ESTIMATING THE TERM STRUCTURE OF INTEREST RATES: A CASE OF TURKEY (1990 – 2001)

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Abstract

This study estimates the term structure of interest rates with treasury bills. A transformed model is used for this measurement. In the empirical model, volatility of 12 month maturity treasury bills are used for dependent variable, and volatility of 1, 3, 6 month maturity treasury bills are used for independent variables. Due to macroeconomic policy, treasury bills interest rates and maturities carry a high volatility but, the evidence of this study supported theory of the term structure suggestions such as equivalent volatility of different maturities.

Keywords **:** *Term structure, Interest rates, Nelson/Siegel model, Swenson model, Treasury bills*

JEL Code **:** E43, E47, F47

FAİZ ORANLARININ VADE YAPISININ HESAPLANMASI: TÜRKİYE İÇİN BİR ÇALIŞMA (1990 – 2001)

Özet

Bu çalışmada, hazine bonosu faiz oranlarının vade yapısı hesaplanmaktadır. Bunun için dönüştürülmüş bir modelden faydalanılmıştır. Ampirik modelde; 12 ay vadeli hazine bonolarının volatilitesi bağımlı değişken, 6, 3 ve 1 ay vadeli hazine bonolarının volatilitesi ise bağımsız değişken olarak kullanılmıştır. Makroekonomik politikalar sonucu olarak hazine bonosu faiz oranları yüksek volatiliteye sahip olmakla birlikte, bu çalışmanın sonuçları farklı vadelerin eşdeğer volatilitesi gibi bir vade yapısı önerileri teorisini destekler niteliktedir.

AnahtarKelimeler**:** *Faiz oranları, vade yapısı, Nelson/Siegel modeli, Swenson modeli, Hazine bonosu*

JEL Sınıflaması **:** E43, E47, F47

INTRODUCTION

Before mentioning prior studies, it will be useful to appreciate all studies related to interest rate term structure. There are two main methods for measuring the term structure of interest rate. One is Nelson and Siegel (1987) and the other is Swenson (1994) model. But these models are means of measuring the relationship in progressive econometric techniques, such as time series models. None of those models is mentioned to time series. But, in part, some studies on term structure and monetary policy relationship created models which best fit to Nelson and Siegel and also Swenson. Some other studies are apart

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from Nelson and Siegel and Swenson, but they used best fit models to expect the term structure of interest rate.

Mankiv and Miron (1986) used data frequency 3 to 6 months, monthly data and found some evidence in favor of expectation theory prior to the founding of the FED in 1915. One of the results of this study was related to macroeconomic regime. The shift in macroeconomic regime which occurred with the founding of the FED led to a remarkable decrease in predictability of short-term interest rate.

Rudebusch (1995) and Balduzzi (1997) expanded the same evidence of Mankiv and Miron, by looking up to more recent data.

Heller (1997) estimated the term structure of interest rate with the Nelson and Siegel model. He illustrated the power of the procedure for monetary purposes and showed how inflation expectation can be extracted by using the expectation hypothesis and Fisher equilibrium.

Meredith and Chinn (1998) found the fact that forward premium anomaly does not exist in long horizon data. They regressed long horizon exchange rate onto long-term interest rate differential for the G-7 countries. All the coefficients on interest rate carry the correct sign, but with a long-term relationship the model of this study is not satisfactory. But this study would be called a central study of the term structure of interest rate within exchange rate.

Geyer and Mader (1999) studied the term structure of interest rate in Austria, Germany, UK, USA and Japan. Due to many differences between countries, they limited the scope of this study. The model of this study would be defined as flexible functional form of Nelson and Siegel model. These studies also compare the result of Nelson and Siegel methods with Swenson model. Some of the results were unsatisfactory for years in some countries. But this study is an acceptable study of interest rate term structure. Both Nelson and Siegel and Swenson model results fit. The aim of the term structure cannot easily predict for some countries unpredictable monetary and fiscal policy.

