**J. Algebra Comb. Discrete Appl.**
**3(1) • 13–30**

**Received: 17 July 2015**
**Accepted: 24 November 2015**

Journal of Algebra Combinatorics Discrete Structures and Applications

# Infinitely many nonsolvable groups whose Cayley graphs are hamiltonian

**Research Article**

**Dave Witte Morris**

**Abstract:** We show there are infinitely many finite groups $G$, such that every connected Cayley graph on $G$ has a hamiltonian cycle, and $G$ is not solvable. Specifically, we show that if $A_5$ is the alternating group on five letters, and $p$ is any prime, such that $p \equiv 1 \pmod{30}$, then every connected Cayley graph on the direct product $A_5 \times \mathbb{Z}_p$ has a hamiltonian cycle.

**2010 MSC:** 05C25, 05C45

**Keywords:** Cayley graph, Hamiltonian cycle, Solvable group, Alternating group

## 1. Introduction

It has been conjectured that every connected Cayley graph on every finite group has a hamiltonian cycle (unless the graph has less than three vertices). In support of this conjecture, the literature provides numerous infinite families of finite groups $G$, for which it is known that every connected Cayley graph on $G$ has a hamiltonian cycle. (See [2] and its references for more information.) However, it seems that the union of these families contains only finitely many groups that are not solvable. This note puts an end to that unsatisfactory state of affairs:

**Proposition 1.1.** *There are infinitely many finite groups $G$, such that every connected Cayley graph on $G$ has a hamiltonian cycle, and $G$ is not solvable.*

Since the alternating group $A_5$ (of order 60) is a nonabelian simple group, and is therefore not solvable, the above is an immediate consequence of the following more specific result.

**Proposition 1.2.** *If $p$ is a prime, such that $p \equiv 1 \pmod{30}$, then every connected Cayley graph on the direct product $A_5 \times \mathbb{Z}_p$ has a hamiltonian cycle.*

*Dave Witte Morris; Department of Mathematics and Computer Science, University of Lethbridge, Canada (email: Dave.Morris@uleth.ca).*

http://dx.doi.org/10.13069/jacodesmath.66457 ISSN 2148-838X

The proof is based on a case-by-case analysis of Cayley graphs of the group $A_5$. Most of the hamiltonian cycles were found by computer search (using a fairly naive backtracking algorithm).

**Remark 1.3.** *Rather than merely groups that are not solvable, it would be much more interesting to find infinitely many finite,* simple *groups $G$, such that every connected Cayley graph on $G$ has a hamiltonian cycle. Regrettably, the known methods seem to be hopelessly inadequate for this problem.*

## 2. Preliminaries

**Definition 2.1.** *Let $S$ be a subset of a finite group $G$. The* Cayley graph *of $G$ with respect to the connection set $S$ is the graph* $\mathrm{Cay}(G;S)$ *whose vertices are the elements of $G$, and with edges $g \,\text{—}\, gs$ and $g \,\text{—}\, gs^{-1}$, for each $g \in G$ and $s \in S$.*

**Notation 2.2.** *Suppose $S$ is a subset of a finite group $G$. For $s_1, \ldots, s_m \in S \cup S^{-1}$, we use $(s_i)_{i=1}^m = (s_1, \ldots, s_m)$ to denote the walk in* $\mathrm{Cay}(G;S)$ *that visits (in order), the vertices*

$$e,\ s_1,\ s_1s_2,\ s_1s_2s_3,\ \ldots,\ s_1s_2\cdots s_m.$$

*We use $(s_1, \ldots, s_m)^k$ to denote the concatenation of $k$ copies of the sequence $(s_i)_{i=1}^m$, and the following illustrates other notations that are often useful:*

$$(a^2, b^{-3}, s_i)_{i=1}^3 = (a, a, b^{-1}, b^{-1}, b^{-1}, s_1, a, a, b^{-1}, b^{-1}, b^{-1}, s_2, a, a, b^{-1}, b^{-1}, b^{-1}, s_3).$$

**Notation 2.3.** *We use $\overline{\phantom{x}}\colon A_5 \times \mathbb{Z}_p \to A_5$ to denote the natural projection (so $\overline{(x,y)} = x$).*

Our argument in Section 3 is based on the same outline as the proof in [3] of the following result.

**Lemma 2.4** ([3]). *Every connected Cayley graph on $A_5$ has a hamiltonian cycle.*

The following result is the reason that the statement of Proposition 1.2 assumes $p \equiv 1 \pmod{30}$. A much weaker hypothesis would suffice in all other parts of the proof.

**Corollary 2.5.** *Let $S$ be a minimal generating set of $A_5 \times \mathbb{Z}_p$, where $p$ is prime, and $p \equiv 1 \pmod{30}$. If there exists $a \in S$, such that $\overline{S \setminus \{a\}}$ generates $A_5$, then* $\mathrm{Cay}(A_5 \times \mathbb{Z}_p; S)$ *has a hamiltonian cycle.*

***Proof.*** Since $\gcd(|A_5|, p) = 1$, the minimality of $S$ implies that $\langle S \setminus \{a\}\rangle = A_5$. (Namely, since $\gcd(|A_5|, p) = 1$, we have $\overline{g} \in \langle g \rangle$ for every $g \in A_5 \times \mathbb{Z}_p$. Therefore $A_5 = \langle \overline{S \setminus \{a\}} \rangle \subseteq \langle S \setminus \{a\}\rangle$. Since the minimality of $S$ implies $a \notin \langle S \setminus \{a\}\rangle$, we conclude that $\langle S \setminus \{a\}\rangle = A_5$.) From Lemma 2.4, we know there is a hamiltonian cycle $(s_i)_{i=1}^{60}$ in $\mathrm{Cay}(A_5; S \setminus \{a\})$. Since, by assumption, $p-1$ is divisible by $30 = 2 \cdot 3 \cdot 5$ (and every element of $A_5$ has order 1, 2, 3, or 5), we know $\overline{a}^{\,p-1}$ is trivial. This means $a^{p-1} \in \mathbb{Z}_p$ (so $a^{p-1}$ centralizes $A_5$), so it is not difficult to verify that

$$\big(s_{2i-1}, a^{p-1}, s_{2i}, a^{-(p-1)}\big)_{i=1}^{30}$$

is a hamiltonian cycle in $\mathrm{Cay}(A_5 \times \mathbb{Z}_p; S)$.

For completeness, we sketch the verification that the given walk is a hamiltonian cycle. Since $\langle S \rangle = A_5 \times \mathbb{Z}_p$, and $S \setminus \{a\} \subseteq A_5$, we know that $a$ projects nontrivially to $\mathbb{Z}_p$. Since $\gcd(|A_5|, p) = 1$, this implies $\mathbb{Z}_p \subseteq \langle a \rangle$, so every element $g$ of $A_5 \times \mathbb{Z}_p$ can be written (uniquely) in the form $g = xa^r$ with $x \in A_5$ and $0 \le r \le p-1$. Since $(s_i)_{i=1}^{60}$ is a hamiltonian cycle in a Cayley graph on $A_5$, we have $x = s_1s_2\cdots s_k$, for some $k$ with $0 \le k < 60$. If $k = 2i-1$ is odd, then

$$g = xa^r = \left(\prod_{j=1}^{i-1} s_{2j-1}s_{2j}\right) s_{2i-1}a^r = \left(\prod_{j=1}^{i-1} (s_{2j-1}a^{p-1}s_{2j}a^{-(p-1)}\right) s_{2i-1}a^r.$$

(because $a^{p-1} \in \mathbb{Z}_p$ is in the center of $A_5 \times \mathbb{Z}_p$). Also, if $k = 2i$ is even, then

$$g = xa^r = \left( \prod_{j=1}^{i-1} s_{2j-1} s_{2j} \right) s_{2i-1} s_{2i} a^r = \left( \prod_{j=1}^{i-1} s_{2j-1} a^{p-1} s_{2j} a^{-(p-1)} \right) s_{2i-1} a^{p-1} s_{2i} a^{-(p-1-r)}.$$

Thus, we see (in either case) that $g$ is one of the vertices on the walk $(s_{2i-1}, a^{p-1}, s_{2i}, a^{-(p-1)})_{i=1}^{30}$. This means that the walk passes through all of the vertices in $\mathrm{Cay}(A_5 \times \mathbb{Z}_p; S)$.

Also, note that the walk has the correct length ($60p$) to be a hamiltonian cycle. Finally, by using once again the fact that $a^{p-1}$ is in the center, we see that the terminal vertex of the walk is

$$\prod_{i=1}^{30} s_{2i-1} a^{p-1} s_{2i} a^{-(p-1)} = \prod_{i=1}^{30} s_{2i-1} s_{2i} = \prod_{i=1}^{60} s_i = e,$$

because $(s_i)_{i=1}^{60}$ is a (hamiltonian) cycle. □

**Remark 2.6.** *For definiteness, we point out that we write our permutations on the left, so $gs(i) = g\big(s(i)\big)$ for $g, s \in A_5$ and $i \in \{1, 2, 3, 4, 5\}$.*

The remainder of this section records a few easy consequences of the following well-known, elementary observation.

**Lemma 2.7** ("Factor Group Lemma" [4, §2.2])**.** *Suppose*

- *$N$ is a cyclic, normal subgroup of $G$,*
- *$(s_i)_{i=1}^m$ is a hamiltonian cycle in $\mathrm{Cay}(G/N; S)$, and*
- *the voltage $\Pi(s_i)_{i=1}^m$ generates $N$.*

*Then $(s_1, s_2, \ldots, s_m)^{|N|}$ is a hamiltonian cycle in $\mathrm{Cay}(G; S)$.*

**Corollary 2.8** ([2, Cor. 2.11])**.** *Suppose*

- *$N$ is a normal subgroup of $G$, such that $|N|$ is prime,*
- *the image of $S$ in $G/N$ is a minimal generating set of $G/N$,*
- *there is a hamiltonian cycle in $\mathrm{Cay}(G/N; S)$, and*
- *$s \equiv t \pmod N$ for some $s, t \in S \cup S^{-1}$ with $s \neq t$.*

*Then there is a hamiltonian cycle in $\mathrm{Cay}(G; S)$.*

**Corollary 2.9.** *Let $S$ be a minimal generating set of $A_5 \times \mathbb{Z}_p$, such that $\overline{S}$ is a minimal generating set of $A_5$. If every element of $\overline{S}$ has order 2, then $\mathrm{Cay}(A_5 \times \mathbb{Z}_p; S)$ has a hamiltonian cycle.*

**Notation 2.10.** *Let $C = (s_i)_{i=1}^m$ be a walk in a Cayley graph $\mathrm{Cay}(G; S)$. For $s \in S$, we use $\mathrm{wt}_C(s)$ to denote the difference between the number of occurrences of $s$ and the number of occurrences of $s^{-1}$ in $C$. (This is the* net weight *of the generator $s$ in $C$.)*

**Lemma 2.11.** *Let $S = \{a_1, \ldots, a_k, b_1, \ldots, b_\ell\}$ be a minimal generating set of $A_5 \times \mathbb{Z}_p$, such that $\overline{S}$ is a minimal generating set of $A_5$. Assume*

- *$\ell \geq 1$, and $|\overline{a_i}| = 2$ for all $i$,*
- *$C_1, \ldots, C_\ell$ are hamiltonian cycles in $\mathrm{Cay}(A_5; \overline{S})$, and*

- $[\mathrm{wt}_{C_i}(b_j)]$ *is the* $\ell \times \ell$ *matrix whose* $(i,j)$ *entry is* $\mathrm{wt}_{C_i}(b_j)$.

*If* $\det[\mathrm{wt}_{C_i}(b_j)] \not\equiv 0 \pmod p$*, then* $\mathrm{Cay}(A_5 \times \mathbb{Z}_p; S)$ *has a hamiltonian cycle.*

***Proof.*** We may assume that $a_i \in A_5$ for $1 \le i \le k$, for otherwise Corollary 2.8 applies with $s = a_i$ and $t = a_i^{-1}$. Write $b_i = (\overline{b_i}, v_i)$ (with $v_i \in \mathbb{Z}_p$) for $1 \le i \le \ell$. Since $S$ generates $A_5 \times \mathbb{Z}_p$, the vector $[v_1, \ldots, v_\ell]$ must be nonzero in $(\mathbb{Z}_p)^\ell$. Then, since, by assumption, the matrix $[\mathrm{wt}_{C_i}(b_j)]$ is invertible over $\mathbb{Z}_p$, this implies $[\mathrm{wt}_{C_i}(b_j)][v_1, \ldots, v_\ell]^T \ne \vec{0}$ in $(\mathbb{Z}_p)^\ell$, so there is some $i$, such that $\sum_{j=1}^{\ell} wt_{C_i}(b_j)\, v_j \ne 0$ in $\mathbb{Z}_p$.

We now show that this sum is precisely the voltage $\Pi C_i$ of the walk $C_i$, so Lemma 2.7 provides the desired hamiltonian cycle in $\mathrm{Cay}(A_5 \times \mathbb{Z}_p; S)$. Write $C_i = (s_k)_{k=1}^n$, let $\pi = \prod_{k=1}^n s_k$ be the voltage of $C_i$, and let $n(s)$ and $n'(s)$, respectively, be the number of occurrences of $s$ and $s^{-1}$ in $C_i$. Since $C_i$ is a (hamiltonian) cycle in $\mathrm{Cay}(A_5; \overline{S})$, we know that $\overline{\pi}$ is trivial, so $\pi = \sum_{k=1}^n s_k^*$, where $s_k^*$ is the projection of $s_k$ to $\mathbb{Z}_p$. Noting that $(s^{-1})^* = -s^*$ for $s \in S$, we have

$$\pi = \sum_{k=1}^{n} s_k^* = \sum_{s \in S \cup S^{-1}} n(s) s^* = \sum_{s \in S} \big(n(s) - n'(s)\big) s^* = \sum_{s \in S} \mathrm{wt}_{C_i}(s)\, s^* = \sum_{j=1}^{\ell} \mathrm{wt}_{C_i}(b_j)\, v_j. \qquad \square$$

## 3. Proof of Proposition 1.2

**Assumptions 3.1.** *Let $S$ be a minimal generating set of $A_5 \times \mathbb{Z}_p$ (and, in accordance with Notation 2.3, let $\overline{S}$ be the image of $S$ in $A_5$). We may assume $\overline{S}$ is a minimal generating set of $A_5$, for otherwise Corollary 2.5 applies. We may also assume, for every element $s$ of $S$ with $|\overline{s}| = 2$, that the projection of $s$ to $\mathbb{Z}_p$ is trivial, for otherwise Corollary 2.8 applies.*

**Case 1.** *Assume $S$ has exactly two elements.* Write $S = \{a, b\}$.

**Subcase 1.1.** *Assume $|\overline{a}| = 2$ and $|\overline{b}| = 3$.* To simplify matters, we show that, by applying an automorphism of $A_5$, we may assume $\overline{a} = (1,2)(3,4)$ and $\overline{b} = (2,4,5)$. First of all, we may assume $\overline{a} = (1,2)(3,4)$, since every element of order 2 in $A_5$ is conjugate to this. Then, in order for $\langle \overline{a}, \overline{b} \rangle$ to be transitive, the support of $\overline{b}$ must contain an element of each cycle of $\overline{a}$ (including the 1-cycle $(5)$). So we may assume $\overline{b} = (2,4,5)$ (after conjugating by $(1,2)$ and/or $(3,4)$, if necessary).

Now, we have the following hamiltonian cycle in $\mathrm{Cay}(A_5; \overline{S})$ (see note 4.1):

$$C_1 = \big((\overline{a}, \overline{b}^{\,2})^3, (\overline{a}, \overline{b}^{\,-2})^3, (\overline{a}, \overline{b}^{\,2}, \overline{a}, \overline{b}^{\,-2})^2\big)^2.$$

By using the fact that each left coset of $\langle \overline{b} \rangle$ appears as consecutive vertices in this cycle, we will show that

$$C_2 = \big((a, b^{3p-1})^3, (a, b^{-(3p-1)})^3, (a, b^{3p-1}, a, b^{-(3p-1)})^2\big)^2$$

passes through all of the vertices in each left coset of $\langle b \rangle$, and is therefore a hamiltonian cycle in $\mathrm{Cay}(A_5 \times \mathbb{Z}_p; S)$.

