International Journal of Engineering and Applied Sciences (IJEAS)

Vol.2, Issue 2(2010)44-54

PARETO-OPTIMALITY AND V.E.G.A. TO MULTI- INPUT MULTI-OUTPUT (M.I.M.O.) SYSTEM REDUCTION

S.K. Mittal, D. Chandra, and B. Dwived
Research Scholar, U. P. Technical University, Lucknow, India

E-mail : mittalsai@rediffmail.com

Accepted Date: 15 October 2009

Abstract

A VEGA (Vector Evaluated Genetic Algorithm) based computer-aided method to derive a reduced order (rth-order) approximant for given (stable) multi-input multi-output (MIMO) linear continuous-time system is presented. In this method, stability and the first r time moments/Markov parameters are preserved as well as the errors between a set of subsequent time moments/Markov parameters of the system and those of the model are minimized. The method is useful as it guarantees improvement as well as alleviates the problems of deciding the values of number of error functions to be minimized and values of weights on the errors which were left unresolved in previous methods. The search area for GA is very wide and it usually converges to a point near global optima.

Keywords : Model reduction**,** Padé approximants, Routh criteria, G.A.

1. Introduction

Approximation of a given high-order (***n***th-order) linear system by a low-order (***r***th-order) one is desirable and sometimes necessary in the simulation and design of control systems. Consequently, a large number of time-domain and frequency-domain systems simplification techniques have been developed to suit different requirement. Reducing the order of multivariable systems in state space has been studied by several authors. Some of the reported methods require the computation of eigenvalues and some others use certain optimization procedures. However, this method is based on a large number of transformation while some others can be evaluated only through numerical procedures. A large number of time-domain and frequency-domain system simplification techniques have been developed to suit different requirement. Amongst them, a frequency domain method is Padé approximation in which 2***r*** terms of the power series expansion (time moments) of the high-order (***n***th-order) transfer function $G_n(s)$ are fully retained in low-order (***r***th-order) model $G_r(s)$. The Padé approximation does not guarantee the stability of the reduced-order model. To overcome the problem of stability, several stable reduction methods such as Routh approximation methods [34] have been proposed. Thus, the basic problem is to match or near match a few terms in excess of ***r*** terms while preserving stability [2,3]. Other closely related problems have also received attention [1,6-12,11-17,24-34,36,37].

Recently, geometric programming based (computer-oriented) methods [29,30] for the solution of the Routh-Padé approximation problem are presented. In these methods [40], Geometric programming based computer-aided methods have been reported recently first *r* time moments/Markov parameters are fully retained and the sum of the weighted squares of errors between a set of subsequent time moments/Markov parameters of the system and those of the model are minimized while preserving stability. These methods [29,30] have the drawback that the question of finding some means (free of hit and trial) of deciding the values of the number of time moments/Markov parameters (say *m*) to be matched or near-matched and the weights to correspond to assured substantial improvement in system approximation as well as the question of establishing the existence of such values are left unresolved.

[36] Proposes a method to construct a state-shared model for multiple-input multiple-output (MIMO) systems. A state-shared model is defined as a linear time invariant state-space structure that is driven by measurement signals-the plant outputs and the manipulated variables, but shared by different multiple input/output models. The genesis of the state-shared model is based on a particular reduced non-minimal realization. Any such realization necessarily fulfills the requirement that the output of the state-shared model is an asymptotically correct estimate of the output of the plant, if the process model is selected appropriately. The approach is demonstrated on a nonlinear MIMO system-a physiological model of calcium fluxes that controls muscle contraction in human cardiac myocytes

In this note, a nonlinear programming based (computer-oriented) method for the solution of Routh-Padé approximation problem is presented. The method is essentially a multi-objective optimization procedure in which not only stability is preserved and the first *r* terms of the power series expansion of $G_n(s)$ are fully retained but also the errors between a set of subsequent time moments/Markov parameters of the system and those of the model are minimized. This alleviates the problem of finding *m* and weights. The applicability of proposed method is shown by means of numerical example. The search area for GA is very wide and it usually converges to a point near global optima [11]. Though Pareto-optimality, which is a key step in the present technique, is well known to the best of author's knowledge, this is the first instance of explicitly showing its usefulness for obtaining reduced-order models for MIMO systems.

