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On A Type Of Faint Continuous Functions

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Abstract - In this paper a new class of functions called strongly faint (τ, μ)-continuous functions has been introduced. Some properties of such functions are studied, furthermore the relationships between strongly faint (τ, μ)-continuous function and its graph are also being investigated here.

Keywords - *μ-open set, θ-open set, strongly faint (τ, μ)-continuous function.*

1 Introduction

For the last one decade or so, a new area of study has emerged and has been rapidly growing. The area is concerned with the investigations of generalized topological spaces and several classes of generalized types of open sets. Our aim here is to study the notion of strongly faint continuous functions by using the concept of generalized topology introduced by *Á*. Cs*á*sz*á*r [7].

We first recall some definitions given in [7]. Let X be a non-empty set and $expX$ denote the power set of X. We call a class $\mu \subseteq expX$ a generalized topology (briefly, GT) [7], if $\varnothing \in \mu$ and μ is closed under arbitrary unions. A set X, with a GT μ on it is said to be a generalized topological space (briefly, GTS) and is denoted by (X, μ). For a GTS (X, μ), the elements of μ are called μ-open sets and the complements of μ-open sets are called μ-closed sets. For $A \subseteq X$, we denote by $c_\mu(A)$ the intersection of all μ-closed sets containing A i.e., the smallest μ-closed set containing A; and by $i_\mu(A)$ the union of all μ-open sets contained in A i.e., the largest μ-open set contained in A (see [5, 7]). A GT μ on X is said to be a quasi topology (briefly QT) [8] if $M, M' \in \mu$ implies $M \cap M' \in \mu$. The pair (X, μ) is said to be a QTS if μ is a QT on X.

Let (X, τ) be a topological space. The δ-closure [24] of a subset A of a topological space (X, τ) is defined as $\{x \in X : A \cap U \neq \varnothing$ for all regular open sets U containing $x\}$, where a subset A is called regular open if $A = int(cl(A))$. A subset A of a topological

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space (X,τ) is called semiopen [12] (resp. preopen [14], α-open [15], β-open [1], b-open [2], δ-preopen [17], δ-semiopen [16], e-open [10]) if $A \subseteq cl(int(A))$ (resp. $A \subseteq int(cl(A))$, $A \subseteq int(cl(int(A)))$, $A \subseteq cl(int(clA))$, $A \subseteq cl(int(A)) \cup int(cl(A))$, $A \subseteq int(cl_\delta(A))$, $A \subseteq cl(int_\delta(A))$, $A \subseteq int(cl_\delta(A)) \cup cl(int_\delta(A)))$. A point $x \in X$ is in $scl(A)$ (resp. $pcl(A)$) if for each semi-open (resp. preopen) set U containing x, $U \cap A \neq \varnothing$. A point $x \in X$ is called a θ-cluster [24] (resp. semi-θ-cluster [13]) point of A denoted by $cl_\theta(A)$ (resp. $scl_\theta(A)$) if $cl(A) \cap U \neq \varnothing$ (resp. $scl(A) \cap U \neq \varnothing$) for every open (resp. semi-open) set U containing x. A subset A is called θ-closed (resp. semi-θ-closed) if $cl_\theta(A) = A$ (resp. $scl_\theta(A) = A$). The complement of a θ-closed (resp. semi-θ-closed) set is called a θ-open (resp. semi-θ-open) set. The family of all θ-open sets in a topological space forms a topology which is weaker than the original topology. For any topological space (X,τ), the collection of all semi-open (resp. preopen, α-open, β-open, b-open, δ-preopen, δ-semiopen, e-open, θ-open, semi-θ-open) sets are denoted by $SO(X)$ (resp. $PO(X), \alpha O(X), \beta O(X), BO(X), \delta PO(X), \delta SO(X), eO(X), \theta O(X), S_\theta O(X)$). We note that each of these collections forms a GT on (X,τ).