Note that $\overline{b^{3p-1}} = \overline{b}^2$ (since $|\overline{b}| = 3$). This implies that if we let $x$ be the terminal vertex of the walk $C_2$, then $\overline{x}$ is the terminal vertex of the hamiltonian cycle $C_1$, so $\overline{x}$ is trivial. The projection of $x$ to $\mathbb{Z}_p$ is also trivial, because $\mathrm{wt}_{C_2}(b) = 0$. Therefore, the walk $C_2$ is closed.

Now, since $C_2$ has the correct length to be a hamiltonian cycle, we need only show that it passes through every element of $A_5 \times \mathbb{Z}_p$. From the fact that $\overline{b^{3p-1}} = \overline{b}^2$, we see that the vertices of $\overline{C_2}$ are precisely the same elements of $A_5$ as the vertices of $C_1$; that is, the walk $\overline{C_2}$ passes through every element of $A_5$. Thus, given any $v \in A_5 \times \mathbb{Z}_p$, the walk $C_2$ visits some vertex $w$ with $\overline{w} = \overline{v}$; that is, $v$ and $w$ are in the same coset of $\mathbb{Z}_p$. Since $\mathbb{Z}_p \subseteq \langle b \rangle$, this implies that $v$ and $w$ are in the same left coset of $\langle b \rangle$. Also, since there are never two consecutive appearances of $a$ in $C_2$, and every occurrence of $b$ is contained in a string $b^{3p-1}$, we know that $C_2$ traverses every element of any left coset of $\langle b\rangle$ that it enters. In particular, $C_2$ traverses every element of the left coset of $w$, so it passes through $v$.

**Subcase 1.2.** *Assume* $|\overline{a}| = 2$ *and* $|\overline{b}| = 5$. We may assume $\overline{b} = (1,2,3,4,5)$, after conjugating by some permutation in $S_5$. Since $|\overline{a}| = 2$, it has a fixed point, which we may assume is 5 (after conjugating by a power of $\overline{b}$). So $|\overline{a}|$ must be either $(1,2)(3,4)$, $(1,3)(2,4)$, or $(1,4)(2,3)$.

- For $\overline{a} = (1,2)(3,4)$, we have the following hamiltonian cycle (see note 4.2):
$$C = \big((\overline{a},\overline{b},\overline{a},\overline{b}^{\,4})^2,\overline{a},\overline{b}^{\,2},\overline{a},\overline{b}^{\,-1},\overline{a},\overline{b}^{\,4},\overline{a},\overline{b},(\overline{a},\overline{b}^{\,2})^3,\\ \overline{a},\overline{b}^{\,-2},\overline{a},\overline{b}^{\,4},\overline{a},\overline{b}^{\,-2},(\overline{a},\overline{b})^2,\overline{a},\overline{b}^{\,-1},\overline{a},\overline{b}^{\,4},\overline{a},\overline{b}^{\,2}\big).$$
  Since $\mathrm{wt}_C(\overline{b}) = 29 \not\equiv 0 \pmod p$, Theorem 2.11 applies.
- For $\overline{a} = (1,3)(2,4)$, we have the following hamiltonian cycle (see note 4.3):
$$C = \big(\overline{a},\overline{b}^{\,4},\overline{a},\overline{b}^{\,-1},\overline{a},\overline{b},\overline{a},\overline{b}^{\,-1},\overline{a},\overline{b}^{\,-4},\overline{a},\overline{b}^{\,-2},(\overline{a},\overline{b}^{\,-4},\overline{a},\overline{b}^{\,2})^2,\\ \overline{a},\overline{b}^{\,-4},\overline{a},\overline{b},\overline{a},\overline{b}^{\,-1},\overline{a},\overline{b}^{\,-4},\overline{a},\overline{b}^{\,-2},\overline{a},\overline{b}^{\,-4},\overline{a},\overline{b}^{\,2}\big).$$
  Since $\mathrm{wt}_C(\overline{b}) = -19 \not\equiv 0 \pmod p$, Theorem 2.11 applies.
- If $\overline{a} = (1,4)(2,3)$, then $\overline{a}$ normalizes $\overline{b}$, so $\langle\overline{a},\overline{b}\rangle \neq A_5$, which contradicts the fact that $S$ is a generating set.

**Subcase 1.3.** *Assume* $|\overline{a}| = |\overline{b}| = 3$. The union of the supports of $\overline{a}$ and $\overline{b}$ must be all of $\{1,2,3,4,5\}$, since $\langle\overline{a},\overline{b}\rangle$ is transitive. Since each support consists of three elements, the intersection must be a single element, which we may assume is 3. Then, by renumbering, we may assume the support of $\overline{a}$ is $\{1,2,3\}$ and the support of $\overline{b}$ is $\{3,4,5\}$. Therefore, either $\overline{a}$ or $\overline{a}^{\,-1}$ is $(1,2,3)$, and either $\overline{b}$ or $\overline{b}^{\,-1}$ is $(3,4,5)$. So we may assume $\overline{a} = (1,2,3)$ and $\overline{b} = (3,4,5)$. We have the following hamiltonian cycle (see note 4.4):
$$C_1 = \big(\overline{b},\overline{a},\overline{b}^{\,2},\overline{a}^{\,2},\overline{b}^{\,-2},\overline{a}^{\,-2},\overline{b}^{\,2},\overline{a}^{\,2},\overline{b}^{\,2},\overline{a}^{\,-2},\overline{b}^{\,-2},\overline{a}^{\,-2},\overline{b}^{\,2},\overline{a}^{\,2},\overline{b}^{\,-2},\overline{a},\overline{b}^{\,-2},\\ \overline{a}^{\,-2},\overline{b}^{\,2},\overline{a},\overline{b},\overline{a}^{\,-1},\overline{b}^{\,-2},\overline{a}^{\,2},\overline{b}^{\,2},\overline{a}^{\,2},\overline{b}^{\,-2},\overline{a}^{\,-2},\overline{b}^{\,-1},\overline{a},\overline{b},\overline{a}^{\,2},\overline{b}^{\,-1},\overline{a}^{\,-2},\overline{b}^{\,-1},\overline{a}\big).$$
Note that $\mathrm{wt}_{C_1}(\overline{a}) = 4$ and $\mathrm{wt}_{C_1}(\overline{b}) = 0$. Conjugation by the permutation $(1,4)(2,5)$ interchanges $\overline{a}$ and $\overline{b}$, and therefore yields a hamiltonian cycle $C_2$ with $\mathrm{wt}_{C_2}(\overline{a}) = 0$ and $\mathrm{wt}_{C_2}(\overline{b}) = 4$. Since $\det\left[\begin{smallmatrix}4&0\\0&4\end{smallmatrix}\right] = 16 \not\equiv 0 \pmod p$, Theorem 2.11 applies.

**Subcase 1.4.** *Assume* $|\overline{a}| = 3$ *and* $|\overline{b}| = 5$. We may assume $\overline{b} = (1,2,3,4,5)$, by replacing it with a conjugate. Then the two fixed points of the 3-cycle $\overline{a}$ are either consecutive or are separated by only one element (in circular order). Therefore, after conjugating by a power of $\overline{b}$, we may assume that one of the fixed points of $\overline{a}$ is 5, and the other is either 3 or 4. Hence, $\overline{a}$ is either $(1,2,4)$ or $(1,2,3)$ (or the inverse of one of these).

- If $\overline{a} = (1,2,4)$, then we have the following two hamiltonian cycles (see notes 4.5 and 4.6):
$$C_1 = \big(\overline{a}^{\,2},\overline{b}^{\,-2},\overline{a}^{\,-2},\overline{b}^{\,-1},\overline{a}^{\,-1},\overline{b},\overline{a}^{\,2},\overline{b}^{\,-1},\overline{a}^{\,-2},\overline{b},\overline{a}^{\,-2},\overline{b}^{\,-2},\overline{a}^{\,-2},\overline{b}^{\,-1},\overline{a}^{\,-1},\overline{b},\overline{a}^{\,2},\overline{b}^{\,-1},\overline{a}^{\,2},\overline{b},\\ \overline{a}^{\,-2},\overline{b}^{\,-1},\overline{a}^{\,2},\overline{b},\overline{a}^{\,2},\overline{b}^{\,-1},\overline{a}^{\,-2},\overline{b},\overline{a}^{\,-2},\overline{b}^{\,-2},\overline{a}^{\,-2},\overline{b}^{\,-1},\overline{a}^{\,-1},\overline{b},\overline{a}^{\,2},\overline{b}^{\,-1},(\overline{a}^{\,-2},\overline{b})^2\big)$$
  and
$$C_2 = \big(\overline{a}^{\,2},\overline{b}^{\,-2},\overline{a}^{\,-2},\overline{b}^{\,-1},\overline{a}^{\,-1},\overline{b},(\overline{a}^{\,2},\overline{b}^{\,-1})^2,\overline{a}^{\,2},\overline{b}^{\,-2},\overline{a}^{\,-2},\overline{b}^{\,-1},\overline{a}^{\,-1},\overline{b},\overline{a}^{\,2},\overline{b}^{\,-1},\overline{a}^{\,2},\overline{b},\\ \overline{a}^{\,-2},\overline{b}^{\,-1},\overline{a}^{\,2},\overline{b},\overline{a}^{\,2},\overline{b}^{\,-1},\overline{a}^{\,-2},\overline{b},\overline{a}^{\,-2},\overline{b}^{\,-2},\overline{a}^{\,-2},\overline{b}^{\,-1},\overline{a}^{\,-1},\overline{b},\overline{a}^{\,2},\overline{b}^{\,-1},(\overline{a}^{\,-2},\overline{b})^2\big)$$
  Then $\big[\mathrm{wt}_{C_1}(\overline{a}),\mathrm{wt}_{C_1}(\overline{b})\big] = [-9,-5]$ and $\big[\mathrm{wt}_{C_2}(\overline{a}),\mathrm{wt}_{C_2}(\overline{b})\big] = [-1,-7]$. Since $\det\left[\begin{smallmatrix}-9&-5\\-1&-7\end{smallmatrix}\right] = 58 \not\equiv 0 \pmod p$, Theorem 2.11 applies.

- If $\overline{a} = (1,2,3)$, then we have the following two hamiltonian cycles (see notes 4.7 and 4.8):

$$\begin{aligned} C_1 = \big(&\overline{a}^{\,2}, \overline{b}^{\,-2}, \overline{a}^{\,-2}, \overline{b}^{\,-1}, \overline{a}^{\,-1}, \overline{b}^{\,3}, (\overline{a}^{\,2}, \overline{b})^2, \overline{a}, \overline{b}^{\,-1}, (\overline{a}^{\,2}, \overline{b})^2, \overline{a}^{\,2}, \overline{b}^{\,-2}, \overline{a}^{\,2}, \overline{b}, \overline{a}, \overline{b}^{\,-1}, \\ &\overline{a}^{\,-2}, \overline{b}, \overline{a}^{\,2}, \overline{b}^{\,-1}, \overline{a}^{\,-2}, \overline{b}, \overline{a}^{\,-1}, \overline{b}^{\,-2}, \overline{a}^{\,-1}, \overline{b}^{\,-1}, \overline{a}^{\,-2}, \overline{b}, \overline{a}^{\,2}, (\overline{b}^{\,-1}, \overline{a}^{\,-2})^2, \overline{b}\big) \end{aligned}$$

and

$$\begin{aligned} C_2 = \big(&\overline{a}^{\,2}, \overline{b}^{\,-1}, (\overline{a}^{\,2}, \overline{b})^2, \overline{a}^{\,2}, \overline{b}^{\,-1}, \overline{a}^{\,-2}, \overline{b}, \overline{a}^{\,-2}, \overline{b}^{\,-1}, \overline{a}^{\,2}, \overline{b}, \overline{a}^{\,2}, \overline{b}^{\,-1}, \overline{a}^{\,-2}, \overline{b}^{\,2}, \\ &(\overline{a}^{\,-2}, \overline{b}^{\,-1})^2, (\overline{a}^{\,-2}, \overline{b})^2, \overline{a}^{\,2}, \overline{b}, \overline{a}^{\,2}, \overline{b}^{\,-1}, \overline{a}, \overline{b}, \overline{a}^{\,-2}, \overline{b}, \overline{a}^{\,2}, (\overline{b}^{\,-1}, \overline{a}^{\,-2})^2, \overline{b}\big) \end{aligned}$$

Then $\big[\mathrm{wt}_{C_1}(\overline{a}), \mathrm{wt}_{C_1}(\overline{b})\big] = [5,-1]$ and $\big[\mathrm{wt}_{C_2}(\overline{a}), \mathrm{wt}_{C_2}(\overline{b})\big] = [-1,3]$. Since $\det \left[\begin{smallmatrix} 5 & -1 \\ -1 & 3 \end{smallmatrix}\right] = 14 \not\equiv 0 \pmod{p}$, Theorem 2.11 applies.

**Subcase 1.5.** *Assume* $|\overline{a}| = |\overline{b}| = 5$. By applying an automorphism of $A_5$, we may assume $\overline{a} = (1,2,3,4,5)$. From Sylow's Theorems, we know that $A_5$ has precisely six Sylow 5-subgroups. One of them is $\langle \overline{a} \rangle$, and $\langle \overline{a} \rangle$ acts transitively on the other 5 by conjugation. So we may assume, after conjugating by a power of $\overline{a}$, that $\langle \overline{b} \rangle = \langle (1,2,3,5,4) \rangle$. Then, by replacing $b$ with its inverse if necessary, we may assume $\overline{b}$ is either $(1,2,3,5,4)$ or $(1,3,4,2,5)$.

- If $\overline{b} = (1,2,3,5,4)$, then we have the following hamiltonian cycle (see note 4.9):

$$\begin{aligned} C_1 = \big(&\overline{b}, \overline{a}^{\,-1}, \overline{b}, \overline{a}, \overline{b}^{\,4}, \overline{a}, \overline{b}^{\,2}, \overline{a}^{\,-1}, \overline{b}^{\,2}, \overline{a}^{\,-1}, \overline{b}^{\,-1}, \overline{a}^{\,-1}, \overline{b}, \overline{a}^{\,-1}, \overline{b}, \overline{a}, \overline{b}^{\,2}, \overline{a}^{\,-1}, \overline{b}^{\,2}, \overline{a}^{\,-1}, \overline{b}^{\,-1}, \\ &\overline{a}, \overline{b}^{\,-2}, \overline{a}^{\,-1}, \overline{b}^{\,-4}, \overline{a}^{\,-1}, \overline{b}^{\,-1}, \overline{a}, \overline{b}^{\,-4}, \overline{a}, \overline{b}^{\,-2}, \overline{a}^{\,-1}, \overline{b}^{\,-1}, \overline{a}, \overline{b}^{\,-2}, \overline{a}^{\,-1}, \overline{b}^{\,4}, \overline{a}, \overline{b}^{\,-2}, \overline{a}\big), \end{aligned}$$

Then $\big[\mathrm{wt}_{C_1}(\overline{a}), \mathrm{wt}_{C_1}(\overline{b})\big] = [-2,0]$. Conjugating by the permutation $(4,5)$ interchanges $\overline{a}$ and $\overline{b}$, and therefore yields a hamiltonian cycle $C_2$ with $\big[\mathrm{wt}_{C_2}(\overline{a}), \mathrm{wt}_{C_2}(\overline{b})\big] = [0,-2]$. Since $\det \left[\begin{smallmatrix} -2 & 0 \\ 0 & -2 \end{smallmatrix}\right] = 4 \not\equiv 0 \pmod{p}$, Theorem 2.11 applies.

