}{14\,641}$	$-\frac{239}{161\,051}$	$\frac{33606}{1771561}$	$-\frac{331844}{19\,487\,171}$	$\frac{303121}{214358881}$
H_n	5	2	10	41	150	542	1831	6435	22574	79052
H_{-n}		$-\frac{7}{11}$	$-\frac{61}{121}$	$-\frac{277}{1331}$	$-\frac{2813}{14\,641}$	$\frac{148\,908}{161\,051}$	$-\frac{727\,195}{1771\,561}$	$-\frac{2234\,183}{19\,487\,171}$	$\frac{5014\,051}{214\,358\,881}$	$-\frac{85824736}{2357947691}$
E_n	0	0	0	1	1	5	18	63	223	771
E_{-n}		$-\frac{1}{11}$	$\frac{18}{121}$	$-\frac{71}{1331}$	$-\frac{130}{14\,641}$	$\frac{943}{161\,051}$	$-\frac{36\,235}{1771561}$	$\frac{701510}{19487171}$	$-\frac{3953405}{214358881}$	$-\frac{2169506}{2357947691}$

For all integers n, 5-primes, Lucas 5-primes and modified 5-primes numbers (using initial conditions in (1.5)) can be expressed using Binet's formulas as

$$\begin{aligned} G_n &= \frac{\alpha^{n+1}}{(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\alpha-\lambda)} + \frac{\beta^{n+1}}{(\beta-\alpha)(\beta-\gamma)(\beta-\delta)(\beta-\lambda)} + \frac{\gamma^{n+1}}{(\gamma-\alpha)(\gamma-\beta)(\gamma-\delta)(\gamma-\lambda)} \\ &\quad + \frac{\delta^{n+1}}{(\delta-\alpha)(\delta-\beta)(\delta-\gamma)(\delta-\lambda)} + \frac{\lambda^{n+1}}{(\lambda-\alpha)(\lambda-\beta)(\lambda-\gamma)(\lambda-\delta)}, \\ H_n &= \alpha^n + \beta^n + \gamma^n + \delta^n + \lambda^n, \\ E_n &= \frac{(\alpha-1)\alpha^n}{(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\alpha-\lambda)} + \frac{(\beta-1)\beta^n}{(\beta-\alpha)(\beta-\gamma)(\beta-\delta)(\beta-\lambda)} + \frac{(\gamma-1)\gamma^n}{(\gamma-\alpha)(\gamma-\beta)(\gamma-\delta)(\gamma-\lambda)} \\ &\quad + \frac{(\delta-1)\delta^n}{(\delta-\alpha)(\delta-\beta)(\delta-\gamma)(\delta-\lambda)} + \frac{(\lambda-1)\lambda^n}{(\lambda-\alpha)(\lambda-\beta)(\lambda-\gamma)(\lambda-\delta)}, \end{aligned}$$

respectively.

2. Generating Functions

Next, we give the ordinary generating function $\sum_{n=0}^{\infty} V_n x^n$ of the sequence V_n.

Lemma 1. *Suppose that $f_{V_n}(x) = \sum_{n=0}^{\infty} V_n x^n$ is the ordinary generating function of the generalized 5-primes sequence $\{V_n\}_{n\geq 0}$. Then, $\sum_{n=0}^{\infty} V_n x^n$ is given by*

$$\sum_{n=0}^{\infty} V_n x^n = \frac{V_0 + (V_1 - 2V_0)x + (V_2 - 2V_1 - 3V_0)x^2 + (V_3 - 2V_2 - 3V_1 - 5V_0)x^3 + (V_4 - 2V_3 - 3V_2 - 5V_1 - 7V_0)x^4}{1 - 2x - 3x^2 - 5x^3 - 7x^4 - 11x^5}. \tag{2.1}$$

Proof. Using the definition of generalized 5-primes numbers, and substracting $2x\sum_{n=0}^{\infty} V_n x^n$, $3x^2\sum_{n=0}^{\infty} V_n x^n$, $5x^3\sum_{n=0}^{\infty} V_n x^n, 7x^4\sum_{n=0}^{\infty} V_n x^n$ and $11x^5\sum_{n=0}^{\infty} V_n x^n$ from $\sum_{n=0}^{\infty} V_n x^n$ we obtain

$$\begin{aligned} (1-2x-3x^2-5x^3-7x^4-11x^5)\sum_{n=0}^{\infty} V_n x^n &= \sum_{n=0}^{\infty} V_n x^n - 2x\sum_{n=0}^{\infty} V_n x^n - 3x^2\sum_{n=0}^{\infty} V_n x^n \\ &\quad -5x^3\sum_{n=0}^{\infty} V_n x^n - 7x^4\sum_{n=0}^{\infty} V_n x^n - 11x^5\sum_{n=0}^{\infty} V_n x^n \\ &= \sum_{n=0}^{\infty} V_n x^n - 2\sum_{n=0}^{\infty} V_n x^{n+1} - 3\sum_{n=0}^{\infty} V_n x^{n+2} \\ &\quad -5\sum_{n=0}^{\infty} V_n x^{n+3} - 7\sum_{n=0}^{\infty} V_n x^{n+4} - 11\sum_{n=0}^{\infty} V_n x^{n+5} \\ &= \sum_{n=0}^{\infty} V_n x^n - 2\sum_{n=1}^{\infty} V_{n-1} x^n - 3\sum_{n=2}^{\infty} V_{n-2} x^n \\ &\quad -5\sum_{n=3}^{\infty} V_{n-3} x^n - 7\sum_{n=4}^{\infty} V_{n-4} x^n - 11\sum_{n=5}^{\infty} V_{n-5} x^n \end{aligned}$$

and so

$$\begin{aligned}(1-2x-3x^2-5x^3-7x^4-11x^5)\sum_{n=0}^{\infty}V_nx^n &= (V_0+V_1x+V_2x^2+V_3x^3+V_4x^4)-2(V_0x+V_1x^2+V_2x^3+V_3x^4)\\ &\quad -3(V_0x^2+V_1x^3+V_2x^4)-5(V_0x^3+V_1x^4)-7V_0x^4\\ &\quad +\sum_{n=5}^{\infty}(V_n-2V_{n-1}-3V_{n-2}-5V_{n-3}-7V_{n-4}-11V_{n-5})x^n\\ &= V_0+(V_1-2V_0)x+(V_2-2V_1-3V_0)x^2+(V_3-2V_2-3V_1-5V_0)x^3\\ &\quad +(V_4-2V_3-3V_2-5V_1-7V_0)x^4.\end{aligned}$$

Rearranging above equation, we obtain

$$\sum_{n=0}^{\infty}V_nx^n=\frac{V_0+(V_1-2V_0)x+(V_2-2V_1-3V_0)x^2+(V_3-2V_2-3V_1-5V_0)x^3+(V_4-2V_3-3V_2-5V_1-7V_0)x^4}{1-2x-3x^2-5x^3-7x^4-11x^5}.$$

The previous lemma gives the following results as particular examples.

COROLLARY 2. *Generated functions of 5-primes, Lucas 5-primes and modified 5-primes numbers are*

$$\sum_{n=0}^{\infty}G_nx^n=\frac{x^3}{1-2x-3x^2-5x^3-7x^4-11x^5},$$

and

$$\sum_{n=0}^{\infty}H_nx^n=\frac{5-8x-9x^2-10x^3-7x^4}{1-2x-3x^2-5x^3-7x^4-11x^5},$$

and

$$\sum_{n=0}^{\infty}E_nx^n=\frac{x^3-x^4}{1-2x-3x^2-5x^3-7x^4-11x^5},$$

respectively.

3. Obtaining Binet Formula From Generating Function

We next find Binet formula of generalized 5-primes numbers $\{V_n\}$ by the use of generating function for V_n.

THEOREM 3. *(Binet formula of generalized 5-primes numbers)*

$$\begin{aligned}V_n &= \frac{d_1\alpha^n}{(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\alpha-\lambda)}+\frac{d_2\beta^n}{(\beta-\alpha)(\beta-\gamma)(\beta-\delta)(\beta-\lambda)}+\frac{d_3\gamma^n}{(\gamma-\alpha)(\gamma-\beta)(\gamma-\delta)(\gamma-\lambda)} \\ &\quad +\frac{d_4\delta^n}{(\delta-\alpha)(\delta-\beta)(\delta-\gamma)(\delta-\lambda)}+\frac{d_5\lambda^n}{(\lambda-\alpha)(\lambda-\beta)(\lambda-\gamma)(\lambda-\delta)}\end{aligned} \tag{3.1}$$

where

$$\begin{aligned}d_1 &= V_0\alpha^4+(V_1-2V_0)\alpha^3+(V_2-2V_1-3V_0)\alpha^2+(V_3-2V_2-3V_1-5V_0)\alpha+(V_4-2V_3-3V_2-5V_1-7V_0),\\ d_2 &= V_0\beta^4+(V_1-2V_0)\beta^3+(V_2-2V_1-3V_0)\beta^2+(V_3-2V_2-3V_1-5V_0)\beta+(V_4-2V_3-3V_2-5V_1-7V_0),\\ d_3 &= V_0\gamma^4+(V_1-2V_0)\gamma^3+(V_2-2V_1-3V_0)\gamma^2+(V_3-2V_2-3V_1-5V_0)\gamma+(V_4-2V_3-3V_2-5V_1-7V_0),\\ d_4 &= V_0\delta^4+(V_1-2V_0)\delta^3+(V_2-2V_1-3V_0)\delta^2+(V_3-2V_2-3V_1-5V_0)\delta+(V_4-2V_3-3V_2-5V_1-7V_0),\\ d_5 &= V_0\lambda^4+(V_1-2V_0)\lambda^3+(V_2-2V_1-3V_0)\lambda^2+(V_3-2V_2-3V_1-5V_0)\lambda+(V_4-2V_3-3V_2-5V_1-7V_0).\end{aligned}$$

Proof. Let

$$h(x) = 1 - 2x - 3x^2 - 5x^3 - 7x^4 - 11x^5.$$

Then for some $\alpha, \beta, \gamma, \delta$ and λ we write

$$h(x) = (1-\alpha x)(1-\beta x)(1-\gamma x)(1-\delta x)(1-\lambda x)$$

i.e.,

$$1 - 2x - 3x^2 - 5x^3 - 7x^4 - 11x^5 = (1-\alpha x)(1-\beta x)(1-\gamma x)(1-\delta x)(1-\lambda x) \quad (3.2)$$

Hence $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}, \frac{1}{\delta}$ and $\frac{1}{\lambda}$ are the roots of $h(x)$. This gives $\alpha, \beta, \gamma, \delta$ and λ as the roots of

$$h(\frac{1}{x}) = 1 - \frac{2}{x} - \frac{3}{x^2} - \frac{5}{x^3} - \frac{7}{x^4} - \frac{11}{x^5} = 0.$$

This implies $x^5 - 2x^4 - 3x^3 - 5x^2 - 7x - 11 = 0$. Now, by (2.1) and (3.2), it follows that

$$\sum_{n=0}^{\infty} V_n x^n = \frac{V_0 + (V_1 - 2V_0)x + (V_2 - 2V_1 - 3V_0)x^2 + (V_3 - 2V_2 - 3V_1 - 5V_0)x^3 + (V_4 - 2V_3 - 3V_2 - 5V_1 - 7V_0)x^4}{(1-\alpha x)(1-\beta x)(1-\gamma x)(1-\delta x)(1-\lambda x)}.$$

Then we write

$$\begin{aligned} & \frac{V_0 + (V_1 - 2V_0)x + (V_2 - 2V_1 - 3V_0)x^2 + (V_3 - 2V_2 - 3V_1 - 5V_0)x^3 + (V_4 - 2V_3 - 3V_2 - 5V_1 - 7V_0)x^4}{(1-\alpha x)(1-\beta x)(1-\gamma x)(1-\delta x)(1-\lambda x)} \quad (3.3) \\ = & \frac{A_1}{(1-\alpha x)} + \frac{A_2}{(1-\beta x)} + \frac{A_3}{(1-\gamma x)} + \frac{A_4}{(1-\delta x)} + \frac{A_5}{(1-\lambda x)}. \end{aligned}$$

So

$$\begin{aligned} & V_0 + (V_1 - 2V_0)x + (V_2 - 2V_1 - 3V_0)x^2 + (V_3 - 2V_2 - 3V_1 - 5V_0)x^3 + (V_4 - 2V_3 - 3V_2 - 5V_1 - 7V_0)x^4 \\ = & A_1(1-\beta x)(1-\gamma x)(1-\delta x)(1-\lambda x) + A_2(1-\alpha x)(1-\gamma x)(1-\delta x)(1-\lambda x) \\ & + A_3(1-\alpha x)(1-\beta x)(1-\delta x)(1-\lambda x) + A_4(1-\alpha x)(1-\beta x)(1-\gamma x)(1-\lambda x) \\ & + A_5(1-\alpha x)(1-\beta x)(1-\gamma x)(1-\delta x). \end{aligned}$$

If we consider $x = \frac{1}{\alpha}$, we get

$V_0 + (V_1 - 2V_0)\frac{1}{\alpha} + (V_2 - 2V_1 - 3V_0)\frac{1}{\alpha^2} + (V_3 - 2V_2 - 3V_1 - 5V_0)\frac{1}{\alpha^3} + (V_4 - 2V_3 - 3V_2 - 5V_1 - 7V_0)\frac{1}{\alpha^4} = A_1(1 - \frac{\beta}{\alpha})(1 - \frac{\gamma}{\alpha})(1 - \frac{\delta}{\alpha})(1 - \frac{\lambda}{\alpha})$.

