GU J Sci 30(4): 623-632 (2017)

Gazi University

Journal of Science





Group-buying Inventory Policy with Imperfect Items under Inspection Errors

Tammarat KLEEBMEK¹, Wuttichai SRISODAPHOL^{2,*}, Angkana BOONYUED¹

¹Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand. ²Department of Statistics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand.

Article Info	Abstract
Received: 18/10/2016 Accepted: 23/09/2017	A recent occurrence in retail selling has been the development of group-buying websites offering products for a limited time and at highly discounted rates. Our objective of this study is to analyze whether the seller should order quantity in the replenishment cycle of how much better to make the maximum profit. This study determines imperfect quality items and while
Keywords	screening the inspectors may mistake errors. We create the group-buying inventory policy for items with imperfect quality under inspection error. This model is under continuous review and
EOQ Group buying Imperfect quality Inspection errors	lead time is assumed to be negligible. A numerical example and sensitivity analysis for the fraction of defective items are illustrated.

1. INTRODUCTION

Misclassification

In recent year, the growth of electronic commercial shop has developed to all over the place in the world. Several new business models for web-based selling appeared together with the advent of the Social network; one of them is web-based group-buying. In 2015, this mechanism is predicted that the overall global group-buying marketplace will influence nearly \$4 billion [13]. The group-buying sites also known as daily deal sites, one of the trade strategies which charms buyers with the similar interest and allow buyers to obtain heavily discount rate from a seller on the product they wish to purchase.

In the group-buying method, the seller determines the fixed number of unit required by the deal, discounted price, starting and ending time. Later, the deal is advertised to offer products for sale on site for invite people to join for a limited time. Finally, joining buyers would success the deal only if the last number of purchasing unit meets the set number at the finish of time. If the last number of the buyer is less than the set number, the deal will be abandon.

Many researchers proposed about several group-buying models. Anand and Aron [1] affirmed that the group-buying strategy was more effective than the posted pricing when demand was indeterminate. Only if the distribution of buyers' demand was known, the sellers were almost always better off by running the posted-price strategy. Chen et al. [4] compared between the group-buying and the posted pricing strategy, the condition within the buyer's arrival under Poisson process, and institute that the group-buying outmatches the posted price mechanism. Chen et al. [2] studied the group-buying in three circumstances, the seller's expected profit, economies of scale, and risk-seeking seller by comparing the group-buying with the fix-price mechanism, and found that all three situations the group-buying outperform the fixprice mechanism. Chen et al. [3] pointed the demand uncertainty in group-buying was more effective where there is larger low-valuation demand than high-valuation demand. Jing and Xie [9] found that the group-buying was optimal when interpersonal communication is very efficient, or when the product valuation of the less-informed consumer segment is high. Chen et al. [5] derived the optimal inventory rationing and replenishment policies for a retailer facing multiple demand classes. Edelman et al. [6] examined two benefits of using services such as Groupon: price discrimination and advertising. They concluded that the sellers who benefit most are those who are patient and unknown and have low marginal costs.

In an aspect of sellers, one of the important features which affect the profit was the inventory system. Typically, the inventory usually prepared under the hypothesis that all items within perfect quality. Actuality, the goods arrive at the inventory comprise a fraction of defective items which may be produced by manufacturing errors or mismanagement of goods. Porteus [15] presented the result of defective items in the basic of the economic order quantity (EOQ) model which found a fixed probability that makes a process out of control. Further, Salameh and Jaber [16] established an economic production quantity (EPQ) model for imperfect quality items with a known probability distribution. Goyal and Cardenas-Barron [7] offered a simpler method to the model of Salameh and Jaber [16]. They suggested the calculation of the expected total annual revenue, the expected total annual cost and modify the expected total annual profit. The result found that lot size calculation was simpler and easier to implement than the classical. Maddah and Jaber [14] mentioned that the expected annual profit in Salameh and Jaber [16] was not accurate, but it could be calculated using the renewal-reward theory. Khan et al. [11] explained the work of Salameh and Jaber [16] by adding the assumption of an error screening process by the inspector. Hsu and Hsu [8] extended the model of Khan et al. [11] by adding the assumption that shortages may be occurring. They proposed the optimal inventory model under the condition of inspection errors, shortage backordering, and sales returns. Khan et al. [10] proposed the inventory model for a two-stage supply chain integrated between vendor and buyer. This model was applied by screening process with errors in quality inspection and learning in production. Sarkar and Saren [17] proposed the inventory model base on an economic production quantity from in-control state and shifts to out-of-control state at any random time under inspection error and warranty cost. They obtained the optimal of the production-run length and non-inspected fraction of the batch. Kleepmek et al. [12] presented group-buying inventory model within demand is distributed as Poisson. They found the optimal solution of order quantity, re-order point and the minimum total cost under lead time is fixed.