- If $\overline{b} = (1,3,4,2,5)$, then we have the following two hamiltonian cycles (see notes 4.10 and 4.11):

$$\begin{aligned} C_1 = \big(&\overline{a}^{\,4}, \overline{b}^{\,-1}, \overline{a}^{\,-4}, \overline{b}, \overline{a}^{\,4}, \overline{b}^{\,-1}, \overline{a}^{\,2}, \overline{b}, \overline{a}^{\,-2}, \overline{b}, \overline{a}^{\,-1}, \overline{b}, \overline{a}^{\,-4}, \overline{b}^{\,-1}, \\ &\overline{a}^{\,-1}, \overline{b}^{\,-1}, \overline{a}^{\,4}, \overline{b}, (\overline{a}^{\,-4}, \overline{b}^{\,-1})^2, (\overline{a}^{\,2}, \overline{b})^2, \overline{a}^{\,4}, (\overline{b}^{\,-1}, \overline{a})^2, \overline{b}\big) \end{aligned}$$

and

$$\begin{aligned} C_2 = \big(&\overline{a}^{\,4}, \overline{b}^{\,-1}, \overline{a}^{\,-4}, \overline{b}, \overline{a}^{\,4}, \overline{b}^{\,-1}, \overline{a}^{\,2}, \overline{b}, \overline{a}^{\,-1}, \overline{b}^{\,-1}, \overline{a}^{\,-4}, \overline{b}^{\,-1}, \overline{a}, \overline{b}^{\,-1}, \\ &\overline{a}^{\,-2}, \overline{b}^{\,-1}, \overline{a}^{\,4}, \overline{b}, (\overline{a}^{\,-4}, \overline{b}^{\,-1})^2, (\overline{a}^{\,2}, \overline{b})^2, \overline{a}^{\,4}, (\overline{b}^{\,-1}, \overline{a})^2, \overline{b}\big) \end{aligned}$$

Then $\big[\mathrm{wt}_{C_1}(\overline{a}), \mathrm{wt}_{C_1}(\overline{b})\big] = [4,0]$ and $\big[\mathrm{wt}_{C_2}(\overline{a}), \mathrm{wt}_{C_2}(\overline{b})\big] = [6,-4]$. Since $\det \left[\begin{smallmatrix} 4 & 0 \\ 6 & -4 \end{smallmatrix}\right] = -16 \not\equiv 0 \pmod{p}$, Theorem 2.11 applies.

**Case 2.** *Assume $S$ has at least three elements.*. We claim that $S$ has exactly three elements, so we may write $S = \{a,b,c\}$ with $|\overline{a}| \le |\overline{b}| \le |\overline{c}|$. To show this, write $\overline{S} = \{s_1, \ldots, s_r\}$, and suppose $r \ge 4$. Let $H_i = \langle s_1, \ldots, s_i \rangle$ for each $i$, and note that, since $\overline{S}$ is a minimal generating set, we have $H_{i-1} \subsetneq H_i$. Since $|A_5| = 2^2 \cdot 3 \cdot 5$ is the product of only 4 primes, we must have $r = 4$ and $|H_i : H_{i-1}|$ is prime for $i = 1, \ldots, 4$. Since $A_4$ is the only subgroup of prime index in $A_5$ (up to conjugacy), we may assume $H_3 = A_4$. Then $H_2$ must be the Sylow 2-subgroup of $A_4$, since that is the only subgroup of prime index. So $H_3 = A_4$ is generated by $s_1$ and $s_3$, contradicting the minimality of $\overline{S}$.

**Subcase 2.1.** *Assume* $|\overline{c}| = 5$. Since $\overline{a}$ and $\overline{b}$ cannot both normalize $\langle \overline{c} \rangle$ (but every proper subgroup of $A_5$ whose order is divisible by 5 has order 5 or 10), we see that either $\langle \overline{a}, \overline{c} \rangle = A_5$ or $\langle \overline{b}, \overline{c} \rangle = A_5$, which contradicts the minimality of $S$.

**Subcase 2.2.** *Assume* $|\overline{a}| = |\overline{b}| = |\overline{c}| = 3$. The minimality of $\overline{S}$ implies that $\langle \overline{a}, \overline{b} \rangle \ne A_5$, so there must be at least two elements in the intersection of the supports of $\overline{a}$ and $\overline{b}$. The supports cannot be equal, since $\overline{a} \neq \overline{b}^{\,\pm 1}$. Therefore, the intersection of the supports consists of two points, which we may assume are 1 and 5. Then we may assume $\overline{a} = (1,2,5)$ and $\overline{b} = (1,3,5)$. Now, for the same reason, the support of $\overline{c}$ must contain exactly two points from the support of $\overline{a}$ and exactly two points from the support of $\overline{b}$, and it must also contain 4 (since 4 is fixed by both $\overline{a}$ and $\overline{b}$, but not by $A_5$). This implies that the support of $\overline{c}$ is $\{1,4,5\}$, so $\overline{c} = (1,4,5)^{\pm 1}$. Therefore (perhaps after replacing $c$ by its inverse), we have

$$\overline{S} = \{(1,2,5),(1,3,5),(1,4,5)\} = \{\,(1,j,n) \mid 1 < j < n\,\} \quad \text{for } n = 5.$$

So [1, App. D] provides the following hamiltonian cycle in $\mathrm{Cay}(A_5;\overline{S})$ (see note 4.12):

$$R_1 = \Big( \big((\overline{a}^{\,2},\overline{b})^2,\overline{a}^{\,2},\overline{c}\big)^2,\overline{a},\overline{b},(\overline{b},\overline{a}^{\,2})^2,\overline{c}^{\,2},\overline{a}^{\,2},\overline{c},\overline{b},\overline{a},\overline{b},\overline{c},\overline{a},(\overline{c},\overline{b}^{\,2})^2,(\overline{a}^{\,2},\overline{b})^2,\overline{a},\overline{c}^{\,2},\overline{a},\overline{b}^{\,2},(\overline{a},\overline{c}^{\,2})^2 \Big).$$

We have $[\mathrm{wt}_{R_1}(\overline{a}),\mathrm{wt}_{R_1}(\overline{b}),\mathrm{wt}_{R_1}(\overline{c})] = [29,17,14]$. Conjugating by $(2,3,4)$ and $(2,3,4)^2$ cyclically permutes $\{\overline{a},\overline{b},\overline{c}\}$, and therefore yields hamiltonian cycles $R_2$ and $R_3$, such that

$$[\mathrm{wt}_{R_2}(\overline{a}),\mathrm{wt}_{R_2}(\overline{b}),\mathrm{wt}_{R_2}(\overline{c})] = [17,14,29] \text{ and } [\mathrm{wt}_{R_3}(\overline{a}),\mathrm{wt}_{R_3}(\overline{b}),\mathrm{wt}_{R_3}(\overline{c})] = [14,29,17].$$

Since

$$\det \begin{bmatrix} 29 & 17 & 14 \\ 17 & 14 & 29 \\ 14 & 29 & 17 \end{bmatrix} = 11{,}340 = 2^2 \cdot 3^4 \cdot 5 \cdot 7 \not\equiv 0 \pmod{p},$$

Theorem 2.11 applies.

**Subcase 2.3.** *Assume* $|\overline{a}| = 2$ *and* $|\overline{b}| = |\overline{c}| = 3$. We claim that we may assume $\overline{S} = \{(12)(45),(1,2,3),(1,2,4)\}$. Arguing as in the first paragraph of Subcase 2.2 (and renumbering), we may assume $\overline{b} = (1,2,3)$ and $\overline{c} = (1,2,4)$. Since $\langle \overline{a},\overline{b},\overline{c}\rangle = A_5$ is transitive on $\{1,2,3,4,5\}$, we know that 5 is in the support of $\overline{a}$. Also, since $\langle \overline{a},\overline{b}\rangle \neq A_5$, we know that the support of $\overline{b}$ does not contain precisely one element of each of the cycles of $\overline{a}$ (and similarly for the support of $\overline{c}$).

If one of the cycles of $\overline{a}$ is disjoint from the support of $\overline{b}$, then the cycle must be $(4,5)$. The support of $\overline{c}$ contains precisely one element of this cycle, and cannot be disjoint from the other cycle, so it must contain the entire cycle. The only 2-element subset of $\{1,2,3\}$ contained in the support of $\overline{c}$ is $\{1,2\}$, so $\overline{a} = (1,2)(4,5)$, as desired.

We may now assume that no cycle of $\overline{a}$ is disjoint from the support of $\overline{b}$ or the support of $\overline{c}$. (This will lead to a contradiction.) This assumption implies that the cycle $(x,5)$ in $\overline{a}$ must be either $(1,5)$ or $(2,5)$. We may assume it is $(1,5)$ (after conjugating by $(1,2)$ and replacing $\overline{b}$ and $\overline{c}$ by their inverses, if necessary). The other cycle either is $(2,3)$ (in which case, $\langle \overline{a},\overline{c}\rangle = A_5$), or is either $(2,4)$ or $(3,4)$ (in which case, $\langle \overline{a},\overline{b}\rangle = A_5$), which contradicts the minimality of $\overline{S}$. This completes the proof of the claim.

We have the following two hamiltonian cycles (see notes 4.13 and 4.14):

$$\begin{aligned} C_1 = \big(&\overline{a},\overline{c}^{\,-1},\overline{a},\overline{b},\overline{a},\overline{c},\overline{a},\overline{b}^{\,2},\overline{a},\overline{b},\overline{c},\overline{b}^{\,-1},\overline{a},\overline{b}^{\,-2},\overline{a},\overline{c}^{\,-1},\overline{a},\overline{c}^{\,-1},\overline{b}^{\,2},\overline{c},\overline{b}^{\,-2},\overline{a},\overline{b}^{\,-2},\overline{c},\overline{a},\overline{b},\overline{c}^{\,-1},\\ &\overline{b}^{\,-1},\overline{a},\overline{c}^{\,-1},\overline{a},\overline{b}^{\,-2},\overline{a},\overline{b}^{\,-1},\overline{c},\overline{a},\overline{b}^{\,2},\overline{a},\overline{b},\overline{c},\overline{b}^{\,-1},\overline{a},\overline{c}^{\,-1},\overline{b}^{\,2},\overline{c},\overline{a},\overline{b},\overline{c}^{\,-1},\overline{a},\overline{b}^{\,-1},\overline{a},\overline{b}\big) \end{aligned}$$

and

$$\begin{aligned} C_2 = \big(&\overline{a},\overline{c}^{\,-1},\overline{a},\overline{b},\overline{a},\overline{c},\overline{a},\overline{b}^{\,2},\overline{a},\overline{b},\overline{c},\overline{b}^{\,-1},\overline{a},\overline{c}^{\,-1},\overline{b}^{\,2},\overline{c},\overline{a},\overline{b},\overline{c}^{\,-1},\overline{b}^{\,-1},\overline{a},\overline{b}^{\,-2},\overline{a},\overline{c}^{\,-1},\overline{b},\overline{a},\overline{b}^{\,2},\\ &\overline{a},\overline{c},\overline{a},\overline{b},\overline{c}^{\,-1},\overline{b}^{\,-1},\overline{a},\overline{c}^{\,-1},\overline{a},\overline{c}^{\,-1},\overline{b}^{\,2},\overline{c},\overline{b}^{\,-2},\overline{a},\overline{b}^{\,-2},\overline{c},\overline{a},\overline{b}^{\,2},\overline{a},\overline{b},\overline{c}^{\,-1},\overline{a},\overline{b}^{\,-1},\overline{a},\overline{b}\big) \end{aligned}$$

Then $\big[\mathrm{wt}_{C_1}(\overline{b}),\mathrm{wt}_{C_1}(\overline{c})\big] = [1,0]$ and $\big[\mathrm{wt}_{C_2}(\overline{b}),\mathrm{wt}_{C_2}(\overline{c})\big] = [7,-2]$. Since $\det\left[\begin{smallmatrix} 1 & 0 \\ 7 & -2 \end{smallmatrix}\right] = -2 \not\equiv 0 \pmod{p}$, Theorem 2.11 applies.

**Subcase 2.4.** *Assume* $|\overline{a}| = |b| = 2$ *and* $|\overline{c}| = 3$. We may assume $\overline{c} = (1,2,3)$.

- Suppose $\overline{a}$ interchanges 4 and 5. This means that one of the 2-cycles in $\overline{a}$ is $(4,5)$. The other 2-cycle must be contained in $\{1,2,3\}$, so, after conjugating by a power of $\overline{c}$, we may assume $\overline{a} = (1,2)(4,5)$. Since $\langle \overline{a}, \overline{b}, \overline{c} \rangle$ is transitive on $\{1,2,3,4,5\}$, the permutation $\overline{b}$ cannot have $(4,5)$ as one of its 2-cycles. So no 2-cycle in $\overline{b}$ is disjoint from the support of $\overline{c}$. Since $\langle \overline{b}, \overline{c} \rangle \neq A_5$ (by the minimality of $\overline{S}$), this implies that one of the 2-cycles must be contained in the support of $\overline{c}$, which is $\{1,2,3\}$. So $\overline{b}$ fixes either 4 or 5. We may assume it is 5 that is fixed (after conjugating by $(4,5)$ if necessary). Then $\overline{b}$ is either $(1,2)(3,4)$ or $(1,3)(2,4)$ or $(2,3)(1,4)$. Since the last two are conjugate by $(1,2)$ (which centralizes $\overline{a}$ and inverts $\overline{b}$), they do not both need to be considered.

  - If $\overline{b} = (1,2)(3,4)$, we have the following hamiltonian cycle (see note 4.15):

$$C = \Big(\overline{a}, \overline{c}^{\,-1}, (\overline{a}, \overline{b})^2, \overline{a}, \overline{c}^{\,-1}, \overline{a}, \overline{b}, \overline{c}^{\,-1}, \overline{b}, \overline{c}, (\overline{b}, \overline{a})^2, \overline{c}, (\overline{a}, \overline{b})^2, \overline{a}, \overline{c}, (\overline{a}, \overline{b})^2,$$
$$\overline{a}, \overline{c}^{\,2}, \overline{b}, \overline{c}^{\,-1}, \overline{b}, (\overline{a}, \overline{b})^2, \overline{c}^{\,-2}, \big((\overline{a}, \overline{b})^2, \overline{a}, \overline{c}^{\,-1}\big)^2, \overline{c}^{\,-1}, \overline{b}, \overline{a}, \overline{c}^{\,-1}, (\overline{a}, \overline{b})^2\Big).$$

    Since $\mathrm{wt}_C(\overline{c}) = -5 \not\equiv 0 \pmod p$, Theorem 2.11 applies.

  - If $\overline{b} = (1,3)(2,4)$, we have the following hamiltonian cycle (see note 4.16):

$$C = \Big((\overline{a}, \overline{b})^4, \overline{c}, (\overline{a}, \overline{b})^4, \overline{a}, \overline{c}^{\,-1}, \overline{b}, \overline{a}, \overline{b}, \overline{c}^{\,-1}, (\overline{b}, \overline{a})^3, \overline{c}^{\,-1},$$
$$\overline{b}, (\overline{a}, \overline{b})^2, \overline{c}, \overline{b}, \overline{a}, \overline{c}^{\,-1}, (\overline{b}, \overline{a})^4, \overline{b}, \overline{c}, (\overline{a}, \overline{b})^4, \overline{a}, \overline{c}^{\,-1}, \overline{b}\Big).$$

    Since $\mathrm{wt}_C(\overline{c}) = -2 \not\equiv 0 \pmod p$, Theorem 2.11 applies.

- We may now assume that neither $\overline{a}$ nor $\overline{b}$ interchanges 4 and 5. Since $\langle \overline{a}, \overline{c} \rangle \neq A_5$, and $\overline{a}$ does not interchange 4 and 5, we see that $\overline{a}$ must fix either 4 or 5. Similarly for $\overline{b}$. Furthermore, $\overline{a}$ and $\overline{b}$ cannot both fix 4 (or 5), since $\langle \overline{a}, \overline{b}, \overline{c} \rangle$ is transitive. So one of them fixes 4, and the other fixes 5.

  We may assume it is $\overline{a}$ that fixes 5. Then we may assume $\overline{a} = (1,2)(3,4)$, after conjugating by a power of $\overline{c}$. Then $\overline{b}$ must be either $(1,2)(3,5)$ or $(1,3)(2,5)$ or $(1,5)(2,3)$. However, the last two are conjugate by $(1,2)$ (which centralizes $\overline{a}$ and inverts $\overline{c}$), so they do not both need to be considered.

  - If $\overline{b} = (1,2)(3,5)$, then we have the hamiltonian cycle (see note 4.17):

$$C = \Big(\overline{a}, \overline{c}^{\,-1}, \overline{a}, \overline{c}, \overline{b}, (\overline{a}, \overline{b})^2, \overline{c}, (\overline{a}, \overline{b})^2, (\overline{a}, \overline{c})^2, \overline{a}, \overline{b}, \overline{a}, \overline{c}^{\,-2}, \overline{a}, \overline{c}^{\,-1}, \overline{b}, \overline{a},$$
$$\overline{c}, \overline{b}, \overline{a}, \overline{b}, \overline{c}^{\,2}, \big((\overline{a}, \overline{b})^2, \overline{a}, \overline{c}\big)^2, (\overline{a}, \overline{c}^{\,-1})^2, (\overline{a}, \overline{b})^2, \overline{a}, \overline{c}^{\,-2}, (\overline{a}, \overline{b})^2\Big).$$

    Since $\mathrm{wt}_C(\overline{c}) = 1 \not\equiv 0 \pmod p$, Theorem 2.11 applies.

