

## HEDGING PERFORMANCE of TURKISH STOCK INDEX FUTURES

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### Abstract

The relation between spot price of the financial instrument and the futures price of the related hedging instrument is defined as hedge ratio. Traditional hedge ratio methods assume time invariance but it might be better to apply dynamic models rather than fixed coefficients model to see time varying aspects of the financial series. At the other hand it's obvious that financial series has time varying aspects. The main objective of this study is to observe time varying hedging effectiveness by using Multivariate GARCH-BEKK methodology.

**Keywords:** Time Varying Hedge Ratio, M-GARCH, TURKDEX, ISE30

## TÜRKİYE HİSSE SENEDİ VADELİ İŞLEMLERİNİN HEDGE PERFORMANSI

### Özet

Finansal enstrümanın spot fiyatı ile aynı enstrümanın vadeli fiyatı arasındaki ilişki hedge oranı ile ifade edilmektedir. Geleneksel olarak hedge oranının zaman içerisinde değişmediği varsayılır. Fakat, sabit katsayılardan ziyade zamanla değişen özellikleri de gösteren dinamik modellerin uygulanması daha iyi sonuçlar verebilir. Şu da gerçektir ki finansal zaman serilerinin zamanla değişen özellikleri vardır. Bu çalışmanın asıl amacı, zamanla değişen hedge etkinliği çok değişkenli GARCH-BEKK modeli ile gözlemlemektir.

**Anahtar Kelimeler:** Zamanla Değişen Hedge Oranı, M-GARCH, TURKDEX, BIST30

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### 1. INTRODUCTION

In the last thirty years global financial system has widened and become more complex, beside the diversity of financial instruments and the number of risk classes has increased. Increasing volume of transactions has caused many corporate treasurers to search for effective and efficient ways to hedge different kind of risks. One way of hedging against market risks is futures. Unlike developed countries, many developing countries has

In practice, portfolio managers should answer the question what should be the optimum spot to futures ratio to maximize the expected utility. Traditional approach to commodity futures hedging adopts a one-to-one ratio (Lien, et. al. 2002). For any given spot position, an equal amount of futures position should be held to hedge against risk. The portfolio approach recognizes the existence of basis risk and determines the optimal futures position by minimizing the variance of the spot futures portfolio. The optimal hedge ratio is then equal to the covariance between the spot and futures returns divided by the variance of

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the futures return. Suppose a linear regression model is constructed with the spot return being the dependent variable and the futures return being the independent variable. The OLS (ordinary least squares) estimate of the slope is the estimated optimal hedge ratio. The conventional OLS approach assumes that the second moments are constant over time. It is well known in the finance literature that asset returns exhibit time-varying conditional heteroscedasticity. That is, hedge ratio is also time varying. Thus, to enhance the estimation results, it is important to take account of the possible time-varying nature of the second moments. The GARCH (generalized autoregressive conditional heteroscedasticity) models proposed by Engle (1982) and Bollerslev (1987) are particularly useful for this purpose and have been extensively applied in the futures market literature.

Time varying asset returns and their effects has been argued many times in the finance literature. Chakraborty and Barkoulas (1999), show that bivariate GARCH models are better than OLS estimations which means dynamic models provides superior out of sample hedging performance compared to static OLS model. Moschini and Aradhyula (1993) again show that optimum hedge ratio is time varying and dynamic bivariate GARCH(1,1) model performs significantly better than a constant one from an economic standpoint.

Floros and Vougas (2004), using OLS regression, simple and vector error correction and multivariate generalized autoregressive heteroscedasticity (M-GARCH) models to estimate corresponding hedge ratios for Greek stock index futures found that M-GARCH models provide best hedging ratios. In an other study, Laws and Thompson (2004) employed OLS and EWMA models to stock index futures. They found that the EWMA method of estimation provide the best estimate of the optimal hedge. Yang (2001) using Ordinary Index and SPI futures on the Australian market, the optimal hedge ratios calculated optimum hedge ratios by OLS, bivariate vector autoregressive model (BVAR), the error -correction model (ECM) and the multivariate diagonal Vec GARCH Model. They found that the GARCH type models provide the greatest portfolio risk reduction, particularly for longer hedging horizons or models optimum hedge ratios better. The results of Bhaduri and Durai (2008) show that the time-varying hedge ratio derived from the multivariate GARCH model has higher mean return and higher average variance reduction across hedged and unhedged positions. Additionally they show that the simple OLS-based strategies perform well at shorter time horizons.

