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Combination of Ratio and Regression Estimator of Population Mean in Presence of Non–response

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Abstract

Singh and Kumar [1] developed regression-cum-ratio estimator for population mean in the presence of non–response for two-phase sampling using information of two auxiliary variables X and Z. In this paper simple generalized regression-cum-ratio estimator in the presence of non–response has been presented with three different situations. Empirical and theoretical study is carried out to compare the efficiency of the suggested estimators over the existing estimators.

1. INTRODUCTION

It has been noted that non-response creates problem mostly in all types of surveys and this cannot be eliminated by simply increasing the sample size. Non-response may be non-ignorable or ignorable which depends whether it is correlated with the target variable or not Little [2] and Glynn *et al.* [3]. Non-response is caused when surveying human population hesitates to respond in surveys and this is increased when sensitive issues are discussed in surveys. It has been observed that non-response increases bias in estimates which is main reason of reducing efficiency of the surveys. Many methods have been considered by different survey statisticians for estimating the population characteristics in the presence of non–response for two-phase sampling. In this aspect the sub-sampling method has been a popular one. We now are going to discuss sub-sampling method along with some popular estimators in the presence of non-response for two-phase sampling.

Suppose a sample of size n is drawn from a population of size N by simple random sample without replacement (SRSWOR). From the n sample, r_1 sample units respond to survey variable Y and r_2 are non-respondent. The population divided in respondents and non-respondent of containing N_1 and N_2 units corresponding to sample respondents and non–respondents. From r_2 non–respondents unit, a sub-sample of k ($k=r_2/h$, $h>1$) sample units is drawn and obtained information about study variable from these k units. The pioneers of this area are Hansen and Hurwitz [4]. They proposed an unbiased estimator of population mean by using sub-sampling technique to overcome the problem of non-response:

$$\bar{y}^* = (r_1/n)\,\bar{y}_{r_1 1} + (r_2/n)\,\bar{y}_{k2}, \tag{1}$$

where $\bar{y}_1 = r_1^{-1}\sum_{i=1}^{r_1} y_i$ and $\bar{y}_{k2} = k^{-1}\sum_{i=1}^{k} y_i$ are means of study variable y. The estimator (1) is unbiased:

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$$Var\left(\bar{y}^*\right) = \lambda S_y^2 + \theta S_{y_2}^2 , \tag{2}$$

where $S_y^2 = \sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2 \Big/ \left(N-1\right)$, $S_{y_2}^2 = \sum_{i=1}^{N_2}\left(y_i - \bar{Y}_2\right)^2 \Big/ \left(N_2 - 1\right)$, $\lambda = \left(1-f\right)/n$, $f = n/N$, $\theta = W_2\left(h-1\right)/n$, $W_2 = N_2/N$, $\bar{Y} = N^{-1}\sum_{i=1}^{N} y_i$ and $\bar{Y}_2 = N_2^{-1}\sum_{i=1}^{N_2} y_i$. Some important and notable references are about in this area are of Cochran [5], Rao [6], Naik and Gupta [7], Khare and Srivastava [8], Khare and Srivastava [9], Tripathi and Khare [10], Tabasum and Khan [11], Singh and Kumar [12], Khare and Srivastava [13], Ismail et al.[14] and Ismail et al.[15]

2. NON-RESPONSE IN TWO-PHASE SAMPLING

The two- phase sampling procedure in the presence of non–response is described as follows:

i) Draw a sample of size n_1 from population of size N by using simple random sampling without replacement (SRSWOR) and record information about the auxiliary variables say X and Z. It is first phase sample.

ii) Draw another sample of size n_2 from the first phase sample of size n_1 by using simple random sampling without replacement (SRSWOR). Suppose r_1 elements are responding and r_2 elements are non-respondents. Collect information on variable of interest Y from responding elements.

iii) Draw a sub-sample of size k where ($k=r_2/h$, $h>1$) and collect information on study variable Y from these selected elements.

Using two-phase sampling techniques, many surveys statisticians suggested different estimator for estimating the population mean in the presence of non–response in two phase sampling. Generally when the population mean of auxiliary variables x and z are known, estimators have been proposed by considering following two situations.