  - If $\overline{b} = (1,3)(2,5)$, then we have the following hamiltonian cycle (see note 4.18):

$$C = \Big(\overline{a}, \overline{b}, \overline{c}^{\,-1}, (\overline{a}, \overline{b})^2, \overline{a}, \overline{c}, \overline{b}, (\overline{a}, \overline{b})^4, \overline{c}, (\overline{a}, \overline{b}, \overline{a}, \overline{c}^{\,-1})^2, \overline{b}, \overline{a}, \overline{b}, \overline{c}^{\,2},$$
$$(\overline{a}, \overline{b})^2, \overline{a}, \overline{c}, \big((\overline{b}, \overline{a})^2, \overline{c}^{\,-1}\big)^2, (\overline{b}, \overline{a})^2, \overline{b}, \overline{c}^{\,-1}, (\overline{b}, \overline{a})^2, \overline{c}^{\,-1}, \overline{b}\Big).$$

    Since $\mathrm{wt}_C(\overline{c}) = -2 \not\equiv 0 \pmod p$, Theorem 2.11 applies.

**Subcase 2.5.** *Assume* $|\overline{a}| = |\overline{b}| = |\overline{c}| = 2$. Since all generators are of order 2, Theorem 2.9 applies. □

# 4. Details of hamiltonian cycles in $A_5$

To aid the reader in validating the many hamiltonian cycles in $A_5$ that appeared in the proof of Theorem 1.2, this final section provides a list of the vertices in the order that they are visited by each cycle.

**4.1.** *A hamiltonian cycle in* $\operatorname{Cay}(A_5; \bar a, \bar b)$ *with* $\bar a = (1,2)(3,4)$ *and* $\bar b = (2,4,5)$.

$$
\begin{array}{cccccccccc}
 & e & \xrightarrow{\bar a} & (1,2)(3,4) & \xrightarrow{\bar b} & (1,2,3,4,5) & \xrightarrow{\bar b} & (1,2,5,3,4) & \xrightarrow{\bar a} & (1,5,3) \\
\xrightarrow{\bar b} & (1,5,2,4,3) & \xrightarrow{\bar b} & (1,5,4,2,3) & \xrightarrow{\bar a} & (1,3,2,5,4) & \xrightarrow{\bar b} & (1,3,2) & \xrightarrow{\bar b} & (1,3,2,4,5) \\
\xrightarrow{\bar a} & (1,4,2,3,5) & \xrightarrow{\bar b^{-1}} & (1,4,3,5,2) & \xrightarrow{\bar b^{-1}} & (1,4)(3,5) & \xrightarrow{\bar a} & (1,2,4,5,3) & \xrightarrow{\bar b^{-1}} & (1,2,3) \\
\xrightarrow{\bar b^{-1}} & (1,2,5,4,3) & \xrightarrow{\bar a} & (1,5,4) & \xrightarrow{\bar b^{-1}} & (1,5)(2,4) & \xrightarrow{\bar b^{-1}} & (1,5,2) & \xrightarrow{\bar a} & (2,5)(3,4) \\
\xrightarrow{\bar b} & (2,3,4) & \xrightarrow{\bar b} & (3,4,5) & \xrightarrow{\bar a} & (1,2)(3,5) & \xrightarrow{\bar b^{-1}} & (1,2,3,5,4) & \xrightarrow{\bar b^{-1}} & (1,2,4,3,5) \\
\xrightarrow{\bar a} & (1,4,5) & \xrightarrow{\bar b} & (1,4)(2,5) & \xrightarrow{\bar b} & (1,4,2) & \xrightarrow{\bar a} & (2,4,3) & \xrightarrow{\bar b^{-1}} & (2,5,3) \\
\xrightarrow{\bar b^{-1}} & (2,3)(4,5) & \xrightarrow{\bar a} & (1,3,5,4,2) & \xrightarrow{\bar b} & (1,3,5) & \xrightarrow{\bar b} & (1,3,5,2,4) & \xrightarrow{\bar a} & (1,4,5,2,3) \\
\xrightarrow{\bar b} & (1,4,2,5,3) & \xrightarrow{\bar b} & (1,4,3) & \xrightarrow{\bar a} & (1,2,4) & \xrightarrow{\bar b} & (1,2)(4,5) & \xrightarrow{\bar b} & (1,2,5) \\
\xrightarrow{\bar a} & (1,5)(3,4) & \xrightarrow{\bar b^{-1}} & (1,5,3,4,2) & \xrightarrow{\bar b^{-1}} & (1,5,2,3,4) & \xrightarrow{\bar a} & (1,3)(2,5) & \xrightarrow{\bar b^{-1}} & (1,3)(4,5) \\
\xrightarrow{\bar b^{-1}} & (1,3)(2,4) & \xrightarrow{\bar a} & (1,4)(2,3) & \xrightarrow{\bar b^{-1}} & (1,4,3,2,5) & \xrightarrow{\bar b^{-1}} & (1,4,5,3,2) & \xrightarrow{\bar a} & (2,4)(3,5) \\
\xrightarrow{\bar b} & (3,5,4) & \xrightarrow{\bar b} & (2,3,5) & \xrightarrow{\bar a} & (1,3,4,5,2) & \xrightarrow{\bar b^{-1}} & (1,3,4) & \xrightarrow{\bar b^{-1}} & (1,3,4,2,5) \\
\xrightarrow{\bar a} & (1,5)(2,3) & \xrightarrow{\bar b} & (1,5,3,2,4) & \xrightarrow{\bar b} & (1,5,4,3,2) & \xrightarrow{\bar a} & (2,5,4) & \xrightarrow{\bar b^{-1}} & (2,4,5) \\
\xrightarrow{\bar b^{-1}} & e & & & & & & & &
\end{array}
$$

**4.2.** *A hamiltonian cycle in* $\operatorname{Cay}(A_5; \bar a, \bar b)$ *with* $\bar a = (1,2)(3,4)$ *and* $\bar b = (1,2,3,4,5)$.

$$
\begin{array}{cccccccccc}
 & e & \xrightarrow{\bar a} & (1,2)(3,4) & \xrightarrow{\bar b} & (2,4,5) & \xrightarrow{\bar a} & (1,4,3,5,2) & \xrightarrow{\bar b} & (2,5,4) \\
\xrightarrow{\bar b} & (1,5)(2,3) & \xrightarrow{\bar b} & (1,3,4) & \xrightarrow{\bar b} & (1,2,4,5,3) & \xrightarrow{\bar a} & (1,4)(3,5) & \xrightarrow{\bar b} & (1,2,5,4,3) \\
\xrightarrow{\bar a} & (1,5,4) & \xrightarrow{\bar b} & (1,2,3) & \xrightarrow{\bar b} & (1,3,4,5,2) & \xrightarrow{\bar b} & (2,4)(3,5) & \xrightarrow{\bar b} & (1,4,3,2,5) \\
\xrightarrow{\bar a} & (1,5)(2,4) & \xrightarrow{\bar b} & (1,4)(2,3) & \xrightarrow{\bar b} & (1,3)(4,5) & \xrightarrow{\bar a} & (1,2,3,5,4) & \xrightarrow{\bar b^{-1}} & (1,4,5) \\
\xrightarrow{\bar a} & (1,2,4,3,5) & \xrightarrow{\bar b} & (1,4)(2,5) & \xrightarrow{\bar b} & (1,5,4,2,3) & \xrightarrow{\bar b} & (1,3,2) & \xrightarrow{\bar b} & (3,4,5) \\
\xrightarrow{\bar a} & (1,2)(3,5) & \xrightarrow{\bar b} & (2,5)(3,4) & \xrightarrow{\bar a} & (1,5,2) & \xrightarrow{\bar b} & (2,3,4) & \xrightarrow{\bar b} & (1,3,2,4,5) \\
\xrightarrow{\bar a} & (1,4,2,3,5) & \xrightarrow{\bar b} & (1,3,2,5,4) & \xrightarrow{\bar b} & (1,5,3) & \xrightarrow{\bar a} & (1,2,5,3,4) & \xrightarrow{\bar b} & (1,5,2,4,3) \\
\xrightarrow{\bar b} & (1,4,2) & \xrightarrow{\bar a} & (2,4,3) & \xrightarrow{\bar b^{-1}} & (1,5,3,4,2) & \xrightarrow{\bar b^{-1}} & (1,3)(2,5) & \xrightarrow{\bar a} & (1,5,2,3,4) \\
\xrightarrow{\bar b} & (1,3)(2,4) & \xrightarrow{\bar b} & (1,4,5,3,2) & \xrightarrow{\bar b} & (3,5,4) & \xrightarrow{\bar b} & (1,2,5) & \xrightarrow{\bar a} & (1,5)(3,4) \\
\xrightarrow{\bar b^{-1}} & (2,5,3) & \xrightarrow{\bar b^{-1}} & (1,3,5,4,2) & \xrightarrow{\bar a} & (2,3)(4,5) & \xrightarrow{\bar b} & (1,3,5) & \xrightarrow{\bar a} & (1,2,3,4,5) \\
\xrightarrow{\bar b} & (1,3,5,2,4) & \xrightarrow{\bar a} & (1,4,5,2,3) & \xrightarrow{\bar b^{-1}} & (1,2,4) & \xrightarrow{\bar a} & (1,4,3) & \xrightarrow{\bar b} & (1,2)(4,5) \\
\xrightarrow{\bar b} & (2,3,5) & \xrightarrow{\bar b} & (1,3,4,2,5) & \xrightarrow{\bar b} & (1,5,3,2,4) & \xrightarrow{\bar a} & (1,4,2,5,3) & \xrightarrow{\bar b} & (1,5,4,3,2) \\
\xrightarrow{\bar b} & e & & & & & & & &
\end{array}
$$

**4.3.** *A hamiltonian cycle in* $\mathrm{Cay}(A_5; \overline{a}, \overline{b})$ *with* $\overline{a} = (1,3)(2,4)$ *and* $\overline{b} = (1,2,3,4,5)$.

$$
\begin{array}{ccccccccccc}
 & e & \xrightarrow{\overline{a}} & (1,3)(2,4) & \xrightarrow{\overline{b}} & (1,4,5,3,2) & \xrightarrow{\overline{b}} & (3,5,4) & \xrightarrow{\overline{b}} & (1,2,5) \\
\xrightarrow{\overline{b}} & (1,5,2,3,4) & \xrightarrow{\overline{a}} & (1,4,3,5,2) & \xrightarrow{\overline{b}^{-1}} & (1,2,4,5,3) & \xrightarrow{\overline{a}} & (2,5,3) & \xrightarrow{\overline{b}} & (1,5)(3,4) \\
\xrightarrow{\overline{a}} & (1,4,2,3,5) & \xrightarrow{\overline{b}^{-1}} & (2,4,5) & \xrightarrow{\overline{a}} & (1,3)(2,5) & \xrightarrow{\overline{b}^{-1}} & (1,2,3,5,4) & \xrightarrow{\overline{b}^{-1}} & (1,4,5) \\
\xrightarrow{\overline{b}^{-1}} & (2,4,3) & \xrightarrow{\overline{b}^{-1}} & (1,5,3,4,2) & \xrightarrow{\overline{a}} & (1,4)(3,5) & \xrightarrow{\overline{b}^{-1}} & (1,3,2,4,5) & \xrightarrow{\overline{b}^{-1}} & (2,3,4) \\
\xrightarrow{\overline{a}} & (1,4,3) & \xrightarrow{\overline{b}^{-1}} & (1,5,3,2,4) & \xrightarrow{\overline{b}^{-1}} & (1,3,4,2,5) & \xrightarrow{\overline{b}^{-1}} & (2,3,5) & \xrightarrow{\overline{b}^{-1}} & (1,2)(4,5) \\
\xrightarrow{\overline{a}} & (1,3,2,5,4) & \xrightarrow{\overline{b}} & (1,5,3) & \xrightarrow{\overline{b}} & (1,2)(3,4) & \xrightarrow{\overline{a}} & (1,4)(2,3) & \xrightarrow{\overline{b}^{-1}} & (1,5)(2,4) \\
\xrightarrow{\overline{b}^{-1}} & (2,5)(3,4) & \xrightarrow{\overline{b}^{-1}} & (1,2)(3,5) & \xrightarrow{\overline{b}^{-1}} & (1,3)(4,5) & \xrightarrow{\overline{a}} & (2,5,4) & \xrightarrow{\overline{b}} & (1,5)(2,3) \\
\xrightarrow{\overline{b}} & (1,3,4) & \xrightarrow{\overline{a}} & (1,4,2) & \xrightarrow{\overline{b}^{-1}} & (1,5,2,4,3) & \xrightarrow{\overline{b}^{-1}} & (1,2,5,3,4) & \xrightarrow{\overline{b}^{-1}} & (1,3,5) \\
\xrightarrow{\overline{b}^{-1}} & (2,3)(4,5) & \xrightarrow{\overline{a}} & (1,2,5,4,3) & \xrightarrow{\overline{b}} & (1,5,2) & \xrightarrow{\overline{a}} & (1,3,5,2,4) & \xrightarrow{\overline{b}^{-1}} & (1,2,3,4,5) \\
\xrightarrow{\overline{a}} & (1,4,3,2,5) & \xrightarrow{\overline{b}^{-1}} & (2,4)(3,5) & \xrightarrow{\overline{b}^{-1}} & (1,3,4,5,2) & \xrightarrow{\overline{b}^{-1}} & (1,2,3) & \xrightarrow{\overline{b}^{-1}} & (1,5,4) \\
\xrightarrow{\overline{a}} & (1,3,5,4,2) & \xrightarrow{\overline{b}^{-1}} & (1,4,5,2,3) & \xrightarrow{\overline{b}^{-1}} & (1,2,4) & \xrightarrow{\overline{a}} & (1,3,2) & \xrightarrow{\overline{b}^{-1}} & (1,5,4,2,3) \\
\xrightarrow{\overline{b}^{-1}} & (1,4)(2,5) & \xrightarrow{\overline{b}^{-1}} & (1,2,4,3,5) & \xrightarrow{\overline{b}^{-1}} & (3,4,5) & \xrightarrow{\overline{a}} & (1,4,2,5,3) & \xrightarrow{\overline{b}} & (1,5,4,3,2) \\
\xrightarrow{\overline{b}} & e & & & & & & & &
\end{array}
$$

**4.4.** *A hamiltonian cycle* $C_1$ *in* $\mathrm{Cay}(A_5; \overline{a}, \overline{b})$ *with* $\overline{a} = (1,2,3)$ *and* $\overline{b} = (3,4,5)$.