This paper is organized as follows. In Sec. 2 we briefly review the results of [29,30]. The improvement is presented in Sec. 3 and numerical example is given in Sec. 4. Finally paper is concluded in Sec. 5.

2. Brief Review of Existing Results

Consider a single-input-single-output system described by the transfer function

$$G_n(s) = \frac{a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_n}{s^n + b_1 s^{n-1} + \ldots + b_n} \tag{1}$$

$$= t_1 + t_2 s + \ldots + t_n s^{n-1} + \ldots \tag{2}$$

(expansion around $s = 0$)

$$= M_1 s^{-1} + M_2 s^{-2} + \ldots + M_n s^{-n} + \ldots \tag{3}$$

(expansion around $s = \infty$)

The problem is to determine its stable reduced-order (*r*th-order) approximant

$$G_r(s) = \frac{\hat{a}_1 s^{r-1} + \hat{a}_2 s^{r-2} + \ldots + \hat{a}_r}{s^r + \hat{b}_1 s^{r-1} + \ldots + \hat{b}_r} \quad (4)$$

$$= \hat{t}_1 + \hat{t}_2 s + \ldots + \hat{t}_r s^{r-1} + \ldots \quad (5)$$

$$= \hat{M}_1 s^{-1} + \hat{M}_2 s^{-2} + \ldots + \hat{M}_r s^{-r} + \ldots \quad (6)$$

A. *Formulation of the objective function*

The formulation of the multiobjective optimization problem will be explained for *r* being even. Formulation for r being odd can be done in a similar way. It is easy to verify that for r even, the following equations hold true:

$$\left.\begin{aligned} \hat{a}_{r+1-i} &= \sum_{j=1}^{i} \hat{t}_j \hat{b}_{r-i+j} \\ \hat{a}_i &= \sum_{j=1}^{i} \hat{M}_j \hat{b}_{i-j} \end{aligned}\right\} \quad i = 1, \ldots, \frac{r}{2} \quad (7)$$

$$\left.\begin{aligned} \hat{t}_i &= \left(-\sum_{j=1}^{i-1} \hat{t}_j \hat{b}_{r-i+j} + \sum_{j=1}^{r+1-i} \hat{M}_j \hat{b}_{r+1-i-j}\right)\hat{b}_r^{-1} \\ \hat{M}_i &= \sum_{j=1}^{r+1-i} \hat{t}_j \hat{b}_{i-1+j} - \sum_{j=1}^{i-1} \hat{M}_j \hat{b}_{i-j} \end{aligned}\right\} \quad i = \frac{r}{2}+1, \frac{r}{2}+2, \ldots, r \quad (8)$$

$$(\hat{b}_0 = 1;\ \hat{b}_i = 0 \text{ for } i \notin \{0, \ldots, r\};\ \hat{t}_i, \hat{M}_i = 0 \text{ for } i < 1).$$

We seek a stable model for which *r* equations given by

$$\left.\begin{aligned} \hat{t}_i - t_i &= 0 \\ \hat{M}_i - M_i &= 0 \end{aligned}\right\} \quad i = 1, \ldots, \frac{r}{2} \quad (9)$$

are satisfied ,which implies, from (7),

$$\left.\begin{aligned} \hat{a}_{r+1-i} &= \sum_{j=1}^{i} t_j \hat{b}_{r-i+j} \\ \hat{a}_i &= \sum_{j=1}^{i} M_j \hat{b}_{i-j} \end{aligned}\right\} \quad i = 1, \ldots, \frac{r}{2}. \quad (10)$$

There exist an infinite number of stable models for which (10) is satisfied. This arbitrariness in stability preservation is exploited by minimizing the sum of the weighted squares of errors.