Meier (1999) studied the term structure of interest rate for Swiss with Nelson and Siegel model and Swenson model among central banks. One of the results of this study is related to pricing short-term derivatives on long-term interest rate that carries problems. He also found that Nelson and Siegel model is attractive on pricing financial instruments under monetary policy anomalies.

Ogaki and Julio (1999) measured the relationship between exchange rate and the term structure of interest rate. The effect of the term structure of interest rate on exchange rate for Mexico is investigated. Data set of this study is 1983 to 1997 with the frequency of month. Treasury bills with the maturity of 28 days and 92 days are used. Empirical results of this study are consistent with the term structure of interest rate for Mexico. Interest rates have counterintuitive effects on exchange rate which may arise from the complimentarily one month domestic and foreign bonds.

Favero and Mosca (2001) studied the reaction function of 3 month rate with the term structure relationship linking to 6 month interest rate to current and expected 3 month interest rate by future rate. Data set of this study is zero coupon bonds with different maturities, between 1984 – 1995 with a frequency of month. Main difference of this study from others go to the fore about uncertainty on monetary policy and expecting the term structure of interest with two simultaneously equation models. The biggest result of this

study is related to the uncertain monetary policy shapes the 6 month linked 3 month interest and expected 3 month interest. These authors also presented evidence that monetary policy uncertainty significantly declined around 1994 when the Federal Open Market Committee began to release its target level for the federal funds rate. Moreover, their result suggested that if the influence of such monetary policy uncertainty is properly controlled, the pure expectation hypothesis cannot be rejected, especially in the low uncertainty era from 1994-1999.

Li and Yu (2006) estimated the interest rate term structures of Treasury and individual corporate bonds using a robust criterion. The Treasury term structure is estimated with Bayesian regression splines based on nonlinear least absolute deviation. The number and locations of the knots in the regression splines are adaptively chosen using the reversible jump Markov chain Monte Carlo method. Due to the small sample size, the individual corporate term structure is estimated by adding a positive parametric credit spread to the estimated Treasury term structure using a Bayesian approach. They presented a case study of U.S. Treasury STRIPS (Separate Trading of Registered Interest and Principal of Securities) and AT&T bonds from April 1994 to December 1996. Compared with several existing term structure estimation approaches, the proposed method is robust to outliers in their case study. In their study, a robust approach is proposed to estimate term structures of both Treasury and individual corporate bonds. The proposed approach is found to be robust against outliers in bond prices.

The usual finance model decomposes the short-term interest rate into unobserved factors that are modeled as autoregressive time series that are unrelated to macroeconomic variation. In contrast, from a macro perspective, the short rate is determined by macroeconomic variables in the context of a monetary policy reaction function.

Rudebusch and Wu (2008) model reconciles these two views in a macro-finance framework that has term structure factors jointly estimated with macroeconomic relationships. In particular, this analysis combines an affine arbitrage-free term structure model with a small New Keynesian rational expectation macroeconomic model with the short-term interest rate related to macroeconomic fundamentals through a monetary policy reaction function. The combined macro-finance model is estimated from the data by maximum likelihood methods and demonstrates empirical fit and dynamics comparable to stand-alone finance or macro models. In this study, they developed and estimated a macro-finance model that combines a canonical affine no-arbitrage finance specification of the term structure of interest rates with standard macroeconomic aggregate relationships for output and inflation. Based on this combination of yield curve and macroeconomic structure and data, they obtained several interesting results: (i) the latent term structure factors from no-arbitrage finance models appear to have important macroeconomic and monetary policy underpinnings, (ii) there is no evidence of a slow partial adjustment of the policy interest rate by the central bank, and (iii) both forward-looking and backward-looking elements play roles in macroeconomic dynamics.

Gasha et.al. (2010) discussed the estimation of models of the term structure of interest rates. After reviewing the term structure models, specifically the Nelson-Siegel Model and Affine Term- Structure Model, their study estimated the terms structure of Treasury bond yields for the United States with pre-crisis data. The study then presented estimations of the terms structure of the U.S. Treasury bond yields from 1972 to 2007. The software which is developed by Fund staff for this purpose made it possible to estimate the

term structure using at least nine models, while opening up the possibility of generating simulated paths of the term structure.