This gives

$$\begin{aligned} A_1 &= \frac{\alpha^4(V_0 + (V_1 - 2V_0)\frac{1}{\alpha} + (V_2 - 2V_1 - 3V_0)\frac{1}{\alpha^2} + (V_3 - 2V_2 - 3V_1 - 5V_0)\frac{1}{\alpha^3} + (V_4 - 2V_3 - 3V_2 - 5V_1 - 7V_0)\frac{1}{\alpha^4})}{(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\alpha-\lambda)} \\ &= \frac{V_0\alpha^4 + (V_1 - 2V_0)\alpha^3 + (V_2 - 2V_1 - 3V_0)\alpha^2 + (V_3 - 2V_2 - 3V_1 - 5V_0)\alpha + (V_4 - 2V_3 - 3V_2 - 5V_1 - 7V_0)}{(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\alpha-\lambda)} \end{aligned}$$

Similarly, we obtain

$$\begin{aligned} A_2 &= \frac{V_0\beta^4 + (V_1 - 2V_0)\beta^3 + (V_2 - 2V_1 - 3V_0)\beta^2 + (V_3 - 2V_2 - 3V_1 - 5V_0)\beta + (V_4 - 2V_3 - 3V_2 - 5V_1 - 7V_0)}{(\beta-\alpha)(\beta-\gamma)(\beta-\delta)(\beta-\lambda)}, \\ A_3 &= \frac{V_0\gamma^4 + (V_1 - 2V_0)\gamma^3 + (V_2 - 2V_1 - 3V_0)\gamma^2 + (V_3 - 2V_2 - 3V_1 - 5V_0)\gamma + (V_4 - 2V_3 - 3V_2 - 5V_1 - 7V_0)}{(\gamma-\alpha)(\gamma-\beta)(\gamma-\delta)(\gamma-\lambda)}, \\ A_4 &= \frac{V_0\delta^4 + (V_1 - 2V_0)\delta^3 + (V_2 - 2V_1 - 3V_0)\delta^2 + (V_3 - 2V_2 - 3V_1 - 5V_0)\delta + (V_4 - 2V_3 - 3V_2 - 5V_1 - 7V_0)}{(\delta-\alpha)(\delta-\beta)(\delta-\gamma)(\delta-\lambda)}, \\ A_5 &= \frac{V_0\lambda^4 + (V_1 - 2V_0)\lambda^3 + (V_2 - 2V_1 - 3V_0)\lambda^2 + (V_3 - 2V_2 - 3V_1 - 5V_0)\lambda + (V_4 - 2V_3 - 3V_2 - 5V_1 - 7V_0)}{(\lambda-\alpha)(\lambda-\beta)(\lambda-\gamma)(\lambda-\delta)}. \end{aligned}$$

Thus (3.3) can be written as

$$\sum_{n=0}^{\infty} V_n x^n = A_1(1-\alpha x)^{-1} + A_2(1-\beta x)^{-1} + A_3(1-\gamma x)^{-1} + A_4(1-\delta x)^{-1} + A_5(1-\lambda x)^{-1}.$$

This gives

$$\begin{aligned}\sum_{n=0}^{\infty} V_n x^n &= A_1\sum_{n=0}^{\infty}\alpha^n x^n + A_2\sum_{n=0}^{\infty}\beta^n x^n + A_3\sum_{n=0}^{\infty}\gamma^n x^n + A_4\sum_{n=0}^{\infty}\delta^n x^n + A_5\sum_{n=0}^{\infty}\lambda^n x^n \\ &= \sum_{n=0}^{\infty}(A_1\alpha^n + A_2\beta^n + A_3\gamma^n + A_4\delta^n + A_5\lambda^n)x^n.\end{aligned}$$

Therefore, comparing coefficients on both sides of the above equality, we obtain

$$V_n = A_1\alpha^n + A_2\beta^n + A_3\gamma^n + A_4\delta^n + A_5\lambda^n$$

and then we get (3.1).

Note that from (1.5) and (3.1) we have

$$\begin{aligned}& V_4-(\beta+\gamma+\delta+\lambda)V_3+(\beta\lambda+\beta\gamma+\lambda\gamma+\beta\delta+\lambda\delta+\gamma\delta)V_2-(\beta\lambda\gamma+\beta\lambda\delta+\beta\gamma\delta+\lambda\gamma\delta)V_1+(\beta\lambda\gamma\delta)V_0 \\ =\ & V_0\alpha^4+(V_1-2V_0)\alpha^3+(V_2-2V_1-3V_0)\alpha^2+(V_3-2V_2-3V_1-5V_0)\alpha+(V_4-2V_3-3V_2-5V_1-7V_0), \\ & V_4-(\alpha+\gamma+\delta+\lambda)V_3+(\alpha\lambda+\alpha\gamma+\alpha\delta+\lambda\gamma+\lambda\delta+\gamma\delta)V_2-(\alpha\lambda\gamma+\alpha\lambda\delta+\alpha\gamma\delta+\lambda\gamma\delta)V_1+(\alpha\lambda\gamma\delta)V_0 \\ =\ & V_0\beta^4+(V_1-2V_0)\beta^3+(V_2-2V_1-3V_0)\beta^2+(V_3-2V_2-3V_1-5V_0)\beta+(V_4-2V_3-3V_2-5V_1-7V_0), \\ & V_4-(\alpha+\beta+\delta+\lambda)V_3+(\alpha\beta+\alpha\lambda+\beta\lambda+\alpha\delta+\beta\delta+\lambda\delta)V_2-(\alpha\beta\lambda+\alpha\beta\delta+\alpha\lambda\delta+\beta\lambda\delta)V_1+(\alpha\beta\lambda\delta)V_0 \\ =\ & V_0\gamma^4+(V_1-2V_0)\gamma^3+(V_2-2V_1-3V_0)\gamma^2+(V_3-2V_2-3V_1-5V_0)\gamma+(V_4-2V_3-3V_2-5V_1-7V_0), \\ & V_4-(\alpha+\beta+\gamma+\lambda)V_3+(\alpha\beta+\alpha\lambda+\alpha\gamma+\beta\lambda+\beta\gamma+\lambda\gamma)V_2-(\alpha\beta\lambda+\alpha\beta\gamma+\alpha\lambda\gamma+\beta\lambda\gamma)V_1+(\alpha\beta\lambda\gamma)V_0 \\ =\ & V_0\delta^4+(V_1-2V_0)\delta^3+(V_2-2V_1-3V_0)\delta^2+(V_3-2V_2-3V_1-5V_0)\delta+(V_4-2V_3-3V_2-5V_1-7V_0), \\ & V_4-(\alpha+\beta+\gamma+\delta)V_3+(\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta)V_2-(\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta)V_1+(\alpha\beta\gamma\delta)V_0 \\ =\ & V_0\lambda^4+(V_1-2V_0)\lambda^3+(V_2-2V_1-3V_0)\lambda^2+(V_3-2V_2-3V_1-5V_0)\lambda+(V_4-2V_3-3V_2-5V_1-7V_0).\end{aligned}$$

Next, using Theorem 3, we present the Binet formulas of 5-primes, Lucas 5-primes and modified 5-primes sequences.

Corollary 4. *Binet formulas of 5-primes, Lucas 5-primes and modified 5-primes sequences are*

$$\begin{aligned}G_n &= \frac{\alpha^{n+1}}{(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\alpha-\lambda)}+\frac{\beta^{n+1}}{(\beta-\alpha)(\beta-\gamma)(\beta-\delta)(\beta-\lambda)}+\frac{\gamma^{n+1}}{(\gamma-\alpha)(\gamma-\beta)(\gamma-\delta)(\gamma-\lambda)} \\ &\quad+\frac{\delta^{n+1}}{(\delta-\alpha)(\delta-\beta)(\delta-\gamma)(\delta-\lambda)}+\frac{\lambda^{n+1}}{(\lambda-\alpha)(\lambda-\beta)(\lambda-\gamma)(\lambda-\delta)},\end{aligned}$$

and

$$H_n = \alpha^n+\beta^n+\gamma^n+\delta^n+\lambda^n,$$

and

$$\begin{aligned}E_n &= \frac{(\alpha-1)\alpha^{n}}{(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\alpha-\lambda)}+\frac{(\beta-1)\beta^{n}}{(\beta-\alpha)(\beta-\gamma)(\beta-\delta)(\beta-\lambda)}+\frac{(\gamma-1)\gamma^{n}}{(\gamma-\alpha)(\gamma-\beta)(\gamma-\delta)(\gamma-\lambda)} \\ &\quad+\frac{(\delta-1)\delta^{n}}{(\delta-\alpha)(\delta-\beta)(\delta-\gamma)(\delta-\lambda)}+\frac{(\lambda-1)\lambda^{n}}{(\lambda-\alpha)(\lambda-\beta)(\lambda-\gamma)(\lambda-\delta)},\end{aligned}$$

respectively.

We can find Binet formulas by using matrix method with a similar technique which is given in [3]. Take $k = i = 5$ in Corollary 3.1 in [3]. Let

$$\Lambda = \begin{pmatrix} \alpha^4 & \alpha^3 & \alpha^2 & \alpha & 1 \\ \beta^4 & \beta^3 & \beta^2 & \beta & 1 \\ \gamma^4 & \gamma^3 & \gamma^2 & \gamma & 1 \\ \delta^4 & \delta^3 & \delta^2 & \delta & 1 \\ \lambda^4 & \lambda^3 & \lambda^2 & \lambda & 1 \end{pmatrix}, \Lambda_1 = \begin{pmatrix} \alpha^{n-1} & \alpha^3 & \alpha^2 & \alpha & 1 \\ \beta^{n-1} & \beta^3 & \beta^2 & \beta & 1 \\ \gamma^{n-1} & \gamma^3 & \gamma^2 & \gamma & 1 \\ \delta^{n-1} & \delta^3 & \delta^2 & \delta & 1 \\ \lambda^{n-1} & \lambda^3 & \lambda^2 & \lambda & 1 \end{pmatrix}, \Lambda_2 = \begin{pmatrix} \alpha^4 & \alpha^{n-1} & \alpha^2 & \alpha & 1 \\ \beta^4 & \beta^{n-1} & \beta^2 & \beta & 1 \\ \gamma^4 & \gamma^{n-1} & \gamma^2 & \gamma & 1 \\ \delta^4 & \delta^{n-1} & \delta^2 & \delta & 1 \\ \lambda^4 & \lambda^{n-1} & \lambda^2 & \lambda & 1 \end{pmatrix}$$

$$\Lambda_3 = \begin{pmatrix} \alpha^4 & \alpha^3 & \alpha^{n-1} & \alpha & 1 \\ \beta^4 & \beta^3 & \beta^{n-1} & \beta & 1 \\ \gamma^4 & \gamma^3 & \gamma^{n-1} & \gamma & 1 \\ \delta^4 & \delta^3 & \delta^{n-1} & \delta & 1 \\ \lambda^4 & \lambda^3 & \lambda^{n-1} & \lambda & 1 \end{pmatrix}, \Lambda_4 = \begin{pmatrix} \alpha^4 & \alpha^3 & \alpha^2 & \alpha^{n-1} & 1 \\ \beta^4 & \beta^3 & \beta^2 & \beta^{n-1} & 1 \\ \gamma^4 & \gamma^3 & \gamma^2 & \gamma^{n-1} & 1 \\ \delta^4 & \delta^3 & \delta^2 & \delta^{n-1} & 1 \\ \lambda^4 & \lambda^3 & \lambda^2 & \lambda^{n-1} & 1 \end{pmatrix}, \Lambda_5 = \begin{pmatrix} \alpha^4 & \alpha^3 & \alpha^2 & \alpha & \alpha^{n-1} \\ \beta^4 & \beta^3 & \beta^2 & \beta & \beta^{n-1} \\ \gamma^4 & \gamma^3 & \gamma^2 & \gamma & \gamma^{n-1} \\ \delta^4 & \delta^3 & \delta^2 & \delta & \delta^{n-1} \\ \lambda^4 & \lambda^3 & \lambda^2 & \lambda & \lambda^{n-1} \end{pmatrix}.$$

Then the Binet formula for 5-primes numbers is

$$\begin{aligned} G_n &= \frac{1}{\det(\Lambda)} \sum_{j=1}^{5} G_{6-j} \det(\Lambda_j) = \frac{1}{\Lambda}(G_5 \det(\Lambda_1) + G_4 \det(\Lambda_2) + G_3 \det(\Lambda_3) + G_2 \det(\Lambda_4) + G_1 \det(\Lambda_5)) \\ &= \frac{1}{\det(\Lambda)}(7 \det(\Lambda_1) + 2 \det(\Lambda_2) + \det(\Lambda_3)) \end{aligned}$$

Similarly, we obtain the Binet formula for Lucas 5-primes and modified 5-primes numbers as

$$\begin{aligned} H_n &= \frac{1}{\det(\Lambda)}(H_5 \det(\Lambda_1) + H_4 \det(\Lambda_2) + H_3 \det(\Lambda_3) + H_2 \det(\Lambda_4) + H_1 \det(\Lambda_5)) \\ &= \frac{1}{\det(\Lambda)}(542 \det(\Lambda_1) + 150 \det(\Lambda_2) + 41 \det(\Lambda_3) + 10 \det(\Lambda_4) + 2 \det(\Lambda_5)) \end{aligned}$$

and

$$\begin{aligned} E_n &= \frac{1}{\det(\Lambda)}(E_5 \det(\Lambda_1) + E_4 \det(\Lambda_2) + E_3 \det(\Lambda_3) + E_2 \det(\Lambda_4) + E_1 \det(\Lambda_5)) \\ &= \frac{1}{\det(\Lambda)}(5 \det(\Lambda_1) + \det(\Lambda_2) + \det(\Lambda_3)) \end{aligned}$$

respectively.

4. Simson Formulas

There is a well-known Simson Identity (formula) for Fibonacci sequence $\{F_n\}$, namely,

$$F_{n+1}F_{n-1} - F_n^2 = (-1)^n$$

which was derived first by R. Simson in 1753 and it is now called as Cassini Identity (formula) as well. This can be written in the form

$$\begin{vmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{vmatrix} = (-1)^n.$$

The following theorem gives generalization of this result to the generalized 5-primes sequence $\{V_n\}_{n \geq 0}$.

THEOREM 5 (Simson Formula of Generalized 5-primes Numbers). *For all integers n, we have*

$$\begin{vmatrix} V_{n+4} & V_{n+3} & V_{n+2} & V_{n+1} & V_{n} \\ V_{n+3} & V_{n+2} & V_{n+1} & V_{n} & V_{n-1} \\ V_{n+2} & V_{n+1} & V_{n} & V_{n-1} & V_{n-2} \\ V_{n+1} & V_{n} & V_{n-1} & V_{n-2} & V_{n-3} \\ V_{n} & V_{n-1} & V_{n-2} & V_{n-3} & V_{n-4} \end{vmatrix} = 11^{n} \begin{vmatrix} V_{4} & V_{3} & V_{2} & V_{1} & V_{0} \\ V_{3} & V_{2} & V_{1} & V_{0} & V_{-1} \\ V_{2} & V_{1} & V_{0} & V_{-1} & V_{-2} \\ V_{1} & V_{0} & V_{-1} & V_{-2} & V_{-3} \\ V_{0} & V_{-1} & V_{-2} & V_{-3} & V_{-4} \end{vmatrix}. \tag{4.1}$$

Proof. (4.1) is given in Soykan [8].

The previous theorem gives the following results as particular examples.

