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Symmetric Bi-Derivation on Hyperrings

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Abstract: In this paper we introduce symmetric bi-derivations in Krasner hyperrings and give its some basic properties.

Keywords: Krasner hyperrings, symmetric bi-derivation

Hiperhalkalarda Simetrik İkili Türevler

Özet. Bu çalışmada Krasner hiperhalkalarda simetrik ikili türevi tanımlayarak bazı temel özelliklerini incelemeye çalıştık.

Anahtar Kelimeler: Krasner hiperhalka, simetrik ikili türev

1. INTRODUCTION

The theory of hyperstructures was introduced in 1934 by Marty at the 8th congress Scandinavian Mathematicians. Then several researchers have worked on this new field and developed it. Mittas introduced the notion of canonical hypergroups. Corsini studied the Canonical Hypergroups, Feebly Canonical Hypergroups, Quasi-Canonical Hypergroups. Krasner introduced the notion of hyperrings and hyperfields. G. G. Massouros introduced the theory of hypercompositional structures into the theory of automata. Asokkumar studied the idempotent elements of Krasner hyperrings. Babaei et al. studied R-parts in hyperrings. The notion of derivations of rings plays a significant role in algebra. The study of derivations in rings got interested after Posner [14], who gave striking results on derivations of prime rings. Then the notion of derivations has been developed by many authors in various directions like Jordan derivation, generalized derivation in rings and near-rings. In 1980, Gy. Maksa [9] introduced the concept of a symmetric biderivation on a ring R (see also [10],where an example can be seen). It was shown in [10] that symmetric biderivations are related to general solution of some functional equations. Some results on a symmetric biderivation in prime and semiprime rings can be found in [16] and [17]. From the motivation of derivations, Vougiouklis introduced a hyperoperation called theta hyperoperation and studied H_ϑ-structures in [15]. Jan Chvalina et al. [8], introduced a hyperoperation * on a differential ring R so that $(R,*)$ is a hypergroup. In [2], the author introduced derivations in Krasner hyperrings and give examples. Also he derived some basic properties of derivations.

In this paper, we introduce symmetric bi-derivations in Krasner hyperrings and give its some basic properties.

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2. PRELIMINARIES

This section explains some basic definitions that have been used in the sequel. A hyperoperation * on a non-empty set H is a mapping of $H \times H$ into the family of non-empty subsets of H (i. e. $x * y \subseteq H$ for every $x, y \in H$). In the sense of Marty, a hypergroup $(H,*)$ is a non-empty set H equipped with a hyperoperation * which satisfies the following axioms:

(i) $x * (y * z) = (x * y) * z$ for every $x, y, z \in H$ (the associative axiom)
(ii) $x * H = H * x = H$ for every $x \in H$ (the reproductive axiom).

The comprehensive review of the theory of hypergroups appears in [4].

Definition 1 A non-empty subset R with a hyperaddition $+$ and a multiplication $\cdot$ is called an additive hyperring or Krasner hyperring if it satisfies the following:

(1) $(R, +)$ is a canonical hypergroup, i. e.,
(i) for every $x, y, z \in R$, $(x + y) + z = x + (y + z)$,
(ii)for every $x, y \in R$, $x + y = y + x$,
(iii)there exists $0 \in R$ such that $0 + x = x$ for all $x \in R$,
(iv)for every $x \in R$ there exists an unique-element denoted by $-x \in R$ such that $0 \in x + (-x)$,
(v)for every $x, y, z \in R$, $z \in x + y$ implies $y \in -x + z$ and $x \in z - y$.
(2) $(R, .)$ is a semigroup having 0 as a bilaterally absorbing elemet, i. e.,
(i)for every $x, y, z \in R$, $(x.y).z = x.(y.z)$,
(ii)$x.0 = 0.x = 0$ for all $x \in R$,
(3) The multiplication $\cdot$ is distributive with respect to the hyperoperation +, i. e., for every $x, y, z \in R$, $x.(y + z) = x.y + x.z$ and $(x + y).z = x.z + y.z$.