It is easy to observe that i_μ and c_μ are idempotent and monotonic, where $\gamma : expX \rightarrow expX$ is said to be idempotent if and only if for each $A \subseteq X$, $\gamma(\gamma(A)) = \gamma(A)$, and monotonic if and only if $\gamma(A) \subseteq \gamma(B)$ whenever $A \subseteq B \subseteq X$. It is also well known from [5, 8] that if μ is a GT on X and $A \subseteq X$, $x \in X$, then $x \in c_\mu(A)$ if and only if for each $M \in \mu$ with $x \in M \Rightarrow M \cap A \neq \varnothing$ and that $c_\mu(X \setminus A) = X \setminus i_\mu(A)$.

Hereafter, throughout the paper we shall use (X,τ) to mean a topological space and (Y,μ), (Y,λ) to be generalized topological spaces unless otherwise stated.

2 Strongly faint (τ,μ)-continuous functions

Definition 2.1. *A function $f : (X,\tau) \rightarrow (Y,\mu)$ is said to be strongly faint (τ,μ)-continuous (resp. (τ,μ)-continuous [19]) if for each $x \in X$ and each μ-open set V of Y containing $f(x)$, there exists a θ-open (resp. open) set U containing x such that $f(U) \subseteq V$.*

Theorem 2.2. *For a function $f : (X,\tau) \rightarrow (Y,\mu)$, the followings are equivalent:*
(i) f is strongly faint (τ,μ)-continuous;
(ii) $f^{-1}(V)$ is θ-open in X for every μ-open set V of Y;
(iii) $f^{-1}(F)$ is θ-closed in X for every μ-closed set V of Y.

Proof: (i) $\Rightarrow$ (ii) : Let V be a μ-open set of Y and $x \in f^{-1}(V)$. Since $f(x) \in V$ and f is strongly (τ,μ)-continuous, there exists a θ-open set U of X containing x such that $f(U) \subseteq V$. It then follows that $x \in U \subseteq f^{-1}(V)$. Hence $f^{-1}(V)$ is θ-open in X.

(ii) $\Rightarrow$ (iii) : Obvious.

(iii) $\Rightarrow$ (i) : Let $x \in X$ and V be a μ-open set of Y containing $f(x)$. Then by (iii), $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is θ-closed in X i.e., $f^{-1}(V)$ is θ-open in X with $x \in f^{-1}(V)$. Thus there exists a θ-open set U containing x such that $f(U) \subseteq V$.

Remark 2.3. *(i) It thus follows from the above theorem that if μ, λ be two GT's on Y such that $\mu \subseteq \lambda$ and if $f : (X,\tau) \rightarrow (Y,\lambda)$ is strongly faint (τ,λ)-continuous then $f : (X,\tau) \rightarrow (Y,\mu)$ is strongly faint (τ,μ)-continuous.*
(ii) Let (Y,σ) be a topological space. Then we have
(a) $\sigma \subseteq \alpha O(Y) \subseteq PO(Y) \subseteq BO(Y) \subseteq \beta O(Y)$;

(b) $\sigma \subseteq \alpha O(Y) \subseteq PO(Y) \subseteq \delta PO(Y) \subseteq eO(Y)$;
(c) $\sigma \subseteq \alpha O(Y) \subseteq SO(Y) \subseteq BO(Y) \subseteq \beta O(Y)$;
(d) $\sigma \subseteq \alpha O(Y) \subseteq SO(Y) \subseteq eO(Y)$;
(e) $\theta O(Y) \subseteq S_\theta O(Y) \subseteq \delta SO(Y) \subseteq SO(Y)$.

Thus from Remark 2.3(i), we can deduce relations between different types of strongly faint (τ, μ)-continuous functions.

Example 2.4. *Let* $X = \{a, b, c\}$, $\tau = \{\varnothing, X, \{a\}, \{b, c\}\}$, $\mu = \{\varnothing, \{a, b\}, \{b, c\}, X\}$ *and* $\lambda = \{\varnothing, \{b, c\}\}$. *Then* μ *and* λ *are two GT's on* X. *The identity map* $f : (X, \tau) \to (Y, \lambda)$ *is strongly faint* (τ, λ)*-continuous but* $f : (X, \tau) \to (Y, \mu)$ *not is strongly faint* (τ, μ)*-continuous.*

We recall that a function $f : ((Y, \mu) \to (Z, \lambda)$ is said to be a (μ, λ)-continuous [5] if $G \in \lambda$ implies that $f^{-1}(G) \in \mu$.