COROLLARY 6. *For all integers n, Simson formula of 5-primes, Lucas 5-primes and modified 5-primes numbers are given as*

$$\begin{vmatrix} G_{n+4} & G_{n+3} & G_{n+2} & G_{n+1} & G_{n} \\ G_{n+3} & G_{n+2} & G_{n+1} & G_{n} & G_{n-1} \\ G_{n+2} & G_{n+1} & G_{n} & G_{n-1} & G_{n-2} \\ G_{n+1} & G_{n} & G_{n-1} & G_{n-2} & G_{n-3} \\ G_{n} & G_{n-1} & G_{n-2} & G_{n-3} & G_{n-4} \end{vmatrix} = 11^{n-3} \tag{4.2}$$

and

$$\begin{vmatrix} H_{n+4} & H_{n+3} & H_{n+2} & H_{n+1} & H_{n} \\ H_{n+3} & H_{n+2} & H_{n+1} & H_{n} & H_{n-1} \\ H_{n+2} & H_{n+1} & H_{n} & H_{n-1} & H_{n-2} \\ H_{n+1} & H_{n} & H_{n-1} & H_{n-2} & H_{n-3} \\ H_{n} & H_{n-1} & H_{n-2} & H_{n-3} & H_{n-4} \end{vmatrix} = 409 \times 431 \times 1103 \times 11^{n-4} \tag{4.3}$$

and

$$\begin{vmatrix} E_{n+4} & E_{n+3} & E_{n+2} & E_{n+1} & E_{n} \\ E_{n+3} & E_{n+2} & E_{n+1} & E_{n} & E_{n-1} \\ E_{n+2} & E_{n+1} & E_{n} & E_{n-1} & E_{n-2} \\ E_{n+1} & E_{n} & E_{n-1} & E_{n-2} & E_{n-3} \\ E_{n} & E_{n-1} & E_{n-2} & E_{n-3} & E_{n-4} \end{vmatrix} = 27 \times 11^{n-4} \tag{4.4}$$

respectively.

5. Some Identities

In this section, we obtain some identities of 5-primes, Lucas 5-primes and modified 5-primes numbers. First, we can give a few basic relations between $\{G_n\}$ and $\{H_n\}$.

LEMMA 7. *The following equalities are true:*

$$\begin{aligned} 1331H_n &= -277G_{n+6} - 117G_{n+5} + 1326G_{n+4} + 11\,747G_{n+3} - 2813G_{n+2}, \qquad (5.1) \\ 121H_n &= -61G_{n+5} + 45G_{n+4} + 942G_{n+3} - 432G_{n+2} - 277G_{n+1}, \\ 11H_n &= -7G_{n+4} + 69G_{n+3} - 67G_{n+2} - 64G_{n+1} - 61G_n, \\ H_n &= 5G_{n+3} - 8G_{n+2} - 9G_{n+1} - 10G_n - 7G_{n-1}, \\ H_n &= 2G_{n+2} + 6G_{n+1} + 15G_n + 28G_{n-1} + 55G_{n-2}, \end{aligned}$$

and

$$
\begin{aligned}
2138793107G_n &= 18571H_{n+6} + 1176092H_{n+5} - 1191254H_{n+4} - 21714675H_{n+3} + 39754441H_{n+2} \\
194435737G_n &= 110294H_{n+5} - 103231H_{n+4} - 1965620H_{n+3} + 3625858H_{n+2} + 18571H_{n+1} \\
194435737G_n &= 117357H_{n+4} - 1634738H_{n+3} + 4177328H_{n+2} + 790629H_{n+1} + 1213234H_n \\
194435737G_n &= -1400024H_{n+3} + 4529399H_{n+2} + 1377414H_{n+1} + 2034733H_n + 1290927H_{n-1} \\
194435737G_n &= 1729351H_{n+2} - 2822658H_{n+1} - 4965387H_n - 8509241H_{n-1} - 15400264H_{n-2}
\end{aligned}
$$

Proof. Note that all the identities hold for all integers n. We prove (5.1). To show (5.1), writing

$$H_n = a \times G_{n+6} + b \times G_{n+5} + c \times G_{n+4} + d \times G_{n+3} + e \times G_{n+2}$$

and solving the system of equations

$$
\begin{aligned}
H_0 &= a \times G_6 + b \times G_5 + c \times G_4 + d \times G_3 + e \times G_2 \\
H_1 &= a \times G_7 + b \times G_6 + c \times G_5 + d \times G_4 + e \times G_3 \\
H_2 &= a \times G_8 + b \times G_7 + c \times G_6 + d \times G_5 + e \times G_4 \\
H_3 &= a \times G_9 + b \times G_8 + c \times G_7 + d \times G_6 + e \times G_5 \\
H_4 &= a \times G_{10} + b \times G_9 + c \times G_8 + d \times G_7 + e \times G_6
\end{aligned}
$$

we find that $a = -\frac{277}{1331}, b = -\frac{117}{1331}, c = \frac{1326}{1331}, d = \frac{11\,747}{1331}, e = -\frac{2813}{1331}$. The other equalities can be proved similarly.

Secondly, we present a few basic relations between $\{G_n\}$ and $\{E_n\}$.

Lemma 8. *The following equalities are true:*

$$
\begin{aligned}
1331E_n &= -71G_{n+6} + 340G_{n+5} - 304G_{n+4} + 3G_{n+3} - 130G_{n+2}, \\
121E_n &= 18G_{n+5} - 47G_{n+4} - 32G_{n+3} - 57G_{n+2} - 71G_{n+1}, \\
11E_n &= -G_{n+4} + 2G_{n+3} + 3G_{n+2} + 5G_{n+1} + 18G_n, \\
E_n &= G_n - G_{n-1},
\end{aligned}
$$

and

$$
\begin{aligned}
297G_n &= -16E_{n+6} + 43E_{n+5} + 37E_{n+4} + 36E_{n+3} + 13E_{n+2}, \\
27G_n &= E_{n+5} - E_{n+4} - 4E_{n+3} - 9E_{n+2} - 16E_{n+1}, \\
27G_n &= E_{n+4} - E_{n+3} - 4E_{n+2} - 9E_{n+1} + 11E_n, \\
27G_n &= E_{n+3} - E_{n+2} - 4E_{n+1} + 18E_n + 11E_{n-1}, \\
27G_n &= E_{n+2} - E_{n+1} + 23E_n + 18E_{n-1} + 11E_{n-2}.
\end{aligned}
$$

Note that all the identities in the above Lemma can be proved by induction as well.

Thirdly, we give a few basic relations between $\{H_n\}$ and $\{E_n\}$.

LEMMA 9. *The following equalities are true:*

$$\begin{aligned}
3267H_n &= 217E_{n+6} - 1864E_{n+5} - 1300E_{n+4} + 23049E_{n+3} + 9866E_{n+2},\\
297H_n &= -130E_{n+5} - 59E_{n+4} + 2194E_{n+3} + 1035E_{n+2} + 217E_{n+1},\\
27H_n &= -29E_{n+4} + 164E_{n+3} + 35E_{n+2} - 63E_{n+1} - 130E_n,\\
27H_n &= 106E_{n+3} - 52E_{n+2} - 208E_{n+1} - 333E_n - 319E_{n-1},\\
27H_n &= 160E_{n+2} + 110E_{n+1} + 197E_n + 423E_{n-1} + 1166E_{n-2},
\end{aligned}$$

and

$$\begin{aligned}
23526724177E_n &= -39550160H_{n+6} + 92241613H_{n+5} + 93222517H_{n+4} - 26985426H_{n+3} + 954441363H_{n+2},\\
2138793107E_n &= 1194\,663H_{n+5} - 2311633H_{n+4} - 20430566H_{n+3} + 61599113H_{n+2} - 39550160H_{n+1},\\
194435737E_n &= 7063H_{n+4} - 1531507H_{n+3} + 6142948H_{n+2} - 2835229H_{n+1} + 1194663H_n,\\
194435737E_n &= -1517381H_{n+3} + 6164137H_{n+2} - 2799914H_{n+1} + 1244104H_n + 77693H_{n-1},\\
194435737E_n &= 3129375H_{n+2} - 7352057H_{n+1} - 6342801H_n - 10543974H_{n-1} - 16691191H_{n-2}.
\end{aligned}$$

We now present a few special identities for the modified 5-primes sequence $\{E_n\}$.

THEOREM 10. *(Catalan's identity) For all integers n and m, the following identity holds*

$$\begin{aligned}
E_{n+m}E_{n-m} - E_n^2 &= (G_{n+m} - G_{n+m-1})(G_{n-m} - G_{n-m-1}) - (G_n - G_{n-1})^2\\
&= (G_n(G_m - G_{m+1}) + G_{n-1}(-G_m + G_{m-2}) + G_{n-2}(-G_m + G_{m-1}))\\
&\quad (G_n(G_{-m} - G_{1-m}) + G_{n-1}(-G_{-m} + G_{-m-2}) + G_{n-2}(-G_{-m} + G_{-m-1}))\\
&\quad -(G_n - G_{n-1})^2
\end{aligned}$$

Proof. We use the identity

$$E_n = G_n - G_{n-1}.$$

Note that for $m = 1$ in Catalan's identity, we get the Cassini identity for the modified 5-primes sequnce

COROLLARY 11. *(Cassini's identity) For all integers numbers n and m, the following identity holds*

$$E_{n+1}E_{n-1} - E_n^2 = (G_{n+1} - G_n)(G_{n-1} - G_{n-2}) - (G_n - G_{n-1})^2.$$

The d'Ocagne's, Gelin-Cesàro's and Melham' identities can also be obtained by using $E_n = G_n - G_{n-1}$.The next theorem presents d'Ocagne's, Gelin-Cesàro's and Melham' identities of modified 5-primes sequence $\{E_n\}$.

THEOREM 12. *Let n and m be any integers. Then the following identities are true:*

(a): *(d'Ocagne's identity)*

$$E_{m+1}E_n - E_mE_{n+1} = (G_{m+1} - G_m)(G_n - G_{n-1}) - (G_m - G_{m-1})(G_{n+1} - G_n).$$

(b): *(Gelin-Cesàro's identity)*

$$E_{n+2}E_{n+1}E_{n-1}E_{n-2} - E_n^4 = (G_{n+2} - G_{n+1})(G_{n+1} - G_n)(G_{n-1} - G_{n-2})(G_{n-2} - G_{n-3}) - (G_n - G_{n-1})^4.$$

(c): *(Melham's identity)*

$$E_{n+1}E_{n+2}E_{n+6}-E_{n+3}^{3}=(G_{n+1}-G_{n})(G_{n+2}-G_{n+1})(G_{n+6}-G_{n+5})-(G_{n+3}-G_{n+2})^{3}.$$

Proof. Use the identity $E_n = G_n - G_{n-1}$.

6. Linear Sums

The following proposition presents some formulas of generalized 5-primes numbers with positive subscripts.

PROPOSITION 13. *If $r=2, s=3, t=5, u=7, v=11$ then for $n\geq 0$ we have the following formulas:*

(a): $\sum_{k=0}^{n} V_k = \frac{1}{27}(V_{n+5}-V_{n+4}-4V_{n+3}-9V_{n+2}-16V_{n+1}-V_4+V_3+4V_2+9V_1+16V_0)$.

(b): $\sum_{k=0}^{n} V_{2k} = \frac{1}{27}(-V_{2n+2}+4V_{2n+1}+25V_{2n}+3V_{2n-1}+22V_{2n-2}+V_4-4V_3+2V_2-3V_1+5V_0)$.

(c): $\sum_{k=0}^{n} V_{2k+1} = \frac{1}{27}(2V_{2n+2}+22V_{2n+1}-2V_{2n}+15V_{2n-1}-11V_{2n-2}-2V_4+5V_3+2V_2+12V_1+11V_0)$.

Proof. Take $r=2, s=3, t=5, u=7, v=11$ in Theorem 2.1 in [9].

As special cases of above proposition, we have the following three corollaries. First one presents some summing formulas of 5-primes numbers (take $V_n = G_n$ with $G_0=0, G_1=0, G_2=0, G_3=1, G_4=2$).

COROLLARY 14. *For $n\geq 0$ we have the following formulas:*

(a): $\sum_{k=0}^{n} G_k = \frac{1}{27}(G_{n+5}-G_{n+4}-4G_{n+3}-9G_{n+2}-16G_{n+1}-1)$.

(b): $\sum_{k=0}^{n} G_{2k} = \frac{1}{27}(-G_{2n+2}+4G_{2n+1}+25G_{2n}+3G_{2n-1}+22G_{2n-2}-2)$.

(c): $\sum_{k=0}^{n} G_{2k+1} = \frac{1}{27}(2G_{2n+2}+22G_{2n+1}-2G_{2n}+15G_{2n-1}-11G_{2n-2}+1)$.

Second one presents some summing formulas of Lucas 5-primes numbers (take $G_n = H_n$ with $H_0=5$, $H_1=2, H_2=10, H_3=41, H_4=150$).

COROLLARY 15. *For $n\geq 0$ we have the following formulas:*

(a): $\sum_{k=0}^{n} H_k = \frac{1}{27}(H_{n+5}-H_{n+4}-4H_{n+3}-9H_{n+2}-16H_{n+1}+29)$.

(b): $\sum_{k=0}^{n} H_{2k} = \frac{1}{27}(-H_{2n+2}+4H_{2n+1}+25H_{2n}+3H_{2n-1}+22H_{2n-2}+25)$.

(c): $\sum_{k=0}^{n} H_{2k+1} = \frac{1}{27}(2H_{2n+2}+22H_{2n+1}-2H_{2n}+15H_{2n-1}-11H_{2n-2}+4)$.

Third one presents some summing formulas of modified 5-primes numbers (take $H_n = E_n$ with $E_0=0, E_1=0, E_2=0, E_3=1, E_4=1$).

COROLLARY 16. *For $n\geq 0$ we have the following formulas:*

(a): $\sum_{k=0}^{n} E_k = \frac{1}{27}(E_{n+5}-E_{n+4}-4E_{n+3}-9E_{n+2}-16E_{n+1})$.

(b): $\sum_{k=0}^{n} E_{2k} = \frac{1}{27}(-E_{2n+2}+4E_{2n+1}+25E_{2n}+3E_{2n-1}+22E_{2n-2}-3)$.

(c): $\sum_{k=0}^{n} E_{2k+1} = \frac{1}{27}(2E_{2n+2}+22E_{2n+1}-2E_{2n}+15E_{2n-1}-11E_{2n-2}+3)$.

The following proposition presents some formulas of generalized 5-primes numbers with negative subscripts.

PROPOSITION 17. *If $r=2, s=3, t=5, u=7, v=11$ then for $n\geq 1$ we have the following formulas:*

(a): $\sum_{k=1}^{n} V_{-k} = \frac{1}{27}(-V_{-n+4}+V_{-n+3}+4V_{-n+2}+9V_{-n+1}+16V_{-n}+V_4-V_3-9V_1-4V_2-16V_0)$.

(b): $\sum_{k=1}^{n} V_{-2k} = \frac{1}{27}(-2V_{-2n+3}+5V_{-2n+2}+2V_{-2n+1}+12V_{-2n}+11V_{-2n-1}-V_4+4V_3-2V_2+3V_1-5V_0)$.

(c): $\sum_{k=1}^{n} V_{-2k+1} = \frac{1}{27}(V_{-2n+3}-4V_{-2n+2}+2V_{-2n+1}-3V_{-2n}-22V_{-2n-1}+2V_4-5V_3-2V_2-12V_1-11V_0)$.

Proof. Take $r = 2, s = 3, t = 5, u = 7, v = 11$ in Theorem 3.1 in [9].

From the above proposition, we have the following corollary which gives sum formulas of 5-primes numbers (take $G_n = G_n$ with $G_0 = 0, G_1 = 0, G_2 = 0, G_3 = 1, G_4 = 2$).

