Nonlinear Buckling Analysis of Cold-Formed Channel Sections

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Abstract

The main objective of this paper is to provide an efficient and accurate finite element model to understand the behavior of cold-formed steel channel columns. The effects of initial local and overall geometric imperfections have been taken into consideration in the analysis. Failure loads and buckling modes as well as load-shortening curves of plain channel columns were investigated in this study. The nonlinear finite element model was verified against experimental results. The finite element analysis was performed on plain channels compressed between pinned ends over different column lengths, and column curves were obtained. An extensive parametric study was carried out using the finite element model to study the load eccentricity on the strength and behavior of channel columns. The column strengths predicted from the finite element model were compared with the design strengths calculated using the European Code, EN 1993-1-3 Eurocode 3: Design of steel structures - Part 1-3: General rules - Supplementary rules for cold-formed members and sheeting, for cold-formed steel structures.

Key words
Cold-Formed Steel Channels, Buckling, Nonlinear Finite Element Analysis, EN 1993-1-3

1. INTRODUCTION

Finite element analysis (FEA) of cold-formed structures plays an increasingly important role in engineering practice, as it is relatively inexpensive and time efficient compared with physical experiments, especially when a parametric study of cross-section geometries is involved. Furthermore, it is difficult to investigate the effects of geometric imperfections and residual stresses of structural members experimentally. Therefore, FEA is more economical than physical experiments, provided that the finite element model (FEM) is accurate. Hence, it is necessary to verify the FEM with experimental results. In general, FEA is a powerful tool in predicting the ultimate loads and complex failure modes of cold-formed structural members. In addition, local and overall geometric imperfections, residual stresses and material non-linearity can be included in the FEM.

The purpose of the paper is to develop an accurate FEM to investigate the strengths of pin-ended cold-formed plain channel columns. The finite element analysis program ABAQUS 6.13 [1] was used for the numerical investigation. The FEM was verified against the cold-formed channel column tests conducted by Young and Rasmussen [2]. The FEM included geometric and material non-linearities.

2. EXPERIMENTAL TEST

The test program described in Young and Rasmussen [2] provided experimental ultimate loads and failure modes for cold-formed plain channel columns compressed between pinned ends. The test specimens were brake-pressed from high strength zinc-coated grade G450 structural steel sheets having nominal yield stress of 450 MPa and specified according to the Australian Standard AS 1397 [3]. The test program comprised two series of plain channels. The channel sections had a nominal thickness of 1.5 mm and a nominal width of the web of 96 mm. The nominal flange width was either 36 or 48 mm and was the only variable in the cross-section geometry. Accordingly, the two test series were labeled P36 and P48 where...
“P” refers to “plain” channels. The average values of measured cross-section dimensions of the pin-ended test specimens are shown in Table 1 using the nomenclature defined in Figure 1. The specimens were tested at various column lengths ranging from 280 to 1565 mm. The measured cross-section dimensions of each specimen are detailed in Young and Rasmussen [2].

Table 1. Average measured specimen dimensions and material properties

<table>
<thead>
<tr>
<th>Test series</th>
<th>Specimen dimensions</th>
<th>Material properties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bf (mm)</td>
<td>Bw (mm)</td>
</tr>
<tr>
<td>P36</td>
<td>36.8</td>
<td>96.9</td>
</tr>
<tr>
<td>P48</td>
<td>49.6</td>
<td>95.4</td>
</tr>
</tbody>
</table>

Figure 1. Definition of symbols

The material properties determined from coupon tests are also summarized in Table 1. The table contains the measured static 0.2% tensile proof stress (σ0.2) and the static ultimate tensile strength (σu) as well as the Young’s modulus (E) and the elongation after fracture (εu) based on a gauge length of 50 mm. The coupons were taken from the center of the web plate in the longitudinal direction of the finished specimen. The coupon dimensions conformed to the Australian Standard AS1391 [4] for the tensile testing of metals using 12.5 mm wide coupons of gauge length 50 mm. The coupons were tested in an Instron TT-KM 250 kN capacity displacement controlled testing machine using friction grips to apply loading at a constant speed of 1 mm/min. The static load was obtained by pausing the applied straining for one minute near the 0.2% proof stress and the ultimate tensile strength. This allowed the stress relaxation associated with plastic straining to take place. The stress–strain curves obtained from the coupon tests are detailed in Young and Rasmussen [2].