To find the improved model, VEGA [11] is used to generate Pareto-optimal solutions by minimizing objective functions $Z^{t}_{\frac{r}{2}+i}, Z^{M}_{\frac{r}{2}+i}$ given by

$$\left.\begin{aligned} z^{t}_{\frac{r}{2}+i} &= \Big(1-\frac{\hat{t}_{\frac{r}{2}+i}}{t_{\frac{r}{2}+i}}\Big)^2 \\ z^{M}_{\frac{r}{2}+i} &= \Big(1-\frac{\hat{M}_{\frac{r}{2}+i}}{M_{\frac{r}{2}+i}}\Big)^2 \end{aligned}\right\} \quad i = 1,\ldots,\frac{r}{2} \tag{11}$$

Using (8) subject to (9), (11) can be expressed as

$$\left.\begin{aligned} z^{t}_{\frac{r}{2}+i} &= \Big(1-\frac{\hat{t}_{\frac{r}{2}+i}}{t_{\frac{r}{2}+i}}\Big)^2 = f(\hat{b}_1,\hat{b}_2,\ldots,\hat{b}_r) \\ z^{M}_{\frac{r}{2}+i} &= \Big(1-\frac{\hat{M}_{\frac{r}{2}+i}}{M_{\frac{r}{2}+i}}\Big)^2 = f(\hat{b}_1,\hat{b}_2,\ldots,\hat{b}_r) \end{aligned}\right\} \quad i = 1,\ldots,\frac{r}{2} \tag{12}$$

B. *Formulation of the stability constraints*

Now following [28], the denominator polynomial of (4) can be expressed as

$$\begin{aligned} & s^r + \hat{\alpha}_1 s^{r-1} + (\hat{\alpha}_2 + \hat{\alpha}_3 + \ldots + \hat{\alpha}_r)s^{r-2} + \hat{\alpha}_1(\hat{\alpha}_3 + \hat{\alpha}_4 + \ldots + \hat{\alpha}_r)s^{r-3} \\ & + [\hat{\alpha}_2(\hat{\alpha}_4 + \hat{\alpha}_5 + \ldots + \hat{\alpha}_r) + \hat{\alpha}_3(\hat{\alpha}_5 + \hat{\alpha}_6 + \ldots + \hat{\alpha}_r) + \hat{\alpha}_4(\hat{\alpha}_6 + \hat{\alpha}_7 + \ldots + \hat{\alpha}_r) + \ldots \\ & + \hat{\alpha}_{r-2}\hat{\alpha}_r]\, s^{r-4} + \ldots + \hat{\alpha}_{1+q}\hat{\alpha}_{3+q}\ldots\hat{\alpha}_{r-2}\hat{\alpha}_r \end{aligned} \tag{13}$$

which is constructed by taking the coefficients of the first two rows of the Routh array with the elements of its first column given [28] by

$$1, \hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_1\hat{\alpha}_3, \hat{\alpha}_2\hat{\alpha}_4, \hat{\alpha}_1\hat{\alpha}_3\hat{\alpha}_5, \ldots, \hat{\alpha}_{1+q}\hat{\alpha}_{3+q}\ldots\hat{\alpha}_{r-2}\hat{\alpha}_r \tag{14}$$

where q =1 for r even and q =0 for r odd. By setting

$$\begin{aligned} & \hat{\alpha}_1\hat{\beta}_1^{-1} = 1, \quad (\hat{\alpha}_2 + \hat{\alpha}_3 + \ldots + \hat{\alpha}_r)\hat{\beta}_2^{-1} = 1, \\ & \hat{\alpha}_1(\hat{\alpha}_3 + \hat{\alpha}_4 + \ldots + \hat{\alpha}_r)\hat{\beta}_3^{-1} = 1, \ldots, (\hat{\alpha}_{1+q}\hat{\alpha}_{3+q}\ldots\hat{\alpha}_{r-2}\hat{\alpha}_r)\hat{\beta}_r^{-1} = 1 \end{aligned} \tag{15}$$