Corollary 18. *For $n \geq 1$, 5-primes numbers have the following properties.*

(a): $\sum_{k=1}^{n} G_{-k} = \frac{1}{27}(-G_{-n+4} + G_{-n+3} + 4G_{-n+2} + 9G_{-n+1} + 16G_{-n} + 1)$.

(b): $\sum_{k=1}^{n} G_{-2k} = \frac{1}{27}(-2G_{-2n+3} + 5G_{-2n+2} + 2G_{-2n+1} + 12G_{-2n} + 11G_{-2n-1} + 2)$.

(c): $\sum_{k=1}^{n} G_{-2k+1} = \frac{1}{27}(G_{-2n+3} - 4G_{-2n+2} + 2G_{-2n+1} - 3G_{-2n} - 22G_{-2n-1} - 1)$.

Taking $G_n = H_n$ with $H_0 = 5$, $H_1 = 2, H_2 = 10, H_3 = 41, H_4 = 150$ in the last proposition, we have the following corollary which presents sum formulas of 5-primes -Lucas numbers.

Corollary 19. *For $n \geq 1$, 5-primes -Lucas numbers have the following properties.*

(a): $\sum_{k=1}^{n} H_{-k} = \frac{1}{27}(-H_{-n+4} + H_{-n+3} + 4H_{-n+2} + 9H_{-n+1} + 16H_{-n} - 29)$.

(b): $\sum_{k=1}^{n} H_{-2k} = \frac{1}{27}(-2H_{-2n+3} + 5H_{-2n+2} + 2H_{-2n+1} + 12H_{-2n} + 11H_{-2n-1} - 25)$.

(c): $\sum_{k=1}^{n} H_{-2k+1} = \frac{1}{27}(H_{-2n+3} - 4H_{-2n+2} + 2H_{-2n+1} - 3H_{-2n} - 22H_{-2n-1} - 4)$.

From the above proposition, we have the following corollary which gives sum formulas of modified 5-primes numbers (take $H_n = E_n$ with $E_0 = 0, E_1 = 0, E_2 = 0, E_3 = 1, E_4 = 1$).

Corollary 20. *For $n \geq 1$, modified 5-primes numbers have the following properties.*

(a): $\sum_{k=1}^{n} E_{-k} = \frac{1}{27}(-E_{-n+4} + E_{-n+3} + 4E_{-n+2} + 9E_{-n+1} + 16E_{-n})$.

(b): $\sum_{k=1}^{n} E_{-2k} = \frac{1}{27}(-2E_{-2n+3} + 5E_{-2n+2} + 2E_{-2n+1} + 12E_{-2n} + 11E_{-2n-1} + 3)$.

(c): $\sum_{k=1}^{n} E_{-2k+1} = \frac{1}{27}(E_{-2n+3} - 4E_{-2n+2} + 2E_{-2n+1} - 3E_{-2n} - 22E_{-2n-1} - 3)$.

7. Matrices Related with Generalized 5-primes Numbers

Matrix formulation of W_n can be given as

$$\begin{pmatrix} W_{n+4} \\ W_{n+3} \\ W_{n+2} \\ W_{n+1} \\ W_n \end{pmatrix} = \begin{pmatrix} r & s & t & u & v \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}^n \begin{pmatrix} W_4 \\ W_3 \\ W_2 \\ W_1 \\ W_0 \end{pmatrix} \tag{7.1}$$

For matrix formulation (7.1), see [2]. In fact, Kalman give the formula in the following form

$$\begin{pmatrix} W_n \\ W_{n+1} \\ W_{n+2} \\ W_{n+3} \\ W_{n+4} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ r & s & t & u & v \end{pmatrix}^n \begin{pmatrix} W_0 \\ W_1 \\ W_2 \\ W_3 \\ W_4 \end{pmatrix}.$$

We define the square matrix A of order 5 as:

$$A=\begin{pmatrix} 2 & 3 & 5 & 7 & 11 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

such that $\det A = 11$. From (1.4) we have

$$\begin{pmatrix} V_{n+4} \\ V_{n+3} \\ V_{n+2} \\ V_{n+1} \\ V_n \end{pmatrix}=\begin{pmatrix} 2 & 3 & 5 & 7 & 11 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} V_{n+3} \\ V_{n+2} \\ V_{n+1} \\ V_n \\ V_{n-1} \end{pmatrix}. \tag{7.2}$$

and from (7.1) (or using (7.2) and induction) we have

$$\begin{pmatrix} V_{n+4} \\ V_{n+3} \\ V_{n+2} \\ V_{n+1} \\ V_n \end{pmatrix}=\begin{pmatrix} 2 & 3 & 5 & 7 & 11 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}^n\begin{pmatrix} V_4 \\ V_3 \\ V_2 \\ V_1 \\ V_0 \end{pmatrix}.$$

If we take $V_n = G_n$ in (7.2) we have

$$\begin{pmatrix} G_{n+4} \\ G_{n+3} \\ G_{n+2} \\ G_{n+1} \\ G_n \end{pmatrix}=\begin{pmatrix} 2 & 3 & 5 & 7 & 11 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} G_{n+3} \\ G_{n+2} \\ G_{n+1} \\ G_n \\ G_{n-1} \end{pmatrix}. \tag{7.3}$$

We also define

$$B_n=\begin{pmatrix} G_{n+3} & 3G_{n+2}+5G_{n+1}+7G_n+11G_{n-1} & 5G_{n+2}+7G_{n+1}+11G_n & 7G_{n+2}+11G_{n+1} & 11G_{n+2} \\ G_{n+2} & 3G_{n+1}+5G_n+7G_{n-1}+11G_{n-2} & 5G_{n+1}+7G_n+11G_{n-1} & 7G_{n+1}+11G_n & 11G_{n+1} \\ G_{n+1} & 3G_n+5G_{n-1}+7G_{n-2}+11G_{n-3} & 5G_n+7G_{n-1}+11G_{n-2} & 7G_n+11G_{n-1} & 11G_n \\ G_n & 3G_{n-1}+5G_{n-2}+7G_{n-3}+11G_{n-4} & 5G_{n-1}+7G_{n-2}+11G_{n-3} & 7G_{n-1}+11G_{n-2} & 11G_{n-1} \\ G_{n-1} & 3G_{n-2}+5G_{n-3}+7G_{n-4}+11G_{n-5} & 5G_{n-2}+7G_{n-3}+11G_{n-4} & 7G_{n-2}+11G_{n-3} & 11G_{n-2} \end{pmatrix}$$

and

$$C_n=\begin{pmatrix} V_{n+3} & 3V_{n+2}+5V_{n+1}+7V_n+11V_{n-1} & 5V_{n+2}+7V_{n+1}+11V_n & 7V_{n+2}+11V_{n+1} & 11V_{n+2} \\ V_{n+2} & 3V_{n+1}+5V_n+7V_{n-1}+11V_{n-2} & 5V_{n+1}+7V_n+11V_{n-1} & 7V_{n+1}+11V_n & 11V_{n+1} \\ V_{n+1} & 3V_n+5V_{n-1}+7V_{n-2}+11V_{n-3} & 5V_n+7V_{n-1}+11V_{n-2} & 7V_n+11V_{n-1} & 11V_n \\ V_n & 3V_{n-1}+5V_{n-2}+7V_{n-3}+11V_{n-4} & 5V_{n-1}+7V_{n-2}+11V_{n-3} & 7V_{n-1}+11V_{n-2} & 11V_{n-1} \\ V_{n-1} & 3V_{n-2}+5V_{n-3}+7V_{n-4}+11V_{n-5} & 5V_{n-2}+7V_{n-3}+11V_{n-4} & 7V_{n-2}+11V_{n-3} & 11V_{n-2} \end{pmatrix}.$$

THEOREM 21. *For all integer $m, n \geq 0$, we have*

(a): $B_n = A^n$

(b): $C_1 A^n = A^n C_1$

(c): $C_{n+m} = C_n B_m = B_m C_n$.

Proof.

(a): By expanding the vectors on the both sides of (7.3) to 5-colums and multiplying the obtained on the right-hand side by A, we get

$$B_n = AB_{n-1}.$$

By induction argument, from the last equation, we obtain

$$B_n = A^{n-1} B_1.$$

But $B_1 = A$. It follows that $B_n = A^n$.

(b): Using (a) and definition of C_1, (b) follows.

(c): We have $C_n = AC_{n-1}$. From the last equation, using induction we obtain $C_n = A^{n-1} C_1$. Now

$$C_{n+m} = A^{n+m-1} C_1 = A^{n-1} A^m C_1 = A^{n-1} C_1 A^m = C_n B_m$$

and similarly

$$C_{n+m} = B_m C_n.$$

Some properties of matrix A^n can be given as

$$A^n = 2A^{n-1} + 3A^{n-2} + 5A^{n-3} + 7A^{n-4} + 11A^{n-5}$$

and

$$A^{n+m} = A^n A^m = A^m A^n$$

and

$$\det(A^n) = 11^n$$

for all integer m and n.

Theorem 22. *For $m, n \geq 0$ we have*

$$\begin{aligned} V_{n+m} &= V_n G_{m+3} + V_{n-1}(3G_{m+2} + 5G_{m+1} + 7G_m + 11G_{m-1}) + V_{n-2}(5G_{m+2} \\ &\quad +7G_{m+1} + 11G_m) + V_{n-3}(7G_{m+2} + 11G_{m+1}) + 11V_{n-4}G_{m+2} \end{aligned} \tag{7.4}$$

Proof. From the equation $C_{n+m} = C_n B_m = B_m C_n$ we see that an element of C_{n+m} is the product of row C_n and a column B_m. From the last equation we say that an element of C_{n+m} is the product of a row C_n and column B_m. We just compare the linear combination of the 2nd row and 1st column entries of the matrices C_{n+m} and $C_n B_m$. This completes the proof.

Remark 23. *By induction, it can be proved that for all integers $m, n \leq 0$, (7.4) holds. So for all integers m, n, (7.4) is true.*

COROLLARY 24. *For all integers m, n, we have*

$$
\begin{aligned}
G_{n+m} &= G_nG_{m+3}+G_{n-1}(3G_{m+2}+5G_{m+1}+7G_m+11G_{m-1})+G_{n-2}(5G_{m+2}+7G_{m+1}+11G_m) \\
&\quad +G_{n-3}(7G_{m+2}+11G_{m+1})+11G_{n-4}G_{m+2}, \\
H_{n+m} &= H_nG_{m+3}+H_{n-1}(3G_{m+2}+5G_{m+1}+7G_m+11G_{m-1})+H_{n-2}(5G_{m+2}+7G_{m+1}+11G_m) \\
&\quad +H_{n-3}(7G_{m+2}+11G_{m+1})+11H_{n-4}G_{m+2}, \\
E_{n+m} &= E_nG_{m+3}+E_{n-1}(3G_{m+2}+5G_{m+1}+7G_m+11G_{m-1})+E_{n-2}(5G_{m+2}+7G_{m+1}+11G_m) \\
&\quad +E_{n-3}(7G_{m+2}+11G_{m+1})+11E_{n-4}G_{m+2}.
\end{aligned}
$$

REFERENCES

[1] Howard, F.T., Saidak, F., Zhou's Theory of Constructing Identities, Congress Numer. 200, 225-237, 2010.

[2] Kalman, D., Generalized Fibonacci Numbers By Matrix Methods, Fibonacci Quarterly, 20(1), 73-76, 1982.

[3] Kiliç, E., Stanica, P., A Matrix Approach for General Higher Order Linear Recurrences, Bulletin of the Malaysian Mathematical Sciences Society, (2) 34(1), 51.67, 2011.

[4] Melham, R. S., Some Analogs of the Identity $F_n^2 + F_{n+1}^2 = F_{2n+1}^2$,Fibonacci Quarterly, 305-311, 1999.

[5] Natividad, L. R., On Solving Fibonacci-Like Sequences of Fourth, Fifth and Sixth Order, International Journal of Mathematics and Computing, 3 (2), 2013.

[6] Rathore, G.P.S., Sikhwal, O., Choudhary, R., Formula for finding nth Term of Fibonacci-Like Sequence of Higher Order, International Journal of Mathematics And its Applications, 4 (2-D), 75-80, 2016.

[7] Sloane, N.J.A., The on-line encyclopedia of integer sequences, http://oeis.org/

[8] Soykan, Y., Simson Identity of Generalized m-step Fibonacci Numbers, Int. J. Adv. Appl. Math. and Mech. 7(2), 45-56, 2019.

[9] Soykan, Y., Sum Formulas For Generalized Fifth-Order Linear Recurrence Sequences, Journal of Advances in Mathematics and Computer Science, 34(5), 1-14, 2019, Article no.JAMCS.53303, ISSN: 2456-9968, DOI: 10.9734/JAMCS/2019/v34i530224.

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Research Article

THE EFFECTS OF POTASSIUM APPLICATIONS ON DROUGHT STRESS IN SUGAR BEET: PART II. PLANT NUTRITION CONTENT

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ABSTRACT

This is the second in a series of papers describing the effects of potassium applications on drought stress in sugar beet. Drought is a natural phenomenon that can affect water resources and agriculture. In this research, the effect of potassium applications under drought stress on some plant nutrition of sugar beet, which is a strategic plant, was investigated. In the experiment, irrigation levels were kept at 33%, 66% and 100% of field capacity. Different doses (10-20-40-80 mg kg-1) of potassium were applied to the plants. The plants were grown in the growth chamber under controlled conditions (day/night 16/8 hours, 25/15 0C, 60-70% humidity). According to the results, the effect of irrigation x potassium interaction on the shoot and root sodium (Na) potassium (K) calcium (Ca) and phosphorus (P) content Na/K and Na/K ratio was found to be statistically significant. Shoot and root sodium content decreased with potassium applications under drought conditions (33%). Shoot and root potassium, phosphorus content increased with potassium applications in both drought and sufficient water conditions. Shoot calcium content change irregular with potassium application while root calcium decreased with potassium application under drought conditions (33%). Shoot and root sodium/potassium ratio decreased with potassium applications in both drought and sufficient water conditions. Shoot and root sodium/calcium ratio change irregular with potassium applications. Therefore, it can be said that potassium may play a critical role in reducing the negative effect of drought stress and uptake plant nutrition in sugar beet.

Key Words: *Drought, Irrigation, Potassium, Sugar Beet.*

1. INTRODUCTION

Drought is a natural phenomenon that can affect water resources and agriculture. Drought is also the slowest growing, most insidious, most dangerous natural disaster that causes the most extensive socio-economic damages (Kadıoğlu 2012). Drought is one of the most costly disasters that do not occur abruptly, but can harm more people than other natural disasters, with an average annual loss of $ 8-10 billion (Wilhite, 2000). Drought stress means that the amount of water present in the soil during vegetation is less than the amount of water required for plant growth and development. Arid and semi-arid areas in Turkey are 51 million hectares. Therefore 37.3% of Turkey said that under the influence of mostly semi-arid climatic conditions (Kadıoğlu, 2012). A large part of the water taken from the soil through plant roots is given to the atmosphere by transpiration from the plant leaves. A small amount is retained in tissues and used for various compounds. In order to obtain high yields and high quality products from plants, this water flow should not slow down or be interrupted. The lack of moisture in the soil causes a decrease in turgor pressure, closure of stomata, slow growth and decrease in yield.