Residual stress measurements were conducted on a companion series of tests of lipped channel specimens by Young and Rasmussen [2, 5]. The plain and lipped channel specimens were cut from the same batch of structural steel sheets and break pressed from the same machine. The membrane and the flexural residual stresses were found to be less than 3% and 7% of the measured 0.2% tensile proof stress, respectively. Hence, the residual stresses were deemed negligible compared with the 0.2% tensile proof stress. Local and overall geometric imperfections were measured prior to testing for the tested columns. The measured maximum local imperfections were found to be of the order of the plate thickness at the tip of the flanges for the two test series. For the pin-ended specimens, the maximum overall minor axis flexural imperfections at mid-length were 1/2200 and 1/1500 of the specimen length for Series P36 and P48, respectively. The measured local and overall geometric imperfection profiles are detailed in Young and Rasmussen [2, 6].

A 250 kN servo-controlled hydraulic actuator was used to apply compressive axial force to the specimen. The tests were controlled by incrementing the shortening of the specimen. This allowed the tests to be continued into the post-ultimate range. Readings of the applied load were taken approximately 1 min after applying an increment of compression, hence allowing the stress relaxation associated with plastic straining to take place. Consequently, the loads recorded were considered to be static loads. The pin-ended bearings were designed to allow rotations about the minor axis while restraining major axis rotations as well as twist rotations and warping. Details of the test rig are given in Young and Rasmussen [7]. The experimental ultimate loads (Pult) of the test specimens are shown in Table 2. The test specimens were labeled such that the test series, type of boundary conditions and specimen length could be identified from the label. For example, the label “P36P0815” defines the specimen belongs to the test Series P36, the fourth letter “P” indicates that the specimen is pin-ended, and the last four digits are the specimen length of 865 mm.

Table 2. Geometric properties and failure loads of U-section members tested by Young and Rasmussen (1998).

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Bf (mm)</th>
<th>Bw (mm)</th>
<th>t (mm)</th>
<th>ri (mm)</th>
<th>L (mm)</th>
<th>A (mm²)</th>
<th>Pult (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P36P0280</td>
<td>36.9</td>
<td>96.6</td>
<td>1.51</td>
<td>0.85</td>
<td>280</td>
<td>247</td>
<td>55.2</td>
</tr>
<tr>
<td>P36P0315</td>
<td>37.0</td>
<td>96.8</td>
<td>1.50</td>
<td>0.85</td>
<td>315</td>
<td>245</td>
<td>52.1</td>
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<tr>
<td>P36P0815</td>
<td>36.8</td>
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<td>0.85</td>
<td>1315</td>
<td>245</td>
<td>27.0</td>
</tr>
<tr>
<td>P48P0300</td>
<td>49.6</td>
<td>94.8</td>
<td>1.51</td>
<td>1.46</td>
<td>300</td>
<td>279</td>
<td>45.2</td>
</tr>
</tbody>
</table>
3. FINITE ELEMENT MODELING

3.1. General
In this study, the finite element program, ABAQUS 6.13 [1] was used in the analysis of cold-formed plain channel columns tested by Young and Rasmussen [2]. The model used the nominal sizes, initial local and overall geometric imperfections and material properties. Finite element analysis for buckling requires two types of analyses. The first is known as eigenvalue analysis that estimates the buckling modes and loads. Such an analysis is a linear elastic analysis performed using the (*BUCKLE) procedure available in the ABAQUS library with the live load applied within the step. The buckling analysis provides the factor by which the live load must be multiplied to reach the buckling load. For practical purposes, only the lowest buckling mode predicted from the eigenvalue analysis is used. The second is called load-displacement nonlinear analysis and follows the eigenvalue prediction. It is necessary to consider whether the postbuckling response is stable or unstable.

3.2. Finite Element Type and Mesh
It is mentioned in the ABAQUS manual that the four-noded doubly curved shell element with reduced integration S4R is suitable for complex buckling behavior [1, 8, 9]. The S4R element has six degrees of freedom per node and provides accurate solutions to most applications. The element also accounts for finite strain and is suitable for large strain analysis. Since buckling of plain channel columns is very sensitive to large strains, the S4R element was used in this study to ensure the accuracy of the results. In order to choose the finite element mesh that provides accurate results with minimum computational time, convergence studies were conducted. It is found that a 10 mm x 10 mm (length by width) ratio provides adequate accuracy in modeling the channel columns.