A non-empty subset I of a canonical hypergroup R is called a canonical subhypergroup of R if I itself is a canonical hypergroup under the same hyperoperation as that of R. Equivalently, a non-empty subset I of a canonical hypergroup R is a canonical subhypergroup of R if for every $x, y \in I$, $x - y \subseteq I$. Here after we denote xy instead of $x.y$. Moreover, for $A, B \subseteq R$ and $x \in R$, by $A + B$ we mean the set $\cup_{a\in A, b\in B}(a + b)$ and $AB = \cup_{a\in A, b\in B}(ab)$, $A + x = A + \{x\}$, $x + B = \{x\} + B$ and also $-A = \{-a: a \in A\}$.

The following elementary facts in a hyperring easily follow from axioms: (i) $-(-a) = a$ for every $a \in R$; (ii) 0 is the unique element such that for every $a \in R$, there is an element $-a \in R$ with the property $0 \in a + (-a)$ and $-0 = 0$; (iii) $-(a + b) = -a - b$ for all $a, b \in R$; (iv) $-(ab) = (-a)b = a(-b)$ for all $a, b \in R$.

In a hyperring R, if there exists an element $1 \in R$, such that $1a = a1 = a$ for every $a \in R$, then the element 1 is the called the identity element of the hyperring R. In fact, the element 1 is unique. Further, if $ab = ba$ for every $a, b \in R$ then the hyperring R is called a commutative hyperring. Throughout this paper, by a hyperring we mean the Krasner hyperring.
Example 1 The set $R = \{0,1\}$ with the following hyperoperations is a hyperring.

+	0	1
0	$\{0\}$	$\{1\}$
1	$\{1\}$	$\{0,1\}$

.	0	1
0	$\{0\}$	$\{0\}$
1	$\{0\}$	$\{1\}$

Definition 2 Let R be a hyperring. A non-empty subset S of R is called a subhyperring of R if $x - y \subseteq S$ and $xy \in S$ for all $x, y \in S$.

Definition 3 Let R be a hyperring and I be a non-empty subset of R. I is called a left (resp. right) hyperideal of R if (i) $(I, +)$ is a canonical subhypergroup of R, i. e., for every $x, y \in I$, $x - y \subseteq I$ and (ii) for every $a \in I$, $r \in R$, $ra \subseteq I$ (resp. $ar \subseteq I$). A hyperideal of R is one which is a left as well as a right hyperideal of R.

Definition 4 A hyperring R is said to be a prime hyperring if $aRb = 0$ for $a, b \in R$ implies either $a = 0$ or $b = 0$.

Definition 5 A hyperring R is said to be a reduced hyperring if it has no nilpotent elements. That is, if $x^n = 0$ for all $x \in R$ and a natural number n, then $x = 0$.

Definition 6 A hyperring R is said to be 2-torsion free if $0 \in x + x$ for $x \in R$ implies $x = 0$.

3. SYMMETRIC BI-DERIVATION OF HYPERRINGS AND EXAMPLES

In this section we define symmetric bi-derivation and strong symmetric bi-derivation of hyperrings and give examples.

Definition 7 Let R be a hyperring. A mapping $D: R \times R \to R$ is called symmetric if $D(x, y) = D(y, x)$ for all $x, y \in R$.

Definition 8 Let R be a hyperring. A map $D: R \times R \to R$ is said to be a symmetric bi-derivation of R if D satisfies: (i) $D(x + z, y) \subseteq D(x, y) + D(z, y)$ and (ii) $D(xz, y) \in D(x, y)z + xD(z, y)$ for all $x, y, z \in R$.

The hyperring R equipped with a symmetric bi-derivation D is called a D-differential hyperring. If the map D is such that $D(x + z, y) = D(x, y) + D(z, y)$ for all $x, y, z \in R$ and satisfies the condition (ii), then D is called a strong symmetric bi-derivation of R. In this case, the hyperring is called strongly D-differential hyperring.