Sugar cane and sugar beet are the two most important sources of sugar production in the world. Sugar beet is one of our strategically important products due to its great contribution to the agriculture and economy of our country. In addition to its contributions to the agricultural sector, sugar beet also contributes to the food and chemical industry with its by-products. The ground and surface water resources of the Central Anatolia Region, where approximately 60% of Turkey's sugar beet cultivation areas are located, are rapidly decreasing. The aim of this research is to determine the effect of potassium applications on some plant nutrition of sugar beet, which is a strategic plant, under drought stress and to try to clarify the relationship between drought stress and potassium.

2. MATERIAL METHOD

2.1 Plant Growth

In this study, washed sand, with a pH of 8.2 and electrical conductivity of 75 µM cm^{-1}, was used. The sand was filled into 25X50 cm plastic sapling production bags. Resistive soil moisture sensors were put inside the sand to control the moisture level. Moisture sensors were calibrated with a device designed using an Arduino developer card, and irrigation was carried out according to the data received from that device (Kızıl et al., 2018). Irrigation levels were kept at 33%, 66% and 100% of field capacity. Serenad varieties of sugar beet (*Beta vulgaris* L.) plants were grown in the climate room under controlled conditions (day/night 16/8 hours, 25/15 ^{0}C, 60-70% humidity). Different doses (10-20-40-80 mg kg^{-1}) of potassium were applied to the plants with a potassium phosphate source. Plants were grown considering the 1: 0.8: 1.2, N: P: K ratio (Adiloglu and Guler, 2002), with 3 replicates for 4 months. Plants were harvested after sampling the leaves for relative water content and membrane damage.

2.2 Ion analysis

The plant shoot and root samples were dried in the oven at 65 ºC for 48 hours and then ground. Samples of 500 mg were taken and nitric acid and hydrogen peroxide was added for wet digestion. The samples were read in flame photometer for Calcium (Ca), potassium (K) and sodium (Na) concentrations while phosphorus (P) concentration was determined by spectrophotometer (Jones et al., 1991).

2.3 Statistical analysis

Analysis of variance (ANOVA) was performed using the general linear model (PROC GLM) procedure of R program. The variance analysis was done based on the following model:

$Yijk = \mu + Mi + Fj + (MF)ij + Gk + eijk$

Where:

Yijk: observed value

μ: grand mean

Mi: effect of irrigation i (i=1, 2, 3)

Fj: effect of potassium j (j=1, 2, 3, 4)

(MF)ij: effect of irrigation x effect of potassium

Gk: effect of replication k (k = 1, 2, 3)

eijk: random error term

Variance analysis (ANOVA) was performed by using the statistical package program using the GLM procedure. Differences between applications were determined by the Tukey multiple comparison test (P <0.05).

4. RESULTS

According to the results of variance analysis, the effect of irrigation x potassium interaction shoot and root sodium (Na) and potassium (K) content were statistically significant ($P \leq 0.01$). In addition, it was determined that the effect of irrigation and potassium applications on shoot and root sodium (Na) and potassium (K) content were statistically significant (Table 1).

Table 1. Mean squares for shoot and root Na and K content

Source of Variation	Df	Shoot Na content	Root Na content	Shoot K content	Root K content
Irrigation	2	0.222714**	0.000006**	4.90103**	3.75618**
Potassium (K)	3	0.004490**	0.000076**	2.69259**	1.55553**
Irrigation * K	6	0.003206**	0.000009*	0.04135**	0.03491**
Error	22	0.000010	0.000001	0.00001	0.00008

* . ** Indicates significant difference at $P \leq 0.05$. $P \leq 0.01$ respectively. Df: Degrees of freedom.

Shoot sodium content increased with the increase of irrigation levels to 0.058, 0.375 and 0.329% respectively. Shoot sodium content increased with potassium applications up until the 80 mg kg^{-1} potassium application (Table 2). The lowest shoot sodium content (0.51%) was obtained at the 33% irrigation level and 20 mg kg^{-1} potassium application, the highest shoot sodium content (0.438%) at the 66% irrigation level and 40 mg kg^{-1} potassium application (Table 2). When the root sodium content is considered, it is observed that root sodium content decreases with increasing irrigation levels and increasing potassium applications. The lowest root sodium content (0.012%) was obtained at the 66% irrigation level and 40 mg kg^{-1} potassium application, the highest root sodium content (0.022%) at the 33% irrigation level and 10 mg kg^{-1} potassium application (Table 2).

Table 2. Mean values of shoot and root Na content

	Irrigation (Field Capacity)				Irrigation (Field Capacity)			
K (mg kg^{-1})	**33%**	**66%**	**100%**	**Mean**	**33%**	**66%**	**100%**	**Mean**
	Shoot Na content (%)				**Root Na content (%)**			
10	0.077 g	0.312 e	0.292 f	0.227 D	0.022 a	0.020 a	0.016 b	0.019 A
20	0.051 h	0.331 d	0.329 d	0.237 C	0.013 bc	0.015 bc	0.014 bc	0.014BC
40	0.052 h	0.438 a	0.350 c	0.280 A	0.013 bc	0.012 bc	0.013 bc	0.013 C
80	0.052 h	0.418 b	0.346 c	0.272 B	0.014 bc	0.014 bc	0.014 bc	0.014 B
Mean	0.058 C	0.375 A	0.329 B		0.016 A	0.015 A	0.014 B	

*The differences between the interaction potassium and Irrigation means having different lower case letters in a column are statistically significant at 0.05. **The differences between the potassium means having different capital letters in a column are statistically significant at 0.05. The differences between the irrigation means having different capital letters in a line are statistically significant at 0.05.

Potassium content of shoots and roots are given in Table 3. Potassium content of shoots and roots increased with the increase of irrigation levels and potassium applications The lowest shoot potassium content (1.552%) was obtained at 33% irrigation level and 10 mg kg^{-1} potassium application, the highest shoot potassium content (4.467%) at 100% irrigation and 80 mg kg^{-1} potassium application. Similarly the lowest root potassium content (0.550%) was obtained at 33% irrigation level and 10 mg kg^{-1} potassium application, the highest root potassium content (2.355%) at 100% irrigation and 80 mg kg^{-1} potassium application.

Table 3. Mean values of shoot and root K content

	Irrigation (Field Capacity)				Irrigation (Field Capacity)			
K (mg kg^{-1})	**33%**	**66%**	**100%**	**Mean**	**33%**	**66%**	**100%**	**Mean**
	Shoot K content (%)				**Root K content (%)**			
10	1.552 l	2.537 ı	2.852 g	2.314D	0.550 l	0.733 j	1.639 e	0.974 D
20	1.922 k	3.027 f	3.262 e	2.737 C	0.611 k	0.809 ı	1.850 c	1.090 C
40	2.240 j	3.499 d	3.761 c	3.166 B	1.053 h	1.250 g	2.131 b	1.478 B
80	2.630 h	4.073 b	4.467 a	3.723 A	1.545 f	1.774 d	2.355 a	1.891 A
Mean	2.086 C	3.284 B	3.585 A		0.940 C	1.141 B	1.994 A	

*The differences between the interaction potassium and Irrigation means having different lower case letters in a column are statistically significant at 0.05. **The differences between the potassium means having different capital letters in a column are statistically significant at 0.05. The differences between the irrigation means having different capital letters in a line are statistically significant at 0.05.

According to the results of variance analysis, the effect of irrigation x potassium interaction shoot and root calcium (Ca) and phosphorus (P) content were statistically significant ($P \leq 0.01$). In addition, it was determined that the effect of irrigation and potassium applications on shoot and root calcium (Ca) and phosphorus (P) content were statistically significant (Table 4).

Table 4. Mean squares for shoot and root Ca and P content

Source of Variation	Df	Shoot Ca content	Root Ca content	Shoot P content	Root P content
Irrigation	2	0.007457**	0.000044**	0.086119**	0.067584**
Potassium (K)	3	0.000424**	0.000091**	0.202020**	0.265640**
Irrigation * K	6	0.000427**	0.000028**	0.014816**	0.012924**
Error	22	0.000002	0.000001	0.000159	0.000016

** Indicates significant difference at $P \leq 0.01$. Df: Degrees of freedom.

Shoot calcium content increased with the increase of irrigation levels to 0.033, 0.077 and 0.092% respectively. Shoot calcium content increased with potassium applications up until the 40 mg kg^{-1} potassium application (Table 5). The lowest shoot calcium content (0.026%) was obtained at the 33% irrigation level and 40 mg kg^{-1} potassium application, the highest shoot calcium content (0.113%) at the 100% irrigation level and 20 mg kg^{-1} potassium application (Table 5). When the root calcium content is considered, it is observed that root calcium content increases with increasing irrigation levels. The lowest root calcium content (0.006%) was obtained at the 33% irrigation level and 40 and 80 mg kg^{-1} potassium application, the highest root calcium content (0.021%) at the 100% irrigation level and 20 mg kg^{-1} potassium application (Table 5).

Table 5. Mean values of shoot and root Ca content

	Irrigation (Field Capacity)				**Irrigation (Field Capacity)**			
K (mg kg^{-1})	**33%**	**66%**	**100%**	**Mean**	**33%**	**66%**	**100%**	**Mean**
	Shoot Ca content (%)				**Root Ca content (%)**			
10	0.036 f	0.076 cde	0.072 e	0.062 C	0.011 de	0.011 de	0.011 de	0.011 B
20	0.037 f	0.079 cd	0.113 a	0.076 A	0.017 b	0.013 cd	0.021 a	0.017 A
40	0.026 g	0.080 c	0.075 cde	0.060 C	0.006 f	0.014 c	0.010 e	0.010 C
80	0.033 f	0.075 de	0.107 b	0.071 B	0.006 f	0.012 de	0.013 cd	0.010 BC
Mean	0.033 C	0.077 B	0.092 A		0.010 C	0.012 B	0.014 A	

*The differences between the interaction potassium and Irrigation means having different lower case letters in a column are statistically significant at 0.05. **The differences between the potassium means having different capital letters in a column are statistically significant at 0.05. The differences between the irrigation means having different capital letters in a line are statistically significant at 0.05.

Shoot phosphorus content increased with the increase of potassium application to 1.179, 1.432, 1.456 and 1.552% respectively (Table 6). The lowest shoot phosphorus content (1.138%) was obtained at the 33% irrigation level and 10 mg kg^{-1} potassium application, the highest shoot phosphorus content (1.607%) at the 33% irrigation level and 40 mg kg^{-1} potassium application (Table 6). When the root phosphorus content is considered, it is observed that root phosphorus content decreases with increasing irrigation levels. The lowest root phosphorus content (0.346%) was obtained at the 100% irrigation level and 20 mg kg^{-1} potassium application, the highest root phosphorus content (0.903%) at the 33% irrigation level and 40 mg kg^{-1} potassium application (Table 6).

Table 6. Mean values of shoot and root P content

K (mg kg^{-1})	Irrigation (Field Capacity)				Irrigation (Field Capacity)			
	33%	**66%**	**100%**	**Mean**	**33%**	**66%**	**100%**	**Mean**
	Shoot P content (%)				Root P content (%)			
10	1.138 f	1.145 f	1.254 e	1.179 D	0.440 g	0.396 h	0.372 ı	0.402 D
20	1.496 bc	1.315 d	1.485 c	1.432 C	0.608 f	0.446 g	0.346 j	0.467 C
40	1.607 a	1.237 e	1.524 b	1.456 B	0.903 a	0.710 c	0.692 d	0.768 A
80	1.597 a	1.498 bc	1.562 a	1.552 A	0.734 b	0.628 e	0.734 b	0.699 B
Mean	1.460 A	1.299 B	1.456 A		0.671 A	0.545 B	0.536 C	

*The differences between the interaction potassium and Irrigation means having different lower case letters in a column are statistically significant at 0.05. **The differences between the potassium means having different capital letters in a column are statistically significant at 0.05. The differences between the irrigation means having different capital letters in a line are statistically significant at 0.05.

According to the results of variance analysis, the effect of irrigation x potassium interaction shoot and root Na/K and Na/K ratio were statistically significant ($P \leq 0.01$). In addition, it was determined that the effect of irrigation and potassium applications on shoot and root Na/K and Na/K ratio were statistically significant (Table 7).

Table 7. Mean squares for shoot and root Na/K and Na/K ratio

Source of Variation	Df	Shoot Na/K ratio	Root Na/K ratio	Shoot Na/Ca ratio	Root Na/K ratio
Irrigation	2	0.015103**	0.000556**	18.9274**	1.55149**
Potassium (K)	3	0.000794**	0.000598**	1.5771**	1.32691**
Irrigation * K	6	0.000134**	0.000114**	0.7987**	0.47641**
Error	22	0.000001	0.000002	0.0070	0.02270

** Indicates significant difference at $P \leq 0.01$. Df: Degrees of freedom.

Shoot Na/K ratio increased with the increase of irrigation levels to 0.030, 0.115 and 0.093 respectively. Shoot Na/K ratio decreased with potassium applications (Table 8). The lowest shoot Na/K ratio (0.020) was obtained at the 33% irrigation level and 80 mg kg^{-1} potassium application, the highest shoot Na/K ratio (0.125) at the 66% irrigation level and 40 mg kg^{-1} potassium application (Table 8). When the root Na/K ratio is considered, it is observed that root Na/K ratio decreases with increasing irrigation levels and potassium applications. The lowest root Na/K ratio (0.006) was obtained at the 100% irrigation level and 40 and 80 mg kg^{-1} potassium application, the highest root Na/K ratio (0.040) at the 33% irrigation level and 10 mg kg^{-1} potassium application (Table 8).

Table 8. Mean values of shoot and root Na/K ratio

	Irrigation (Field Capacity)				Irrigation (Field Capacity)			
K (mg kg^{-1})	**33%**	**66%**	**100%**	**Mean**	**33%**	**66%**	**100%**	**Mean**
	Shoot Na/K ratio				**Root Na/K ratio**			
10	0.050 f	0.123 a	0.102 c	0.092 A	0.040 a	0.028 b	0.010 def	0.026 A
20	0.027 g	0.109 b	0.101 c	0.079 B	0.021 c	0.018 c	0.008 ef	0.016 B
40	0.023 gh	0.125 a	0.093 d	0.080 B	0.013 d	0.010 de	0.006 ef	0.010 C
80	0.020 h	0.103 c	0.077 e	0.067 C	0.009 def	0.008 ef	0.006 f	0.008 D
Mean	0.030 C	0.115 A	0.093 B		0.021 A	0.016 B	0.007 C	

*The differences between the interaction potassium and Irrigation means having different lower case letters in a column are statistically significant at 0.05. **The differences between the potassium means having different capital letters in a column are statistically significant at 0.05. The differences between the irrigation means having different capital letters in a line are statistically significant at 0.05.