Situation 1: The population mean of both auxiliary variables x and z, are unknown, and incomplete information on study variable y, and incomplete information x and z.

Situation 2: The population mean of both auxiliary variables x and z, are known, and incomplete information on study variable y, and complete information x and z.

Situation 3: We propose a new situation when the population mean of both auxiliary variables x and z, are known, incomplete information is available on study variable and incomplete information is one auxiliary variable x and complete information in z.

Cochran 5 by using situation 1 proposed regression estimator i.e.

$$t_c = \left[\bar{y}^* + \hat{\beta}_{yx}^*\left(\bar{X} - \bar{x}^*\right)\right], \tag{3}$$

The mean square error of (3) is

$$MSE\left(t_c\right) \approx \lambda_2\left(S_y^2 + \beta_{yx}^2 S_x^2 - 2\beta_{yx}S_{xy}\right) + \theta\left(S_{y(2)}^2 + \beta_{yx}^2 S_{x(2)}^2 - 2\beta_{yx}S_{xy(2)}\right). \tag{4}$$

Rao 6 by using situation 2 proposed regression estimator i.e.

$$t_R = \left[\bar{y}^* + \hat{\beta}_{yx}\left(\bar{X} - \bar{x}\right)\right], \tag{5}$$

The mean square error of (5) is:

$$MSE\left(t_R\right) \approx \lambda_2\left(S_y^2+\beta_{yx}^2 S_x^2-2\beta_{yx}S_{xy}\right)+\theta S_{y(2)}^2. \quad (6)$$

Singh and Kumar 1 by using the above two situations proposed first following regression-cum-ratio estimator for Situation 1:

$$t_{(1)sk} = \left\{\bar{y}^* + \hat{\beta}_{yx}^*\left(\bar{X}-\bar{x}^*\right)\right\}\left(\frac{\bar{Z}}{\bar{Z}+\alpha_{sk}\left(\bar{z}^*-\bar{Z}\right)}\right). \quad (7)$$

where $\bar{x}^* = w_1\bar{x}_1 + w_1\bar{x}_{k2}$ and $\bar{z}^* = w_1\bar{z}_1 + w_1\bar{z}_{k2}$ are unbiased estimator of the population mean $\bar{X}$ and $\bar{Z}$ respectively

$$\hat{\beta}_{xy}^* = (s_{xy}^*/s_x^{*2}),\ s_{xy}^* = \frac{1}{n_1-1}\left(\sum_{i=1}^{n_1}x_iy_i + h\sum_{i=1}^{k}x_iy_i - n_1\overline{xy}^*\right), s_x^{*2} = \frac{1}{n_1-1}\left(\sum_{i=1}^{n_1}x_i^2 + h\sum_{i=1}^{k}x_i^2 - n_1\overline{xx}^*\right).$$

In (7) α_{sk} is a suitably chosen scalar.

The mean square error of (7) up to first degree approximation is

$$MSE\left(t_{(1)sk}\right) \approx \lambda_2\left\{S_y^2\left(1-\rho_{xy}^2\right)+\alpha_{sk}R_2\left(\alpha_{sk}R_2-2A\right)S_z^2\right\}$$
$$+\ \theta\left\{S_{y(2)}+\beta_{yx}S_{x2}^2\left(\beta_{yx}-2\beta_{yx2}\right)+\alpha_{sk}R_2\left(\alpha_{sk}R_2-2B\right)S_{z2}^2\right\}, \quad (8)$$

where $\lambda_2 = n_2^{-1}-N^{-1}$, $\lambda_1 = n_1^{-1}-N^{-1}$, $R_1 = \bar{Y}/\bar{X}$, $R_2 = \bar{Y}/\bar{Z}$,

$A = \left(\beta_{yz}-\beta_{yx}\beta_{xz}\right), B = \left(\beta_{yz(2)}-\beta_{yx}\beta_{xz(2)}\right)$

$C_y = S_y/\bar{Y}, C_{y2} = S_{y2}/\bar{Y}, C_x = S_x/\bar{X}, C_{x2} = S_{x2}/\bar{X}$ and $C_z = S_z/\bar{Z}, C_{z2} = S_{z2}/\bar{Z}$.