This research designs the new inventory system by the concept of modern business which is groupbuying, so the sellers can sell the most profitable. We create a situation under items with imperfect quality by the fraction of defective items distributed as uniform distribution. Moreover, the model is assumed that the items will be sold every deal within the time for each deal that distributed as uniform distribution. Under the proposed model, we would like to find the optimal order quantity by maximizing the total profit. This paper is organized as follows. The model is formulated in Section2. Section 3 presents numerical examples, followed by a conclusion in Section 4.

2. GROUP-BUYING INVENTORY MODEL

In this section, we develop an inventory model corresponding to the group-buying process with imperfect items. First of all, let us introduce the following notations that are used in this paper.

- D : demand rate
- N: the fixed number of unit required by the deal
- Q : order size
- p: fraction of defective items
- A, B : positive integer
- T : cycle length
- x : screening rate
- c: unit variable cost
- v: unit price of defective items
- s: unit price of non-defective items
- t: total screening time
- d : unit screening cost

- h : unit holding cost
- m_1 : probability of non-defective items are classified to defective
- m_2 : probability of defective items are classified to non-defective
- c_a : cost of accepting defective items
- c_r : cost of rejecting non-defective items

We now assume the following assumptions for developing our model.

- 1. Inventory level is under continuous review.
- 2. Order quantity of size Q per cycle is placed every time.
- 3. Shortage is not allowed.
- 4. Lead time of inventory replenishment is assumed to be negligible.
- 5. Time for each deal is uniformly distributed.
- 6. Inspection error can occur.

From the assumptions mentioned above, the inventory level can be showed in Figure 1.

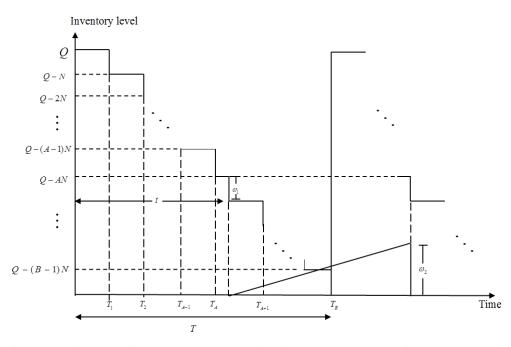


Figure 1. The group-buying inventory level with imperfect items and inspection error.

From Figure 1, Once we obtain a lot of sizes Q, the goods will be sold at volume N within time T_d where $T_d = T_i - T_{i-1}$ for i = 0, 1, 2, ..., B and, $T_0 = 0$ and the inventory will decrease N by the deal. We know that there will be some defective items which contained a fraction of defective (p) in the lot. We have proceeded the inspection process of which determine the inspection error rate, it is possible that the inspector can make two cases of error (i) the non-defective items are classified as defective and (ii) the defective items are classified as non-defective. At the same time, items are checked along with sales at the rate x items/unit of time and take t time to cover the quantity by the inspector. After the screening process is done, the inventory will decrease to Q - AN, we have the number of defective items (ω_1) from two sources including non-defective item is classified as defective and defective items is classified as defective. These items will be sold at a discount price in the market immediately. General group-buying sale, the buyer may receive the items with the case (ii) error. The buyer would like to return items; it is acceptable for us to get those returned items back. These returned items or defective item is classified as non-defective will be sold together with ω_1 in the next cycle. Finally, the inventory level will decrease to zero; the volume of items will be immediately replenished.