**2.METHODOLOGY**

There are many measures about hedging effectiveness in the risk literature including Markowitz (1959), Ederington (1979), Howard and D’antonio (1987) and Lindahl’s measures (1991). Markowitz version of hedging effectiveness is based on the reduction of standard deviation of a portfolio. As the standard deviation decreases, hedging effectiveness increases. According to Ederington (1979), hedging effectiveness is the R – square of the OLS regression.

$$\Delta \ln P_t = c + \beta \Delta \ln F_t + \varepsilon_t$$

Where  $P_t$  and  $F_t$  are spot price and future price at time t respectively. The level of the R square reflects the hedging effectiveness of the future market.

A simple estimator for the above equation, which is widely used by practitioners, is the slope coefficient in the OLS regression of portfolio against futures returns. Despite its

popularity, the OLS hedge ratio has several limitations. Besides disregarding the effects of serial correlation in returns, OLS hedge ratios are biased downwards if the index and the futures price are cointegrated and the latter corrects deviations from the equilibrium equation.

Data employed in this study is composed of 1118 daily observations between February 2, 2005 and July 7, 2009 on the ISE30 of Istanbul Stock Exchange (ISE). Daily closing prices are obtained from ISE and TURKDEX. ISE 30 comprises 30 Turkish companies quoted at ISE with the highest market capitalizations. Futures contracts are quoted on the TURKDEX and the price of a contract price is measured by an index and the size of the contract is calculated by multiplying by index multiplier. There are 12 delivery months. The nearest two delivery months are traded. Price Quotation is determined by dividing ISE National-100 Index valued by 1.000. Minimum price fluctuation is 0.025 or 25 index points.

Return series of both spot market variables and futures market variables are calculated by taking difference of natural logarithm of the series. Table 1 summarizes the descriptive statistics of the variables.

$$r_{p_t} = \log p_t - \log p_{t-1} \times 100 \quad \text{and} \quad r_{f_t} = \log f_t - \log f_{t-1} \times 100$$

According to descriptive statistics ISE30 index returns have higher standard deviation. Jarque Berra statistics are higher for \$/TL than ISE30 values. This may be the result of international markets on domestic foreign exchange markets. Jarque Berra and Skewness statistics state that return series are not normally distributed and there is excess kurtosis which means that it's better to use student's t distribution.

**Table 1 :** Descriptive Statistics of ISE30 index and \$/TL exchange rate in the Spot and Futures markets

	PISE30	FISE30
Mean	0.025	0.025
Median	-0.003	0.000
Maximum	12.726	9.657
Minimum	-9.745	-9.972
Std. Dev.	2.178	2.170
Skewness	-0.001	-0.095
Kurtosis	5.474	5.508
Jarque-Bera	286.96*	294.71*
Observations	1118	1118

**\* Denotes 1 % significance**

The results of unit root tests for return series of ISE30 and \$/TL spot and futures for the level are reported in Table 2. Notice that apart from the augmented Dicky-Fuller (ADF) tests, which attempt to account for temporally dependent and heterogeneously distributed errors by including lagged sequences of first differences of the variable in its set of regressors, the Philips Perron test is also used. The null hypothesis for ADF test is that the variables contain a unit root or they are non-stationary at a certain significant level. However, the power of standard unit root tests which have null hypothesis of non-stationarity has recently been questioned in the literature. (Schwert (1987) and DeJong and

Whiteman (1991)) These tests often tend to accept the null too frequently against a stationary alternative. (Yang, 2001).

**Table 2 : ADF and Philips Perron Stationarity Tests**

		ADF TEST RESULTS	
		PISE30	FISE30
Intercept	t-Statistic	-31.541*	-31.819*
trend & intercept	t-Statistic	-31.545*	-31.814*
		PHILIPS PERRON	
Intercept	t-Statistic	-31.498*	-31.794*
trend & intercept	t-Statistic	-31.493*	-31.789*