$\rho_{xy} = S_{xy}/S_xS_y, \rho_{xy2} = S_{xy2}/S_{x2}S_{y2}, \rho_{yz} = S_{yz}/S_zS_y, \rho_{zy2} = S_{yz2}/S_{z2}S_{y2}$ and
$\rho_{xz} = S_{xz}/S_xS_z, \rho_{xz2} = S_{xz2}/S_{x2}S_{z2}$.

$\beta_{yx} = S_{yx}/S_x^2, \beta_{yx2} = S_{yx2}/S_{x2}^2, \beta_{yz} = S_{yz}/S_z^2, \beta_{yz2} = S_{yz2}/S_{z2}^2$ and
$\beta_{xz} = S_{xz}/S_z^2, \beta_{xz2} = S_{xz2}/S_{z2}^2$.

$S_x^2 = \sum_{i=1}^{N}\left(x_i-\bar{X}\right)^2/\left(N-1\right), S_{x_2}^2 = \sum_{i=1}^{N_2}\left(x_i-\bar{X}_2\right)^2/\left(N-1\right), \bar{X}_2 = N_2^{-1}\sum_1^{N_2}x_i$,
$S_{xy} = \sum_{i=1}^{N}\left(x_i-\bar{X}\right)\left(y_i-\bar{Y}\right)/\left(N-1\right)$ and $S_{xy(2)} = \sum_{i=1}^{N_2}\left(x_i-\bar{X}_2\right)\left(y_i-\bar{Y}_2\right)/\left(N_2-1\right)$.

$S_z^2 = \sum_{i=1}^{N}\left(z_i-\bar{Z}\right)^2/\left(N-1\right), S_{z_2}^2 = \sum_{i=1}^{N_2}\left(z_i-\bar{Z}_2\right)^2/\left(N-1\right), Z_2 = N_2^{-1}\sum_1^{N_2}z_i$,
$S_{xz} = \sum_{i=1}^{N}\left(x_i-\bar{X}\right)\left(z_i-\bar{Z}\right)/\left(N-1\right)$ and $S_{xz(2)} = \sum_{i=1}^{N_2}\left(x_i-\bar{X}_2\right)\left(z_i-\bar{Z}_2\right)/\left(N_2-1\right)$.

$S_{yz} = \sum_{i=1}^{N}\left(z_i-\bar{Z}\right)\left(y_i-\bar{Y}\right)/\left(N-1\right)$ and $S_{yz(2)} = \sum_{i=1}^{N_2}\left(z_i-\bar{Z}_2\right)\left(y_i-\bar{Y}_2\right)/\left(N_2-1\right)$.

Now the optimum value of α_{sk} is

$$\alpha_{sk} = \left\{N^* / \left(R^* D^*\right)\right\}. \tag{9}$$

where $$N^* = \left\{\left(\frac{1-f}{n_2}\right) A S_z^2 + \left(\frac{W_2(h-1)}{n_2}\right) B S_{z_2}^2\right\}, \tag{10}$$

$$D^* = \left\{\left(\frac{1-f}{n_2}\right) S_z^2 + \left(\frac{W_2(h-1)}{n_2}\right) S_{z_2}^2\right\} \tag{11}$$

By putting the values of (10) and (11) in (9), and simplify, we get

$$\alpha_{sk} = \frac{\left\{\lambda_2\left(\beta_{yz} - \beta_{yx}\beta_{xz}\right) S_z^2 + \theta\left(\beta_{zy2} - \beta_{yx}\beta_{xz2}\right) S_{z_2}^2\right\}}{R_2\left(\lambda_2 S_z^2 + \theta S_{z_2}^2\right)} \tag{12}$$

Secondly Singh and Kumar [1] also proposed following regression-cum-ratio estimator for Situation 2:

$$t_{(2)sk} = \left\{\bar{y}^* + \hat{\beta}_{yx}\left(\bar{X} - \bar{x}\right)\right\}\left(\frac{\bar{Z}}{\bar{Z} + \alpha_{sk}^*\left(\bar{z} - \bar{Z}\right)}\right). \tag{13}$$

where α_{sk}^* is a suitably chosen scalar.

They considered the situation in which population means $\bar{X}$ and $\bar{Z}$ are known and information on auxiliary variables x and z are obtained from all the sample units n_1.