(15) is matched with the denominator polynomial of the model in (4), namely, with

$$\hat{\beta}_r + \hat{\beta}_{r-1}s + \ldots + \hat{\beta}_1 s^{r-1} + s^r \tag{16}$$

and the necessary and the sufficient condition that all the roots of (16) be strictly in the left half plane is [28]

$$\hat{a}_1 > 0, \hat{a}_2 > 0, \ldots, \hat{a}_r > 0 \tag{17a}$$

which, of course, implies

$$\hat{b}_1 > 0, \hat{b}_2 > 0, \ldots, \hat{b}_r > 0\,. \tag{17b}$$

Note that, for a given $r, \hat{b}_i, i = 1, \ldots, r$, can easily be expressed in terms of $\hat{a}_i$, $i = 1, \ldots, r$, by constructing an inverse Routh array (i.e., with the element of its first column given by (14)) in a manner analogous to [28].Thus, pertaining to *r*=4, (15) becomes

$$\hat{b}_1 = \hat{a}_1,\ \hat{b}_2 = \hat{a}_3 + \hat{a}_3 + \hat{a}_4,\ \hat{b}_3 = \hat{a}_1(\hat{a}_3 + \hat{a}_4),\ \hat{b}_4 = \hat{a}_2\hat{a}_4\,. \tag{18}$$

3. Application of VEGA

Now, the problem is to minimize (12), satisfying (17a). The vector evaluated genetic algorithm (VEGA) [11] is proposed herein for solving the above stated problem. VEGA is the simplest possible multi-objective GA [11] and is straightforward extension of a single-objective extension of multi-objective optimization. Since a number of objectives (say *Q*) have to be handled, GA population is divided at every generation into *Q* equal subpopulations randomly. Each subpopulation is assigned a fitness value based on different objective function.

After each solution is assigned a fitness value, the selection operator restricted among solutions of each subpopulation, is applied until the complete subpopulation is filled [11]. The following VEGA procedure is used [11].

Step* 1:** Set, for population size N, an objective function counter ***i = 1 and define $x = N / Q$

***Step* 2:** For all solution, $j = 1 + (i-1) * x$ to $j = i * x$, assign fitness as: $Z(\hat{\mathbf{b}}^{(j)}) = z_i(\hat{\mathbf{b}}^{(j)})$.

***Step* 3:** Perform proportionate selection on all x solutions to create a mating pool P_i.

Step* 4:** If $i = Q$, go to *Step* 5. Otherwise, increment ***i by one and go to *Step* 2.

***Step* 5:** Combine all mating pools together: $P = \bigcup_{i=1}^{Q} P_i$. Perform crossover and mutation on *P* to create a new population [11].

In this VEGA, linear crossover operator is used. It creates three solutions, $0.5(\hat{b}_i^{(1,t)} + \hat{b}_i^{(2,t)})$, $(1.5\hat{b}_i^{(1,t)} - 0.5\hat{b}_i^{(2,t)})$, $(-0.5\hat{b}_i^{(1,t)} + 1.5\hat{b}_i^{(2,t)})$ from two parent solutions $\hat{b}_i^{(1,t)}$ and $\hat{b}_i^{(2,t)}$ at generation *t*, with the best two solutions being chosen as offspring. For performing mutation, random mutation is used. Instead of creating a solution from the entire search space, a solution in the vicinity of parent solution with a uniform probability distribution is chosen: $y_i^{(1,t+1)} = \hat{b}_i^{(1,t)} + (r_i - 0.5)\Delta_i$ where r_i is a random number in [0,1].

The Routh-Padé approximants by Pareto-Optimality and V.E.G.A. can be extended to reduce a class of (common denominator type) multi-input multi-output (M.I.M.O.) system. In this case the method is successively applied to each of the scalar transfer functions $G_{ij}(s)$ of transfer matrix $G(s) = [G(s)]_{lxm}$ to form reduced-order matrix. The steps to be followed are:

Step 1: Compute the time-moments and Markov-parameters of each scalar transfer function .$G_{ij}(s)$

Step 2: Reduce the common denominator of MIMO system by the proposed method for which the objective function is formulated pertaining to individual $\hat{G}_{ij}(s)$.