Theorem 2.5. *If* $f : (X, \tau) \to (Y, \mu)$ *is strongly* (τ, μ)*-continuous and* $g : (Y, \mu) \to (Z, \lambda)$ *is* (μ, λ)*-continuous, then* $g \circ f : (X, \tau) \to (Z, \lambda)$ *is* (τ, λ)*-continuous.*

Proof: Let $G \in \lambda$. Then $g^{-1}(G)$ is μ-open in Y. Hence $(g \circ f)^{-1}(G) = f^{-1}(g^{-1}(G))$ is θ-open in X.

We recall that for any subset A of a topological space (X, τ), the θ-frontier of A [11] is denoted by $Fr_\theta(A)$ and defined by $Fr_\theta(A) = cl_\theta(A) \cap cl_\theta(X \setminus A)$.

Theorem 2.6. *Let* (X, τ) *be a regular space. Then the set of all points* $x \in X$ *at which the function* $f : (X, \tau) \to (Y, \mu)$ *is not strongly* (τ, μ)*-continuous is identical with the union of* θ*-frontier of the inverse images of* μ*-open subsets of* Y *containing* $f(x)$.

Proof: Suppose that f is not strongly (τ, μ)-continuous at $x \in X$. Then there exists a μ-open set V of Y containing $f(x)$ such that for each θ-open set U of X containing x, $f(U) \nsubseteq V$. So $U \cap (X \setminus f^{-1}(V)) \neq \varnothing$ for each θ-open set U of X containing x. Since X is regular, $x \in cl_\theta(X \setminus f^{-1}(V))$. Also, $x \in f^{-1}(V) \subseteq cl_\theta(f^{-1}(V))$. Thus $x \in Fr_\theta(f^{-1}(V))$.

Conversely, suppose that $x \in Fr_\theta(f^{-1}(V))$ for some μ-open set V of Y containing $f(x)$. If possible, let f be strongly (τ, μ)-continuous at x. Then there exists a θ-open set U of X containing x such that $f(U) \subseteq V$. Thus $U \subseteq f^{-1}(V)$ and hence $x \in int_\theta(f^{-1}(V)) \subseteq X \setminus Fr_\theta(f^{-1}(V))$. This is a contradiction. Thus f is strongly (τ, μ)-continuous at x.

Definition 2.7. *(i) A GTS* (X, μ) *is called*
(i) μ*-connected [22] if* X *cannot be written as union of two nonempty disjoint* μ*-open sets.*
(ii) μ*-compact [20] if every* μ*-open cover of* X *has a finite subcover. A subset* A *of* X *is said to be* μ*-compact relative to* (X, μ) *if every cover of* A *by* μ*-open sets of* X *has a finite subcover.*

Definition 2.8. *A topological space* (X, τ) *is said to be*
(i) θ*-connected [11] if it can not be written as union of two nonempty disjoint* θ*-open sets.*
(ii) A subset A *of a space* (X, τ) *is said to be* θ*-compact relative to* (X, τ) *[21] if every cover of* A *by* θ*-open subsets of* A *has a finite subcover.*

It should be mentioned that θ-connected is equivalent with connected (see [11].

Theorem 2.9. *If $f : (X, \tau) \to (Y, \mu)$ is a strongly faint (τ, μ)-continuous surjection and (X, τ) is θ-connected, then (Y, μ) is μ-connected.*

Proof: Let us assume that (Y, μ) be not μ-connected. Then there exist nonempty disjoint μ-open sets G_1 and G_2 such that $Y = G_1 \cup G_2$. Since f is strongly (τ, μ)-continuous, $f^{-1}(G_1)$ and $f^{-1}(G_2)$ are θ-open in X (by Theorem 2.2). Also $f^{-1}(G_1) \cap f^{-1}(G_2) = \varnothing$ and $X = f^{-1}(G_1) \cup f^{-1}(G_2)$. Thus X is not θ-connected - a contradiction and hence (Y, μ) is μ-connected.