Define the cases of the classification from the screening of items before the items are sold by an inspector; all four cases are possible [11],

- (1) A non-defective item is classified as non-defective.
- (2) A non-defective item is classified as defective.
- (3) A defective item is classified as non-defective.
- (4) A defective item is classified as defective.

We consider a number of items are classified according to the classification from the screening of items in four cases, which can be explained in Table 1.

Table 1. The numbers of items are classified as four possibilities in the inspection process

Item	Inspection		
	Non-defective	Defective	
Non-defective	$Q(1-p)(1-m_1)$	$Q(1-p)m_1$	
Defective	Qpm ₂	$Qp(1-m_2)$	

Suppose that ω_1 and ω_2 is a number of items that are classified as defective, and defective items that are returned from the market, respectively. So ω_1 and ω_2 are obtained by

Let N(Q, p) be a number of non-defective items with the shortage is not allowed, then N(Q, p) must be equal to the total between the actual demand and items are returned from the market, so N(Q, p) is given as

$$N(Q, p) = BN + Qpm_{2}$$

$$Q - Q(1-p)m_{1} - Qp(1-m_{2}) = BN + Qpm_{2}$$

$$B = Q(1-m_{1})(1-p)/N.$$
(3)

Define the total revenue TR(Q) is the sum of the revenue from salvaging items (R_1) and the revenue from the selling non-defective items (R_2) , where

$$R_{1} = \nu(\omega_{1} + \omega_{2}) = \nu(Q(1 - p)m_{1} + Qp(1 - m_{2}) + Qpm_{2})$$
(4)

$$R_2 = s(Q(1-p)(1-m_1) + \omega_2) = s(Q(1-p)(1-m_1) + Qpm_2).$$
(5)

Therefore,

$$TR(Q) = R_1 + R_2$$

= $v(Q(1-p)m_1 + Qp(1-m_2) + Qpm_2) + s(Q(1-p)(1-m_1) + Qpm_2).$ (6)

The total cost TC(Q) of the group-buying inventory model could be expressed as

TC(Q) = Setup cost + Unit cost + Screening cost + Holding cost (7)

The total cost TC(Q) is obtained from the following components.

The setup cost per cycle is K.

The unit cost for producing per cycle is cQ.

The screening cost (SC) is the sum of inspection cost, and cost caused by misclassification,

(8)

$$SC = dQ + c_r Q(1-p)m_1 + c_a Qpm_2.$$

The holding cost (*HC*) per cycle is given by

$$HC = hQ\omega_1 / x + hBT_d (Q - \omega_1 - N(B - 1)/2) + hT\omega_2 / 2,$$
(9)

where $T = BT_d$, $T_d = T_i - T_{i-1}$ for i = 0, 1, 2, ..., B and, $T_0 = 0$.

Therefore, TC(Q) can be written as

$$TC(Q) = K + cQ + dQ + c_r Q(1-p)m_1 + c_a Qpm_2 + hQ\omega_1 / x + hBT_d [Q - \omega_1 - N(B-1)/2] + hT\omega_2 / 2.$$
(10)

Hence, we have the total profit TP(Q) = TR(Q) - TC(Q) as follow.

$$TP(Q) = v(\omega_1 + \omega_2) + s[Q(1-p)(1-m_1) + \omega_2] - K - cQ - dQ - c_rQ(1-p)m_1 - c_aQpm_2 - hQ\omega_1 / x - hBT_d[Q - \omega_1 - N(B-1)/2] - hBT_d\omega_2 / 2.$$