Step 3: Compute numerator coefficients of each individual $\hat{G}_{ij}(s)$ by retaining r terms of each scalar transfer function.

4. Example

Consider the MIMO system reported by Shamash [37],

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \tag{19}$$

where:

$$G_{11}(s) = \frac{s^2 + 12s + 20}{D(s)}$$

Time-moments and Markov parameters of $G_{11}(s)$ are following:

$t_{121} = 1.00,\quad t_{211} = -1.00,\quad t_{321} = 1.0, M_{111} = 1.00,\quad M_{211} = -1.00,\quad M_{311} = 1.0$

$$G_{12}(s) = \frac{11s^2 + 40s + 20}{D(s)}$$

Time-moments and Markov-parameters of $G_{12}(s)$ are following:

$t_{121} = 1.0,\quad t_{211} = 0.40,\quad t_{321} = -0.74, M_{121} = 11.0,\quad M_{321} = -103,\quad M_{321} = 1007$

$$G_{21}(s) = \frac{s^2 + 12s + 20}{D(s)}$$

Time-moments and Markov-parameters of $G_{11}(s)$ are following:

$t_{131} = 1.0,\quad t_{231} = -1.0,\quad t_{331} = 1.0, M_{131} = 1.0,\quad M_{231} = -1.0,\quad M_{331} = 1.0$

$$G_{22}(s) = \frac{33s^2 + 122s + 80}{D(s)}$$

Time-moments and Markov-parameters of $G_{22}(s)$ are following:

$t_{141} = 4.00,\quad t_{241} = -0.30,\quad t_{341} = -0.47, M_{141} = 33.00,\quad M_{241} = -307,\quad M_{341} = -1.00$

$$D(s) = s^3 + 13s^2 + 32s + 20$$

Second-order model be represented by:

$$\hat{G}_2(s) = \begin{bmatrix} \hat{G}_{11}(s) & \hat{G}_{12}(s) \\ \hat{G}_{21}(s) & \hat{G}_{22}(s) \end{bmatrix} = \frac{\begin{bmatrix} \hat{a}_{111}s + \hat{a}_{112} & \hat{a}_{211}s + \hat{a}_{212} \\ \hat{a}_{311}s + \hat{a}_{312} & \hat{a}_{411}s + \hat{a}_{412} \end{bmatrix}}{s^2 + \hat{b}_1 s + \hat{b}_2} \tag{20}$$

Fully retaining of first Time-moment and first Markov-parameter of each $G_{ij}(s)$, we obtain:

$$\left.\begin{array}{llll}\hat{t}_{111}=t_{111} & \Rightarrow a_{112}=t_{111}\hat{b}_2, & \hat{M}_{111}=M_{111}\Rightarrow a_{111}=M_{111}\\ \hat{t}_{121}=t_{121} & \Rightarrow a_{212}=t_{121}\hat{b}_2, & \hat{M}_{121}=M_{121}\Rightarrow a_{211}=M_{121}\\ \hat{t}_{131}=t_{131}, & \Rightarrow a_{312}=t_{131}\hat{b}_2, & \hat{M}_{131}=M_{131}\Rightarrow a_{311}=M_{131}\\ \hat{t}_{141}=t_{141}, & \Rightarrow a_{412}=t_{141}\hat{b}_2, & \hat{M}_{141}=M_{141}\Rightarrow a_{411}=M_{141}\end{array}\right\} \quad (21)$$

$$\hat{t}_{211}=\frac{\hat{a}_{111}-t_{111}\hat{b}_1}{\hat{b}_2}=\frac{\hat{a}_{111}-1.00\hat{b}_1}{\hat{b}_2},\ \hat{t}_{221}=\frac{\hat{a}_{211}-t_{121}\hat{b}_1}{\hat{b}_2}=\frac{\hat{a}_{211}-1.00\hat{b}_1}{\hat{b}_2}$$

$$\hat{t}_{231}=\frac{\hat{a}_{311}-t_{131}\hat{b}_1}{\hat{b}_2}=\frac{\hat{a}_{311}-1.00\hat{b}_1}{\hat{b}_2},\ \hat{t}_{241}=\frac{\hat{a}_{141}-t_{141}\hat{b}_1}{\hat{b}_2}=\frac{\hat{a}_{141}-4.00\hat{b}_1}{\hat{b}_2}$$