Shoot Na/Ca ratio increased with the increase of irrigation levels to 1.786, 4.841 and 3.706 respectively (Table 9). The lowest shoot Na/Ca ratio (1.408) was obtained at the 33% irrigation level and 20 mg kg^{-1} potassium application, the highest shoot Na/Ca ratio (5.609) at the 66% irrigation level and 80 mg kg^{-1} potassium application (Table 9). When the root Na/Ca ratio is considered, it is observed that root Na/Ca ratio decreases with increasing irrigation levels (Table 9). The lowest root Na/Ca ratio (0.676) was obtained at the 100% irrigation level and 20 mg kg^{-1} potassium application, the highest root Na/Ca ratio (2.269) at the 33% irrigation level and 80 mg kg^{-1} potassium application (Table 9).

Table 9. Mean values of shoot and root Na/Ca ratio

	Irrigation (Field Capacity)				Irrigation (Field Capacity)			
K (mg kg^{-1})	**33%**	**66%**	**100%**	**Mean**	**33%**	**66%**	**100%**	**Mean**
	Shoot Na/Ca ratio				**Root Na/Ca ratio**			
10	2.148 f	4.088 c	4.036 c	3.424 B	1.993 a	1.833 ab	1.416 bc	1.748 A
20	1.408 g	4.185 c	2.906 e	2.833 C	0.776 e	1.101 cde	0.676 e	0.851 C
40	2.030 f	5.484 a	4.641 b	4.052 A	2.177 a	0.881 de	1.333 c	1.464 B
80	1.558 g	5.609 a	3.240 d	3.469 B	2.269 a	1.248 cd	1.061 cde	1.526 B
Mean	1.786 C	4.841 A	3.706 B		1.804 A	1.266 B	1.122 B	

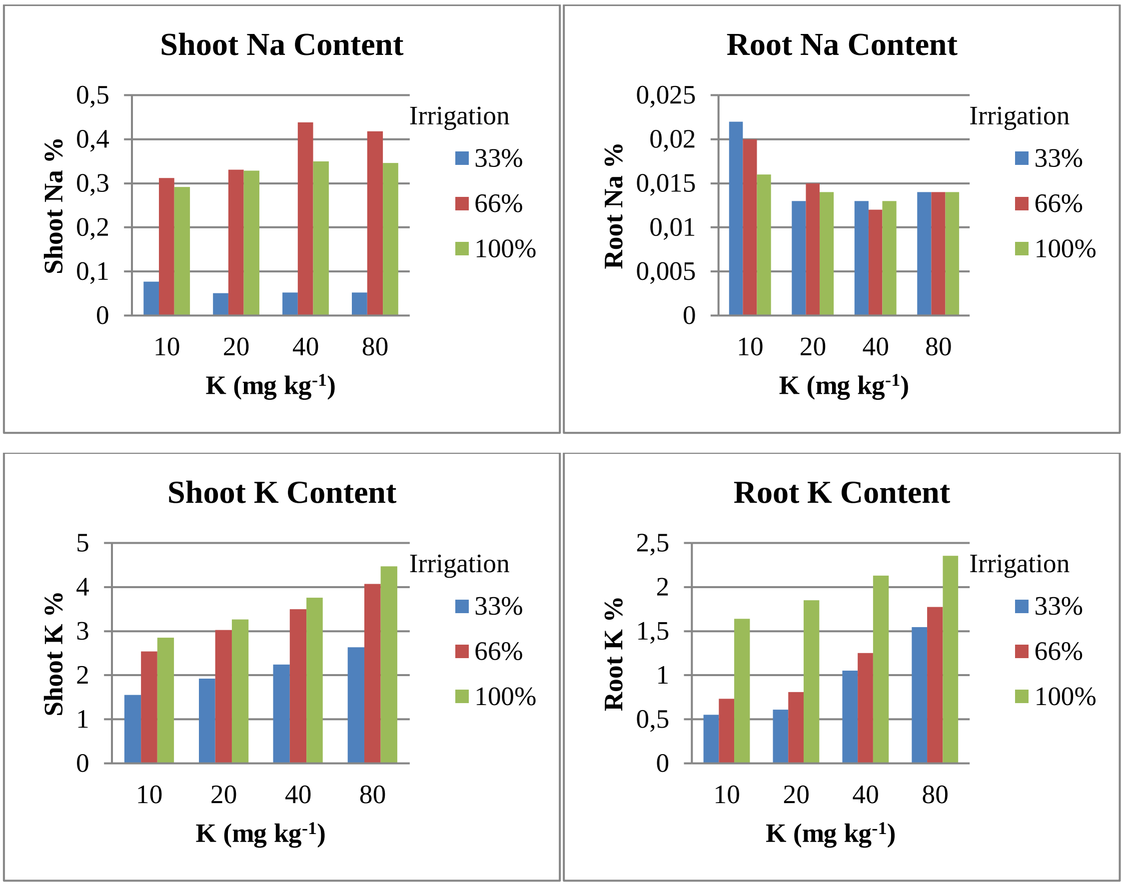
*The differences between the interaction potassium and Irrigation means having different lower case letters in a column are statistically significant at 0.05. **The differences between the potassium means having different capital letters in a column are statistically significant at 0.05. The differences between the irrigation means having different capital letters in a line are statistically significant at 0.05.

5. DISCUSSION

Sodium and potassium ions are very important in sugar beet cells for osmotic adjustment they are major vacuolar osmolyte factors (Choluj et al. 2008). According to the results, shoot sodium content has increased with increasing irrigation levels (Figure 1). Shoot sodium content is lower in stressed plants than plants grown under normal conditions. Contrarily root sodium content decreased with irrigation levels. Wu et. al., (2013) stated in his study that drought condition decreased shoot and root sodium content the results obtained were in partly compatible with this study. In Raza's study (2013) drought increased the sodium content in contrast to this study drought decreased shoot sodium content in our study (Figure 1). According to Abd-El-Motagally et.al., (2004) sodium is translocated to the shoots, where it replaces potassium in various metabolic functions in sugar beet. It has been suggested that sugar beet plants increase potassium and sodium content in the leaves than in the roots under drought stress for protects itself (Ghoulam et al., 2002; Li et. al., 2009).When we compare the shoot and root sodium contents, it is seen that there is more sodium in the shoot than the root (Table 2).

When Table 2 and Figure 1 are examined, it is seen that shoot sodium content increases with increasing potassium application which is not in parallel with previous studies. According to Bee et al.,(1997), potassium did not affect sodium content of sugar beet and on the other hand according to many researchers, potassium application decreased sodium content of sugar beet (Huijbregts et. al., 1996; Turhan and Piskin, 2005; Mubarek et.al., 2016). In this study root sodium content decreased with potassium application (Table 2) in parallel with previous studies. When results are examined in terms of the interaction between irrigation levels and potassium the lowest shoot sodium content (0.51%) was obtained at the 33% irrigation level and 20 mg kg^{-1} potassium application, the highest shoot sodium content (0.438%) at the 66% irrigation level and 40 mg kg^{-1} potassium application (Table 2). The lowest root sodium content (0.012%) was obtained at the 66% irrigation level and 40 mg kg^{-1} potassium application, the highest root sodium content (0.022%) at the 33% irrigation level and 10 mg kg^{-1} potassium application (Table 2).

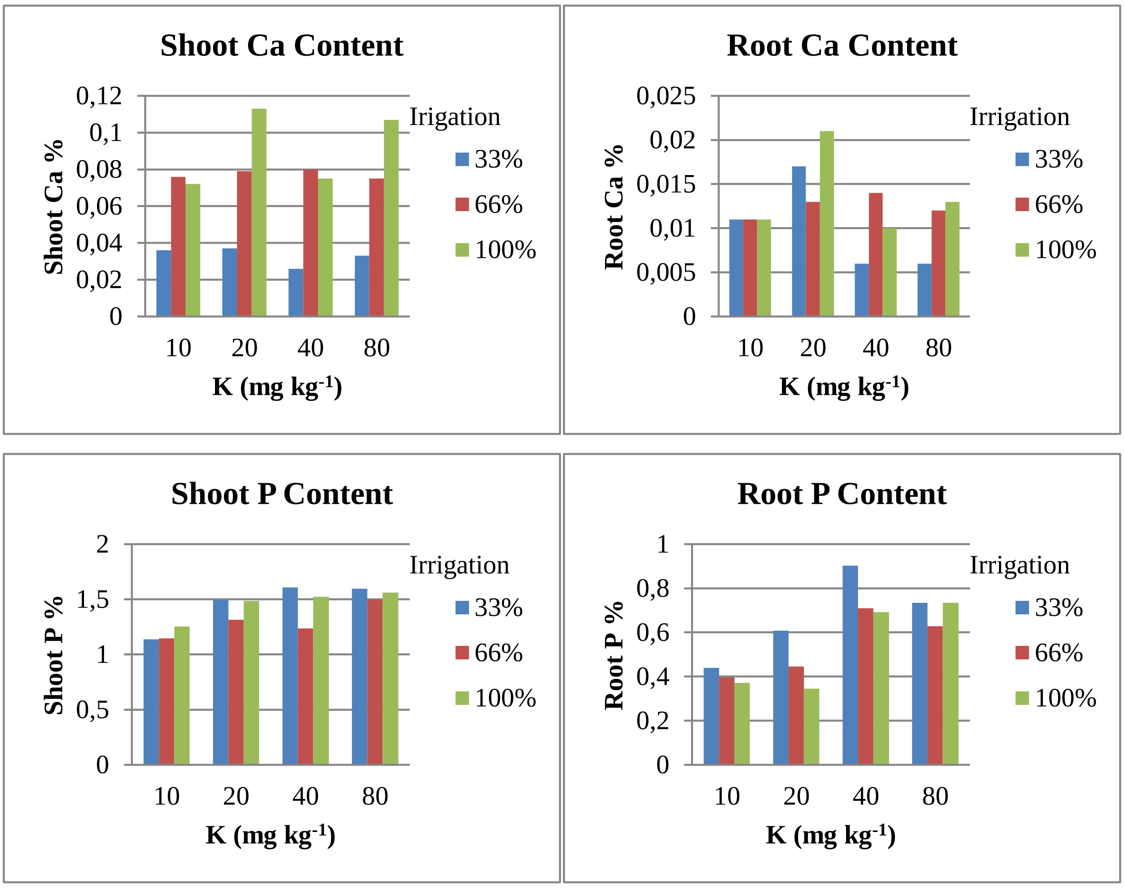
Figure 1. Shoot and root sodium (Na) and potassium (K) content (%) changes



According to the results, shoot and root potassium content has increased with increasing irrigation levels (Table 3). In this study, the highest shoot and root potassium content (3.585% and 1.994% respectively) was obtained at the 100% irrigation level, while the lowest (2.086% and 0.940% respectively) was obtained at the 33% irrigation level. According to Hu and Schimidhalter (2005) the lack of water under salt and drought stress conditions causes reduces the turgor pressure thus the potassium concentration reduce under these stress conditions. Previous studies have emphasized that potassium accumulation is very important under drought and salt stress condition for protect osmotic adjustment (Ashraf et. al., 2003). Contrary to our study plants can absorb 2-3 times of potassium under stress (Nejad et. al., 2010). Increasing the dose of potassium application resulted in significant increase shoot and root potassium content (Figure 1). Similarly our results other researcher (Huijbregts et. al., 1996; Bee et al.,1997; Mubarek et. al., 2016; Abd-El-Motagally 2004) stated that plant potassium content increased with potassium application dose.

When results are examined in terms of the interaction between irrigation levels and potassium the lowest shoot potassium content (1.552%) was obtained at the 33% irrigation level and 10 mg kg^{-1} potassium application, the highest shoot potassium content (4.467%) at the 100% irrigation level and 80 mg kg^{-1} potassium application (Table 3). The lowest root potassium content (0.550%) was obtained at the 33% irrigation level and 10 mg kg^{-1} potassium application, the highest root potassium content (2.355%) at the 100% irrigation level and 80 mg kg^{-1} potassium application (Table 3). According to Ashraf (1998) potassium application increased potassium content of plan under drought condition.

Figure 2. Shoot and root calcium (Ca) and phosphorus (P) content (%) changes



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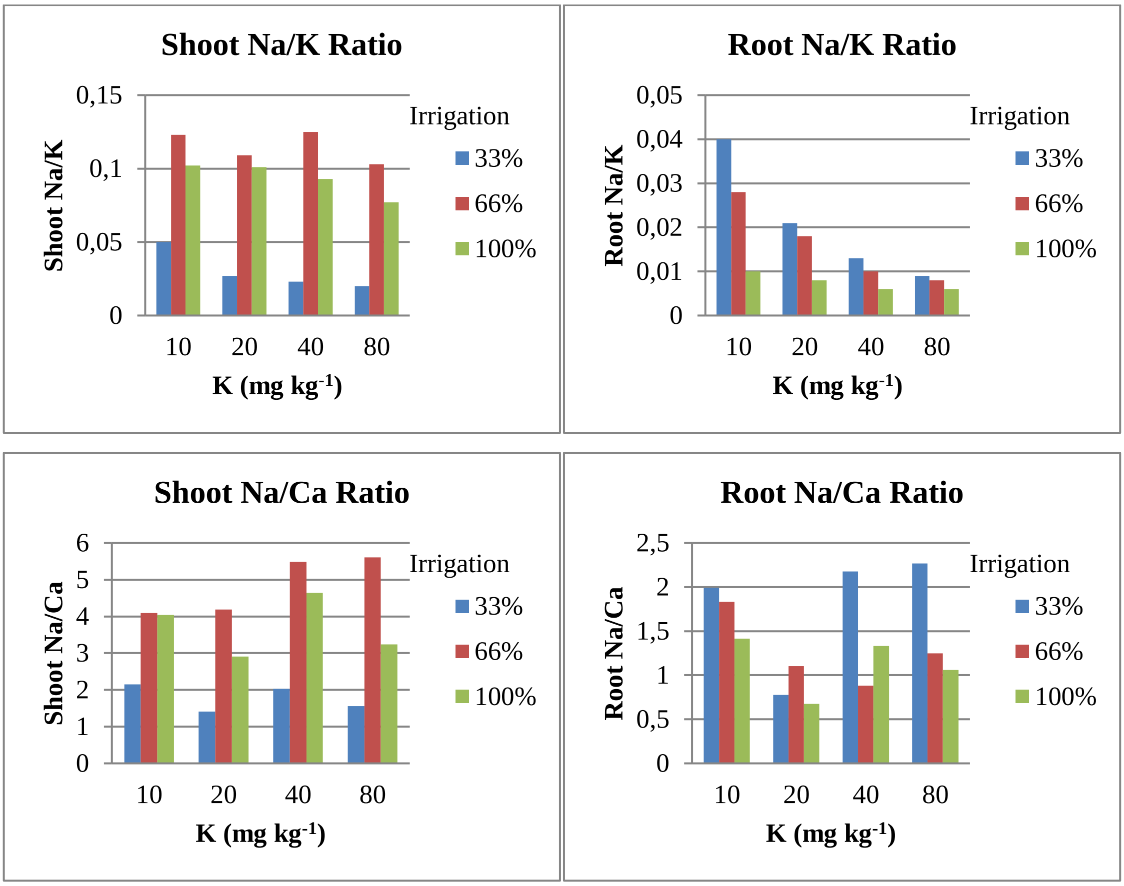
Calcium is a macronutrient and absolutely necessary for plant growth. According to the results, shoot and root calcium content has increased with increasing irrigation levels (Figure 2). Shoot and root calcium content is lower in stressed plants than plants grown under normal conditions. According to Abdalla and El-Khohiban (2007), drought causes a decrease in calcium concentration in wheat plants. Kusvuran (2010) stated in her study that drought condition decreased plant calcium content depending on the duration of stress and plants increase calcium content in the leaves than in the roots under drought stress for ion balance. When we compare the shoot and root calcium contents in our study, it is seen that there is more calcium in the shoot than the root (Table 5). In this study contrary to some studies shoot calcium content increased under drought condition (Wu et.al., 2013). According to Abd-El-Motagally (2004) shoot and root calcium content of sugar beet decreased by increasing potassium our results are partially parallel with this study. Shoot and root calcium content increased in parallel with the increasing potassium doses while it decreased at 40 mg kg^{-1} (Figure 2).