Substituting
$$B = Q(1-m_1)(1-p)/N$$
, ω_1 and ω_2 in $TP(Q)$, so
 $TP(Q) = v[Q(1-p)m_1 + Qp(1-m_2) + Qpm_2] + s[Q(1-p)(1-m_1) + Qpm_2] - K - cQ - dQ - c_rQ(1-p)m_1 - c_aQpm_2 - hQ[Q(1-p)m_1 + Qp(1-m_2)]/x - h(Q(1-m_1)(1-p)/N)T_d \{Q - [Q(1-p)m_1 + Qp(1-m_2)] - N((Q(1-m_1)(1-p)/N) - 1)/2\} - h(Q(1-m_1)(1-p)/N)T_dQpm_2/2.$
(11)

Since p, m_1 and m_2 are random variables with probability density function (pdf) f(p), $f(m_1)$ and $f(m_2)$, respectively. Therefore, the expected total profit (*ETP(Q)*) is the expected value of Eq. (11). So, *ETP(Q)* can be written as

$$ETP(Q) = v(Q(1 - E[p])E[m_1] + QE[p](1 - E[m_2]) + QE[p]E[m_2]) + s(Q(1 - E[p])(1 - E[m_1]) + QE[p]E[m_2]) + s(Q(1 - E[p])(1 - E[m_1]) + QE[p]E[m_2]) - K - cQ - dQ - c_rQ(1 - E[p])E[m_1] - c_aQE[p]E[m_2] - hQ[Q(1 - E[p])E[m_1] + QE[p](1 - E[m_2])]/x - h(Q(1 - E[m_1])(1 - E[p])/N)E[T_d](Q - [Q(1 - E[p])E[m_1] + QE[p](1 - E[m_2])]) + h(Q(1 - E[m_1])(1 - E[p])/N)E[T_d](N((Q(1 - E[m_1])(1 - E[p])/N) - 1)/2) - h(Q(1 - E[m_1])(1 - E[p])/N)E[T_d]QE[p]E[m_2]/2.$$
(12)

Then, we can find the expected total profit per unit of time (ETPU(Q)) by using renewal reward theorem [14],

$$ETPU(Q) = ETP(Q) / E[T], \qquad (13)$$

where $E[T] = E[BT_d] = E[Q(1-m_1)(1-p)T_d/N]$ by the time during successive replenishments of inventory is BT_d . T_d is uniformly distributed with parameters $(0,\beta)$ where $T_d = T_i - T_{i-1}$ for i = 0, 1, 2, ..., B and, $T_0 = 0$, then the expected value of T_d is $\beta/2$.

Hence we use the formula in Eq. (13) for calculating ETPU(Q), it can be written as follow, $ETPU(Q) = v(Q(1 - E[p])E[m_1] + QE[p](1 - E[m_2]) + QE[p]E[m_2]) / E[BT_d] + s(Q(1 - E[p])(1 - E[m_1]) + QE[p]E[m_2]) / E[BT_d] + (-K - cQ - dQ - c_rQ(1 - E[p])E[m_1] - c_aQE[p]E[m_2]) / E[BT_d] + (-hQ[Q(1 - E[p])E[m_1] + QE[p](1 - E[m_2])] / x) / E[BT_d] + (-h(Q(1 - E[m_1])(1 - E[p]) / N)E[T_d](Q - [Q(1 - E[p])E[m_1] + QE[p](1 - E[m_2])]) / E[BT_d] + (-h(Q(1 - E[m_1])(1 - E[p]) / N)E[T_d] - N((Q(1 - E[m_1])(1 - E[p]) / N) - 1) / 2)) / E[BT_d] + (-h(Q(1 - E[m_1])(1 - E[p]) / N)E[T_d]QE[p]E[m_2] / 2) / E[BT_d].$ (14)