\* Denotes significance at 1 %

### 2.1.The Multivariate GARCH-BEKK Model

A perfect hedge is not possible unless the equity portfolio has the same composition as the index and the hedging horizon exactly matches the contract maturity. This is the situation that investors commonly face and exposes them to basis risk: losses may be incurred because of differences between the returns on the portfolio and on the futures position. A common remedy is to set the relative size of the futures position with respect to the portfolio, the ‘hedge ratio’, so that the variance of the covered portfolio return is minimized. The optimal hedge ratio may be defined as the quantities of the spot instrument and the hedging instrument that ensure that the total value of the hedged portfolio does not change (Hatemi-J and Roca, 2006). A typical hedging model involves a decision maker who allocates wealth between a risk-free asset and two risky assets: the physical asset and the corresponding futures. Let  $Q_S$  and  $Q_F$  are the optimum levels of assets bought and futures sold. Each position has been taken at time t and held until time t+1. Then the optimal hedge ratio can be defined as;

$$HR = \frac{Q_f}{Q_p}$$

Assumptions about preferences and/or the distribution of cash and futures prices are typically necessary to characterize this ratio. But a useful result obtains when the futures price  $f_t$  and spot price  $P_t$  are conditionally jointly normally distributed and the futures market is unbiased. Then the optimum hedge ratio is;

$$HR_t = \frac{Cov(p_t, f_t | \Omega_{t-1})}{Var(f_t | \Omega_{t-1})} = \frac{Corr(p_t, f_t | \Omega_{t-1}) \sigma_{p_t} | \Omega_{t-1}}{\sigma_{p_t} | \Omega_{t-1}}$$

$\Omega_{t-1}$  is the information set implying that the hedge ratio of interest is independent of risk preferences. If P and F are for the same assets it is likely that  $p_t$  and  $f_t$  are close to the same value and  $HR$  is close to 1. Then the optimal hedge ratio is near 1.

Where this becomes more interesting is where you are hedging one product with a different products future contract. If the joint distribution of cash and futures prices changes over time, then  $HR_t$  defined above may also change over time. The time path of  $HR_t$  can be calculated given knowledge of the (time-dependent) covariance matrix for cash and futures prices, which can be estimated with GARCH models. The optimum hedge ratio can be constant over time if both covariance and variance terms vary at the same rate but it is a legitimate possibility (Moschini, Myers, 2001). Assuming a constant hedge ratio would simplify implementation of an optimal hedging strategy. But application of Multivariate GARCH models with time varying conditional correlations would be more realistic.

The actual superiority of time-varying hedge ratios is essentially an empirical issue, which has been investigated extensively on several markets. The dynamics of conditional variances and covariances are usually represented with bivariate GARCH-type models for returns innovations. A common specification is the constant correlation GARCH that restricts the changes in the covariance between spot and futures innovations to be driven by standard deviations (Pattarin and Ferretti, 2004).

Multivariate GARCH models are very similar to univariate GARCH models except that they also allow to measure dynamic relationships. Several multivariate GARCH models have been proposed including BEKK – GARCH. The BEKK model which was proposed by Engle and Kroner (1995) has some simple solutions for the problems of previous models like VECH and DCC models. First of all, the requirement of positive definite H matrix is ensured by BEKK parameterization (Syriopoulos and Roumpis, 2008). The first market is the spot market and the second market is the futures market.

$$H_t = C' C A' \varepsilon'_{t-1} \varepsilon_{t-1} A G' H_{t-1} G$$

or,

$$H_t = C' C \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1} \varepsilon_{2,t-1} \\ \varepsilon_{1,t-1} \varepsilon_{2,t-1} & \varepsilon_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} H_{t-1}$$

Where,  $a$  is the ARCH parameter,  $g$  is the GARCH parameter and  $\varepsilon$  is the error term. If  $H_t$  representation is enlarged by matrix multiplications following equations can be obtained.

$$h_{11t} = c_{11} + a_{11}^2 \varepsilon_{1,t-1}^2 + 2a_{11}a_{12} \varepsilon_{1,t-1} \varepsilon_{2,t-1} + a_{21}^2 \varepsilon_{2,t-1}^2 + g_{11}^2 h_{11,t-1} + 2g_{11}g_{21}h_{12,t-1} + g_{21}^2 h_{22,t-1}$$

$$h_{22t} = c_{21} + c_{22} + a_{12}^2 \varepsilon_{1,t-1}^2 + 2a_{12}a_{22} \varepsilon_{1,t-1} \varepsilon_{2,t-1} + a_{22}^2 \varepsilon_{2,t-1}^2 + g_{12}^2 h_{11,t-1} + 2g_{12}g_{22}h_{12,t-1} + g_{22}^2 h_{22,t-1}$$

$$h_{12t} = c_{11}c_{21} + a_{11}a_{12} \varepsilon_{1,t-1}^2 + a_{21}a_{12} \varepsilon_{1,t-1} \varepsilon_{2,t-1} + a_{11}a_{22} \varepsilon_{1,t-1} \varepsilon_{2,t-1} + a_{21}a_{22} \varepsilon_{2,t-1}^2 + g_{11}g_{12}h_{11,t-1} + g_{11}g_{22}h_{12,t-1} + g_{12}g_{22}h_{22,t-1}$$