2. METHODOLOGY

Term structure is related with the continuous function of maturity. This allows for assigning spot rates to any maturity in order to price ay any date in future. The term structure can be estimated, however from observed coupon bonds by assuming parametric function relating spot rates and time to maturity. Nelson and Siegel (1987) have suggested a flexible function for the forward rate that can be used to obtain a corresponding function for the spot rate. The starting point for the Nelson and Siegel (1987) model is the formulation of the process for the instantaneous forward rate.

$$F(m,\beta)=\beta_0+\beta_1\exp(-m/\tau)+\beta_2(m/\tau)\exp(-m/\tau)$$

β parameters are used to determine shape for the forward rate curve and need to be estimated from observed prices. F is used for to predict forward prices, but what if spot rates corresponding function. The corresponding function is;

$$r(m,\beta)=\beta_0+\beta_1\left(\frac{1-\exp(-m/\tau)}{(-m/\tau)}\right)+\beta_2\frac{1-\exp(-m/\tau)}{(-m/\tau)}-\exp(-m/\tau)$$

β_0 is the limit of spot rate as the maturity tends to infinity, we can call it long-term interest rate. If the maturity tends to zero spot rate converges to the sum $\beta_0 + \beta_1$. This further implies that $-\beta_1$ can be interpreted as spread between long and short term interest rates.

Swenson (1994) has proposed an extension of Nelson and Siegel model that allows flexibility. Whereas, the Nelson and Siegel model can have only one local maximum or local minimum. The Swenson extensions allows for two humps. This gives Nelson and Siegel model more flexibility and fit into the short-term rate to long-term rate.

Another implication is formed in study of Ogaki and Santanella (1999). They design the econometric model of the study with the similar observations and assumptions. But the topic was a little bit different from term structure; they investigated the term structure of interest rate with exchange rates.

$$St = \alpha + \beta_1(r_{t-3} - r_{t-3*}) + \beta_2 (r_{1-t} - r_{1-t*}) + e_t$$

Where $(r_{t-3} - r_{t-3*})$ is the three month interest rate differential and $(r_{1-t} - r_{1-t*})$ is one month interest rate differential. St is the natural log of domestic currency in terms of foreign currency. But the methodology of this study differs from Nelson and Siegel and Swenson models.

In part of the equivalent assumptions and accepting spot rate – time to maturity graphic, Meier (1999) study offers another implication to this relationship in generating final expectation model. Similar relationship is repeated in Mosca and Favero (2001) study. Basic of this relationship is related to equivaliating the long-term maturities to short-term maturities differences.

$$R_{t-t6} = b_1 + b_2R_{t-t3} + b_3ER_{t-t3,\ t-t6}$$

This model is one of the simultaneous equation systems of Mosca and Favero (2001). This model implies that (R_{t-t6}) 6 month bonds are related to (R_{t-t3}) three month bonds, and E($R_{t-t3,\ t-t6}$), expected 6 month maturity bonds linked to three months. This form of interest rates does not carry a structural relationship between low and high maturities. Nevertheless, basis of this study is to constitute a formal relationship between maturities structural changes. The best way to show this in parametrical form is to fit volatilities of different maturities as mentioned in Nelson and Siegel (1997).

3. DATA SET

Data set of this study is derived from the Central Bank of the Republic of Turkey (TCMB) and National Treasury Ministry (http://www.tcmb.gov.tr, http://www.hazine.gov.tr). Sample period is quarterly from January 1990 to December 2001. With the quarterly period of time some data were missing due to government debt privacy policies. On some years there were neither 12 month data nor 6 month data and 1 month data. This problem is exceeded by recalculating and generating the recent data to others by the following equations derived from basic finance compound interest.