Substituting
$$E[BT_d] = Q(1 - E[m_1])(1 - E[p])E[T_d]/N$$
 in Eq. (14), so
 $ETPU(Q) = vQ(1 - E[p])E[m_1]/(Q(1 - E[m_1])(1 - E[p])E[T_d]/N)$
 $+vQE[p](1 - E[m_2])/(Q(1 - E[m_1])(1 - E[p])E[T_d]/N)$
 $+vQE[p]E[m_2]/(Q(1 - E[m_1])(1 - E[p])E[T_d]/N)$
 $+s(Q(1 - E[p])(1 - E[m_1]) + QpE[m_2])/(Q(1 - E[m_1])(1 - E[p])E[T_d]/N)$
 $-(K + cQ)/(Q(1 - E[m_1])(1 - E[p])E[T_d]/N)$
 $-(dQ + c_rQ(1 - E[p])E[m_1] + c_aQE[p]E[m_2])/(Q(1 - E[m_1])(1 - E[p])E[T_d]/N)$
 $-hQ[Q(1 - E[p])E[m_1] + QE[p](1 - E[m_2])]/(x(Q(1 - E[m_1])(1 - E[p])E[T_d]/N)))$
 $-h[Q - [Q(1 - E[p])E[m_1] + QE[p](1 - E[m_2])] - N((Q(1 - E[m_1])(1 - E[p])E[T_d]/N) - 1)/2]$
 $-hQE[p]E[m_2]/2.$ (15)

We calculate the optimal value of Q^* can be obtained by considering ETPU(Q). The first derivative of Eq. (15) with respect to Q as follow

$$ETPU'(Q) = K / \left(\left(Q^2 (1 - E[m_1])(1 - E[p]) / N \right) E[T_d] \right) - h \left[(1 - E[p]) E[m_1] + E[p](1 - E[m_2]) \right] / \left(x \left(\left((1 - E[m_1])(1 - E[p]) / N \right) E[T_d] \right) \right) - h \left[1 - \left[(1 - E[p]) E[m_1] + E[p](1 - E[m_2]) \right] - \left((1 - E[m_1])(1 - E[p]) \right) / 2 \right] - h E[p] E[m_2] / 2.$$
(16)

The second derivative of Eq. (15) with respect to Q, we obtain

$$ETPU''(Q) = -K / \left(\left(Q^3 (1 - E[m_1])(1 - E[p]) / N \right) E[T_d] \right).$$
(17)

Since K > 0, Q > 0, $E[T_d] > 0$, N > 0, 0 < E[p] < 1 and $0 < E[m_1] < 1$ then ETPU''(Q) < 0 such that ETPU(Q) is a concave function.

Hence, ETPU(Q) is concave. We can find the optimal value Q^* by maximizing of ETPU(Q). Setting the first derivative of ETPU(Q) with respect to Q equal to zero and solving yields the solution

$$Q^{*} = \sqrt{\frac{KN}{\left(A_{1}E[T_{d}]\right)\left(h\left(\frac{N\left(A_{2}+A_{3}\right)}{(xA_{1}E[T_{d}])}+1-A_{2}-A_{3}-\frac{A_{1}}{2}+\frac{A_{4}}{2}\right)\right)}}$$
(18)

where $A_1 = (1 - E[m_1])(1 - E[p]), A_2 = (1 - E[p])E[m_1], A_3 = E[p](1 - E[m_2]) \text{ and } A_4 = E[p]E[m_2].$

The corresponding the number of sales in cycle length, say B^* , is

$$B^* = Q^* (1 - E[m_1])(1 - E[p]) / N$$
⁽¹⁹⁾

In case B^* is not an integer, one has to compute both $ETPU([B^*])$ and $ETPU([B^*])$, and take one of $[B^*]$ (the smallest integer greater than or equal to B^*) and $[B^*]$ (the largest integer less than or equal to B^*) for which $ETPU(\cdot)$ is upper.