Theorem 2.10. *If $f : (X, \tau) \to (Y, \mu)$ is strongly faint (τ, μ)-continuous then $f(K)$ is μ-compact relative to (Y, μ) for each subset K which is θ-compact relative to (X, τ).*

Proof: Let $\{V_\alpha : \alpha \in \Lambda\}$ be a cover of $f(K)$ by μ-open sets of $f(K)$. Then for each $x \in K$, there exists $\alpha_x \in \Lambda$ such that $f(x) \in V_{\alpha_x}$. Since f is strongly (τ, μ)-continuous, there exists a θ-open set U_x containing x such that $f(U_x) \subseteq V_{\alpha_x}$. Then the family $\{U_x : x \in K\}$ is a cover of K by θ-open sets of (X, τ). Since K is θ-compact relative to X, there exists a finite subset K_0 of K such that $K \subseteq \cup\{U_x : x \in K_0\}$. Thus $f(K) \subseteq \cup\{f(U_x) : x \in K_0\} \subseteq \cup\{V_{\alpha_x} : x \in K_0\}$. Thus $f(K)$ is μ-compact relative to (Y, μ).

Theorem 2.11. *The surjective strongly faint (τ, μ)-continuous image of a θ-compact space is μ-compact.*

Proof: Follows from the above theorem.

3 Separation axioms

Definition 3.1. *(a) A GTS (X, μ) is said to be*
(i) μ-T_1 [18] if $x, y \in X$, $x \neq y$ implies the existence of $K, K^1 \in \mu$ such that $x \in K$, $y \notin K$ and $x \notin K^1$, $y \in K^1$.
(ii) μ-T_2 [9] if for any pair of distinct points $x, y \in X$, there exist two disjoint μ-open sets U and V such that $x \in U$ and $y \in V$.
(b) A topological space (X, τ) is called
(i) θ-T_1 [6] if for any pair of distinct points $x, y \in X$ there exist θ-open sets U and V in X containing x and y respectively such that $x \notin V$ and $y \notin U$.
(ii) θ-T_2 [23] if for any pair of distinct points $x, y \in X$ there exist disjoint θ-open sets U and V in X containing x and y respectively.

We recall [6] that a space (X, τ) is T_2 if and only if it is θ-T_1.

Theorem 3.2. *If $f : (X, \tau) \to (Y, \mu)$ is a strongly faint (τ, μ)-continuous injection and (Y, μ) is a μ-T_1 space then X is a θ-T_1 (i.e., T_2) space.*

Proof: Suppose that (Y, μ) is a μ-T_1 space. Then for any pair of distinct points x and y in X, there exist μ-open sets V_1, V_2 of Y such that $f(x) \in V_1$, $f(y) \in V_2$, $f(x) \notin V_2$, $f(y) \notin V_1$. Since f is strongly faint (τ, μ)-continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are θ-open subsets of (X, τ) such that $x \in f^{-1}(V_1)$, $y \in f^{-1}(V_2)$, $x \notin f^{-1}(V_2)$ and $y \notin f^{-1}(V_1)$. This shows that X is a θ-T_1 space (equivalently a Hausdorff space).

It is known from [6] that If (X, τ) is θ-T_2 then it is Urysohn.

Theorem 3.3. *If $f : (X, \tau) \to (Y, \mu)$ is a strongly faint (τ, μ)-continuous injection and (Y, μ) is a μ-T_2 space then X is a θ-T_2 space (and hence Urysohn).*

Proof: Let us assume that Y be μ-T_2. For any pair of distinct points x and y in X, there exist μ-open sets V_1 and V_2 in Y such that $f(x) \in V_1$ and $f(y) \in V_2$ with $V_1 \cap V_2 = \varnothing$. Since f is strongly faint (τ, μ)-continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are θ-open in X containing x and y, respectively. Therefore, $f^{-1}(V_2) \cap f^{-1}(V_2) = \varnothing$ because $V_1 \cap V_2 = \varnothing$. This shows that X is θ-T_2.