Decreasing Maturity $[(1 + R_{t-n})^{t/n} - 1]$

Increasing Maturity $[(1 + R_{t-n})^{n/t} - 1]$

This kind of a solution can create multicollinearity problem, if the absent data periods carry long-term and same with other maturities. All data are quarterly because when the period frequency decreases it becomes so difficult to find related data with the similar maturity.

On some periods, due to macroeconomic policy, maturity of treasury bills changed with unsuitable maturities. This problem is exceeded by recalculating the similar maturity for the related maturity.

4. MODELING THE TERM STRUCTURE OF INTEREST RATE

Modeling the term structure of interest rates is a highly complicated treatment. Most of the term structure studies use treasury bill or t-bonds for measurement. Basic principle of interest rate is the validity of no arbitrage equation. No arbitrage equation is valid under perfect capital market hypothesis. With a simple equality no arbitrage position would be defined as follows;

(1) $r_{t-6} = r_{t-3}[r_{t-3}]$

Parameters of this simple model are 6 month basis treasury bill interest and three month basis interest. This equation simply means, yield to maturity of different maturity based bonds are equivalent. While this assumption in model 1 is valid it with the treasury bill rate of different maturities. Assuming model 1 as a proxy volatility degree of different maturity based treasury bill rate can be shown as model 2.

(2) $(Vol)^o\ r_{t-6} \equiv (Volr)^o_{t-3}$

With model 2's assumption of volatile interest rates in different maturities, the covariance matrices would be interpreted as equal.

$$(3) \qquad [\text{Cov } r_{t-6} = \begin{vmatrix} \Omega_{11}\Omega_{12} \\ \Omega_{21}\Omega_{22} \end{vmatrix}] \equiv [\text{Cov } r_{t-3} = \begin{vmatrix} \Omega_{11}\Omega_{12} \\ \Omega_{21}\Omega_{22} \end{vmatrix}]$$

Interpreting the 12 month based treasury bill interest volatility as dependent variable, independent variables will be the lower level maturities.

$$(4) \qquad \text{Vol}(r_{t-12}) = \text{Vol}(r_{t-11}) + \text{Vol}(r_{t-10}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \text{Vol}(r_{t-1})$$

Equation 4 gives an idea of designing a new model to term structure of t-bill interest rate. But before designing a new model, it will be significant to notice that these assumptions can easily be changed by government debt strategies and macroeconomic policies. Since the maturity of t-bills are neither longer than 36 months nor smaller than 1 month, the variables in model 4 for Turkey need to be redefined.

$$(5) \qquad \text{Vol}(r_{t-12}) = \text{Vol}(r_{t-6}) + \text{Vol}(r_{t-3}) + \text{Vol}(r_{t-1})$$

With more parametric approach to model 5, it will be much simpler to design a new model by developing past model and derivatives. So, a new approach can be defined as model 6.

$$(6) \qquad r_{t-12} = \alpha + \beta_1 r_{t-6} + \beta_2 r_{t-3} + \beta_3 r_{t-1}$$

Dependent and independent variables of model 6 are treasury bill interest rate of 12, 6, 3 and 1 month maturities. 9 month matured treasury bill interests are excluded from this model because of missing data. By the econometric methodology it is a need to linearize the series with natural logarithm (ln).

$$(7) \qquad [LNr_{t-12}] = \alpha + \beta_1 [LNrt_{-6}] + \beta_2 [LNrt_{-3}] + \beta_3 [LNrt_{-1}]$$

Model 7 carries the relationship of linear data but not the volatile of different maturities. To attain volatility, it will be better to differentiate the variables with 1 lagged ones of same maturity.

$$(8) \qquad [(LNr_{t-12}) - (LNr_{(t-1)-12})] = \alpha + \beta_1 [(LNr_{t-6}) - (LNr_{(t-1)-6})] + \beta_2 [(LNr_{t-3}) - (LNr_{(t-1)-3})] + \beta_3 [(LNr_{t-1}) - (LNr_{(t-1)-1})]$$

Simplifying model 8 with less complicated variable definitions would help to calibrate and explain.

$$(9) \qquad Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + e_t$$

Y is the volatility of 12 month maturity based treasury bills. X_1 is the volatility of 6 month maturity based treasury bills. X_2 is the volatility of 3 month maturity based treasury bills. X_3 is the volatility of 1 month maturity based treasury bills. E_t is the error term of the model.