For the special case, when $D = N/E[T_d]$, by Eq. (18), $p \to 0$, $m_1 \to 0$ and $m_2 \to 0$. Then, we have

$$Q^* = \sqrt{\frac{2KN}{E[T_d]h}} = \sqrt{\frac{2KD}{h}}.$$
(20)

This is the well-known EOQ formula. Also, we get

$$B^* = Q^* / N \, .$$

(21)

3. NUMERICAL EXAMPLE

In this section, we consider the inventory system that replenishes the orders instantly, the items with imperfect quality and inspection errors. The probability density function for the fraction of defective items is generally taken from the history of a supplier and worker. We adjust the data from Salameh and Jaber [16] for this numerical study. We illustrate the model developed in this paper by the following numerical example.

$$N = 274 \text{unit}$$

$$x = 175200 \text{unit/year}$$

$$c = \$25/\text{unit}$$

$$v = \$20/\text{unit}$$

$$s = \$50/\text{unit}$$

$$d = \$0.5/\text{unit}$$

$$h = \$5/\text{unit}$$

$$c_a = \$500/\text{unit}$$

$$c_r = \$100/\text{unit}$$

$$f(p) = \begin{cases} 25, 0 \le p \le 0.05 \\ 0, \text{ otherwise} \end{cases} \Rightarrow E[p] = 0.02$$

$$f(T_d) = \begin{cases} 0.25, 0 \le T_d \le 4 \\ 0, \text{ otherwise} \end{cases} \Rightarrow E[T_d] = 2$$

$$f(m_1) = \begin{cases} 25, 0 \le m_1 \le 0.05 \\ 0, \text{ otherwise} \end{cases} \Rightarrow E[m_1] = 0.02$$

$$f(m_2) = \begin{cases} 25, 0 \le m_2 \le 0.05 \\ 0, \text{ otherwise} \end{cases} \Rightarrow E[m_2] = 0.02.$$

From above data, the optimal value Q^* and $ETPU(Q^*)$ are obtained by Eq. (18) and Eq. (15), respectively. We obtain the following optimal solution $Q^* = 1,454$ units and B^* from Eq. (19) as 5.097. We have $ETPU(Q^*) = 1,094,603.88$ /year. These results are also shown in Figure 2.

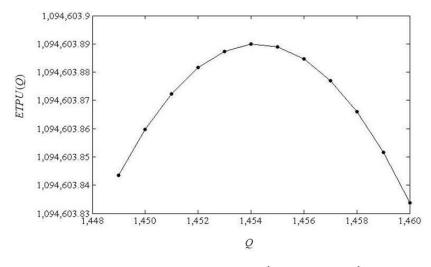


Figure 2. The optimal values of Q^* and $ETPU(Q^*)$.

Figure 2 demonstrates that the optimal values of Q^* is 1,454 units and $B^* = 5.097$. Since the value of B^* should be integer, we get $ETPU(\lfloor B^* \rfloor = 5) = 1,115,903.14$ and $ETPU(\lceil B^* \rceil = 6) = 929,896.35$; so we choose $B^* = \mid B^* \mid =5$.

The replenishment policy for the group-buying inventory system would be as follows. For the replenishment cycle, we obtain the order quantity 1,454 units. The products will be sold all 5 times, each time being sold per 274 units by takes an average about 2 days. Hence, the seller will be obtained the profit about \$1,115,903.14 per year.

We consider the maximum ETPU(Q) with the selected value of each the fraction of defective while a fixed level of both inspection errors $(m_1 \text{ and } m_2)$ as shown in Figure 3.

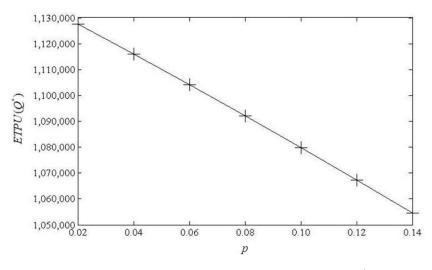


Figure 3. The relationship between p and $ETPU(Q^*)$.

As depicted in Figure 3, this curve is obtained by varying the upper bound of the uniform distribution of the fraction of defectives at 0.02, 0.04, 0.06, 0.08, 0.10, 0.12 and 0.14. Clearly, the expected total profit will be decreased when the fraction of defective increases.