$h_{11t}$  , and  $h_{22t}$  are the variance of spot market and future market respectively and  $h_{12t}$  is the covariance between spot and futures markets. To see the causality effect of spot market to future market  $a_{12}$  and  $g_{12}$  and the causality effect of future market to spot market  $a_{21}$  and  $g_{21}$  are equalised to zero hence, the following simple form of multivariate GARCH with two dimensions take place.

$$h_{11t} = c_{11} + a_{11}^2 \varepsilon_{1,t-1}^2 + g_{11}^2 h_{11,t-1}$$

$$h_{22t} = c_{21} + c_{22} + a_{22}^2 \varepsilon_{2,t-1}^2 + g_{22}^2 h_{22,t-1}$$

$$h_{12t} = c_{11}c_{21} + a_{11}a_{22} \varepsilon_{1,t-1} \varepsilon_{2,t-1} + g_{11}g_{22}h_{12,t-1}$$

This form allow to measure causality relationship between return series of spot and future markets in variance equations. This variance modeling, allow us to see dynamic or time varying conditional variances and correlations.

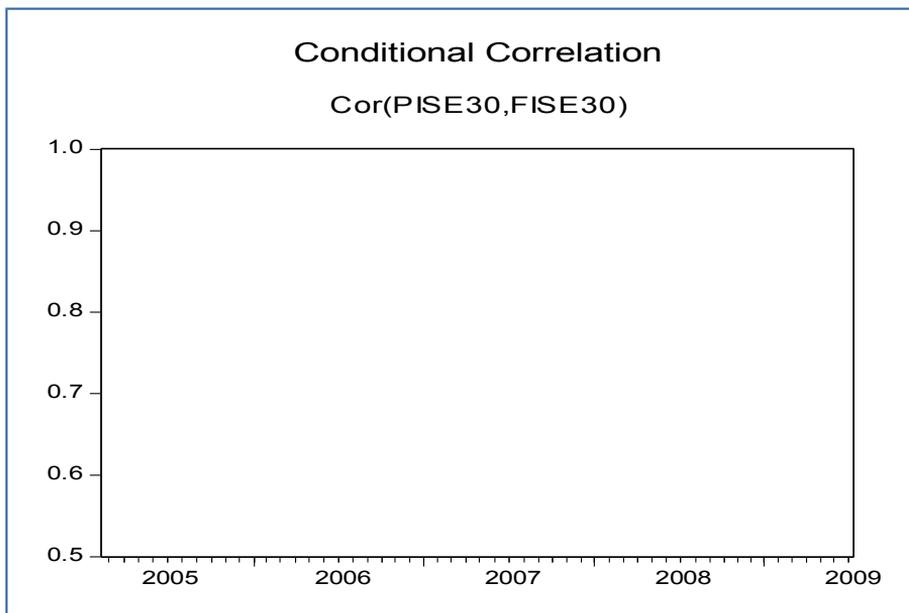
Before implementing GARCH model we employ VAR model to determine the mean equation of multivariate GARCH-BEKK models. Using the Akaike Information Criterion (AIC) optimum lag lengths are determined 7 for ISE30. Autocorrelation – LM test and portmanteau autocorrelation tests signed that there are no autocorrelations up to those lag lengths. In the next step these equations used to set up systems with two dimensions under multivariate GARCH methodology. Maximum Likelihood Estimations for Var(7) – GARCH BEKK(1,1) are summarized in table 3. The Ljung Box statistics for the residuals are above 5 %. Hence, the null hypotheses that the residuals from each estimated conditional mean equation are white noise is not rejected, implying that the simple conditional models of ISE30 index and ISE 30 futures index seem to fit the data adequately. The ARCH and GARCH parameter estimates in Table 3 are highly significant.