Proposition 1 Let R be a hyperring and $D: R \times R \to R$ be a symmetric bi-derivation of R. Then
(i) $D(a, 0) = 0$, $\forall a \in R$
(ii) $D(-a, b) = -D(a, b)$, $\forall a, b \in R$
(iii) if 1 is the identity element of R, then $D(1, a) \in D(1, a) + D(1, a)$, $\forall a \in R$

Proof (i) $D(a, 0) = D(a, 0.0) \in D(a, 0)0 + 0D(a, 0) = 0 + 0 = 0$, and so $D(a, 0) = 0$.
(ii) $\forall a, b \in R$, $0 = D(a, 0) = D(a, b - b) \subseteq D(a, b) + D(a, -b)$. That is, $D(a, b) \in 0 - D(a, -b)$. Hence $D(a, b) = -D(a, -b)$. Therefore, $-D(a, b) = -\big(-D(a, -b)\big) = D(a, -b)$.
(iii) $D(1, a) = D(1.1, a) \in D(1, a).1 + 1.D(1, a) = D(1, a) + D(1, a)$. That is, $D(1, a) \in D(1, a) + D(1, a)$.

Example 2 Consider the hyperring $R = \{0, a, b\}$ with the hyperaddition and the multiplication defined as follow.

$+$	0	a	b
0	$\{0\}$	$\{a\}$	$\{b\}$
a	$\{a\}$	$\{a,b\}$	R
b	$\{b\}$	R	$\{a,b\}$

$\cdot$	0	a	b
0	0	0	0
a	0	b	a
b	0	a	b

Define a map $D: R \times R \to R$ by $D(0,0) = 0$, $D(a,0) = D(0,a) = 0$, $D(b,0) = D(0,b) = 0$, $D(a,a) = a$, $D(a,b) = D(b,a) = b$, $D(b,b) = a$. Clearly, D is a symmetric bi-derivation of R. Here D is a strong symmetric bi-derivation of R.

Example 3 Let R be a commutative hyperring and $M(R) = \left\{\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} : a, b \in R\right\}$ be a collection of 2×2 matrices over R. A hyperaddition $\oplus$ is defined on $M(R)$ by $\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \oplus \begin{pmatrix} c & 0 \\ d & 0 \end{pmatrix} = \left\{\begin{pmatrix} x & 0 \\ y & 0 \end{pmatrix} : x \in a + c, y \in b + d\right\}$ for all $\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix}, \begin{pmatrix} c & 0 \\ d & 0 \end{pmatrix} \in M(R)$. Clearly, this hyperaddition is well-defined and $(M(R),\oplus)$ is a canonical hypergroup. The matrix $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ is the additive identity of $M(R)$. Also, for each matrix $\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \in M(R)$, there exists a unique matrix $\begin{pmatrix} -a & 0 \\ -b & 0 \end{pmatrix} \in M(R)$ such that $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \in \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \oplus \begin{pmatrix} -a & 0 \\ -b & 0 \end{pmatrix}$.

Now, a multiplication $\otimes$ is defined on $M(R)$ by $\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \otimes \begin{pmatrix} c & 0 \\ d & 0 \end{pmatrix} = \begin{pmatrix} ac & 0 \\ bc & 0 \end{pmatrix}$ for all $\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix}, \begin{pmatrix} c & 0 \\ d & 0 \end{pmatrix} \in M(R)$. Clearly, the multiplication $\otimes$ is well-defined and associative. Therefore, $(M(R),\otimes)$ is a semigroup.

Let $\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix}, \begin{pmatrix} x & 0 \\ y & 0 \end{pmatrix}, \begin{pmatrix} p & 0 \\ q & 0 \end{pmatrix} \in M(R)$. Then

$$\begin{aligned}\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \otimes \left\{\begin{pmatrix} x & 0 \\ y & 0 \end{pmatrix} \oplus \begin{pmatrix} p & 0 \\ q & 0 \end{pmatrix}\right\} &= \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \otimes \left\{\begin{pmatrix} r & 0 \\ s & 0 \end{pmatrix} : r \in x + p, s \in y + q\right\} \\ &= \left\{\begin{pmatrix} ar & 0 \\ br & 0 \end{pmatrix} : r \in x + p, s \in y + q\right\}\end{aligned}$$

and

$$\begin{aligned}\left\{\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \otimes \begin{pmatrix} x & 0 \\ y & 0 \end{pmatrix}\right\} \oplus \left\{\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \otimes \begin{pmatrix} p & 0 \\ q & 0 \end{pmatrix}\right\} &= \begin{pmatrix} ax & 0 \\ bx & 0 \end{pmatrix} \oplus \begin{pmatrix} ap & 0 \\ bp & 0 \end{pmatrix} \\ &= \left\{\begin{pmatrix} l & 0 \\ m & 0 \end{pmatrix} : l \in ax + ap, m \in bx + bp\right\}\end{aligned}$$