4. CONCLUSION

In this research, we formulate and solve a problem to define the group-buying inventory model with imperfect quality under inspection errors by avoiding a shortage. We determine the duration of the deal, a fraction of defective and probability of misclassification are distributed as uniform distribution. We can calculate the optimal solution $ETPU(Q^*)$, Q^* and B^* . The results from numerical example illustrate that the fraction of defective affects the seller's profit. The expected total profit per unit will decrease when the fraction of defective increases. Additional potential researchers of the group-buying inventory model improve the condition of having a product shortage, backorder, varying cost etc.

ACKNOWLEDGMENTS

This study is supported by the Science Achievement Scholarship of Thailand (SAST).

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

REFERENCES

- [1] Anand, K. S. and Aron, R., "Group buying on the web: a comparison of price-discovery mechanisms", Management Science, 49: 1546-1562, (2003).
- [2] Chen, J., Chen, X. and Song, X., "Comparison of the group-buying auction and the fixed-pricing mechanism", Decision Support Systems, 43: 445-459, (2007).
- [3] Chen, J., Kauffman, R. J., Liu, Y. and Song, X., "Segmenting uncertain demand in group-buying auction", Electronic Commerce Research and Applications, 9: 126-147, (2010).
- [4] Chen, J., Liu, Y. and Song, X., "Comparison of group-buying online auction and posted pricing mechanism in uncertain market", Journal of Electronic Science and Technology of China, 2: 1-5, (2004).
- [5] Chen, Z., Liang, X., Wang, H. and Yan, H., "Inventory rationing with multiple demand classes: The case of group buying", Operations Research letters, 40: 404-408, (2012).
- [6] Edelman, B., Jaffe, S. and Kominers, S. D., "To Groupon or Not to Groupon: The profitability of deep discounts", Harvard Business School, Working Paper, (2014).
- [7] Goyal, S. K. and Cardenas-Barron, L. E., "Note on: Economic production quantity model for items with imperfect quality-a practical approach", International Journal of Production Economics, 77: 85– 87, (2002).
- [8] Hsu, J.T. and Hsu, L.F. "An EOQ model with imperfect quality items, inspection errors, shortage backordering, and sales returns", International Journal of Production Economics, 143: 162–170, (2013).
- [9] Jing, X. and Xie, J., "Group-Buying: A New Mechanism for Selling through Social Interactions", Management Science, 57: 1354-1372, (2011).
- [10] Khan, M., Jaber, M.Y. and Ahmad, A. "An integrated supply chain model with errors in quality inspection and learning in production". Omega, 42: 16-24, (2014).
- [11] Khan, M., Jaber, M. Y. and Bonney, M., "An economic order quantity (EOQ) for items with imperfect quality and inspection errors", International Journal of Production Economics, 133: 113–118, (2011).
- [12] Kleebmek, T., Boonyued, A. and Srisodaphol, W., "Group-buying inventory policy with demand under Poisson process", Songklanakarin Journal Science and Technology, 38: 113-117, (2016).
- [13] Liang, X., Ma, L., Xie, L. and Yan, H., "The informational aspect of the group-buying mechanism", European Journal of Operational Research, 234: 331-340, (2014).
- [14] Maddah, B. and Jaber, M. Y., "Economic order quantity for items with imperfect quality: Revisited", International Journal of Production Economics, 112: 808–815, (2008).
- [15] Porteus, E. L., "Optimal lot sizing process quality improvement and setup cost reduction", Operation Research, 34: 137-144, (1986).
- [16] Salameh, M. K. and Jaber, M. Y., "Economic production quantity model for items with imperfect quality", International Journal of Production Economics, 64: 59–64, (2000).

[17] Sarkar, B. and Sharmila, S. "Product inspection policy for an imperfect production system with inspection errors and warranty cost", European Journal of Operation Research, 248: 263-271, (2016).