$$\hat{t}_{341}=\frac{-\hat{t}_{141}-\hat{t}_{241}\hat{b}_1}{\hat{b}_2},\ \hat{t}_{321}=\frac{-\hat{t}_{111}-\hat{t}_{211}\hat{b}_1}{\hat{b}_2},\ \hat{t}_{231}=\frac{-\hat{t}_{121}-\hat{t}_{221}\hat{b}_1}{\hat{b}_2},\ \hat{t}_{211}=\frac{-\hat{t}_{131}-\hat{t}_{231}\hat{b}_1}{\hat{b}_2}$$

$$\begin{array}{l}\hat{M}_{211}=\hat{a}_{112}-M_{111}\hat{b}_1=\hat{a}_{112}-1.00\hat{b}_1,\ \hat{M}_{221}=\hat{a}_{212}-M_{121}\hat{b}_1=\hat{a}_{212}-11.00\hat{b}_1,\\ \hat{M}_{231}=\hat{a}_{312}-M_{131}\hat{b}_1=\hat{a}_{312}-1.00\hat{b}_1,\ \hat{M}_{241}=\hat{a}_{412}-M_{141}\hat{b}_1\hat{a}_{412}-33.00\hat{b}_1,\\ \hat{M}_{311}=-M_{111}\hat{b}_2-\hat{M}_{211}\hat{b}_1=-1.00\hat{b}_2-\hat{M}_{211}\hat{b}_1,\ \hat{M}_{321}=-M_{121}\hat{b}_2-\hat{M}_{221}\hat{b}_1=-11.00\hat{b}_2-\hat{M}_{221}\hat{b}_1,\\ \hat{M}_{331}=-M_{131}\hat{b}_2-\hat{M}_{231}\hat{b}_1=-1.00\hat{b}_2-\hat{M}_{231}\hat{b}_1,\\ \hat{M}_{341}=-M_{141}\hat{b}_2-\hat{M}_{241}\hat{b}_1=-33.00\hat{b}_2-\hat{M}_{241}\hat{b}_1\end{array} \quad (22)$$

Following VEGA [11] parameters has been used to obtain the optimal values of $\hat{b}_1$ and $\hat{\mathrm{b}}_2$.

$$\left.\begin{array}{ll}\text{Population size} & :6\\ \text{Selection} & :\text{Roulette - wheel selection operator}\\ \text{Crossover} & :\text{Linear crossover (Elite preserving)}\\ \text{Mutation} & :\text{Random mutation } (\Delta_i=0.1)\end{array}\right\}$$

For the following population of initial conditions, the population after crossover and mutation operators are shown in following table:

Table 1. The population after crossover and mutation operators

Sl. No.	Initial Population		Population after Selection Operator		Population after Crossover & Mutation Operator		Assigned Fitness Value
	b_1	**b_2**	**b_1**	**b_2**	**b_1**	**b_2**	
1.	8.293	7.2343	8.3231	7.3323	8.291056	7.293356	0.000000
2.	8.2893	7.3123	8.2893	7.3123	8.240355	7.263356	0.000010
3.	8.3231	7.3323	8.3231	7.3323	8.319006	7.271307	0.000043
4.	8.233	7.5668	8.2334	7.3564	8.184456	7.307456	0.015129
5.	8.2334	7.3564	8.2334	7.3564	8.185506	7.308506	0.015129
6.	8.2313	7.3543	8.2313	7.3543	8.182356	7.305356	0.015129

Applying Pareto-Optimality and V.E.G.A., algorithm converges to the following optimal solution:

$$\hat{b}_1 = 8.291056, \qquad \hat{b}_2 = 7.293356. \tag{23}$$

Finally, the model of $\hat{G}_2(s)$ takes the form:

$$\hat{G}_2(s) = \frac{\begin{bmatrix} s+7.293356 & 11s+7.293356 \\ s+7.293356 & 33s+29.173424 \end{bmatrix}}{s^2+8.291056s+7.293356} \tag{24}$$