3.3. Boundary Conditions and Load Application
Following the experimental tests, the ends of the columns were free to rotate and fixed to translate in any direction except for the displacement at the loaded end in the direction of the applied load. The nodes other than the two ends were free to translate and rotate in any direction. The load was applied in increments using the modified RIKS method available in the ABAQUS library. The RIKS method is generally used to predict unstable and nonlinear collapse of a structure such as postbuckling analysis. It uses the load magnitude as an additional unknown and solves simultaneously for loads and displacements. The load was applied as static uniform loads at each node of the loaded end which is identical to the experimental investigation. The nonlinear geometry parameter (*NLGEOM) was included to deal with the large displacement analysis.

3.4. Material Modeling
The material behavior provided by ABAQUS allows for a multilinear stress-strain curve to be used. The first part of the multilinear curve represents the elastic part up to the proportional limit stress with measured Young’s modulus and Poisson’s ratio equal to 0.3. Since the analysis of postbuckling involves large in-elastic strains, the nominal (engineering) static stress-strain curve was converted to a true stress and logarithmic plastic strain curve. The true stress ($\sigma_{true}$) and plastic true strain ($\varepsilon_{pl, true}$) were calculated using Equations 1. and 2.

$$\sigma_{true} = \sigma (1 + \varepsilon)$$  \hspace{1cm} (1)

$$\varepsilon_{true}^{pl} = \ln (1 + \varepsilon) - \sigma_{true} / E$$  \hspace{1cm} (2)

where $E$ = Young’s modulus, and $\sigma$ and $\varepsilon$ = measured nominal (engineering) stress and strain based on the original cross-section area of the coupon specimens as detailed in Young and Rasmussen [2]. The engineering stresses and strains were obtained from tensile coupon tests. The coupon specimens were loaded at a constant speed of 1 mm/min.

Figure 2 shows the measured engineering and true stress–strain curves for the test Series P36. The incremental plasticity model required the portion of the true stress–strain curve from the point corresponding to the last value of the linear range of the engineering stress–strain curve to the ultimate point of the true stress–strain curve, as shown in Figure 2. The Poisson’s ratio was taken as 0.3 and the measured Young’s modulus as shown in Table 1 was used in the FEM.
3.5. Modeling of Initial Local and Overall Geometric Imperfections

The geometric imperfections were included in the FEM by using a linear perturbation analysis. The main purpose of the perturbation analysis was to establish probable buckling modes (eigenmode) of the column. The eigenmode was then scaled by a factor (scale factor) to obtain a perturbed mesh of the column for the non-linear analysis. Eigenmode 1 was used in the FEM, in which local or overall buckling mode was predicted from the analysis.

4. RESULTS AND DISCUSSIONS

4.1. Comparison of experimental results with finite element analysis results

In the verification of the finite element model, a total of 8 cold-formed steel plain channel columns were analyzed. The incremental plasticity models obtained from the true stress–strain curves were used in the FEM for the corresponding test series. A scale factor of 25% of the plate thickness was used in modeling the geometric imperfections of the columns. A comparison between the experimental results and the results of the finite element model is carried out. The main objective of this comparison is to verify and check the accuracy of the finite element model. The ultimate loads (P_{fem}) predicted by the FEA are compared with the experimental ultimate loads (P_{test}) as shown in Table 3 for Series P36 and P48, respectively. The mean values of the P_{test}/P_{fem} ratio are 1.014 and 1.046 with the corresponding coefficients of variation (COV) of 0.0016 and 0.0096 for Series P36 and P48, respectively. Generally, good agreement has been achieved for most of the columns. Three modes of failure were reported by Young and Rasmussen [7] and verified by the finite element model. The failure modes are the local buckling, flexural buckling and flexural-torsional buckling.