The signs and significance levels of β_1, β_2, and β_3 will be explanatory to the term structure relation of 12 month maturity based treasury bills. The measurement will indicate the term structure of treasury bills.

5. ECONOMETRIC APPLICATION

The application of model 9 starts with multicollinearity test, in order to support basic OLS assumptions. Second step can be defined as unit root test for series; this will also show possible cointegration process. Third step is OLS estimation. Fourth step is autocorrelation test for error terms. Fifth step is heteroscedasticity.

Results section will summarize the signs and significance of the coefficients. This will help to understand the structural change of treasury bill interests, relationship with different maturities.

5.1. MULTICOLLINEARITY

Multicollinearity can be defined as relationship between dependent and independent variables. The best way to test this problem is to create a correlation matrix for variables. The table below shows the correlation matrix for the variables used in model 9.

Table 1: The Correlation Coefficients for Testing Multicollinearity

R	Y	X_1	X_2	X_3
Y	1	0.55	0.54	0.54
X_1	0.55	1	0.51	0.34
X_2	0.54	0.51	1	0.43
X_3	0.54	0.34	0.43	1

Results show that all correlation coefficient squares are lower than medium multicollinearity, this eposes that there is no multicollinearity problem in the variables.

5.2. UNIT ROOT TESTS

To avoid other problems which would occur in next analysis; it will be better to check for unit root following the multicollinearity test. Table 2 indicates the ADF unit root test results.

Table 2: Unit Root Test Results

	ADF value	McKinnon	Lag*
Y	-7.677	-3.41	1
X_1	-9.001	-3.41	1
X_2	-8.861	-3.41	1
X_2	-7.186	-3.41	2
X_3	-7.624	-3.41	1

*Schwartz Information criteria is being used, 95% Significance Level. ($\alpha = 0.05$)

Results show that there is no unit root problem in the series because of ADF > Mc Kinnon (95%) in all series.

5.3. OLS ESTIMATION

If the significance level is 90%, all the variables are statistically significant except for constant α. Explanatory power of the independent variables is 48% and F-statistic supports the meaning of the model. All the coefficients are positive.

These circumstances cannot be satisfactory for the results because econometric tests have not been completed yet. Before generalizing the results tests of autocorrelation and heteroscedasticity tests must be applied.

Table 3: OLS Results

	Coefficient	t-Statistic
α	1.03E-05	0.000499
β_1	0.166845	2.386258
β_2	0.171939	1.718346
β_3	0.232300	2.539301
R-squared	0.483193	
Durbin-Watson stat	2.277931	
F-statistic	12.46609	

5.4. AUTOCORRELATION TEST

There are many methods to test autocorrelation. This study will use Breusch Godfrey Serial Correlation LM Test and probability of F-values. Hypothesis and error term relationship is shown below. Autocorrelation test is going to measure the 1 lagged and 4 lagged differences as mentioned in econometric theory for autocorrelation in quarterly data.

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + e_t$$

$$Et = a + \delta e_{t-1} + U_t$$

$$Et = a + \delta_1 e_{t-1} + \delta_2 e_{t-4} + U_t$$

Related hypothesis for 1 lagged variable is ;

$H_o : \delta_1 = 0$; (No Autocorrelation) (Probability of F-Stat. > 0.10)

$H_a : \delta_1 \neq 0$; (Autocorrelation Situation) (Probability of F-Stat. < 0.10)

Related hypothesis for 1 – 4 lagged variable is ;

$H_o : \delta_1, \delta_2 = 0$; (No Autocorrelation) (Probability of F-Stat. > 0.10)

$H_a : \delta_1, \delta_2 \neq 0$; (Autocorrelation Situation) (Probability of F-Stat. < 0.10)

Table 4: Autocorrelation Results

Breusch – Godfrey Serial Correlation LM Test Results:			
Lags Included	**F Statistic**	**Probability**	**Accepted Hypothesis**
1	1.1264	0.2950	H_o
1-4	3.9018	0.8142	H_o

5.5. HETEROSCEDASTICITY

White test can be used for heteroscedasticity, related hypothesis is given below.

