# APPLYING GENETIC ALGORITHM FOR OPTIMIZATION OF SIX-BAR MECHANISM OF PROSTHETIC KNEE JOINT 

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#### Abstract

Six-bar linkages have been used in some prosthetic knees in the past years, but only a few publications have been written on the special functions of the mechanism as used in transfemoral prosthesis. This paper investigates the advantages of the mechanism as used in the prosthetic knee from the kinematic and instant inactive joints points of view. Computer simulation and an optimization method were used in the investigation. The results show that the six-bar mechanism, as compared to the four-bar mechanism, can be designed to better achieve the expected trajectory of the ankle joint in swing phase. Moreover, a six-bar linkage can be designed to have more instant inactive joints than a four-bar linkage, hence making the prosthetic knee more stable in the standing phase


Keywords: Genetic Algorithm, Six-Bar Mechanism, Prosthetic Knee

## 1. Introduction

Four-bar mechanisms have been widely used in the prosthetic knee for many years and are a subject of investigation by Zarrugh, Radcliffe, Hobson, and other scientists and researchers [1-4]. Six-bar mechanisms have been successfully used in some knee joints, such as Total Knee and 3R60 Knee produced by the Otto Bock Company; a few publications on kinematic performance of the six-bar knee mechanism have been reported [5, 6]. The general constitution of multiple-bar linkage for the prosthetic knee was outlined by Van de Veen [5], but no further investigations have been reported. Patil and Chakraborty designed a particular six-bar knee-ankle mechanism to provide coordinate motion between knee and ankle joint during walking and squatting [6]. Compared with four-bar mechanisms, six-bar mechanisms have much more design variables. Therefore, with appropriate design, six-bar mechanisms can provide advantages that are more functional. The basic concerns with kinematic of a prosthetic knee include the gait pattern (especially the trajectory of ankle joint in swing phase, which provides enough foot ground clearance), angular displacement of the shank, and stability in the standing phase. Moreover, with the intelligent knee developed in the last several years, the desire has been to adapt the prosthesis to walking speed and terrain [7, 8].
In this paper, the kinematic performance of the six-bar mechanism used in the prosthetic knee is investigated by optimization method. First, the constitutions of six-bar linkages with total revolute joints are stated. Second, the optimum design procedure is adopted for kinematic design to realize the expected trajectory (spatio-temporal curve) of the ankle joint. Moreover, because more Instant Inactive Joints can exist in six-bar mechanisms than can exist in four-bar mechanisms [9], the stability in the standing phase can be ensured even under some disturbance.

## 2. Methods

Fundamental types of six-bar mechanisms are the Watt type and Stephenson type as shown in Figure 1. Based on these two types, the knee joint has four configurations (see Figure 2(a-c)). The design parameters of these configurations are the same. The particular objective is to constitute the six-bar knee mechanism so that he shank is fixed to link 5 or 6 while the thigh is fixed to ink 1 . Otherwise, for example, if the shank is connected to link 3, then the function of the six-bar knee mechanism will be the same as that of four-bar mechanisms.


Figure 1. Basic six-bar mechanism: (a) Watt type and (b) Stephenson type


Figure 2. Configurations (a) 1, (b) 2, (c) 3, and (d) 4 of six-bar mechanism for prosthetic kn
The kinematic design aims to achieve the expected trajectory of the ankle joint and the locus of the geometric center of the knee mechanism and to ensure the stability in the extended position of the knee. Meanwhile, the dimensions of links should be within an acceptable range. The geometric center of the knee mechanism can be calculated by the equations (1), where xgc, ygc are the coordinates of the geometric center of the knee mechanism and xi, yi are the coordinates of the seven joints of the mechanism.

$$
\begin{align*}
& x_{g c}=\frac{1}{7} \sum_{i=1}^{7} x_{i}  \tag{1}\\
& y_{g c}=\frac{1}{7} \sum_{i=1}^{7} y_{i}
\end{align*}
$$

To meet the requirements just mentioned, we adopted the optimum procedure. The optimization is based on the expected relative motion of thigh and shank. As an example, taking the configurations shown in Figure 2(a) with the shank and link 5 connected (Figure 3) the optimization problem is expressed in the subsequent paragraphs.