The reduced -order model derived by the technique of [25,27] is:

$$\hat{G}_2(s) = \frac{\begin{bmatrix} 0.8047s+1.5385 & 2.9586s+1.5385 \\ 0.8047s+1.5385 & 8.6036s+6.1538 \end{bmatrix}}{s^2+2.3432s+1.5385} \tag{25}$$

Alternatively, the Hurwitz polynomial method by Appiah [1] yields:

$$\hat{G}_2(s) = \frac{\begin{bmatrix} 0.9697s+1.61616 & 3.232324s+1.61616 \\ 0.9697s+1.61616 & 9.53536s+6.46464 \end{bmatrix}}{s^2+2.58586s+1.61616} \tag{26}$$

The input responses of the high order system (19) and various reduced-order models (24), (25) and (26) to unit step and impulse applied to the two inputs are compared. Step and impulse responses of (24) show an improvement over (25) and (26).

5. Conclusions

In this note, the problem of finding Routh-Padé approximants has been viewed as a multi-objective optimization problem for Multi-input multi-output systems. It is shown that using VEGA [ ], the denominator of the model can be chosen so as to minimize errors between the (r+1)th, (r+2)th,…,2rth time moments and Markov parameters of the model and the corresponding time moments and Markov-parameters of the system while preserving stability. Having obtained the denominator in this manner, the numerator can be determined by retaining first r time-moments/Markov-parameters of the system. VEGA [11] is used to generate multiple Pareto-optimal solutions and the final solution is chosen based on best fitness value. This eliminates the use of weights (w) in the objective functions paving way for greater degree of freedom in optimization. The present approach, therefore, leads to an improved approximant. Further, approximant is obtained by matching or near-matching of 2r time moments/Markov parameters i.e. within the ambit of standard Padé approximation.

Acknowledgement

The author wishes to thank Dr. N.L. Prajapati for his assistance.

References

[1] Appiah, R.K, Pade methods of Hurwitz polynomial approximation with application to linear system reduction, *Int. J.Control.* 29: 39-48, 1979.

[2] Ashoor, N. and Singh, V., A note on low order modeling, *IEEE Trans. Automat. Contr.*, 27, 1124-1126, 1982.

[3] Ashoor, N. and Singh, V., Remarks on system simplification under consideration of time response, *Electron. Lett.,* 18, 496-497, 1982.

[4] Choo, Y. Improvement to modified Routh approximation method., *Electron. Lett.* , 35, 606-607. 1999.

[5] Choo, Y., Improvement to modified Routh approximation method (correction), *Electron. Lett.,* 1119, 1999.

[6] Choo, Y., Direct method for obtaining modified Routh approximants., *Electron. Lett.,* 35, 627–1628 1999.

[7] Choo, Y., Improved bilinear Routh approximation method for discrete time systems, *Trans. ASME J. Dyn. Syst. Meas. Control.,* 123, 125–127, 2001.

[8] Choo, Y., Equivalence of bilinear Routh and Schwarz approximation methods for discrete-time systems, Electron. Lett., 38, 761–762, 2002.

[9] Choo, Y. and Dongmin, K., SISO Continuous System Reduction via impulse response Gramian by iterative formulae, *Trans. ASME J. Dyn. Syst. Meas. Contr.,* 128: 391-393, 2006.

[10] Dolgin, Y. and Zeheb, E., On Routh-Pade model reduction of interval systems, *IEEE Trans. Autom. Control,* 48, 1610–1612,2003.

[11] Deb., K., Multi-objective Optimization using Evolutionary Algorithm., *New York, John Wiley and Sons Ltd.,* 2002.

[12] Hsieh, C.S and Hwang, C., Model reduction of continuous-time system using a modified Routh-approximation method, IEE Proc. D. Control Theory Appl., 136, 151–156 1989.