Theorem 3.4. *If $f, g : (X, \tau) \to (Y, \mu)$ are strongly faint (τ, μ)-continuous function and (Y, μ) is a μ-T_2 space, then $E = \{x \in X : f(x) = g(x)\}$ is closed in X.*

Proof: Suppose that $x \notin E$. Then $f(x) \neq g(x)$. Since (Y, μ) is μ-T_2, there exist disjoint μ-open sets V_1 and V_2 in Y containing $f(x)$ and $g(x)$ respectively. Since f and g are strongly faint (τ, μ)-continuous, there exist θ-open sets U_1 and U_2 of X containing x such that $f(U_1) \subseteq V_1$ and $g(U_2) \subseteq V_2$. Set $D = U_1 \cap U_2$. Then $D \cap E = \varnothing$, with D a θ-open subset and hence an open subset containing x. Then $x \notin cl(E)$ and thus E is closed in X.

Definition 3.5. *A GTS (X, μ) is said to be*
(i) μ-regular [18] if for each μ-closed set F and each point $x \notin F$, there exist disjoint μ-open sets U and V such that $x \in U$, $F \subseteq V$.
(ii) μ-normal [18] if for any two disjoint μ-closed subsets F and K, there exist disjoint μ-open sets U and V such that $F \subseteq U$, $K \subseteq V$.

Definition 3.6. *A topological space (X, τ) is said to be*
(i) θ-regular [4] if for each θ-closed set F and each point $x \notin F$, there exist disjoint θ-open sets U and V such that $x \in U$, $F \subseteq V$.
(ii) θ-normal [4] if for any two disjoint θ-closed subsets F and K, there exist disjoint θ-open sets U and V such that $F \subseteq U$, $K \subseteq V$.

Definition 3.7. *A function $f : (X, \mu) \to (Y, \sigma)$ is called (τ_θ, μ)-open if for each θ-open set V in X, $f(V)$ is μ-open in (Y, μ).*

Theorem 3.8. *If $f : (X, \tau) \to (Y, \mu)$ is a strongly faint (τ, μ)-continuous (τ_θ, μ)-open injection from a regular space (X, τ) onto a space (Y, μ), then (Y, μ) is μ-regular.*

Proof: Let F be a μ-closed subset of Y and $y \notin F$. Take $y = f(x)$. Since f is strongly faint (τ, μ)-continuous, $f^{-1}(F)$ is θ-closed in X such that $f^{-1}(y) = x \notin f^{-1}(F)$. Take $G = f^{-1}(F)$. Thus we have $x \notin G$. Since X is θ-regular, then there exist disjoint θ-open sets U and V in X such that $G \subseteq U$ and $x \in V$. So $F = f(G) \subseteq f(U)$ and $y = f(x) \in f(V)$ such that $f(U)$ and $f(V)$ are disjoint μ-open sets. This shows that Y is μ-regular.

Theorem 3.9. *If $f : (X, \tau) \to (Y, \mu)$ is a strongly faint (τ, μ)-continuous (τ_θ, μ)-open injection from a θ-normal space (X, τ) onto a space (Y, μ), then Y is μ-normal.*

Proof: Let F_1 and F_2 be disjoint μ-closed subsets of Y. Since f is strongly faint (τ, μ)-continuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are θ-closed sets. Take $V_1 = f^{-1}(F_1)$ and $V_2 = f^{-1}(F_2)$. So we have $V_1 \cap V_2 = \varnothing$. Since X is θ-normal, there exist disjoint θ-open sets G_1 and G_2 such that $V_1 \subseteq G_1$ and $V_2 \subseteq G_2$. Thus $F_1 = f(V_1) \subseteq f(G_1)$ and $F_2 = f(V_2) \subseteq f(G_2)$ such that $f(G_1)$ and $f(G_2)$ are disjoint μ-open sets and hence Y is μ-normal.