$$
\begin{array}{ccccccccccc}
 & e & \xrightarrow{\overline{b}} & (3,4,5) & \xrightarrow{\overline{a}} & (1,2,4,5,3) & \xrightarrow{\overline{b}} & (1,2,4,3,5) & \xrightarrow{\overline{b}} & (1,2,4) \\
\xrightarrow{\overline{a}} & (1,4)(2,3) & \xrightarrow{\overline{a}} & (1,3,4) & \xrightarrow{\overline{b}^{-1}} & (1,3,5) & \xrightarrow{\overline{b}^{-1}} & (1,3)(4,5) & \xrightarrow{\overline{a}^{-1}} & (2,3)(4,5) \\
\xrightarrow{\overline{a}^{-1}} & (1,2)(4,5) & \xrightarrow{\overline{b}} & (1,2)(3,5) & \xrightarrow{\overline{b}} & (1,2)(3,4) & \xrightarrow{\overline{a}} & (2,4,3) & \xrightarrow{\overline{a}} & (1,4,3) \\
\xrightarrow{\overline{b}} & (1,4,5) & \xrightarrow{\overline{b}} & (1,4)(3,5) & \xrightarrow{\overline{a}^{-1}} & (1,5,3,2,4) & \xrightarrow{\overline{a}^{-1}} & (1,2,5,3,4) & \xrightarrow{\overline{b}^{-1}} & (1,2,5) \\
\xrightarrow{\overline{b}^{-1}} & (1,2,5,4,3) & \xrightarrow{\overline{a}^{-1}} & (3,5,4) & \xrightarrow{\overline{a}^{-1}} & (1,5,4,3,2) & \xrightarrow{\overline{b}} & (1,5,2) & \xrightarrow{\overline{b}} & (1,5,3,4,2) \\
\xrightarrow{\overline{a}} & (2,4)(3,5) & \xrightarrow{\overline{a}} & (1,4,2,5,3) & \xrightarrow{\overline{b}^{-1}} & (1,4)(2,5) & \xrightarrow{\overline{b}^{-1}} & (1,4,3,2,5) & \xrightarrow{\overline{a}} & (1,5)(3,4) \\
\xrightarrow{\overline{b}^{-1}} & (1,5,3) & \xrightarrow{\overline{b}^{-1}} & (1,5,4) & \xrightarrow{\overline{a}^{-1}} & (1,3,2,5,4) & \xrightarrow{\overline{a}^{-1}} & (1,2,3,5,4) & \xrightarrow{\overline{b}} & (1,2,3) \\
\xrightarrow{\overline{b}} & (1,2,3,4,5) & \xrightarrow{\overline{a}} & (1,3,2,4,5) & \xrightarrow{\overline{b}} & (1,3,5,2,4) & \xrightarrow{\overline{a}^{-1}} & (1,5,2,3,4) & \xrightarrow{\overline{b}^{-1}} & (1,5)(2,3) \\
\xrightarrow{\overline{b}^{-1}} & (1,5,4,2,3) & \xrightarrow{\overline{a}} & (1,3,5,4,2) & \xrightarrow{\overline{a}} & (2,5,4) & \xrightarrow{\overline{b}} & (2,5,3) & \xrightarrow{\overline{b}} & (2,5)(3,4) \\
\xrightarrow{\overline{a}} & (1,5,2,4,3) & \xrightarrow{\overline{a}} & (1,4,3,5,2) & \xrightarrow{\overline{b}^{-1}} & (1,4,5,3,2) & \xrightarrow{\overline{b}^{-1}} & (1,4,2) & \xrightarrow{\overline{a}^{-1}} & (1,3)(2,4) \\
\xrightarrow{\overline{a}^{-1}} & (2,3,4) & \xrightarrow{\overline{b}^{-1}} & (2,3,5) & \xrightarrow{\overline{a}} & (1,3)(2,5) & \xrightarrow{\overline{b}} & (1,3,4,2,5) & \xrightarrow{\overline{a}} & (1,5)(2,4) \\
\xrightarrow{\overline{a}} & (1,4,2,3,5) & \xrightarrow{\overline{b}^{-1}} & (1,4,5,2,3) & \xrightarrow{\overline{a}^{-1}} & (2,4,5) & \xrightarrow{\overline{a}^{-1}} & (1,3,4,5,2) & \xrightarrow{\overline{b}^{-1}} & (1,3,2) \\
\xrightarrow{\overline{a}} & e & & & & & & & &
\end{array}
$$

**4.5.** *A hamiltonian cycle $C_1$ in* $\mathrm{Cay}(A_5;\overline{a},\overline{b})$ *with* $\overline{a}=(1,2,4)$ *and* $\overline{b}=(1,2,3,4,5)$.

$$\begin{array}{cccccccccc}
 & e & \xrightarrow{\overline{a}} & (1,2,4) & \xrightarrow{\overline{a}} & (1,4,2) & \xrightarrow{\overline{b}^{-1}} & (1,5,2,4,3) & \xrightarrow{\overline{b}^{-1}} & (1,2,5,3,4) \\
\xrightarrow{\overline{a}^{-1}} & (3,4,5) & \xrightarrow{\overline{a}^{-1}} & (1,5,3,4,2) & \xrightarrow{\overline{b}^{-1}} & (1,3)(2,5) & \xrightarrow{\overline{a}^{-1}} & (1,4,5,2,3) & \xrightarrow{\overline{b}} & (1,3,5,4,2) \\
\xrightarrow{\overline{a}} & (3,5,4) & \xrightarrow{\overline{a}} & (1,2,3,5,4) & \xrightarrow{\overline{b}^{-1}} & (1,4,5) & \xrightarrow{\overline{a}^{-1}} & (1,5)(2,4) & \xrightarrow{\overline{a}^{-1}} & (1,2,5) \\
\xrightarrow{\overline{b}} & (1,5,2,3,4) & \xrightarrow{\overline{a}^{-1}} & (2,5)(3,4) & \xrightarrow{\overline{a}^{-1}} & (1,3,4,5,2) & \xrightarrow{\overline{b}^{-1}} & (1,2,3) & \xrightarrow{\overline{b}^{-1}} & (1,5,4) \\
\xrightarrow{\overline{a}^{-1}} & (2,5,4) & \xrightarrow{\overline{a}^{-1}} & (1,2)(4,5) & \xrightarrow{\overline{b}^{-1}} & (1,4,3) & \xrightarrow{\overline{a}^{-1}} & (1,3)(2,4) & \xrightarrow{\overline{b}} & (1,4,5,3,2) \\
\xrightarrow{\overline{a}} & (2,5,3) & \xrightarrow{\overline{a}} & (1,5,3,2,4) & \xrightarrow{\overline{b}^{-1}} & (1,3,4,2,5) & \xrightarrow{\overline{a}} & (1,5)(3,4) & \xrightarrow{\overline{a}} & (1,2,3,4,5) \\
\xrightarrow{\overline{b}} & (1,3,5,2,4) & \xrightarrow{\overline{a}^{-1}} & (2,3,5) & \xrightarrow{\overline{a}^{-1}} & (1,4,3,5,2) & \xrightarrow{\overline{b}^{-1}} & (1,2,4,5,3) & \xrightarrow{\overline{a}} & (1,4,2,5,3) \\
\xrightarrow{\overline{a}} & (1,5,3) & \xrightarrow{\overline{b}} & (1,2)(3,4) & \xrightarrow{\overline{a}} & (2,3,4) & \xrightarrow{\overline{a}} & (1,3,4) & \xrightarrow{\overline{b}^{-1}} & (1,5)(2,3) \\
\xrightarrow{\overline{a}^{-1}} & (1,4,3,2,5) & \xrightarrow{\overline{a}^{-1}} & (1,3,2,4,5) & \xrightarrow{\overline{b}} & (1,4)(3,5) & \xrightarrow{\overline{a}^{-1}} & (2,4)(3,5) & \xrightarrow{\overline{a}^{-1}} & (1,2)(3,5) \\
\xrightarrow{\overline{b}^{-1}} & (1,3)(4,5) & \xrightarrow{\overline{b}^{-1}} & (1,4)(2,3) & \xrightarrow{\overline{a}^{-1}} & (2,4,3) & \xrightarrow{\overline{a}^{-1}} & (1,3,2) & \xrightarrow{\overline{b}^{-1}} & (1,5,4,2,3) \\
\xrightarrow{\overline{a}^{-1}} & (1,2,5,4,3) & \xrightarrow{\overline{b}} & (1,5,2) & \xrightarrow{\overline{a}} & (2,4,5) & \xrightarrow{\overline{a}} & (1,4)(2,5) & \xrightarrow{\overline{b}^{-1}} & (1,2,4,3,5) \\
\xrightarrow{\overline{a}^{-1}} & (1,3,5) & \xrightarrow{\overline{a}^{-1}} & (1,4,2,3,5) & \xrightarrow{\overline{b}} & (1,3,2,5,4) & \xrightarrow{\overline{a}^{-1}} & (2,3)(4,5) & \xrightarrow{\overline{a}^{-1}} & (1,5,4,3,2) \\
\xrightarrow{\overline{b}} & e & & & & & & & &
\end{array}$$

**4.6.** *A second hamiltonian cycle $C_2$ in* $\mathrm{Cay}(A_5;\overline{a},\overline{b})$ *with* $\overline{a}=(1,2,4)$ *and* $\overline{b}=(1,2,3,4,5)$.

$$\begin{array}{cccccccccc}
 & e & \xrightarrow{\overline{a}} & (1,2,4) & \xrightarrow{\overline{a}} & (1,4,2) & \xrightarrow{\overline{b}^{-1}} & (1,5,2,4,3) & \xrightarrow{\overline{b}^{-1}} & (1,2,5,3,4) \\
\xrightarrow{\overline{a}^{-1}} & (3,4,5) & \xrightarrow{\overline{a}^{-1}} & (1,5,3,4,2) & \xrightarrow{\overline{b}^{-1}} & (1,3)(2,5) & \xrightarrow{\overline{a}^{-1}} & (1,4,5,2,3) & \xrightarrow{\overline{b}} & (1,3,5,4,2) \\
\xrightarrow{\overline{a}} & (3,5,4) & \xrightarrow{\overline{a}} & (1,2,3,5,4) & \xrightarrow{\overline{b}^{-1}} & (1,4,5) & \xrightarrow{\overline{a}} & (1,2,5) & \xrightarrow{\overline{a}} & (1,5)(2,4) \\
\xrightarrow{\overline{b}^{-1}} & (2,5)(3,4) & \xrightarrow{\overline{a}} & (1,5,2,3,4) & \xrightarrow{\overline{a}} & (1,3,4,5,2) & \xrightarrow{\overline{b}^{-1}} & (1,2,3) & \xrightarrow{\overline{b}^{-1}} & (1,5,4) \\
\xrightarrow{\overline{a}^{-1}} & (2,5,4) & \xrightarrow{\overline{a}^{-1}} & (1,2)(4,5) & \xrightarrow{\overline{b}^{-1}} & (1,4,3) & \xrightarrow{\overline{a}^{-1}} & (1,3)(2,4) & \xrightarrow{\overline{b}} & (1,4,5,3,2) \\
\xrightarrow{\overline{a}} & (2,5,3) & \xrightarrow{\overline{a}} & (1,5,3,2,4) & \xrightarrow{\overline{b}^{-1}} & (1,3,4,2,5) & \xrightarrow{\overline{a}} & (1,5)(3,4) & \xrightarrow{\overline{a}} & (1,2,3,4,5) \\
\xrightarrow{\overline{b}} & (1,3,5,2,4) & \xrightarrow{\overline{a}^{-1}} & (2,3,5) & \xrightarrow{\overline{a}^{-1}} & (1,4,3,5,2) & \xrightarrow{\overline{b}^{-1}} & (1,2,4,5,3) & \xrightarrow{\overline{a}} & (1,4,2,5,3) \\
\xrightarrow{\overline{a}} & (1,5,3) & \xrightarrow{\overline{b}} & (1,2)(3,4) & \xrightarrow{\overline{a}} & (2,3,4) & \xrightarrow{\overline{a}} & (1,3,4) & \xrightarrow{\overline{b}^{-1}} & (1,5)(2,3) \\
\xrightarrow{\overline{a}^{-1}} & (1,4,3,2,5) & \xrightarrow{\overline{a}^{-1}} & (1,3,2,4,5) & \xrightarrow{\overline{b}} & (1,4)(3,5) & \xrightarrow{\overline{a}^{-1}} & (2,4)(3,5) & \xrightarrow{\overline{a}^{-1}} & (1,2)(3,5) \\
\xrightarrow{\overline{b}^{-1}} & (1,3)(4,5) & \xrightarrow{\overline{b}^{-1}} & (1,4)(2,3) & \xrightarrow{\overline{a}^{-1}} & (2,4,3) & \xrightarrow{\overline{a}^{-1}} & (1,3,2) & \xrightarrow{\overline{b}^{-1}} & (1,5,4,2,3) \\
\xrightarrow{\overline{a}^{-1}} & (1,2,5,4,3) & \xrightarrow{\overline{b}} & (1,5,2) & \xrightarrow{\overline{a}} & (2,4,5) & \xrightarrow{\overline{a}} & (1,4)(2,5) & \xrightarrow{\overline{b}^{-1}} & (1,2,4,3,5) \\
\xrightarrow{\overline{a}^{-1}} & (1,3,5) & \xrightarrow{\overline{a}^{-1}} & (1,4,2,3,5) & \xrightarrow{\overline{b}} & (1,3,2,5,4) & \xrightarrow{\overline{a}^{-1}} & (2,3)(4,5) & \xrightarrow{\overline{a}^{-1}} & (1,5,4,3,2) \\
\xrightarrow{\overline{b}} & e & & & & & & & &
\end{array}$$

**4.7.** *A hamiltonian cycle $C_1$ in* $\mathrm{Cay}(A_5;\overline{a},\overline{b})$ *with* $\overline{a}=(1,2,3)$ *and* $\overline{b}=(1,2,3,4,5)$.

$$
\begin{array}{cccccccccc}
 & e & \xrightarrow{\overline{a}} & (1,2,3) & \xrightarrow{\overline{a}} & (1,3,2) & \xrightarrow{\overline{b}^{-1}} & (1,5,4,2,3) & \xrightarrow{\overline{b}^{-1}} & (1,4)(2,5) \\
\xrightarrow{\overline{a}^{-1}} & (1,3,5,2,4) & \xrightarrow{\overline{a}^{-1}} & (1,5,2,3,4) & \xrightarrow{\overline{b}^{-1}} & (1,2,5) & \xrightarrow{\overline{a}^{-1}} & (1,3,5) & \xrightarrow{\overline{b}} & (1,2,5,3,4) \\
\xrightarrow{\overline{b}} & (1,5,2,4,3) & \xrightarrow{\overline{b}} & (1,4,2) & \xrightarrow{\overline{a}} & (2,3,4) & \xrightarrow{\overline{a}} & (1,3)(2,4) & \xrightarrow{\overline{b}} & (1,4,5,3,2) \\
\xrightarrow{\overline{a}} & (3,4,5) & \xrightarrow{\overline{a}} & (1,2,4,5,3) & \xrightarrow{\overline{b}} & (1,4,3,5,2) & \xrightarrow{\overline{a}} & (2,5)(3,4) & \xrightarrow{\overline{b}^{-1}} & (1,2)(3,5) \\
\xrightarrow{\overline{a}} & (2,5,3) & \xrightarrow{\overline{a}} & (1,5,3) & \xrightarrow{\overline{b}} & (1,2)(3,4) & \xrightarrow{\overline{a}} & (2,4,3) & \xrightarrow{\overline{a}} & (1,4,3) \\
\xrightarrow{\overline{b}} & (1,2)(4,5) & \xrightarrow{\overline{a}} & (2,3)(4,5) & \xrightarrow{\overline{a}} & (1,3)(4,5) & \xrightarrow{\overline{b}^{-1}} & (1,4)(2,3) & \xrightarrow{\overline{b}^{-1}} & (1,5)(2,4) \\
\xrightarrow{\overline{a}} & (1,4,2,3,5) & \xrightarrow{\overline{a}} & (1,3,4,2,5) & \xrightarrow{\overline{b}} & (1,5,3,2,4) & \xrightarrow{\overline{a}} & (1,4)(3,5) & \xrightarrow{\overline{b}^{-1}} & (1,3,2,4,5) \\
\xrightarrow{\overline{a}^{-1}} & (1,2,3,4,5) & \xrightarrow{\overline{a}^{-1}} & (1,4,5) & \xrightarrow{\overline{b}} & (1,2,3,5,4) & \xrightarrow{\overline{a}} & (1,3,2,5,4) & \xrightarrow{\overline{a}} & (1,5,4) \\
\xrightarrow{\overline{b}^{-1}} & (1,4,3,2,5) & \xrightarrow{\overline{a}^{-1}} & (1,2,4,3,5) & \xrightarrow{\overline{a}^{-1}} & (1,5)(3,4) & \xrightarrow{\overline{b}} & (1,2,4) & \xrightarrow{\overline{a}^{-1}} & (1,3,4) \\
\xrightarrow{\overline{b}^{-1}} & (1,5)(2,3) & \xrightarrow{\overline{b}^{-1}} & (2,5,4) & \xrightarrow{\overline{a}^{-1}} & (1,3,5,4,2) & \xrightarrow{\overline{b}^{-1}} & (1,4,5,2,3) & \xrightarrow{\overline{a}^{-1}} & (2,4,5) \\
\xrightarrow{\overline{a}^{-1}} & (1,3,4,5,2) & \xrightarrow{\overline{b}} & (2,4)(3,5) & \xrightarrow{\overline{a}} & (1,4,2,5,3) & \xrightarrow{\overline{a}} & (1,5,3,4,2) & \xrightarrow{\overline{b}^{-1}} & (1,3)(2,5) \\
\xrightarrow{\overline{a}^{-1}} & (2,3,5) & \xrightarrow{\overline{a}^{-1}} & (1,5,2) & \xrightarrow{\overline{b}^{-1}} & (1,2,5,4,3) & \xrightarrow{\overline{a}^{-1}} & (3,5,4) & \xrightarrow{\overline{a}^{-1}} & (1,5,4,3,2) \\
\xrightarrow{\overline{b}} & e & & & & & & & &
\end{array}
$$

**4.8.** *A second hamiltonian cycle $C_2$ in* $\mathrm{Cay}(A_5;\overline{a},\overline{b})$ *with* $\overline{a}=(1,2,3)$ *and* $\overline{b}=(1,2,3,4,5)$.