Figure 3 plotted the relationship between the ultimate load and the column effective length L_{eff}=L for channels reported by Young and Rasmussen [2] where L actual column length. The column curves show the experimental ultimate loads together with that obtained by the finite element method. It can be seen that good agreement has been achieved between both results for most of the columns.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>B_f (mm)</th>
<th>B_w (mm)</th>
<th>t (mm)</th>
<th>r_i (mm)</th>
<th>L (mm)</th>
<th>A (mm^2)</th>
<th>P_{test} (kN)</th>
<th>P_{fem} (kN)</th>
<th>P_{test}/P_{fem}</th>
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<tr>
<td>P36P0280</td>
<td>36.9</td>
<td>96.6</td>
<td>1.51</td>
<td>0.85</td>
<td>280</td>
<td>247</td>
<td>55.2</td>
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<td>96.8</td>
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<td>0.85</td>
<td>315</td>
<td>245</td>
<td>52.1</td>
<td>53.2</td>
<td>0.98</td>
</tr>
<tr>
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<td>36.8</td>
<td>97.5</td>
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<td>815</td>
<td>249</td>
<td>40.9</td>
<td>38.3</td>
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<td>0.85</td>
<td>1315</td>
<td>245</td>
<td>27.0</td>
<td>27.3</td>
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<td>0.0016</td>
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<td>45.2</td>
<td>47.0</td>
<td>0.96</td>
</tr>
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<td>49.8</td>
<td>94.5</td>
<td>1.53</td>
<td>1.48</td>
<td>565</td>
<td>283</td>
<td>38.6</td>
<td>39.5</td>
<td>0.98</td>
</tr>
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<td>P48P1065</td>
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<td>1.52</td>
<td>1.48</td>
<td>1065</td>
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<td>31.6</td>
<td>1.07</td>
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<td>P48P1565</td>
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<td>1.47</td>
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<td>1.17</td>
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<td>1.046</td>
<td>1.046</td>
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<tr>
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<td>0.0096</td>
<td>0.0096</td>
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</tbody>
</table>
Figure 3. Comparison of experimental results with FEM results.

Figure 4 shows the ultimate load against the axial shortening behavior of column P48P0565 that has a length of 565 mm. The curve has been predicted by the finite element model and compared with the test results. The experimental ultimate load was 38.6 kN compared with 39.5 kN predicted by the finite element analysis. The failure mode of the test specimen P48P0565 was reported as a flexural buckling. The same failure mode has been confirmed numerically by the model as shown in Figure 5.

Figure 4. Load-axial shortening curve for P48P0565.

Figure 5. Failure mode of column P48P0565.

4.2. Design Rules

EN1993-1-3 [10] represents the unified European Code for cold-formed steel design, and contains specific provisions for structural applications using cold-formed steel products made from coated or uncoated thin gauge hot or cold-rolled sheet
and strip. In EN1993-1-3, cross sections subject to combined axial compression \( N_{Ed} \) and bending moments \( M_{y,Ed} \) and \( M_{z,Ed} \) should satisfy the criterion:

\[
\frac{N_{Ed}}{N_{c,Rd}} + \frac{M_{y,Ed} + \Delta M_{y,Ed}}{M_{cy,Rd,com}} + \frac{M_{z,Ed} + \Delta M_{z,Ed}}{M_{cz,Rd,com}} \leq 1.0 \tag{3}
\]

in which

- \( N_{Ed} \): design value of the compression force,
- \( N_{c,Rd} \): design compression resistance of a cross section,
- \( M_{y,Ed} \): applied bending moment about the major axis,
- \( M_{z,Ed} \): applied bending moment about the minor axis,
- \( M_{cy,Rd,com} \): moment resistances for the maximum compressive stress in a cross section that is subject only to moment about the y-y axis
- \( M_{cz,Rd,com} \): moment resistances for the maximum compressive stress in a cross section that is subject only to moment about the z-z axis

The additional moments \( \Delta M_{y,Ed} \) and \( \Delta M_{z,Ed} \) due to shifts of the effective centroidal axes should be taken as:

\[
\Delta M_{y,Ed} = N_{Ed} \cdot e_{Ny} \tag{4}
\]
\[
\Delta M_{z,Ed} = N_{Ed} \cdot e_{Nz} \tag{5}
\]

in which \( e_{Ny} \) and \( e_{Nz} \) are the shifts of y-y and z-z centroidal axis of the effective cross section relative to the gross cross section.