Definition 3.10. *A graph $G(f)$ of a function $f : (X, \tau) \to (Y, \mu)$ is said to be (μ, θ)-closed if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist a θ-open U set of X containing x and a μ-open set V of Y containing y such that $(U \times V) \cap G(f) = \varnothing$.*

Lemma 3.11. *A graph $G(f)$ of a function $f : (X, \tau) \to (Y, \mu)$ is (μ, θ)-closed in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist a θ-open set U of X containing x and a μ-open set V of Y containing y such that $f(U) \cap V = \varnothing$.*

Theorem 3.12. *If $f : (X, \tau) \to (Y, \mu)$ is a strongly faint (τ, μ)-continuous function and (Y, μ) is μ-T_2, then $G(f)$ is (μ, θ)-closed.*

Proof: Let $(x, y) \in (X \times Y) \setminus G(f)$, then $f(x) \neq y$. Since Y is μ-T_2, there exist disjoint μ-open sets V_1 and V_2 in Y such that $f(x) \in V_1$ and $y \in V_2$. Since f is strongly faint (τ, μ)-continuous, $f^{-1}(V_1)$ is θ-open in X containing x. Take $U_1 = f^{-1}(V_1)$. We then have $f(U_1) \subseteq V_1$. Hence $f(U_1) \cap V_2 = \varnothing$. This shows that $G(f)$ is (μ, θ)-closed.

Theorem 3.13. *Let $f : (X, \tau) \to (Y, \mu)$ be such that it has a (μ, θ)-closed graph $G(f)$. If f is a strongly faint (τ, μ)-continuous injection, then (X, τ) is θ-T_2 (and hence Urysohn).*

Proof: Let x and y be any two distinct points of X. Then since f is injective, we have $f(x) \neq f(y)$. Thus $(x, f(y)) \in (X \times Y) \setminus G(f)$. By Lemma 3.11, there exist a θ-open set U of X and a μ-open set V of Y such that $(x, f(y)) \in U \times V$ and $f(U) \cap V = \varnothing$. Hence $U \cap f^{-1}(V) = \varnothing$ and $y \notin U$. Since f is strongly faint (τ, μ)-continuous, there exists a θ-open set W of X containing y such that $f(W) \subseteq V$. Therefore, we have $f(U) \cap f(W) = \varnothing$. Since f is injective, we obtain $U \cap W = \varnothing$. This implies that X is θ-T_2.

Theorem 3.14. *Let (Y, μ) be QTS. If $f : (X, \tau) \to (Y, \mu)$ has a (μ, θ)-closed graph, then $f(K)$ is μ-closed in (Y, μ) for each subset K which is θ-compact relative to X.*

Proof: Suppose that $y \notin f(K)$. Then $(x, y) \notin G(f)$ for each $x \in K$. Since $G(f)$ is (μ, θ)-closed, there exist a θ-open U_x of X containing x and a μ-open V_x of Y containing y such that $f(U_x) \cap V_x = \varnothing$ (by Lemma 3.11). Then the family $\{U_x : x \in K\}$ is a cover of K by θ-open sets. Since K is θ-compact relative to (X, τ), there exists a finite subset K_0 of K such that $K \subseteq \cup\{U_x : x \in K_0\}$. Set $V = \cap\{V_x : x \in K_0\}$. Then V is a μ-open set in Y containing y. Therefore we have $f(K) \cap V \subseteq \cup[f(U_x) : x \in K_0] \cap V \subseteq \cup[f(U_x) \cap V : x \in K_0] = \varnothing$. It then follows that $y \notin c_\mu(f(K))$. Therefore, $f(K)$ is μ-closed in (Y, μ).

Corollary 3.15. *Let (Y, μ) be a QTS. If $f : (X, \tau) \to (Y, \mu)$ is strongly faint (τ, μ)-continuous and (Y, μ) is μ-T_2, then $f(K)$ is μ-closed in (Y, μ) for each subset K which is θ-compact relative to (X, τ).*

Proof: The proof follows from Theorems 3.12 and 3.14.

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