[13] Householder, A.S., *The numerical treatment of single non-linear equation, McGraw-Hill Book Co., New -Newyork 1970.*

[14] Hwang, C. and Hwang, J.H. and Guo, T.Y., Multifrequency Routh Approximants for linear systems, *IEE Proc. Control Theory Applicat.* , 142, 351-358, 1975.

[15] Hwang, C.Y. and Lee, Y.C., A new family of Routh approximants. Circuits Syst. Signal Process. , 16, 1-25, 1997.

[16] Hwang, C. and Yang, S.F., Comments on the computation of interval Routh approximants., *IEEE Trans. Autom. Control.,* 44, 1782–1787, 1999.

[17] Kelley, K.J., Aircraft manoeuvre optimization by reduced order approximation control and dynamic system (Ed:C.T. Leondes), *Academic Press London*, 132-174, 1973.

[18] Krishnamurthy V. and V. Sheshadri , A simple and direct method of reduction order of linear systems using routh approximation in frequency domain IEEE Trans. Autom. Control. 21, 797-799, 1976.

[19] Luss, R.and Jaakola, T., Direct search and systematic reduction of size of search region. *AICHE J.,* 19, 760-766, 1973.

[20] Lucas, T.N., The bilinear method: a new stability-preserving order reduction approach. *Proc. Inst. Mech. Engg. I. J. Syst. Contr. Engg.,* 216, 429-436, 2002.

[21] Lucas, T.N., Constrained optimal Pade model reduction., ASME J .Dyna. Syst. Meas. Control, 119, 685-690, 1997.

[22] Lee, Y.C. and Hwang, C.and Hwang J.H., Model-reduction of SISO systems by Routh expansion and balancing method., J. Franklin Inst., 331B, 367–380, 1994.

[23] Lucas, T.N., 1988. Scaled impulse energy approximation for model reduction, *IEEE Trans.Automat. Contr.,* 133, 791-793, 1998.

[24] Manigandan T., Devarajan N. and Svanandam S.N. Design of PID controller using reduced order model, *Academic Open Internet Journal,* 15, 1-15, 2005.

[25] Pal, J., State reduced-order Padé approximants using Routh-Hurwitz array, *Electron. Lett.* , 15, 25-26, 1979.

[26] Parks, P.C, A new proof of the Routh-Hurwitz criterion using the second method of Lyapunov, *Proc. Camb. Philos. Soc.*, 694-702, 1962.

[27] Pal, J., Improved Pade approximants using stability equation method, *Electron. Lett.*, 19, 426-427, 1983.

[28] Puri, V. and Lim, D.P. , Stable model reduction by impulse response error minimization using Michailov criterion and Pade´ approximation, *Trans. ASME J. Dyn. Syst. Meas.Control*, 110, 389–394, 1988.

[29] Singh, V., Obtaining Routh-Pade approximants using Luss-Jaakola algorithm, *IEE Proc. Part I,* 152.. 129-132, 2005.

[30] Singh, V. and Dinesh Chandra, and Kar, H., Improved Routh-Pade approximants: A computer –aided approach., *IEEE Trans. Automat. Contr.,* 49, 292-295, 2004 .

[31] Singh V. Stable approximants for stable systems: A new approach, *Proc. IEEE.,* 69, 1155-1156, 1981.

[32] Singh, V., Nonuniqueness of model reduction using the Routh approach, *IEEE Trans. Autom. Control,* 24, 650-651, 1979.

[33] Shamash, Y., Truncation method of reduction: a viable alternative, *Electron. Lett.* , 17, 97-98, 1981.

[34] Shamash, Y., Model reduction usng the Routh stability criterion and the Pade approximation technique, *Int. J. Control,* 21, 475-484, 1975.

[35] Shamash, Y., Stable biased reduced-order models using the Routh method of Reduction, *Int. J. Syst. Sci..,* 11, 641-654, 1980.

[36] Tian, Z, Hoo, K., Stable Shared model for MIMO systems, Journal of Control Theory and applications, 4, 348-356, 2005.

[37] Shamash, Y., The viability of analytical methods for the reduction of multivariable systems, Proc. of IEEE, 69, 1163-1164, 1981.