$$
\begin{array}{cccccccccc}
 & e & \xrightarrow{\overline{a}} & (1,2,3) & \xrightarrow{\overline{a}} & (1,3,2) & \xrightarrow{\overline{b}^{-1}} & (1,5,4,2,3) & \xrightarrow{\overline{a}} & (1,3,5,4,2) \\
\xrightarrow{\overline{a}} & (2,5,4) & \xrightarrow{\overline{b}} & (1,5)(2,3) & \xrightarrow{\overline{a}} & (1,3,5) & \xrightarrow{\overline{a}} & (1,2,5) & \xrightarrow{\overline{b}} & (1,5,2,3,4) \\
\xrightarrow{\overline{a}} & (1,3,5,2,4) & \xrightarrow{\overline{a}} & (1,4)(2,5) & \xrightarrow{\overline{b}^{-1}} & (1,2,4,3,5) & \xrightarrow{\overline{a}^{-1}} & (1,5)(3,4) & \xrightarrow{\overline{a}^{-1}} & (1,4,3,2,5) \\
\xrightarrow{\overline{b}} & (1,5,4) & \xrightarrow{\overline{a}^{-1}} & (1,3,2,5,4) & \xrightarrow{\overline{a}^{-1}} & (1,2,3,5,4) & \xrightarrow{\overline{b}^{-1}} & (1,4,5) & \xrightarrow{\overline{a}} & (1,2,3,4,5) \\
\xrightarrow{\overline{a}} & (1,3,2,4,5) & \xrightarrow{\overline{b}} & (1,4)(3,5) & \xrightarrow{\overline{a}} & (1,2,5,3,4) & \xrightarrow{\overline{a}} & (1,5,3,2,4) & \xrightarrow{\overline{b}^{-1}} & (1,3,4,2,5) \\
\xrightarrow{\overline{a}^{-1}} & (1,4,2,3,5) & \xrightarrow{\overline{a}^{-1}} & (1,5)(2,4) & \xrightarrow{\overline{b}} & (1,4)(2,3) & \xrightarrow{\overline{b}} & (1,3)(4,5) & \xrightarrow{\overline{a}^{-1}} & (2,3)(4,5) \\
\xrightarrow{\overline{a}^{-1}} & (1,2)(4,5) & \xrightarrow{\overline{b}^{-1}} & (1,4,3) & \xrightarrow{\overline{a}^{-1}} & (2,4,3) & \xrightarrow{\overline{a}^{-1}} & (1,2)(3,4) & \xrightarrow{\overline{b}^{-1}} & (1,5,3) \\
\xrightarrow{\overline{a}^{-1}} & (2,5,3) & \xrightarrow{\overline{a}^{-1}} & (1,2)(3,5) & \xrightarrow{\overline{b}} & (2,5)(3,4) & \xrightarrow{\overline{a}^{-1}} & (1,4,3,5,2) & \xrightarrow{\overline{a}^{-1}} & (1,5,2,4,3) \\
\xrightarrow{\overline{b}} & (1,4,2) & \xrightarrow{\overline{a}} & (2,3,4) & \xrightarrow{\overline{a}} & (1,3)(2,4) & \xrightarrow{\overline{b}} & (1,4,5,3,2) & \xrightarrow{\overline{a}} & (3,4,5) \\
\xrightarrow{\overline{a}} & (1,2,4,5,3) & \xrightarrow{\overline{b}^{-1}} & (1,3,4) & \xrightarrow{\overline{a}} & (1,2,4) & \xrightarrow{\overline{b}} & (1,4,5,2,3) & \xrightarrow{\overline{a}^{-1}} & (2,4,5) \\
\xrightarrow{\overline{a}^{-1}} & (1,3,4,5,2) & \xrightarrow{\overline{b}} & (2,4)(3,5) & \xrightarrow{\overline{a}} & (1,4,2,5,3) & \xrightarrow{\overline{a}} & (1,5,3,4,2) & \xrightarrow{\overline{b}^{-1}} & (1,3)(2,5) \\
\xrightarrow{\overline{a}^{-1}} & (2,3,5) & \xrightarrow{\overline{a}^{-1}} & (1,5,2) & \xrightarrow{\overline{b}^{-1}} & (1,2,5,4,3) & \xrightarrow{\overline{a}^{-1}} & (3,5,4) & \xrightarrow{\overline{a}^{-1}} & (1,5,4,3,2) \\
\xrightarrow{\overline{b}} & e & & & & & & & &
\end{array}
$$

**4.9.** *A hamiltonian cycle $C_1$ in* $\mathrm{Cay}(A_5;\overline{a},\overline{b})$ *with* $\overline{a}=(1,2,3,4,5)$ *and* $\overline{b}=(1,2,3,5,4)$.

$$
\begin{array}{cccccccccc}
 & e & \xrightarrow{\overline{b}} & (1,2,3,5,4) & \xrightarrow{\overline{a}^{-1}} & (1,4,5) & \xrightarrow{\overline{b}} & (1,2,3) & \xrightarrow{\overline{a}} & (1,3,4,5,2) \\
\xrightarrow{\overline{b}} & (2,4,3) & \xrightarrow{\overline{b}} & (1,4)(3,5) & \xrightarrow{\overline{b}} & (1,2,5) & \xrightarrow{\overline{b}} & (1,5,4,2,3) & \xrightarrow{\overline{a}} & (1,3,2) \\
\xrightarrow{\overline{b}} & (3,5,4) & \xrightarrow{\overline{b}} & (1,2,5,3,4) & \xrightarrow{\overline{a}^{-1}} & (1,3,5) & \xrightarrow{\overline{b}} & (1,2,5,4,3) & \xrightarrow{\overline{b}} & (1,5,3,4,2) \\
\xrightarrow{\overline{a}^{-1}} & (1,3)(2,5) & \xrightarrow{\overline{b}^{-1}} & (1,4,2,3,5) & \xrightarrow{\overline{a}^{-1}} & (2,4,5) & \xrightarrow{\overline{b}} & (1,4)(2,3) & \xrightarrow{\overline{a}^{-1}} & (1,5)(2,4) \\
\xrightarrow{\overline{b}} & (1,4,5,2,3) & \xrightarrow{\overline{a}} & (1,3,5,4,2) & \xrightarrow{\overline{b}} & (2,5)(3,4) & \xrightarrow{\overline{b}} & (1,5,3,2,4) & \xrightarrow{\overline{a}^{-1}} & (1,3,4,2,5) \\
\xrightarrow{\overline{b}} & (1,5,2,4,3) & \xrightarrow{\overline{b}} & (1,4,5,3,2) & \xrightarrow{\overline{a}^{-1}} & (1,3)(2,4) & \xrightarrow{\overline{b}^{-1}} & (1,2,3,4,5) & \xrightarrow{\overline{a}} & (1,3,5,2,4) \\
\xrightarrow{\overline{b}^{-1}} & (2,3,4) & \xrightarrow{\overline{b}^{-1}} & (1,2)(4,5) & \xrightarrow{\overline{a}^{-1}} & (1,4,3) & \xrightarrow{\overline{b}^{-1}} & (1,3,2,4,5) & \xrightarrow{\overline{b}^{-1}} & (1,5,2,3,4) \\
\xrightarrow{\overline{b}^{-1}} & (2,5,4) & \xrightarrow{\overline{b}^{-1}} & (1,2)(3,5) & \xrightarrow{\overline{a}^{-1}} & (1,3)(4,5) & \xrightarrow{\overline{b}^{-1}} & (1,5)(2,3) & \xrightarrow{\overline{a}} & (1,3,4) \\
\xrightarrow{\overline{b}^{-1}} & (2,3)(4,5) & \xrightarrow{\overline{b}^{-1}} & (1,5,2) & \xrightarrow{\overline{b}^{-1}} & (1,4,2,5,3) & \xrightarrow{\overline{b}^{-1}} & (1,2,4,3,5) & \xrightarrow{\overline{a}} & (1,4)(2,5) \\
\xrightarrow{\overline{b}^{-1}} & (2,4)(3,5) & \xrightarrow{\overline{b}^{-1}} & (1,2)(3,4) & \xrightarrow{\overline{a}^{-1}} & (1,5,3) & \xrightarrow{\overline{b}^{-1}} & (1,4,3,2,5) & \xrightarrow{\overline{a}} & (1,5,4) \\
\xrightarrow{\overline{b}^{-1}} & (2,5,3) & \xrightarrow{\overline{b}^{-1}} & (1,4,3,5,2) & \xrightarrow{\overline{a}^{-1}} & (1,2,4,5,3) & \xrightarrow{\overline{b}} & (1,4,2) & \xrightarrow{\overline{b}} & (2,3,5) \\
\xrightarrow{\overline{b}} & (1,3,2,5,4) & \xrightarrow{\overline{b}} & (1,5)(3,4) & \xrightarrow{\overline{a}} & (1,2,4) & \xrightarrow{\overline{b}^{-1}} & (3,4,5) & \xrightarrow{\overline{b}^{-1}} & (1,5,4,3,2) \\
\xrightarrow{\overline{a}} & e & & & & & & & &
\end{array}
$$

**4.10.** *A hamiltonian cycle $C_1$ in* $\mathrm{Cay}(A_5;\overline{a},\overline{b})$ *with* $\overline{a}=(1,2,3,4,5)$ *and* $\overline{b}=(1,3,4,2,5)$.

$$
\begin{array}{cccccccccc}
 & e & \xrightarrow{\overline{a}} & (1,2,3,4,5) & \xrightarrow{\overline{a}} & (1,3,5,2,4) & \xrightarrow{\overline{a}} & (1,4,2,5,3) & \xrightarrow{\overline{a}} & (1,5,4,3,2) \\
\xrightarrow{\overline{b}^{-1}} & (1,4,2,3,5) & \xrightarrow{\overline{a}^{-1}} & (2,4,5) & \xrightarrow{\overline{a}^{-1}} & (1,2)(3,4) & \xrightarrow{\overline{a}^{-1}} & (1,5,3) & \xrightarrow{\overline{a}^{-1}} & (1,3,2,5,4) \\
\xrightarrow{\overline{b}} & (1,2,4,5,3) & \xrightarrow{\overline{a}} & (1,4,3,5,2) & \xrightarrow{\overline{a}} & (2,5,4) & \xrightarrow{\overline{a}} & (1,5)(2,3) & \xrightarrow{\overline{a}} & (1,3,4) \\
\xrightarrow{\overline{b}^{-1}} & (1,5,2) & \xrightarrow{\overline{a}} & (2,3,4) & \xrightarrow{\overline{a}} & (1,3,2,4,5) & \xrightarrow{\overline{b}} & (1,2)(3,5) & \xrightarrow{\overline{a}^{-1}} & (1,3)(4,5) \\
\xrightarrow{\overline{a}^{-1}} & (1,4)(2,3) & \xrightarrow{\overline{b}} & (1,2,5,4,3) & \xrightarrow{\overline{a}^{-1}} & (1,4)(3,5) & \xrightarrow{\overline{b}} & (1,5,4,2,3) & \xrightarrow{\overline{a}^{-1}} & (1,4)(2,5) \\
\xrightarrow{\overline{a}^{-1}} & (1,2,4,3,5) & \xrightarrow{\overline{a}^{-1}} & (3,4,5) & \xrightarrow{\overline{a}^{-1}} & (1,3,2) & \xrightarrow{\overline{b}^{-1}} & (1,5)(2,4) & \xrightarrow{\overline{a}^{-1}} & (2,5)(3,4) \\
\xrightarrow{\overline{b}^{-1}} & (1,2,3) & \xrightarrow{\overline{a}} & (1,3,4,5,2) & \xrightarrow{\overline{a}} & (2,4)(3,5) & \xrightarrow{\overline{a}} & (1,4,3,2,5) & \xrightarrow{\overline{a}} & (1,5,4) \\
\xrightarrow{\overline{b}} & (1,3)(2,4) & \xrightarrow{\overline{a}^{-1}} & (1,5,2,3,4) & \xrightarrow{\overline{a}^{-1}} & (1,2,5) & \xrightarrow{\overline{a}^{-1}} & (3,5,4) & \xrightarrow{\overline{a}^{-1}} & (1,4,5,3,2) \\
\xrightarrow{\overline{b}^{-1}} & (1,3,4,2,5) & \xrightarrow{\overline{a}^{-1}} & (2,3,5) & \xrightarrow{\overline{a}^{-1}} & (1,2)(4,5) & \xrightarrow{\overline{a}^{-1}} & (1,4,3) & \xrightarrow{\overline{a}^{-1}} & (1,5,3,2,4) \\
\xrightarrow{\overline{b}^{-1}} & (1,3,5,4,2) & \xrightarrow{\overline{a}} & (2,5,3) & \xrightarrow{\overline{a}} & (1,5)(3,4) & \xrightarrow{\overline{b}} & (1,4,2) & \xrightarrow{\overline{a}} & (2,3)(4,5) \\
\xrightarrow{\overline{a}} & (1,3,5) & \xrightarrow{\overline{b}} & (1,5,3,4,2) & \xrightarrow{\overline{a}} & (2,4,3) & \xrightarrow{\overline{a}} & (1,4,5) & \xrightarrow{\overline{a}} & (1,2,3,5,4) \\
\xrightarrow{\overline{a}} & (1,3)(2,5) & \xrightarrow{\overline{b}^{-1}} & (1,2,4) & \xrightarrow{\overline{a}} & (1,4,5,2,3) & \xrightarrow{\overline{b}^{-1}} & (1,2,5,3,4) & \xrightarrow{\overline{a}} & (1,5,2,4,3) \\
\xrightarrow{\overline{b}} & e & & & & & & & &
\end{array}
$$

**4.11.** *A second hamiltonian cycle $C_2$ in* $\mathrm{Cay}(A_5; \overline{a}, \overline{b})$ *with* $\overline{a} = (1,2,3,4,5)$ *and* $\overline{b} = (1,3,4,2,5)$.

$$\begin{array}{cccccccccc}
 & e & \xrightarrow{\overline{a}} & (1,2,3,4,5) & \xrightarrow{\overline{a}} & (1,3,5,2,4) & \xrightarrow{\overline{a}} & (1,4,2,5,3) & \xrightarrow{\overline{a}} & (1,5,4,3,2) \\
\xrightarrow{\overline{b}^{-1}} & (1,4,2,3,5) & \xrightarrow{\overline{a}^{-1}} & (2,4,5) & \xrightarrow{\overline{a}^{-1}} & (1,2)(3,4) & \xrightarrow{\overline{a}^{-1}} & (1,5,3) & \xrightarrow{\overline{a}^{-1}} & (1,3,2,5,4) \\
\xrightarrow{\overline{b}} & (1,2,4,5,3) & \xrightarrow{\overline{a}} & (1,4,3,5,2) & \xrightarrow{\overline{a}} & (2,5,4) & \xrightarrow{\overline{a}} & (1,5)(2,3) & \xrightarrow{\overline{a}} & (1,3,4) \\
\xrightarrow{\overline{b}^{-1}} & (1,5,2) & \xrightarrow{\overline{a}} & (2,3,4) & \xrightarrow{\overline{a}} & (1,3,2,4,5) & \xrightarrow{\overline{b}} & (1,2)(3,5) & \xrightarrow{\overline{a}^{-1}} & (1,3)(4,5) \\
\xrightarrow{\overline{b}^{-1}} & (1,4)(2,5) & \xrightarrow{\overline{a}^{-1}} & (1,2,4,3,5) & \xrightarrow{\overline{a}^{-1}} & (3,4,5) & \xrightarrow{\overline{a}^{-1}} & (1,3,2) & \xrightarrow{\overline{a}^{-1}} & (1,5,4,2,3) \\
\xrightarrow{\overline{b}^{-1}} & (1,4)(3,5) & \xrightarrow{\overline{a}} & (1,2,5,4,3) & \xrightarrow{\overline{b}^{-1}} & (1,4)(2,3) & \xrightarrow{\overline{a}^{-1}} & (1,5)(2,4) & \xrightarrow{\overline{a}^{-1}} & (2,5)(3,4) \\
\xrightarrow{\overline{b}^{-1}} & (1,2,3) & \xrightarrow{\overline{a}} & (1,3,4,5,2) & \xrightarrow{\overline{a}} & (2,4)(3,5) & \xrightarrow{\overline{a}} & (1,4,3,2,5) & \xrightarrow{\overline{a}} & (1,5,4) \\
\xrightarrow{\overline{b}} & (1,3)(2,4) & \xrightarrow{\overline{a}^{-1}} & (1,5,2,3,4) & \xrightarrow{\overline{a}^{-1}} & (1,2,5) & \xrightarrow{\overline{a}^{-1}} & (3,5,4) & \xrightarrow{\overline{a}^{-1}} & (1,4,5,3,2) \\
\xrightarrow{\overline{b}^{-1}} & (1,3,4,2,5) & \xrightarrow{\overline{a}^{-1}} & (2,3,5) & \xrightarrow{\overline{a}^{-1}} & (1,2)(4,5) & \xrightarrow{\overline{a}^{-1}} & (1,4,3) & \xrightarrow{\overline{a}^{-1}} & (1,5,3,2,4) \\
\xrightarrow{\overline{b}^{-1}} & (1,3,5,4,2) & \xrightarrow{\overline{a}} & (2,5,3) & \xrightarrow{\overline{a}} & (1,5)(3,4) & \xrightarrow{\overline{b}} & (1,4,2) & \xrightarrow{\overline{a}} & (2,3)(4,5) \\
\xrightarrow{\overline{a}} & (1,3,5) & \xrightarrow{\overline{b}} & (1,5,3,4,2) & \xrightarrow{\overline{a}} & (2,4,3) & \xrightarrow{\overline{a}} & (1,4,5) & \xrightarrow{\overline{a}} & (1,2,3,5,4) \\
\xrightarrow{\overline{a}} & (1,3)(2,5) & \xrightarrow{\overline{b}^{-1}} & (1,2,4) & \xrightarrow{\overline{a}} & (1,4,5,2,3) & \xrightarrow{\overline{b}^{-1}} & (1,2,5,3,4) & \xrightarrow{\overline{a}} & (1,5,2,4,3) \\
\xrightarrow{\overline{b}} & e & & & & & & & &
\end{array}$$