Hence, we get

$$\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \otimes \left\{\begin{pmatrix} x & 0 \\ y & 0 \end{pmatrix} \oplus \begin{pmatrix} p & 0 \\ q & 0 \end{pmatrix}\right\} = \left\{\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \otimes \begin{pmatrix} x & 0 \\ y & 0 \end{pmatrix}\right\} \oplus \left\{\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \otimes \begin{pmatrix} p & 0 \\ q & 0 \end{pmatrix}\right\}.$$

Similarly, we have

$$\left\{\begin{pmatrix} x & 0 \\ y & 0 \end{pmatrix} \oplus \begin{pmatrix} p & 0 \\ q & 0 \end{pmatrix}\right\} \otimes \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} = \left\{\begin{pmatrix} x & 0 \\ y & 0 \end{pmatrix} \otimes \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix}\right\} \oplus \left\{\begin{pmatrix} p & 0 \\ q & 0 \end{pmatrix} \otimes \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix}\right\}.$$

Thus, $M(R)$ is a Krasner hyperring.

Now define a function D on $M(R)$ by $D\left(\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix}, \begin{pmatrix} c & 0 \\ d & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 \\ ac & 0 \end{pmatrix}$. Clearly, this map is well-defined and symmetric. Let us show that D is a symmetric bi-derivation. For all $\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix}, \begin{pmatrix} c & 0 \\ d & 0 \end{pmatrix}, \begin{pmatrix} e & 0 \\ f & 0 \end{pmatrix} \in M(R)$,

$$D\left(\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \oplus \begin{pmatrix} e & 0 \\ f & 0 \end{pmatrix}, \begin{pmatrix} c & 0 \\ d & 0 \end{pmatrix}\right) = D\left(\left\{\begin{pmatrix} r & 0 \\ s & 0 \end{pmatrix} : r \in a + e, s \in b + f\right\}, \begin{pmatrix} c & 0 \\ d & 0 \end{pmatrix}\right)$$

$$=\left\{\begin{pmatrix} 0 & 0 \\ rc & 0 \end{pmatrix} : r \in a+e\right\}$$

and

$$D\left(\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix}, \begin{pmatrix} c & 0 \\ d & 0 \end{pmatrix}\right) \oplus D\left(\begin{pmatrix} e & 0 \\ f & 0 \end{pmatrix}, \begin{pmatrix} c & 0 \\ d & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 \\ ac & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & 0 \\ ec & 0 \end{pmatrix}$$
$$=\left\{\begin{pmatrix} 0 & 0 \\ l & 0 \end{pmatrix} : l \in ac+ec\right\}.$$

Also,

$$D\left(\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \otimes \begin{pmatrix} e & 0 \\ f & 0 \end{pmatrix}, \begin{pmatrix} c & 0 \\ d & 0 \end{pmatrix}\right) = D\left(\begin{pmatrix} ae & 0 \\ be & 0 \end{pmatrix}, \begin{pmatrix} c & 0 \\ d & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 \\ (ae)c & 0 \end{pmatrix}$$

and

$$\left\{D\left(\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix}, \begin{pmatrix} c & 0 \\ d & 0 \end{pmatrix}\right) \otimes \begin{pmatrix} e & 0 \\ f & 0 \end{pmatrix}\right\} \oplus \left\{\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \otimes D\left(\begin{pmatrix} e & 0 \\ f & 0 \end{pmatrix}, \begin{pmatrix} c & 0 \\ d & 0 \end{pmatrix}\right)\right\}$$
$$= \left\{\begin{pmatrix} 0 & 0 \\ ac & 0 \end{pmatrix} \otimes \begin{pmatrix} e & 0 \\ f & 0 \end{pmatrix}\right\} \oplus \left\{\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 \\ ec & 0 \end{pmatrix}\right\} = \begin{pmatrix} 0 & 0 \\ (ac)e & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 0 & 0 \\ (ac)e & 0 \end{pmatrix}.$$

Definition 9 Let R be a hyperring and $D: R \times R \to R$ be a symmetric map. A mapping $d: R \to R$ defined by $d(x) = D(x,x)$ is called the trace of D.