Figure 3. Design parameters for optimization

### 2.1. Objective Function

$$
\begin{equation*}
F(x)=\binom{C 1 \sum_{i=1}^{n} \sqrt{\left|\tilde{x}_{p i}-x_{p i}\right|^{2}+\left|\tilde{y}_{p i}-y_{p i}\right|^{2}}}{+C 2 \sum_{i=1}^{n} \sqrt{\left|\tilde{x}_{k i}-x_{k i}\right|^{2}+\left|\tilde{y}_{k i}-y_{k i}\right|^{2}}} \tag{2}
\end{equation*}
$$

where $n$ is the number of selected points in a gait cycle, $n=25 ; x_{p i} y_{p i}$ are the calculated coordinates of the trajectory of the ankle joint during the optimum process; $\tilde{x}_{p i}, \tilde{y}_{p i}$ are the coordinates of the expected trajectory of the ankle joint; $\mathrm{x}_{\mathrm{k}}$, $\mathrm{y}_{\mathrm{k}}$ are the calculated coordinates of the trajectory of the geometrical center of the knee joint during the optimum process; $\tilde{x}_{k i}, \tilde{y}_{k i}$ are the coordinates of the expected trajectory of the knee joint; and $C 1, C 2$ are the weight factors and $C 1+C 2=0.9+0.1=1 . C 1$ is much larger than $C 2$ here, because emphasis is put on the locus of the ankle joint.
How to choose the expected trajectory is a problem needed to make further studies. What we used here is based on the gait analysis of the sound side of a transfemoral prosthesis user while walking at a normal speed ( $1.2 \mathrm{~m} / \mathrm{s}$ ), because we hope to increase the level of symmetry of gait parameters.

By defining a frame $x O y$ fixed on the thigh, shown in Figure 3, the design parameters can be expressed as a vector $X$ such that

$$
\begin{equation*}
X=\left[x_{1}, x_{2}, x_{3}, \ldots, x_{16}\right]=\left[l_{1}, l_{2}, l_{3}, \ldots, l_{14}, \theta, \beta\right] \tag{3}
\end{equation*}
$$

The variables in the vector are as indicated in Figure 3. There are, in total, 16 elements, including 14 dimensions of links and two angular positions of the thigh and shank $\theta$ and $\beta$, respectively. The coordinates of the points A, B, C, D, E, F, G, I, J, and P in the frame are expressed as functions of the design parameters in the following equations:

$$
\begin{gather*}
x_{I}=l_{12} \cos (\theta)  \tag{4}\\
y_{I}=l_{12} \sin (\theta)  \tag{5}\\
x_{A}=x_{I}-l_{12} \sin (\theta)  \tag{6}\\
y_{A}=x_{I}+l_{12} \cos (\theta)  \tag{7}\\
x_{B}=x_{I}+\left(l_{1}-l_{11}\right) \sin (\theta)  \tag{8}\\
y_{B}=x_{I}-\left(l_{1}-l_{11}\right) \cos (\theta)  \tag{9}\\
x_{C}=F_{x}\left(x_{A}, y_{A}, x_{B}, y_{B}, \beta, l_{3}\right)  \tag{10}\\
y_{C}=F_{y}\left(x_{A}, y_{A}, x_{B}, y_{B}, \beta, l_{3}\right)  \tag{11}\\
\eta_{d}=\cos ^{-1}\left(\frac{l_{5}^{2}+l_{1}^{2}+l_{3}^{2}-l_{4}^{2}-2 l_{1} l_{3} \cos \beta}{\left.2 l_{5} \sqrt{l_{1}^{2}+l_{3}^{2}-2 l_{1} l_{3} \cos \beta}\right)}\right.  \tag{12}\\
x_{D}=F_{x}\left(x_{C}, y_{C}, x_{B}, y_{B}, \eta_{D}, l_{5}\right)  \tag{13}\\
y_{D}=F_{y}\left(x_{C}, y_{C}, x_{B}, y_{B}, \eta_{D}, l_{5}\right)  \tag{14}\\
\eta_{E}=\cos ^{-1}\left(\frac{l_{2}^{2}+l_{3}^{2}-l_{1}^{2}}{2 l_{2} l_{3}}\right)  \tag{15}\\
\eta_{G}=\cos ^{-1}\left(\frac{l_{5}^{2}+l_{8}^{2}-l_{6}^{2}}{2 l_{5} l_{8}}\right)  \tag{16}\\
y_{E}=F_{y}\left(x_{A}, y_{A}, x_{C}, y_{C}, \eta_{E}, l_{2}\right)  \tag{18}\\
x_{G}=F_{x}\left(x_{C}, y_{C}, x_{D}, y_{D}, \eta_{G}, l_{8}\right)  \tag{19}\\
y_{G}=F_{y}\left(x_{C}, y_{C}, x_{D}, y_{D}, \eta_{G}, l_{8}\right)  \tag{20}\\
\eta_{G}=\cos ^{-1}\left(\frac{l_{5}^{2}+l_{8}^{2}-l_{6}^{2}}{2 l_{5} l_{8}}\right)  \tag{21}\\
x_{F}=F_{x}\left(x_{E}, y_{E}, x_{G}, y_{G}, \eta_{F}, l_{9}\right) \tag{22}
\end{gather*}
$$

$$
\begin{gather*}
y_{F}=F_{y}\left(x_{E}, y_{E}, x_{G}, y_{G}, \eta_{F}, l_{9}\right)  \tag{23}\\
y_{J}=x_{F}+\frac{l_{13}}{l_{10}}\left(y_{G}-y_{F}\right)  \tag{25}\\
x_{P}=x_{J}+l_{14} \cos \left(\tan ^{-1}\left(\frac{y_{G}-y_{F}}{x_{G}-x_{F}}\right)+\alpha\right)  \tag{26}\\
y_{P}=y_{J}+l_{14} \sin \left(\tan ^{-1}\left(\frac{y_{G}-y_{F}}{x_{G}-x_{F}}\right)+\alpha\right) \tag{27}
\end{gather*}
$$

where "Funxy" is defined in Equation (16) as:

$$
\begin{align*}
& F_{x}\left(x_{1}, y_{1}, x_{2}, y_{2}, \eta, l\right)=x_{1}+\frac{\left(x_{2}-x_{1}\right) \cos \eta+\left(y_{2}-y_{1}\right) \sin \eta}{\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}} l  \tag{28}\\
& F_{y}\left(x_{1}, y_{1}, x_{2}, y_{2}, \eta, l\right)=y_{1}+\frac{\left(x_{1}-x_{2}\right) \sin \eta+\left(y_{2}-y_{1}\right) \cos \eta}{\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}} l \tag{29}
\end{align*}
$$

where $l$ is the distance between points $R$ and $S$ and $\eta$ is the angle between the two lines $S-R$ and T-R. Equations (28) and (29) is used to calculate the coordinates of an arbitrary point $\mathrm{R}(x, y)$ based on coordinates of the other two known points $\mathrm{S}(\mathrm{x} 1, \mathrm{y} 1)$ and $T(x 2, y 2)$.

### 2.2. Constraints

Self-locking condition in the extended knee positions given by

$$
\begin{align*}
& \left(y_{I}-y_{H}\right)\left(x_{P}-x_{J}\right)=\left(y_{P}-y_{J}\right)\left(x_{I}-x_{H}\right)  \tag{30}\\
& \left(y_{E}-y_{F}\right)\left(x_{F}-x_{G}\right)=\left(y_{F}-y_{G}\right)\left(x_{E}-x_{F}\right) \tag{31}
\end{align*}
$$

Dimensional limitation of links is $l_{\text {imin }}<l_{i}<l_{\text {imax }}(i=1,2, \ldots, 14)$ where $l i$ is the same as defined in Equation (3) and limin and limax are the dimension limitation to the length of each bar. The minimum and maximum dimension of $\mathrm{l}_{\mathrm{i}}$ is displayed in Table 1.