**4.12.** *A hamiltonian cycle $R_1$ in* $\mathrm{Cay}(A_5; \overline{a}, \overline{b}, \overline{c})$ *with* $\overline{a} = (1,2,5)$, $\overline{b} = (1,3,5)$, *and* $\overline{c} = (1,4,5)$.

$$\begin{array}{cccccccccc}
 & e & \xrightarrow{\overline{a}} & (1,2,5) & \xrightarrow{\overline{a}} & (1,5,2) & \xrightarrow{\overline{b}} & (1,3,2) & \xrightarrow{\overline{a}} & (2,5,3) \\
\xrightarrow{\overline{a}} & (1,5)(2,3) & \xrightarrow{\overline{b}} & (1,2,3) & \xrightarrow{\overline{a}} & (1,3)(2,5) & \xrightarrow{\overline{a}} & (1,5,3) & \xrightarrow{\overline{c}} & (1,4,3) \\
\xrightarrow{\overline{a}} & (1,2,5,4,3) & \xrightarrow{\overline{a}} & (1,5,2,4,3) & \xrightarrow{\overline{b}} & (2,4,3) & \xrightarrow{\overline{a}} & (1,4,3,2,5) & \xrightarrow{\overline{a}} & (1,5,4,3,2) \\
\xrightarrow{\overline{b}} & (1,2)(3,4) & \xrightarrow{\overline{a}} & (2,5)(3,4) & \xrightarrow{\overline{a}} & (1,5)(3,4) & \xrightarrow{\overline{c}} & (1,3,4) & \xrightarrow{\overline{a}} & (1,2,5,3,4) \\
\xrightarrow{\overline{b}} & (1,4)(2,5) & \xrightarrow{\overline{b}} & (1,3,2,5,4) & \xrightarrow{\overline{a}} & (1,5,3,2,4) & \xrightarrow{\overline{a}} & (1,4)(2,3) & \xrightarrow{\overline{b}} & (1,2,3,5,4) \\
\xrightarrow{\overline{a}} & (1,3,5,2,4) & \xrightarrow{\overline{a}} & (1,4)(3,5) & \xrightarrow{\overline{c}} & (3,5,4) & \xrightarrow{\overline{c}} & (1,3,5) & \xrightarrow{\overline{a}} & (1,2)(3,5) \\
\xrightarrow{\overline{a}} & (2,3,5) & \xrightarrow{\overline{c}} & (1,4,2,3,5) & \xrightarrow{\overline{b}} & (1,5,4,2,3) & \xrightarrow{\overline{a}} & (1,3)(2,4) & \xrightarrow{\overline{b}} & (2,4)(3,5) \\
\xrightarrow{\overline{c}} & (1,2,4,3,5) & \xrightarrow{\overline{a}} & (1,4,3,5,2) & \xrightarrow{\overline{c}} & (1,3,5,4,2) & \xrightarrow{\overline{b}} & (1,5,3,4,2) & \xrightarrow{\overline{b}} & (1,4,2) \\
\xrightarrow{\overline{c}} & (1,2)(4,5) & \xrightarrow{\overline{b}} & (1,3,4,5,2) & \xrightarrow{\overline{b}} & (1,4,5,3,2) & \xrightarrow{\overline{a}} & (2,3)(4,5) & \xrightarrow{\overline{a}} & (1,3,2,4,5) \\
\xrightarrow{\overline{b}} & (1,2,4,5,3) & \xrightarrow{\overline{a}} & (1,4,5,2,3) & \xrightarrow{\overline{a}} & (1,3)(4,5) & \xrightarrow{\overline{b}} & (3,4,5) & \xrightarrow{\overline{a}} & (1,2,3,4,5) \\
\xrightarrow{\overline{c}} & (1,5,2,3,4) & \xrightarrow{\overline{c}} & (2,3,4) & \xrightarrow{\overline{a}} & (1,3,4,2,5) & \xrightarrow{\overline{b}} & (1,4,2,5,3) & \xrightarrow{\overline{b}} & (2,5,4) \\
\xrightarrow{\overline{a}} & (1,5)(2,4) & \xrightarrow{\overline{c}} & (1,2,4) & \xrightarrow{\overline{c}} & (2,4,5) & \xrightarrow{\overline{a}} & (1,4,5) & \xrightarrow{\overline{c}} & (1,5,4) \\
\xrightarrow{\overline{c}} & e & & & & & & & &
\end{array}$$

**4.13.** *A hamiltonian cycle $C_1$ in* $\mathrm{Cay}(A_5; \overline{a}, \overline{b}, \overline{c}, \overline{c})$ *with* $\overline{a} = (1,2)(4,5)$, $\overline{b} = (1,2,3)$, *and* $\overline{c} = (1,2,4)$.

$$\begin{array}{cccccccccc}
 & e & \xrightarrow{\overline{a}} & (1,2)(4,5) & \xrightarrow{\overline{c}^{-1}} & (1,5,4) & \xrightarrow{\overline{a}} & (1,2,5) & \xrightarrow{\overline{b}} & (1,5)(2,3) \\
\xrightarrow{\overline{a}} & (1,3,2,5,4) & \xrightarrow{\overline{c}} & (1,5,4,3,2) & \xrightarrow{\overline{a}} & (2,5,3) & \xrightarrow{\overline{b}} & (1,5,3) & \xrightarrow{\overline{b}} & (1,2)(3,5) \\
\xrightarrow{\overline{a}} & (3,5,4) & \xrightarrow{\overline{b}} & (1,2,5,4,3) & \xrightarrow{\overline{c}} & (1,5,4,2,3) & \xrightarrow{\overline{b}^{-1}} & (2,5,4) & \xrightarrow{\overline{a}} & (1,5,2) \\
\xrightarrow{\overline{b}^{-1}} & (1,3)(2,5) & \xrightarrow{\overline{b}^{-1}} & (2,3,5) & \xrightarrow{\overline{a}} & (1,3,5,4,2) & \xrightarrow{\overline{c}^{-1}} & (1,2,3,5,4) & \xrightarrow{\overline{a}} & (1,3,5) \\
\xrightarrow{\overline{c}^{-1}} & (1,4,2,3,5) & \xrightarrow{\overline{b}} & (1,3,4,2,5) & \xrightarrow{\overline{b}} & (1,5)(2,4) & \xrightarrow{\overline{c}} & (1,4,5) & \xrightarrow{\overline{b}^{-1}} & (1,3,2,4,5) \\
\xrightarrow{\overline{b}^{-1}} & (1,2,3,4,5) & \xrightarrow{\overline{a}} & (1,3,4) & \xrightarrow{\overline{b}^{-1}} & (1,4)(2,3) & \xrightarrow{\overline{b}^{-1}} & (1,2,4) & \xrightarrow{\overline{c}} & (1,4,2) \\
\xrightarrow{\overline{a}} & (2,4,5) & \xrightarrow{\overline{b}} & (1,4,5,2,3) & \xrightarrow{\overline{c}^{-1}} & (1,5,2,4,3) & \xrightarrow{\overline{b}^{-1}} & (2,5)(3,4) & \xrightarrow{\overline{a}} & (1,5,3,4,2) \\
\xrightarrow{\overline{c}^{-1}} & (1,2,5,3,4) & \xrightarrow{\overline{a}} & (1,5)(3,4) & \xrightarrow{\overline{b}^{-1}} & (1,4,3,2,5) & \xrightarrow{\overline{b}^{-1}} & (1,2,4,3,5) & \xrightarrow{\overline{a}} & (1,4)(3,5) \\
\xrightarrow{\overline{b}^{-1}} & (1,5,3,2,4) & \xrightarrow{\overline{c}} & (1,4,5,3,2) & \xrightarrow{\overline{a}} & (2,4,3) & \xrightarrow{\overline{b}} & (1,4,3) & \xrightarrow{\overline{b}} & (1,2)(3,4) \\
\xrightarrow{\overline{a}} & (3,4,5) & \xrightarrow{\overline{b}} & (1,2,4,5,3) & \xrightarrow{\overline{c}} & (1,4,2,5,3) & \xrightarrow{\overline{b}^{-1}} & (2,4)(3,5) & \xrightarrow{\overline{a}} & (1,4,3,5,2) \\
\xrightarrow{\overline{c}^{-1}} & (1,3,5,2,4) & \xrightarrow{\overline{b}} & (1,4)(2,5) & \xrightarrow{\overline{b}} & (1,5,2,3,4) & \xrightarrow{\overline{c}} & (1,3,4,5,2) & \xrightarrow{\overline{a}} & (2,3,4) \\
\xrightarrow{\overline{b}} & (1,3)(2,4) & \xrightarrow{\overline{c}^{-1}} & (1,2,3) & \xrightarrow{\overline{a}} & (1,3)(4,5) & \xrightarrow{\overline{b}^{-1}} & (2,3)(4,5) & \xrightarrow{\overline{a}} & (1,3,2) \\
\xrightarrow{\overline{b}} & e & & & & & & & &
\end{array}$$

**4.14.** *A second cycle $C_2$ in* $\mathrm{Cay}(A_5; \overline{a}, \overline{b}, \overline{c})$ *with* $\overline{a} = (1,2)(4,5)$, $\overline{b} = (1,2,3)$, *and* $\overline{c} = (1,2,4)$.

$$\begin{array}{cccccccccc}
 & e & \xrightarrow{\overline{a}} & (1,2)(4,5) & \xrightarrow{\overline{c}^{-1}} & (1,5,4) & \xrightarrow{\overline{a}} & (1,2,5) & \xrightarrow{\overline{b}} & (1,5)(2,3) \\
\xrightarrow{\overline{a}} & (1,3,2,5,4) & \xrightarrow{\overline{c}} & (1,5,4,3,2) & \xrightarrow{\overline{a}} & (2,5,3) & \xrightarrow{\overline{b}} & (1,5,3) & \xrightarrow{\overline{b}} & (1,2)(3,5) \\
\xrightarrow{\overline{a}} & (3,5,4) & \xrightarrow{\overline{b}} & (1,2,5,4,3) & \xrightarrow{\overline{c}} & (1,5,4,2,3) & \xrightarrow{\overline{b}^{-1}} & (2,5,4) & \xrightarrow{\overline{a}} & (1,5,2) \\
\xrightarrow{\overline{c}^{-1}} & (1,4)(2,5) & \xrightarrow{\overline{b}} & (1,5,2,3,4) & \xrightarrow{\overline{b}} & (1,3,5,2,4) & \xrightarrow{\overline{c}} & (1,4,3,5,2) & \xrightarrow{\overline{a}} & (2,4)(3,5) \\
\xrightarrow{\overline{b}} & (1,4,2,5,3) & \xrightarrow{\overline{c}^{-1}} & (1,2,4,5,3) & \xrightarrow{\overline{b}^{-1}} & (3,4,5) & \xrightarrow{\overline{a}} & (1,2)(3,4) & \xrightarrow{\overline{b}^{-1}} & (1,4,3) \\
\xrightarrow{\overline{b}^{-1}} & (2,4,3) & \xrightarrow{\overline{a}} & (1,4,5,3,2) & \xrightarrow{\overline{c}^{-1}} & (1,5,3,2,4) & \xrightarrow{\overline{b}} & (1,4)(3,5) & \xrightarrow{\overline{a}} & (1,2,4,3,5) \\
\xrightarrow{\overline{b}} & (1,4,3,2,5) & \xrightarrow{\overline{b}} & (1,5)(3,4) & \xrightarrow{\overline{a}} & (1,2,5,3,4) & \xrightarrow{\overline{c}} & (1,5,3,4,2) & \xrightarrow{\overline{a}} & (2,5)(3,4) \\
\xrightarrow{\overline{b}} & (1,5,2,4,3) & \xrightarrow{\overline{c}^{-1}} & (1,3)(2,5) & \xrightarrow{\overline{b}^{-1}} & (2,3,5) & \xrightarrow{\overline{a}} & (1,3,5,4,2) & \xrightarrow{\overline{c}^{-1}} & (1,2,3,5,4) \\
\xrightarrow{\overline{a}} & (1,3,5) & \xrightarrow{\overline{c}^{-1}} & (1,4,2,3,5) & \xrightarrow{\overline{b}} & (1,3,4,2,5) & \xrightarrow{\overline{b}} & (1,5)(2,4) & \xrightarrow{\overline{c}} & (1,4,5) \\
\xrightarrow{\overline{b}^{-1}} & (1,3,2,4,5) & \xrightarrow{\overline{b}^{-1}} & (1,2,3,4,5) & \xrightarrow{\overline{a}} & (1,3,4) & \xrightarrow{\overline{b}^{-1}} & (1,4)(2,3) & \xrightarrow{\overline{b}^{-1}} & (1,2,4) \\
\xrightarrow{\overline{c}} & (1,4,2) & \xrightarrow{\overline{a}} & (2,4,5) & \xrightarrow{\overline{b}} & (1,4,5,2,3) & \xrightarrow{\overline{b}} & (1,3,4,5,2) & \xrightarrow{\overline{a}} & (2,3,4) \\
\xrightarrow{\overline{b}} & (1,3)(2,4) & \xrightarrow{\overline{c}^{-1}} & (1,2,3) & \xrightarrow{\overline{a}} & (1,3)(4,5) & \xrightarrow{\overline{b}^{-1}} & (2,3)(4,5) & \xrightarrow{\overline{a}} & (1,3,2) \\
\xrightarrow{\overline{b}} & e & & & & & & & &
\end{array}$$

**4.15.** *A hamiltonian cycle in* $\mathrm{Cay}(A_5;\overline{a},\overline{b},\overline{c})$ *with* $\overline{a}=(1,2)(4,5)$, $\overline{b}=(1,2)(3,4)$, *and* $\overline{c}=(1,2,3)$.