It is obvious that, in case $D: R \times R \to R$ be a symmetric mapping which is also bi-additive (i. e. hyperadditive in both arguments), the trace of D satisfies the relation

$$d(x+y) = D(x+y, x+y) \subseteq d(x) + D(x,y) + D(x,y) + d(y)$$

and $d(0) = D(0,0) = 0$. If D is strong symmetric bi-derivation, we have

$$d(x+y) = d(x) + D(x,y) + D(x,y) + d(y)$$

Also $d(-x) = -d(x)$. Indeed,
$0 = d(0) = d\big(x + (-x)\big) \subseteq d(x) + D(x,-x) + D(x,-x) + d(-x) = -d(x) + d(-x),$ hence $d(-x) \in 0 - \big(-d(x)\big)$. That is, $d(-x) = d(x)$.

Proposition 2 Let R be a hyperring, D be a symmetric bi-derivation of R and a be a fixed element of R. Then $S = \{x \in R: D(x,a) = 0\}$ is a subhyperring of R.

Proof Since $D(0,a) = 0$, we get S is non-empty. Let $x, y \in S$. Then $D(x,a) = 0$ and $D(y,a) = 0$. Now, $D(x+y,a) \subseteq D(x,a) + D(y,a) = 0$. Further, for any $x \in S$, $D(-x,a) = -D(x,a) = 0$. Also, $D(xy,a) \in D(x,a)y + xD(y,a) = 0$. Thus for any $x, y \in S$, $x+y \subseteq S$, $-x \in S$, $xy \in S$. So S is a subhyperring of R.

Proposition 3 Let D be a symmetric bi-derivation of a prime hyperring R and $a \in R$ such that $aD(x,y) = 0$ (or $D(x,y)a = 0$) for all $x, y \in R$. Then either $a = 0$ or $D = 0$.

Proof Let $x, y, z \in R$. Suppose $aD(x,y) = 0$ for all $x, y \in R$, then
$0 = aD(xz,y) \in axD(z,y) + aD(x,y)z = axD(z,y)$.
Thus, $axD(z,y) = 0$. Since R be a prime hyperring, $a = 0$ or $D(z,y) = 0$. If $a \neq 0$, then $D(z,y) = 0$. That is, $D = 0$.
Suppose $D(x,y)a = 0$ for all $x, y \in R$, then
$0 = D(xz,y)a \in xD(z,y)a + D(x,y)za = D(x,y)za$.
Thus, $D(x,y)za = 0$. Since R be a prime hyperring, $a = 0$ or $D(x,y) = 0$. If $a \neq 0$, then $D(x,y) = 0$. That is, $D = 0$.

Proposition 4 Let D be a strong symmetric bi-derivation with trace d of 2-torsion free prime hyperring R and $a \in R$ such that $ad(x) = 0$ (or $d(x)a = 0$) for all $x \in R$. Then either $a = 0$ or $D = 0$.

Proof Suppose $ad(x) = 0$ for all $x \in R$. Then for all $y \in R$,
$0 = ad(x + y) = ad(x) + aD(x, y) + aD(x, y) + ad(y) = aD(x, y) + aD(x, y)$.
For all $z \in R$,

$$\begin{aligned} 0 &= aD(xz, y) + aD(xz, y) \in aD(x, y)z + axD(z, y) + aD(x, y)z + axD(z, y) \\ &= axD(z, y) + axD(z, y) \end{aligned}$$

Since R is 2-torsion free, we get $axD(z, y) = 0$ for all $x, y, z \in R$. Since R be a prime hyperring, $a = 0$ or $D(z, y) = 0$. If $a \neq 0$, then $D(z, y) = 0$. That is, $D = 0$.

Theorem 1 Let D be a strong symmetric bi-derivation of 2-torsion free reduced hyperring R. If $D(D(x, y), y) = 0$ for all $x, y \in R$, then $D = 0$.