Other researcher stated before that plants can uptake more phosphorus in under drought condition (Khondakar et al. 1983) and our results in parallel with this study. Shoot and root phosphorus content decreased with the increase of irrigation levels to 1.460, 1.299, 1.456% and 0.671, 0.545, 0.536% respectively. It has been suggested that potassium application increased wheat phosphorus uptakes (Raza et. al., 2013). Similar results were obtained in our study shoot and root phosphorus content increased with the increase of potassium application (Table 6).

Figure 3. Shoot and root Na/K and Na/Ca ratio changes



Shoot Na/K ratio increased with the increase of irrigation levels to 0.030, 0.115 and 0.093 respectively (Table 8). According to Al-Jbawi and Abbas (2013) depend on the duration of stress Na decreases while K increases, as a result Na/K ratio decreases. It has been suggested that root Na/K ratio significantly increased under drought (Wu et. al., 2013) and our results not in parallel with this study. When the root Na/K ratio is considered, it is observed that root Na/K ratio decreases with increasing irrigation levels. It has been emphasized that the impaired intracellular electrolyte balance is improved, the amount of potassium to compete with sodium is increasing and the disturbed intracellular Na/K balance is readjusted by additional potassium (Kabay and Sensoy, 2017). Our results are consistent with previous studies it is observed that root Na/K ratio decreases with increasing potassium applications (Figure 3).

Plants prefer calcium instead of sodium under stress condition to balance ion regulation and osmotic pressure for increase the tolerance level. Therefore Na/Ca ratio is an important parameter under stress (Koc, 2005; Dasgan et. al., 2006). Under stress, sodium replaces calcium in the cell membrane, thereby Na/Ca ratio in the apoplast increasing (Kaya and Tuna, 2010). Drought increased shoot Na/Ca ratio while reducing root Na/Ca ratio (Wu et. al., 2013). Contrary to this study shoot Na/Ca ratio increased with the increase of irrigation levels to 1.786, 4.841 and 3.706 respectively (Table 9) while the root Na/Ca ratio decreases with increasing irrigation levels. The lowest root Na/Ca ratio (0.676) was obtained at the 100% irrigation level and 20 mg kg^{-1} potassium application, the highest root Na/Ca ratio (2.269) at the 33% irrigation level and 80 mg kg^{-1} potassium application (Table 9).

6. CONCLUSION

According to the results of variance analysis, the effect of irrigation x potassium interaction on the shoot and root sodium (Na) potassium (K) calcium (Ca) and phosphorus (P) content Na/K and Na/K ratio was found to be statistically significant.

Shoot and root sodium content decreased with potassium applications under drought conditions (33%). Shoot and root potassium content increased with potassium applications in both drought and sufficient water conditions. Shoot calcium content change irregular with potassium application while root calcium decreased with potassium application under drought conditions (33%). Shoot phosphorus content increased with potassium applications in both drought and sufficient water conditions. Root phosphorus content increased with potassium applications up until the 40 mg kg^{-1} potassium application under drought condition. Shoot and root sodium/potassium ratio decreased with potassium applications in both drought and sufficient water conditions. Shoot and root sodium/calcium ratio change irregular with potassium applications.

In summary, applying potassium to the plants under drought stress led to an increase in the shoot and root potassium and phosphorus content while decrease in the shoot and root sodium, sodium/potassium ratio and root calcium. Thus, it can be said that potassium may play a critical role in reducing the negative effects of drought stress in sugar beet. Therefore, it is thought that keeping the K nutrition at a sufficient level for the plants grown in the regions where irrigation may be a problem can be beneficial in reducing the damage of drought stress.

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REFERENCES

ABDALLA, M.M., EL-KHOSHIBAN, N.H., 2007. The Influence of Water Stress on Growth, Relative Water Content, Photosynthetic Pigments, Some Metabolic and Hormonal Contents of Two Triticium aestivum Cultivars. Journal of Applied Sciences Research, 3(12): 2062-2074.

ABD-EL-MOTAGALLY, F. M. F., 2004, Evaluation of two sugar beet cultivars (*Beta vulgaris* L.) for growth and yield under drought and heat conditions, A thesis for PhD. Institute of Plant Nutrition, University Giessen, Germany, 143 pp.

ADILOGLU, A. and GULER, M., 2002, Tekirdag-hayrabolu yöresinde yetiştirilen seker pancarının *(Beta Vulgaris L.)* beslenme durumunun belirlenmesi. S. Ü. Ziraat Fakültesi Dergisi, 16 (29):26–30.

AL-JBAWI, E. and ABBAS, F., 2013, The Effect of Length during Drought Stress on Sugar Beet (*Beta vulgaris* L.) Yield and Quality, Persian Gulf Crop Protection ISSN: 2251-9343, Volume 2 Issue 1, Pages 35-43.

ASHRAF, M., 1998, Yield and yield components response of wheat (*Tritcium aestivum* L.) genotypes grown under different soil water deficit conditions, Acta Agron. Hung., 46: 45-51.

ASHRAF, M., ARFAN, M., AHMAD, A., 2003, Salt Tolerance in Okra: Ion Relations and Gas Exchanges Characteristics, Journal of Plant Nutrition, 26 (1): 63-79.

BEE, P.M., JARVIS, P.J., ARMSTRONG, M.J. 1997, The Effect of potassium and sodium fertiliser on sugar beet yield and quality, Procedings of the 60 th International Institute for Beet Research Congress, July, Cambridge (UK).

CHOLUJ, D., KARWOWSKA, R, CISZEWSKA, A, JASINSKA, M., 2008, Influence of long-term drought stress on osmolyte accumulation in sugar beet (*Beta vulgaris* L.) plants. Acta Physiol. Plant, 30, 679–687.

DASGAN, H.Y., KOÇ, S., EKICI, B., AKTAŞ, H., ABAK, K., 2006. Bazı Fasulye ve Börülce Genotiplerinin Tuz Stresine Tepkileri. Alatarım, 5(1): 23-31.

GHOULAM C., FOURSY, A., FARES, K., 2002, Effects of salt stress on growth, inorganic ions and proline accumulation in relation to osmotic adjustment in five sugar beet cultivars, Environ Exp Bot 47:39–50.

HU, Y.C., SCHMIDHALTER, U., 2005. Drought and salinity: A comparison of their effects on mineral nutrition of plants, J Plant Nutr Soil Sci 168: 541-549.

HUIJBREGTS, A.W.M., GLATTKOWSKI, H., HOUGHTON, B.J. and HADJIANTONIOU, D., 1996, Effect of agronomic factors on parametersused in formulas to estimate extractable sugar in sugar beets, Institut Internationalde Recherches Betteravieres Proceedings, 353-368.

JONES Jr., J.B., WOLF, B. and MILLS, H.A., 1991. Plant Analysis Handbook: A Practical Sampling, Preparation, Analysis, and Interpretation Guide. Micro-Macro Publishing, Athens.

KABAY, T., SENSOY, S., 2017, Enzyme, chlorophyll and ion changes in some common bean genotypes by high temperature stress. Ege Üniversitesi Ziraat Fakültesi Dergisi, 54(4): 429-437.

KADIOGLU, M., Türkiye'de İklim Değişikliği Risk Yönetimi. Türkiye'nin Birleşmiş Milletler İklim Değişikliği Çerçeve Sözleşmesi'ne İlişkin İkinci Ulusal Bildirimi Hazırlık Faaliyetlerinin Desteklenmesi Projesi. T.C. Çevre ve Şehircilik Bakanlığı, Birleşmiş Milletler Kalkınma Programı (UNDP), 2012.

KAYA, C., TUNA, A. L., 2010. Potasyumun tuz stresinde yetişen bitkilerde rolü ve önemi. http://www.ipipotash.org

KHONDAKAR, Z.H., ASLAM, A., REHMAN, S., KHAN, T.H., 1983, Influence of soil moisture stress on yield, grain quality, availability and uptake of N, P and K by wheat, Int. J. Tropical Agric. 1, 211-220.

KIZIL, U., AKSU, S. and CAMOGLU G., 2018, Kontrollü ortamda bitkisel yetiştiricilik için arduino uyumlu bir toprak nemi izleme sistemi tasarımı, Comü Ziraat Fakültesi Dergisi, vol.6:131-139.

KOC, S., 2005. Fasulyelerde Tuzluluğa Tolerans Bakımından Genotipsel Farklılıkların Erken Bitki Gelişimi Aşamasında Belirlenmesi, Çukurova Üniversitesi, Fen Bilimleri Enst., Yüksek Lisans Tezi.

KUSVURAN, S., 2010, Kavunlarda Kuraklık ve Tuzluluğa Toleranslı Fizyolojik Mekanizmaları Arasındaki Bağlantılar, Çukurova Üniversitesi, Fen Bilimleri Enstitüsü, Doktora tezi, Adana.

LI, Y., 2009. Physiological Responses of Tomato Seedlings (*Lycopersicon esculentum)* to Salt Stres. Modern Applied Science, 3 (3): 171-176.

MUBARAK, M.U., ZAHIR, M., AHMAD, S., WAKEEL, A., 2016, Sugar beet yield and industrial sugar contents improved by potassium fertilization under scarce and adequate moisture conditions, Journal of Integrative Agriculture 2016, 15(11): 2620–2626.

NEJAD, T. S., BAKHSHANDE, A. and JAZAYERI, A. 2010, Calculated linear regression equations of motion K^+ and Na^+ ions and compare moving process these elements in corn roots, Report and Opinion, 2(3): 15-22.

RAZA SAMAR, M. A., FARRUKH SALEEM, M., MUSTAFA SHAH, G., JAMIL M. and HAIDER KHAN, I., 2013, Potassium applied under drought improves physiological and nutrient uptake performances of wheat (*Triticum Aestivun* L.), Journal of Soil Science and Plant Nutrition, 2013, 13(1), 175-185.

TURHAN, M. and PISKIN, A, 2005, Farklı dozlarda uygulanan potasyumun şeker pancarının verim ve kalitesine etkisi, Potassium and its Importance in Agriculture Symposium. Eskisehir. Turkey.

WILHITE, D.A., 2000, Drought: A Global Assessment. Natural Hazards and Disasters Series. London: Routledge Publishers.

WU, G., LIANG, N., FENG, R., and ZHANG, J., 2013, Evaluation of salinity tolerance in seedlings of sugar beet (*Beta vulgaris* L.) cultivars using proline, soluble sugars and cation accumulation criteria, Acta Physiologiae Plantarum 35:2665–2674.

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Research Article

CHIRP-ENGINEERING SEISMIC METHOD FOR EXPLORING SEABED AND UNDERWATER STRUCTURES: OFF-SHORE WESTERN ANATOLIA[1]

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ABSTRACT

CHIRP systems are widely used in seabed sediment classification, submarine faults, positioning of marine engineering structures, pipeline geotechnical studies, platform and well area assessments, archaeological and environmental impact assessments. The resolution of the system is in the order of decimeter. In this study, the characteristics of submarine active faults, buried faults, seabed and underlying layers in the region were analyzed and interpreted by CHIRP data collected off-shore Seferihisar, Teke Peninsula and Alaçatı.

Keywords: *Submarine Active Faults, Buried Faults, CHIRP-Engineering Seismic.*

[1] *This study is the revised form of the manuscript, presented at "3rd International Conference on Awareness"on 5 - 7 December 2019, Çanakkale / TURKEY*

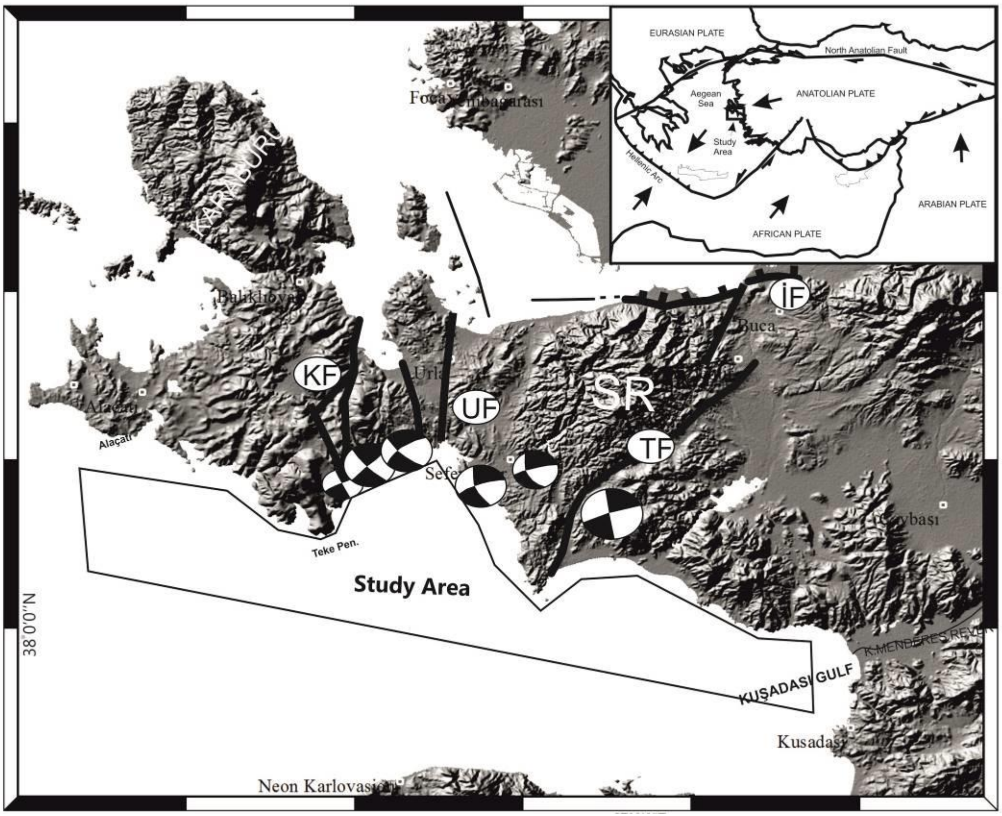
1. INTRODUCTION

Active faults and similar structures under the sea generally protect their structural features better since they are not exposed to negative external factors that occur over time as much as those on land. In addition, the seawater layer on it allows high resolution marine seismic methods (CHIRP, multi-beam bathymetry, side scan sonar etc.) to be applied.