**Table 3 :** Var(p) – GARCH BEKK estimation tables

	PISE30 - FISE30		
VAR(p)	7		
	Coef.	Std Error	Prob.
c <sub>11</sub>	0.070	0.0179	0.00
c <sub>12</sub>	0.069	0.0149	0.00

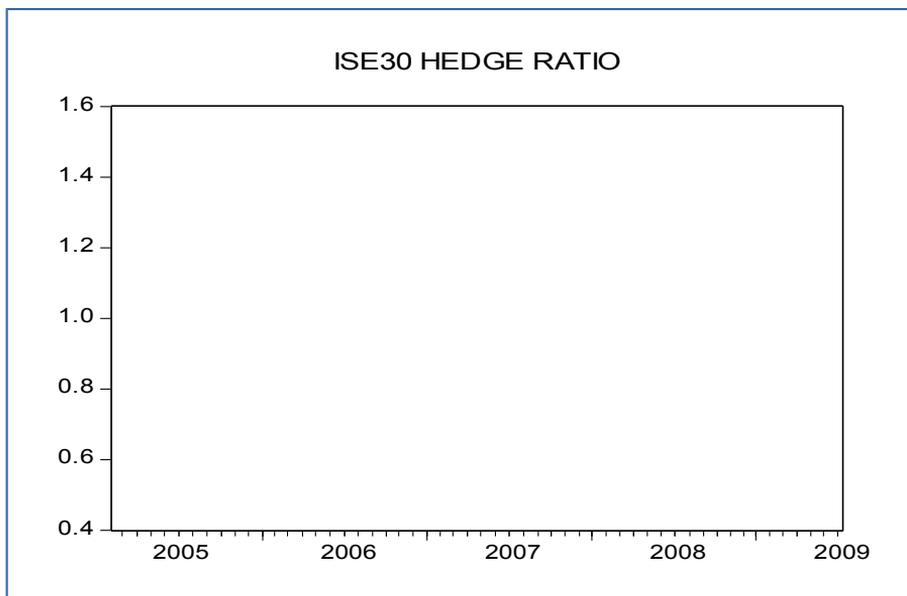
<b>c<sub>22</sub></b>	0.081	0.0155	0.00
<b>a<sub>11</sub></b>	0.271	0.0153	0.00
<b>a<sub>22</sub></b>	0.260	0.0149	0.00
<b>g<sub>11</sub></b>	0.954	0.0047	0.00
<b>g<sub>22</sub></b>	0.955	0.0045	0.00
<b>Q(12)</b>	43.621		0.65
<b>Q(24)</b>	105.219		0.24
<b>Log. Li.</b>	-3577.51		
<b>Iterations</b>	48		

Time varying conditional correlations are estimated from the GARCH-BEKK models conditional variance covariance matrices implying that there is low correlation between spot and futures markets during the first year of Turkish Derivatives Exchange (Figure 1). This may be a result of inefficiency of TURKDEX and lack of trade. After the second half of 2006 conditional correlations are above 0.9.



**Figure 1 :** Conditional Correlations between ISE30 index and futures ISE30 index

The estimated hedging ratio for ISE30 under the diagonal BEKK specification is plotted in Figure 2. OHRs are computed from the estimated GARCH models using the in sample estimates of the time varying conditional variance covariance matrices. Estimated conditional OHR's under BEKK specification are illustrated in Figure 2. Figure clearly demonstrates that OHRs are time varying and change from day to day as new information is obtained.



**Figure 2 :** Time varying hedge ratio of ISE30 index

### 3.CONCLUSIONS

There are various approaches for risk minimization which calculate different hedge ratios. First, one-to-one hedge assumes that the correlation between the spot and futures is perfect (hedge ratio=1). This hedge ratio fails when the real correlation between spot and futures prices is less than perfect and ignores the stochastic nature of futures and spot prices, as well as time variation of hedge ratios. A second approach estimates the hedge ratio as the OLS coefficient of a regression of spot returns on futures return like Ederington (1979). This methodology imposes constant correlation between variables. It's obvious that financial variables exhibit time varying conditional heteroscedasticity, time varying concept should be added to optimum hedge ratio methodology. M-GARCH models are one of the techniques that handle time variance.

In this paper we show that optimal hedge ratio is not constant over time. We have used BEKK parameterization of the multivariate GARCH(1,1) model that nest the hypothesis of constancy of the ratio of conditional covariance to conditional variance of one of the variables. GARCH – BEKK model estimated using daily data for ISE30 index. In the first year of TURKDEX optimum hedge ratio is highly volatile implying that there should be informational inefficiency related to structure of the new futures market. Especially lack of trade might stem the valuation of new information. As it has seen on time varying conditional correlation graphs, during the first year of TURKDEX correlations are low.

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