Proof Let $D(D(x, y), y) = 0$ for all $x, y \in R$. Replacing x by xz, $z \in R$, we get

$$\begin{aligned} 0 = D(D(xz, y), y) &\in D(D(x, y)z + xD(z, y), y) = D(D(x, y)z, y) + D(xD(z, y), y) \\ &\in D(x, y)D(z, y) + D(D(x, y), y)z + xD(D(z, y), y) + D(x, y)D(z, y), \end{aligned}$$

From here, $0 \in D(x, y)D(z, y) + D(x, y)D(z, y)$. Since R is 2-torsion free hyperring, we have $D(x, y)D(z, y) = 0$ for all $x, y, z \in R$. If we take x instead of z, we get $\left(D(x, y)\right)^2 = 0$ for all $x, y \in R$. Since R is reduced hyperring, we have $D(x, y) = 0$ for all $x, y \in R$. That is, $D = 0$.

Definition 10 Let D be a non-trivial symmetric bi-derivation (resp. strong symmetric bi-derivation) of a hyperring R. A hyperideal I of R is said to be a D-differential (resp. strongly D-differential) hyperideal of R, if $D(I, I) \subseteq I$.

Remark 1 ([2]) Let S be a non-empty subset of a hyperring R. The set $Ann_l(S) = \{x \in R: xS = 0\}$ is called the left annihilator of S in R. Similarly, we have the right annihilator $Ann_r(S)$ of S in R. In a reduced hyperring R, if $ab = 0$ for all $a, b \in R$, then $ba = 0$ and therefore, there is no distinction from a left annihilator of S and a right annihilator of S in R. In this case, we just call it by the annihilator of S in R and is denoted by $Ann(S)$. The following results of reduced hyperrings follows from [1].

Proposition 5 ([2]) Let R be a reduced hyperring. (i) If S is a non-empty subset of R, then $Ann(S)$ is a hyperideal of R. (ii) If S_1 and S_2 are subsets of R such that $S_1 \subseteq S_2$, then $Ann(S_2) \subseteq Ann(S_1)$.
Corollary 1 Let R be a reduced hyperring and I be a D-differential hyperideal of R, then $Ann(I) \subseteq Ann\big(D(I, I)\big)$.

Proof Since I is a D-differential hyperideal of R, we have $D(I, I) \subseteq I$. From Proposition 5, we have $Ann(I) \subseteq Ann\big(D(I, I)\big)$.

Theorem 2 Let D be a symmetric bi-derivation of a reduced hyperring R. Then for any subset S of R, $D\big(Ann(S), Ann(S)\big) \subseteq Ann(S)$.

Proof If $x, y \in Ann(S)$, $Sx = 0$ and $Sy = 0$. Now for $s \in S$, $0 = D(sx, y) \in D(s, y)x + sD(x, y)$. Multiplying by s from the right, we get $0 \in D(s, y)xs + sD(x, y)s$. Hence we have $sD(x, y)s = 0$. And so, $\left(sD(x, y)\right)^2 = 0$. Since R is reduce hyperring, we get $sD(x, y) = 0$. That is, $D(x, y) \in Ann(S)$. This means that $D\big(Ann(S), Ann(S)\big) \subseteq Ann(S)$.

Example 4 Consider the reduced hyperring $R = \{0, a, b, c\}$ with the hyperaddition $\oplus$ and the multiplication $\odot$ defined as follows.

$+$	0	a	b	c
0	$\{0\}$	$\{a\}$	$\{b\}$	$\{c\}$
a	$\{a\}$	$\{0,b\}$	$\{a,c\}$	$\{b\}$
b	$\{b\}$	$\{a,c\}$	$\{0,b\}$	$\{a\}$
c	$\{c\}$	$\{b\}$	$\{a\}$	$\{0\}$

$.$	0	a	b	c
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{0\}$	$\{a\}$	$\{b\}$	$\{c\}$
b	$\{0\}$	$\{b\}$	$\{b\}$	$\{0\}$
c	$\{0\}$	$\{c\}$	$\{0\}$	$\{c\}$

It is clear that the map $D: R \times R \to R$ defined by $D(a,a) = D(b,b) = D(a,b) = D(b,a) = b$ and $D(x,y) = 0$ if x, y in the other cases, is a symmetric bi-derivation of R.
Now, $Ann(0,b) = \{0,c\}$ is a hyperideal of R. Since $D\big(Ann(0,b), Ann(0,b)\big) = D(\{0,c\},\{0,c\}) = \{0\} \subseteq Ann(0,b)$, we see that $Ann(0,b)$ is a D-differential hyperideal of R.

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