Table 1. The limited values of the design variables (mm)

|  | $l_{1}$ | $l_{2}$ | $l_{3}$ | $l_{4}$ | $l_{5}$ | $l_{6}$ | $l_{7}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $l_{i \max }$ | 35 | 90 | 60 | 55 | 40 | 35 | 50 |


| $l_{i \min }$ | 5 | 50 | 30 | 20 | 15 | 10 | 25 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | $l_{8}$ | $l_{9}$ | $l_{10}$ | $l_{11}$ | $l_{12}$ | $l_{13}$ | $l_{14}$ |
| $l_{i \max }$ | 30 | 25 | 50 | 52 | 55 | 25 | 45 |
| $l_{i \min }$ | 12 | 10 | 20 | 20 | 18 | 8 | 15 |

## 3. Genetic Algorithm

The discovery of genetic algorithm (GA) was dated to the 1960s by Holland and further described by Goldberg [10]. The GAs have been applied successfully to problems in many fields such as optimization design, fuzzy logic control, neural networks, expert systems, scheduling, and many others [11]. For a specific problem, the GA codes a solution as an individual chromosome. It then defines an initial population of those individuals that represent a part of the solution space of the problem. The search space therefore, is defined as the solution space in which each feasible solution is represented by a distinct chromosome. Before the search starts, a set of chromosomes is randomly chosen from the search space to form the initial population. Next, through computations the individuals are selected in a competitive manner, based on their fitness as measured by a specific objective function.
The genetic search operators such as selection, mutation and crossover are then applied one after another to obtain a new generation of chromosomes in which the expected quality over all the chromosomes is better than that of the previous generation. This process is repeated until the termination criterion is met, and the best chromosome of the last generation is reported as the final solution.

## 4. Results and Discussion

The After the optimization method of genetic algorithm Function is applied, the design parameters are obtained as:

$$
\begin{aligned}
& X=\left[x_{1}, x_{2}, \ldots, x_{16}\right]=\left[l_{1}, l_{2}, \ldots, l_{14}, \theta, \alpha\right] \\
& =\left[25,71,40.6,38,28.2,21.8,32,18.9,6.85,35,32,38.5,14,26.9,85^{0}, 10^{0}\right]
\end{aligned}
$$

Then the six-bar knee mechanism was designed, and the trajectory generated by the mechanism can be obtained by the kinematic analysis being applied. The comparison of the generated trajectory of the ankle joint with expected ones is shown in Figure 4. The mean square errors for ankle and knee are Errankle $=1.96 \%$ and Errknee $=11.43 \%$, respectively. The comparison of the trajectory of the ankle joint in swing phase of the six-bar linkage knee is also made and given in Figure 5. The dimensions of the four bar linkage were designed with the use of the same procedure as that used for the six-bar linkage knee. The mean square error of ankle joint trajectories of the four-bar mechanism is 6.71 percent, while that of the six-bar mechanism is 1.92 percent.


Figure 4. Trajectory of ankle joint by optimal six-bar linkage


Figure 5. Trajectory of ankle joint by optimal six-bar in swing phase

## 5. Conclusion

The six-bar prosthetic knee mechanism has been investigated from kinematic point of view in this paper. The performance of the knee mechanism is shown in the following aspects:

- The trajectory of the ankle joint and the movement of the shank can be much closer to that expected than to that of the four-bar linkage if one were to apply the optimum design procedure proposed in this paper.
- Since more IIJs exist in a six-bar linkage than in a four-bar linkage, a six-bar is more capable of maintaining stability in standing phase under interference.
The mean square error of ankle joint trajectories of the four-bar mechanism is 6.71 percent, while that of the six-bar mechanism is 1.92 percent.


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