$$\begin{array}{cccccccccc}
 & e & \xrightarrow{\overline{a}} & (1,2)(4,5) & \xrightarrow{\overline{c}^{-1}} & (1,3)(4,5) & \xrightarrow{\overline{a}} & (1,2,3) & \xrightarrow{\overline{b}} & (1,3,4) \\
\xrightarrow{\overline{a}} & (1,2,3,4,5) & \xrightarrow{\overline{b}} & (1,3,5) & \xrightarrow{\overline{a}} & (1,2,3,5,4) & \xrightarrow{\overline{c}^{-1}} & (1,5,4) & \xrightarrow{\overline{a}} & (1,2,5) \\
\xrightarrow{\overline{b}} & (1,5)(3,4) & \xrightarrow{\overline{c}^{-1}} & (1,4,3,2,5) & \xrightarrow{\overline{b}} & (1,5)(2,4) & \xrightarrow{\overline{c}} & (1,4,2,3,5) & \xrightarrow{\overline{b}} & (1,3,2,4,5) \\
\xrightarrow{\overline{a}} & (1,4)(2,3) & \xrightarrow{\overline{b}} & (1,3)(2,4) & \xrightarrow{\overline{a}} & (1,4,5,2,3) & \xrightarrow{\overline{c}} & (1,3,4,5,2) & \xrightarrow{\overline{a}} & (2,3,4) \\
\xrightarrow{\overline{b}} & (1,3,2) & \xrightarrow{\overline{a}} & (2,3)(4,5) & \xrightarrow{\overline{b}} & (1,3,5,4,2) & \xrightarrow{\overline{a}} & (2,3,5) & \xrightarrow{\overline{c}} & (1,3)(2,5) \\
\xrightarrow{\overline{a}} & (1,5,4,2,3) & \xrightarrow{\overline{b}} & (1,3,2,5,4) & \xrightarrow{\overline{a}} & (1,5)(2,3) & \xrightarrow{\overline{b}} & (1,3,4,2,5) & \xrightarrow{\overline{a}} & (1,5,2,3,4) \\
\xrightarrow{\overline{c}} & (1,3,5,2,4) & \xrightarrow{\overline{c}} & (1,4)(2,5) & \xrightarrow{\overline{b}} & (1,5,2,4,3) & \xrightarrow{\overline{c}^{-1}} & (2,5)(3,4) & \xrightarrow{\overline{b}} & (1,5,2) \\
\xrightarrow{\overline{a}} & (2,5,4) & \xrightarrow{\overline{b}} & (1,5,4,3,2) & \xrightarrow{\overline{a}} & (2,5,3) & \xrightarrow{\overline{b}} & (1,5,3,4,2) & \xrightarrow{\overline{c}^{-1}} & (1,4,2,5,3) \\
\xrightarrow{\overline{c}^{-1}} & (2,4)(3,5) & \xrightarrow{\overline{a}} & (1,4,3,5,2) & \xrightarrow{\overline{b}} & (2,4,5) & \xrightarrow{\overline{a}} & (1,4,2) & \xrightarrow{\overline{b}} & (2,4,3) \\
\xrightarrow{\overline{a}} & (1,4,5,3,2) & \xrightarrow{\overline{c}^{-1}} & (1,2,4,5,3) & \xrightarrow{\overline{a}} & (1,4,3) & \xrightarrow{\overline{b}} & (1,2,4) & \xrightarrow{\overline{a}} & (1,4,5) \\
\xrightarrow{\overline{b}} & (1,2,4,3,5) & \xrightarrow{\overline{a}} & (1,4)(3,5) & \xrightarrow{\overline{c}^{-1}} & (1,5,3,2,4) & \xrightarrow{\overline{c}^{-1}} & (1,2,5,3,4) & \xrightarrow{\overline{b}} & (1,5,3) \\
\xrightarrow{\overline{a}} & (1,2,5,4,3) & \xrightarrow{\overline{c}^{-1}} & (3,5,4) & \xrightarrow{\overline{a}} & (1,2)(3,5) & \xrightarrow{\overline{b}} & (3,4,5) & \xrightarrow{\overline{a}} & (1,2)(3,4) \\
\xrightarrow{\overline{b}} & e & & & & & & & &
\end{array}$$

**4.16.** *A hamiltonian cycle in* $\mathrm{Cay}(A_5;\overline{a},\overline{b},\overline{c})$ *with* $\overline{a}=(1,2)(4,5)$, $\overline{b}=(1,3)(2,4)$, *and* $\overline{c}=(1,2,3)$.

$$\begin{array}{cccccccccc}
 & e & \xrightarrow{\overline{a}} & (1,2)(4,5) & \xrightarrow{\overline{b}} & (1,3,2,5,4) & \xrightarrow{\overline{a}} & (1,5)(2,3) & \xrightarrow{\overline{b}} & (1,2,4,3,5) \\
\xrightarrow{\overline{a}} & (1,4)(3,5) & \xrightarrow{\overline{b}} & (1,5,3,4,2) & \xrightarrow{\overline{a}} & (2,5)(3,4) & \xrightarrow{\overline{b}} & (1,4,5,2,3) & \xrightarrow{\overline{c}} & (1,3,4,5,2) \\
\xrightarrow{\overline{a}} & (2,3,4) & \xrightarrow{\overline{b}} & (1,4,3) & \xrightarrow{\overline{a}} & (1,2,4,5,3) & \xrightarrow{\overline{b}} & (2,5,3) & \xrightarrow{\overline{a}} & (1,5,4,3,2) \\
\xrightarrow{\overline{b}} & (1,2,3,5,4) & \xrightarrow{\overline{a}} & (1,3,5) & \xrightarrow{\overline{b}} & (1,5)(2,4) & \xrightarrow{\overline{a}} & (1,4)(2,5) & \xrightarrow{\overline{c}^{-1}} & (1,3,5,2,4) \\
\xrightarrow{\overline{b}} & (1,5,2) & \xrightarrow{\overline{a}} & (2,5,4) & \xrightarrow{\overline{b}} & (1,3)(4,5) & \xrightarrow{\overline{c}^{-1}} & (2,3)(4,5) & \xrightarrow{\overline{b}} & (1,2,5,4,3) \\
\xrightarrow{\overline{a}} & (1,5,3) & \xrightarrow{\overline{b}} & (2,4)(3,5) & \xrightarrow{\overline{a}} & (1,4,3,5,2) & \xrightarrow{\overline{b}} & (1,5,2,3,4) & \xrightarrow{\overline{a}} & (1,3,4,2,5) \\
\xrightarrow{\overline{c}^{-1}} & (1,4,2,3,5) & \xrightarrow{\overline{b}} & (1,5)(3,4) & \xrightarrow{\overline{a}} & (1,2,5,3,4) & \xrightarrow{\overline{b}} & (1,4,5,3,2) & \xrightarrow{\overline{a}} & (2,4,3) \\
\xrightarrow{\overline{b}} & (1,2,3) & \xrightarrow{\overline{c}} & (1,3,2) & \xrightarrow{\overline{b}} & (1,2,4) & \xrightarrow{\overline{a}} & (1,4,5) & \xrightarrow{\overline{c}^{-1}} & (1,3,2,4,5) \\
\xrightarrow{\overline{b}} & (1,2,5) & \xrightarrow{\overline{a}} & (1,5,4) & \xrightarrow{\overline{b}} & (1,3,5,4,2) & \xrightarrow{\overline{a}} & (2,3,5) & \xrightarrow{\overline{b}} & (1,5,2,4,3) \\
\xrightarrow{\overline{a}} & (1,4,2,5,3) & \xrightarrow{\overline{b}} & (3,4,5) & \xrightarrow{\overline{a}} & (1,2)(3,4) & \xrightarrow{\overline{b}} & (1,4)(2,3) & \xrightarrow{\overline{c}} & (1,3,4) \\
\xrightarrow{\overline{a}} & (1,2,3,4,5) & \xrightarrow{\overline{b}} & (1,4,3,2,5) & \xrightarrow{\overline{a}} & (1,5,3,2,4) & \xrightarrow{\overline{b}} & (1,2)(3,5) & \xrightarrow{\overline{a}} & (3,5,4) \\
\xrightarrow{\overline{b}} & (1,5,4,2,3) & \xrightarrow{\overline{a}} & (1,3)(2,5) & \xrightarrow{\overline{b}} & (2,4,5) & \xrightarrow{\overline{a}} & (1,4,2) & \xrightarrow{\overline{c}^{-1}} & (1,3)(2,4) \\
\xrightarrow{\overline{b}} & e & & & & & & & &
\end{array}$$

**4.17.** *A hamiltonian cycle in* $\operatorname{Cay}(A_5; \overline{a}, \overline{b}, \overline{c})$ *with* $\overline{a} = (1,2)(3,4)$, $\overline{b} = (1,2)(3,5)$, *and* $\overline{c} = (1,2,3)$.

$$e \xrightarrow{\overline{a}} (1,2)(3,4) \xrightarrow{\overline{c}^{-1}} (1,4,3) \xrightarrow{\overline{a}} (1,2,4) \xrightarrow{\overline{c}} (1,4)(2,3)$$
$$\xrightarrow{\overline{b}} (1,3,5,2,4) \xrightarrow{\overline{a}} (1,4,5,2,3) \xrightarrow{\overline{b}} (1,3,2,4,5) \xrightarrow{\overline{a}} (1,4,2,3,5) \xrightarrow{\overline{b}} (1,3)(2,4)$$
$$\xrightarrow{\overline{c}} (1,4,2) \xrightarrow{\overline{a}} (2,4,3) \xrightarrow{\overline{b}} (1,4,3,5,2) \xrightarrow{\overline{a}} (2,4,5) \xrightarrow{\overline{b}} (1,4,5,3,2)$$
$$\xrightarrow{\overline{a}} (2,4)(3,5) \xrightarrow{\overline{c}} (1,4,2,5,3) \xrightarrow{\overline{a}} (1,5,3,2,4) \xrightarrow{\overline{c}} (1,4)(3,5) \xrightarrow{\overline{a}} (1,2,4,5,3)$$
$$\xrightarrow{\overline{b}} (1,4,5) \xrightarrow{\overline{a}} (1,2,4,3,5) \xrightarrow{\overline{c}^{-1}} (1,5)(3,4) \xrightarrow{\overline{c}^{-1}} (1,4,3,2,5) \xrightarrow{\overline{a}} (1,5)(2,4)$$
$$\xrightarrow{\overline{c}^{-1}} (1,3,4,2,5) \xrightarrow{\overline{b}} (1,5,4,2,3) \xrightarrow{\overline{a}} (1,3,2,5,4) \xrightarrow{\overline{c}} (1,5,4) \xrightarrow{\overline{b}} (1,2,5,3,4)$$
$$\xrightarrow{\overline{a}} (1,5,3) \xrightarrow{\overline{b}} (1,2,5) \xrightarrow{\overline{c}} (1,5)(2,3) \xrightarrow{\overline{c}} (1,3,5) \xrightarrow{\overline{a}} (1,2,3,4,5)$$
$$\xrightarrow{\overline{b}} (1,3)(4,5) \xrightarrow{\overline{a}} (1,2,3,5,4) \xrightarrow{\overline{b}} (1,3,4) \xrightarrow{\overline{a}} (1,2,3) \xrightarrow{\overline{c}} (1,3,2)$$
$$\xrightarrow{\overline{a}} (2,3,4) \xrightarrow{\overline{b}} (1,3,5,4,2) \xrightarrow{\overline{a}} (2,3)(4,5) \xrightarrow{\overline{b}} (1,3,4,5,2) \xrightarrow{\overline{a}} (2,3,5)$$
$$\xrightarrow{\overline{c}} (1,3)(2,5) \xrightarrow{\overline{a}} (1,5,2,3,4) \xrightarrow{\overline{c}^{-1}} (1,4)(2,5) \xrightarrow{\overline{a}} (1,5,2,4,3) \xrightarrow{\overline{c}^{-1}} (2,5)(3,4)$$
$$\xrightarrow{\overline{a}} (1,5,2) \xrightarrow{\overline{b}} (2,5,3) \xrightarrow{\overline{a}} (1,5,3,4,2) \xrightarrow{\overline{b}} (2,5,4) \xrightarrow{\overline{a}} (1,5,4,3,2)$$
$$\xrightarrow{\overline{c}^{-1}} (1,2,5,4,3) \xrightarrow{\overline{c}^{-1}} (3,5,4) \xrightarrow{\overline{a}} (1,2)(4,5) \xrightarrow{\overline{b}} (3,4,5) \xrightarrow{\overline{a}} (1,2)(3,5)$$
$$\xrightarrow{\overline{b}} e$$

**4.18.** *A hamiltonian cycle in* $\operatorname{Cay}(A_5; \overline{a}, \overline{b}, \overline{c})$ *with* $\overline{a} = (1,2)(3,4)$, $\overline{b} = (1,3)(2,5)$, *and* $\overline{c} = (1,2,3)$.

$$e \xrightarrow{\overline{a}} (1,2)(3,4) \xrightarrow{\overline{b}} (1,4,3,2,5) \xrightarrow{\overline{c}^{-1}} (1,2,4,3,5) \xrightarrow{\overline{a}} (1,4,5)$$
$$\xrightarrow{\overline{b}} (1,3,4,5,2) \xrightarrow{\overline{a}} (2,3,5) \xrightarrow{\overline{b}} (1,5,3) \xrightarrow{\overline{a}} (1,2,5,3,4) \xrightarrow{\overline{c}} (1,5,3,2,4)$$
$$\xrightarrow{\overline{b}} (1,2,3,5,4) \xrightarrow{\overline{a}} (1,3)(4,5) \xrightarrow{\overline{b}} (2,4,5) \xrightarrow{\overline{a}} (1,4,3,5,2) \xrightarrow{\overline{b}} (1,5)(3,4)$$
$$\xrightarrow{\overline{a}} (1,2,5) \xrightarrow{\overline{b}} (1,3,2) \xrightarrow{\overline{a}} (2,3,4) \xrightarrow{\overline{b}} (1,4,2,5,3) \xrightarrow{\overline{c}} (1,5,3,4,2)$$
$$\xrightarrow{\overline{a}} (2,5,3) \xrightarrow{\overline{b}} (1,2,3) \xrightarrow{\overline{a}} (1,3,4) \xrightarrow{\overline{c}^{-1}} (1,4)(2,3) \xrightarrow{\overline{a}} (1,3)(2,4)$$
$$\xrightarrow{\overline{b}} (2,5,4) \xrightarrow{\overline{a}} (1,5,4,3,2) \xrightarrow{\overline{c}^{-1}} (1,2,5,4,3) \xrightarrow{\overline{b}} (2,4,3) \xrightarrow{\overline{a}} (1,4,2)$$
$$\xrightarrow{\overline{b}} (1,3,4,2,5) \xrightarrow{\overline{c}} (1,5)(2,4) \xrightarrow{\overline{c}} (1,4,2,3,5) \xrightarrow{\overline{a}} (1,3,2,4,5) \xrightarrow{\overline{b}} (1,2)(4,5)$$
$$\xrightarrow{\overline{a}} (3,5,4) \xrightarrow{\overline{b}} (1,5,2,4,3) \xrightarrow{\overline{a}} (1,4)(2,5) \xrightarrow{\overline{c}} (1,5,2,3,4) \xrightarrow{\overline{b}} (1,4)(3,5)$$
$$\xrightarrow{\overline{a}} (1,2,4,5,3) \xrightarrow{\overline{b}} (2,3)(4,5) \xrightarrow{\overline{a}} (1,3,5,4,2) \xrightarrow{\overline{c}^{-1}} (1,5,4,2,3) \xrightarrow{\overline{b}} (2,4)(3,5)$$
$$\xrightarrow{\overline{a}} (1,4,5,3,2) \xrightarrow{\overline{b}} (1,2,3,4,5) \xrightarrow{\overline{a}} (1,3,5) \xrightarrow{\overline{c}^{-1}} (1,5)(2,3) \xrightarrow{\overline{b}} (1,2)(3,5)$$
$$\xrightarrow{\overline{a}} (3,4,5) \xrightarrow{\overline{b}} (1,4,5,2,3) \xrightarrow{\overline{a}} (1,3,5,2,4) \xrightarrow{\overline{b}} (1,5,4) \xrightarrow{\overline{c}^{-1}} (1,3,2,5,4)$$
$$\xrightarrow{\overline{b}} (1,2,4) \xrightarrow{\overline{a}} (1,4,3) \xrightarrow{\overline{b}} (2,5)(3,4) \xrightarrow{\overline{a}} (1,5,2) \xrightarrow{\overline{c}^{-1}} (1,3)(2,5)$$
$$\xrightarrow{\overline{b}} e$$

## References

[1] R. Gould, R. Roth, Cayley digraphs and $(1, j, n)$-sequencings of the alternating groups $A_n$, Discrete Math. 66(1-2) (1987) 91–102.

[2] K. Kutnar, D. Marušič, D. W. Morris, J. Morris, P. Šparl, Hamiltonian cycles in Cayley graphs whose order has few prime factors, Ars Math. Contemp. 5(1) (2012) 27–71.
[3] K. Kutnar, D. Marušič, D. W. Morris, J. Morris, P. Šparl, Cayley graphs on $A_5$ are hamiltonian, unpublished appendix to [2], http://arxiv.org/src/1009.5795/anc/A5.pdf.
[4] D. Witte, J. A. Gallian, A survey: Hamiltonian cycles in Cayley graphs, Discrete Math. 51(3) (1984) 293–304.