This study covers off-shore Seferihisar, Teke Peninsula and Alaçatı, south of Izmir Bay. The study area and its surroundings are an important region in terms of active faults both on land and submarine. Karaburun Fault, Tuzla (Orhanlı) Fault, Urla Fault and İzmir Fault are the most important faults in the terrestrial area around the study area (Figure 1). All these faults can be traced on land to Kuşadası Bay and the surrounding coasts. There are a few previous studies involving the study area and its surroundings, using marine seismic methods. Aksu et al. (1987; 1990), mapped normal faults using the shallow marine seismic study carried out in İzmir Bay, Kuşadası Bay and its surroundings. In the marine seismic study carried out in the same area (Ocakoğlu et al., 2004; 2005), it was stated that the active tectonics of the study area is developing under the control of strike-slip faults.

Within the scope of this study, it has been tried to reveal how effective the method is in determining and analyzing active faults, buried faults and layers under the sea, as a result of processing and interpretation of CHIRP high resolution marine seismic reflection data collected off-shore Seferihisar, Teke Peninsula and Alaçatı.

Figure 1. Map of the area where the study area and the seismic profiles are located, active faults (Ocakoğlu et al., 2004; 2005) and focus mechanism of strong earthquakes (Benetatos et al., 2006, Tan and Taymaz 2001; Tan and Taymaz 2003). UF: Urla Fault, KF: Karaburun Fault, IF: İzmir Fault, TF: Tuzla Fault. The inner map shows the regional tectonics of the Anatolian microplate.



2. METHOD

During operation, high resolution engineering seismic (CHIRP) data was collected on the starboard side of the ship, with a 3.5 kHz Bathy 2010 CHIRP system. Photos and schematic representation of the CHIRP system are given in Figure 2 and Figure 3. The signal emitted from the transducer is reflected from the seabed and the units underneath, and it is perceived by the transducer in the same way (Figure 2 and Figure 3).

The vertical resolution of CHIRP systems is dependent upon the bandwidth of the source. For example a 2–8 kHz source equates to a theoretical vertical resolution of 0.125 m, assuming compressional wave velocity of 1500 m s^{-1}. The horizontal resolution of CHIRP systems is primarily dependent upon the source characteristics (beam angle, dominant frequency), compressional wave velocity of the sediments, towfish altitude and pulse rate of the system; with characteristic horizontal resolutions of 1 to 2 m (Quinn et al., 1998). For the processing of digitally recorded CHIRP data, Quinn et al. (1998) suggested a flow chart. Accordingly, processing of CHIRP data is carried out in 2 stages. In the 1st stage, correlation and deconvolution and in the 2nd stage, filtering operations are performed.

Figure 2. CHIRP high resolution engineering seismic recorder and transducers

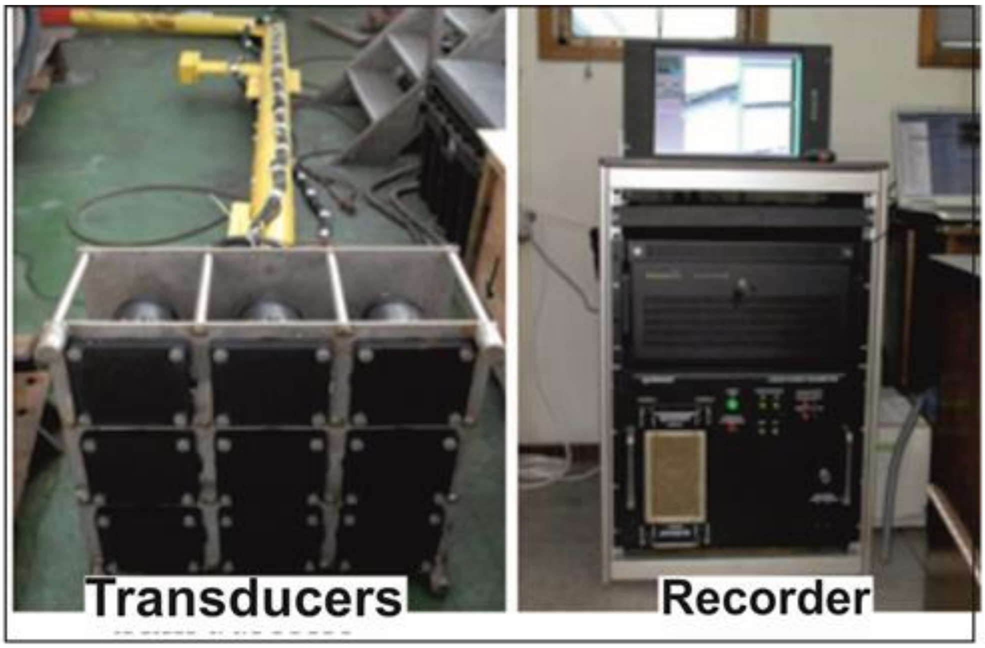
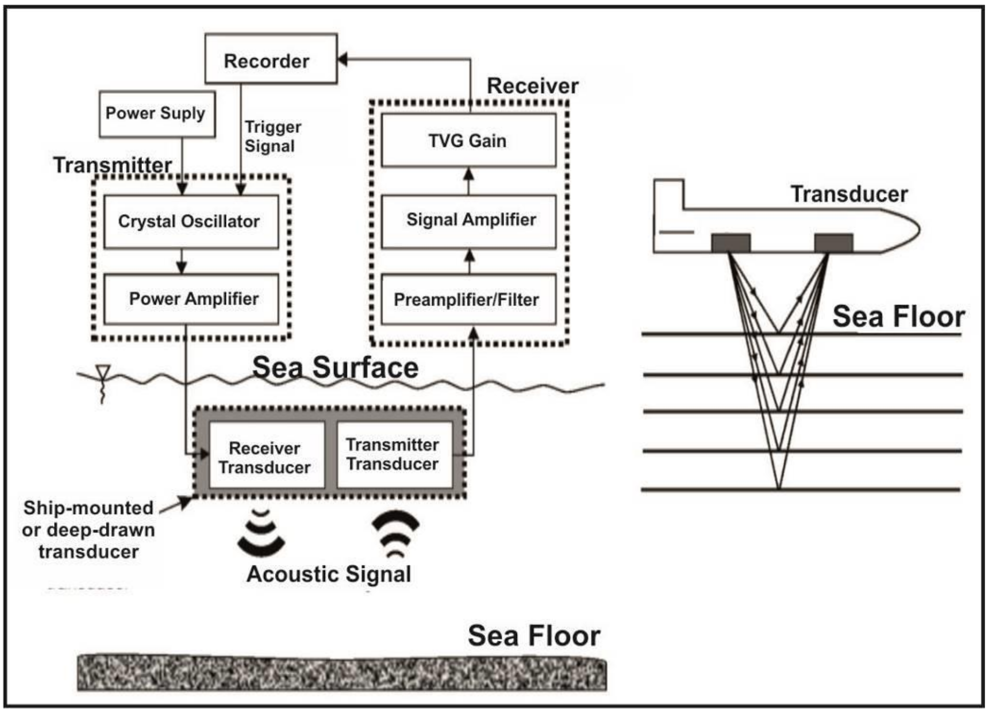


Figure 3. Block structure and working principle of high resolution engineering seismic systems (Çifçi et al., 2005)

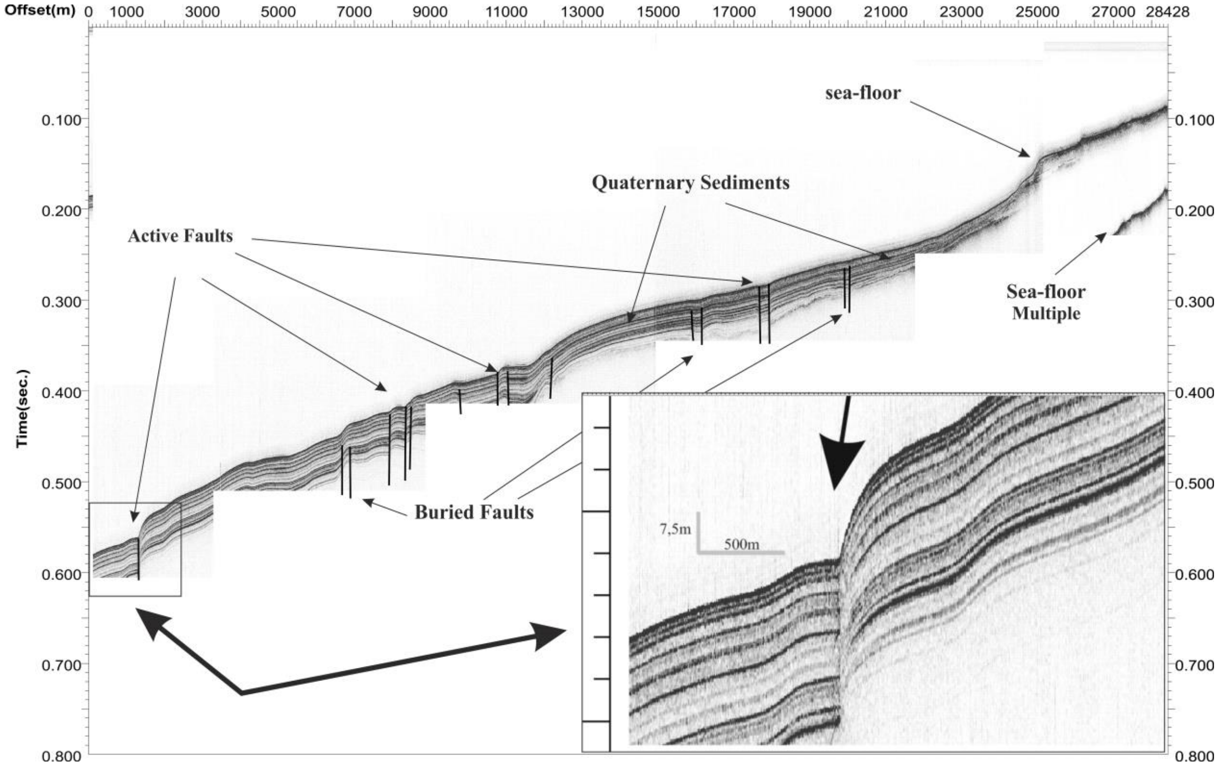


3. INTERPRETATION OF CHIRP DATA

From analyzed high resolution CHIRP marine seismic data the seabed, layers under the seabed, active faults cutting the layers, buried faults, slip distance on active faults can be clearly distinguished. In determining active and buried faults in sections, it is taken into account whether the layers under the sea are cut along these faults to the seabed. Just below the sea water column, the sea floor consisting of the young sediments and the faults reaching very close are interpreted as active fault in this study. Faults that could not reach the seabed were evaluated as buried faults (Figure 4).

Figure 4 shows a CHIRP section obtained from the study area. Here, it is seen that the sea deepens from the right to the left of the section. The young and Quaternary sediments that form the sea floor and the underlying layers show a parallel and uniform sedimentation. When the section is analyzed in terms of active faults, it is observed that these faults cut structural layers and sea-floor at the bottom of the sea and below. It is understood that the most important reason for these structural changes is due to the vertical slip on the active faults here (Figure 4). In Figure 4, it is seen that the location of the largest slip of the active faults is approximately at 1500m offset to the left of the section and 550 milliseconds two-way traveltime depth.

Figure 4. Active faults, buried faults and Quaternary Deposits observed on the CHIRP-Marine Engineering Seismic section obtained from the study area.



4. CONCLUSIONS

In this study, as a result of processing and interpretation of CHIRP data collected in Seferihisar, Tekeburnu and Alaçatı, the characteristics of submarine active faults, buried faults, seabed and underlying layers were investigated. Quaternary sediments generally show a parallel and regular sedimentation structure, while active and buried faults seem to cause lateral discontinuities and generally vertical slip in these sediments. In addition, as well as active faults, buried faults caused structural changes in the sea floor. This suggests that these buried faults may still keep their activity.

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REFERENCES

AKSU, A.E., PİPER, D.J.W. ve KONUK, T., (1987). Late Quaternary tectonic and Sedimentary history of outer İzmir ans Çandarlı bays, western Turkey. *Marine Geophysics,* (76), 89-104.

AKSU, A.E., KONUK, T., ULUĞ, A., DUMAN, M., PİPER, D.J.W., (1990). Quaternary tectonic and sedimentary history of Eastern Aegean Sea shelf area. *Geophysics,* (4), 3-35.

BENETATOS, C., KIRATZI, A., GANAS, A., ZIAZIA, M., PLESSA, A. & DRAKATOS, G. (2006). Strike Slip Motions in the Gulf of Sığacık (western Turkey): Properties of the 17 October 2005 earthquake seismic sequence. *Tectonophysics,* 426, *263-279*

ÇİFÇİ, G., DONDURUR, D., OKAY, S. (2005), *Yüksek ayrımlı deniz jeofiziği yöntemleri*, Jeofizik Mühendisleri Odası

OCAKOĞLU, N., DEMİRBAĞ, E. & KUŞÇU, I. (2004). Neotectonic structures in the area offshore of Alaçati, Doğanbey and Kuşadasi (western Turkey): evidence of strike-slip faulting in the Aegean extensional province. *Tectonophysics, 391*, 67- 83.

OCAKOĞLU, N., DEMIRBAĞ, E. & KUŞÇU, I. (2005). Neotectonic structures in Izmir Gulf and surrounding regions (western Turkey): Evidences of strike-slip faulting with compression in the Aegean extensional regime. *Marine Geology*, *219*, 155-171.

QUINN, R., BULL, J.M. and DIX, J.K. (1998). Optimal Processing of Marine High-Resolution Seismic Reflection (CHIRP) Data, *Marine Geophysical Researches, 20,* 13-20.

TAN, O. & TAYMAZ, T. (2001). Source parameters of November 6, 1992 Doganbey (İzmir) earthquake (Mw=6.0) obtained from inversion of teleseismic body-waveforms. *4th International Turkish Geology Symposium,* 171.

TAN, O. & TAYMAZ, T. (2003). Seismotectonics of Karaburun Peninsula and Kusadası Gulf: source parameters of April 2, 1996 Kusadası Gulf and April 10, 2003 Seferihisar (İzmir) earthquakes. *International Workshop on the North Anatolian, East Anatolian and Dead Sea Fault Systems: Recent Progress in Tectonics and Paleoseismology and Field Training Course in Paleoseismology,* Middle East Technical University, 147, Ankara.