If \( M_{yz,Rd,ten} \leq M_{cy,Rd,com} \) or \( M_{xz,Rd,ten} \leq M_{cz,Rd,com} \) the following criterion should also be satisfied:

\[
\frac{M_{y,Ed} + \Delta M_{y,Ed}}{M_{cy,Rd,ten}} + \frac{M_{z,Ed} + \Delta M_{z,Ed}}{M_{cz,Rd,ten}} \leq \frac{N_{Ed}}{N_{c,Rd}} \tag{6}
\]

in which

- \( M_{yz,Rd,ten} \): design moment resistance of a cross section for maximum tensile stress if subject only to moment about the y-y axis,
- \( M_{xz,Rd,ten} \): design moment resistance of a cross section for maximum tensile stress if subject only to moment about the z-z axis.

### 4.3. Parametric Study and Discussions

It is shown that the FE model closely predicted the behavior of plain U-section columns compared with the test results. Hence parametric studies were carried out to study the effects of load eccentricity on the strength and behavior of U-section columns. A total of 8 plain U-section columns was performed in the parametric study. Two series of columns P36 and P48 were studied. All U-sections had same geometric properties as the test specimens. The load eccentricity with respect to the major principal axis, \( e_3 \), were selected as 10.27 and 11.97 for series P36 and P48, respectively. P36 series of columns consists of four column lengths of 280, 315, 815, and 1315 mm, whereas P48 series of columns consists of four column lengths of 300, 565, 1065, and 1565 mm. A scale factor of 25% of the plate thickness was used in modeling the geometric imperfections of the columns. The residual stresses were not considered since its effect on the column capacity and load-shortening behavior is negligible. The measured stress strain curves of series P36 and P48 were used in all parametric studies. A summary of the parametric study results is presented in Table 4. Slenderness (\( \lambda \)) and the ultimate loads (\( P_{Ult-e} \)) of the U-sections are given in Table 4.

The results of the parametric study are compared with the design strengths obtained using the European Code, EN1993-1-3. It can be seen that the EN1993-1-3 design strengths (\( P_{En-1993-1,3} \)) are generally quite conservative for U-section columns as shown in Table 4.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>L (mm)</th>
<th>( \lambda )</th>
<th>( e_3 ) (mm)</th>
<th>( P_{Ult-e} ) (kN)</th>
<th>( P_{En-1993-1,3} ) (kN)</th>
<th>( P_{Ult-e} / P_{En-1993-1,3} )</th>
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<tbody>
<tr>
<td>P36P0280</td>
<td>280</td>
<td>42</td>
<td>10.27</td>
<td>25.50</td>
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<td>2.11</td>
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<tr>
<td>P36P0315</td>
<td>315</td>
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<td>10.27</td>
<td>25.20</td>
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<td>2.08</td>
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<td>815</td>
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<td>10.27</td>
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<td>25.20</td>
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<td>11.97</td>
<td>24.90</td>
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<td>1.89</td>
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</table>
5. CONCLUSIONS

This paper provides an efficient nonlinear finite element model for understanding the behavior of eccentrically loaded single U-sections. Theoretical buckling and the experimental failure loads of pin ended, concentrically loaded U-sections were predicted by eigenvalue and load-deformation analyses of various models developed in ABAQUS 6.13. The U-sections were modeled by shell elements considering geometrically and materially nonlinear behavior. Initial imperfections, end support conditions, geometry and material property variation of the U-sections were included differently in each model. The load-carrying capacity of eccentrically loaded single U-sections are investigated by performing an extensive parametric study obtaining the most realistic estimations. The results of the parametric study are compared with the design strengths obtained using the European Code, EN1993-1-3. It is seen that the EN1993-1-3 design strengths are quite conservative for U-section columns.

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BIOGRAPHY

Assist. Prof. Dr. Mustafa DURMAZ was born on 1st September, 1977 in Gebze, Kocaeli. He studied his elementary studies at the Esrefbey Primary School, his secondary school studies at the Sarkuysan Secondary School and completed his high school at the Gebze Technical and Industrial Vocational High School. Mustafa DURMAZ graduated from the Department of Civil Engineering in 2000 (B.Sc.) of the Karadeniz Technical University (KTU), and obtained his M.Sc. Degree from the Department of Civil Engineering in 2003. He obtained his Ph.D. in 2011 at the Department of Civil Engineering of the KTU. He started his academic career as Research Assistant in Department of Civil Engineering of KTU Engineering Faculty in 2000. In the Department of Civil Engineering of Gümüşhane University Engineering Faculty, he was promoted to the rank of Assistant Professor in 2012. He has been serving as Vice Head of Department at the Department of Civil Engineering since 2013. He is married and has two children.