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + e_t$$

$H_o : Var(E_{t-1}) = Var(E_{t-n})$ (No heteroscedasticity) (Prob. F-Stat. > 0.10)

$H_a : Var(E_{t-1}) \neq Var(E_{t-n})$ (Heteroscedasticity) (Prob. F-Stat. < 0.10)

Table 5: Heteroscedasticity Test Results

White Heteroscedasticity Test		
F-Statistic	Probability	Accepted Hypothesis
0.2384	0.9860	H_o

White test results shows that there is no heteroscedasticity problem in this estimation.

CONCLUSION

In this study, we have aimed to estimate the structure of interest rates with the structural relationship of different maturities. Econometric results show that long-term Treasury bill interest rate differentials are supported by short-term treasury bills. This implication is related to the signs of all β coefficients. All the coefficients are statistically

significant (α=0.10). This means that all the different maturities of treasury bills support high maturities. R square is 48% and F-statistics show that model is working properly as a whole. The assumption of equivalent volatility of different maturities is also supported by these results. The findings of this study is compliant with of Mankiv and Miron (1986), Geyer and Mader (1999), Favero and Mosca (2001) and Rudebusch and Wu (2008).

REFERENCES

Balduzzi, P. (1997). A model of Target Changes and the Term Structure of Interest Rates. Journal of Monetary Economics 24, 371 – 399.

Favero, A. C. and F. Mosca (2000). Uncertainty on Monetary Policy and the Expectations Model of the Term Structure of Interest Rates. Economics Letters (71), 369 – 375

Geyer, A. and R. Mader (1999). Estimation of the Term Structure of Interest Rates. Working paper (37) http://www.oenb.co.at/ workpaper/pubwork.htm

Heller D. (1997). Monetary Policy and Estimating the Term Structure of Interest Rate. Working Paper of Switzerland National Bank, 97-2.

Mankiv, N. G. and J. A. Miron (1986). The Changing Behavior of the Term Structure of Interest Rates. Quarterly Journal of Economics (101), 221 – 221.

Meier, I. (1999). Estimating the Term Structure of Interest Rates. University of Bern.

Meredith G and M.D. Chinn (1998). Long – Horizon Uncovered Interest Rate Parity. National Bureau of Economic Research Working Paper 6797.

Li, M. and Y. Yan (2006). A Robust Approach to the Interest Rate Term Structure Estimation, Journal of Data Science, v.4, no.2, p. 189-205.

Gasha, G., H.Y., Medeiros, C., Rodriguez, M., Salvati, J. and Yi, J. (2010). On the Estimation of Term Structure Models and An Application to the United States, IMF Working Paper WP/10/258, International Monetary Fund.

Nelson, C. R. S. and Siegel A. F. (1987). Parsimonious Modelling of Yield Curves. Journal of Business, (60), 47-55.

Ogaki M. and J. A. Santanella, (1999). The Exchange Rate and the Term Structure of Interest Rates in Mexico. Ohio State University Department of Economics Working Paper 99-21.

Rudebusch, G.D. (1995). Federal Reserve Interest Rate Targeting, Rational Expectations and the Term Structure. Journal of Monetary Economics 35, 245 - 274.

Rudebusch, G. D. and Wu, T. (2008). 'Macro-finance Model of the Term Structure, Monetary Policy and the Economy', *Economic Journal*, (118), 906–926.

Swenson, L.E.O. (1994). Estimating and Interpreting Forward Interest Rates. NBER Working paper No:4871

http://www.tcmb.gov.tr

http://www.hazine.gov.tr