But few units are failed to supply information on the study variable y so we use $(r1 + k)$ responding units on the study variable y .and we have complete information on the both auxiliary variables x and z . So the estimator that they defined as:

The mean square error of (13) up to first degree approximation is

$$MSE\left(t_{(2)sk}\right) \approx \lambda_2\left\{S_y^2\left(1-\rho_{xy}^2\right) + \alpha_{sk}^* R_2\left(\alpha_{sk}^* R_2 - 2A\right) S_z^2\right\} + \theta S_{y(2)}^2. \tag{14}$$

The optimum value of α_{sk}^* is

$$\alpha_{sk}^* = \left\{A / R_2\right\}. \tag{15}$$

3. THE PROPOSED ESTIMATORS

In this section we have suggested an estimator of population mean for two phase sampling in the presence of non–response. The estimators have been suggested for two different situations discussed in Section 2.

3.1 Proposed Estimators for Situation 1

In incomplete information case we now propose following combination of regression- cum- ratio estimator for Situation 1:

$$t_{(1)d}=\left[\bar{y}^{*}+\hat{\beta}_{yx}^{*}\left(\bar{X}-\bar{x}^{*}\right)\right]\left(\frac{\bar{z}^{*}}{\bar{Z}}\right)^{\alpha_d}, \quad (16)$$

where α_d is a suitably chosen scalar. Using $\bar{y}^{*}=\bar{Y}+\bar{e}_y^{*}$, $\bar{x}^{*}=\bar{X}+\bar{e}_x^{*}$ and $\bar{z}^{*}=\bar{Z}+\bar{e}_z^{*}$ the above estimator can be written as:

$$t_{(1)d}-\bar{Y}=\bar{e}_y^{*}-\hat{\beta}_{yx}^{*}\bar{e}_x^{*}+\alpha_d R_2\bar{e}_z^{*}.$$

Squaring above equation, applying expectation and using following results:

$$\left.\begin{array}{l} E\left(\bar{e}_y^{*2}\right)=\lambda_2 S_y^2+\theta S_{y_2}^2\ ;\ E\left(\bar{e}_x^{*2}\right)=\lambda_2 S_x^2+\theta S_{x_2}^2\ ;E\left(\bar{e}_z^{*2}\right)=\lambda_2 S_z^2+\theta S_{z_2}^2 \\ E\left(\bar{e}_y^{*}\bar{e}_x^{*}\right)=\lambda_2 S_{xy}+\theta S_{xy(2)}\ ;E\left(\bar{e}_y^{*}\bar{e}_z^{*}\right)=\lambda_2 S_{yz}+\theta S_{yz(2)}\ ;E\left(\bar{e}_x^{*}\bar{e}_z^{*}\right)=\lambda_2 S_{xz}+\theta S_{xz(2)} \end{array}\right\} \quad (17)$$

The mean square error of (16) may be written as:

$$MSE\left(t_{(1)d}\right)\approx\lambda_2\left(S_y^2+\beta_{yx}^2S_x^2+\alpha_d^2R_2^2S_z^2-2\beta_{yx}S_{xy}+2\alpha_d R_2S_{zy}-2\alpha_d\beta_{yx}R_2S_{xz}\right)$$
$$+\theta\left(S_{y_2}^2+\beta_{yx}^2S_{x_2}^2+\alpha_d^2R_2^2S_{z_2}^2-2\beta_{yx}S_{xy2}+2\alpha_d R_2S_{zy2}-2\alpha_d\beta_{yx}R_2S_{xz2}\right). \quad (18)$$

Now the optimum value of α_d is

$$\alpha_d=\frac{-\lambda_2\left(\beta_{yz}-\beta_{yx}\beta_{xz}\right)S_z^2-\theta\left(\beta_{zy2}-\beta_{yx}\beta_{xz2}\right)S_{z_2}^2}{R_2\left(\lambda_2S_z^2+\theta S_{z_2}^2\right)}$$

or

$$\alpha_d=-\left\{\frac{\left\{\lambda_2\left(\beta_{yz}-\beta_{yx}\beta_{xz}\right)S_z^2+\theta\left(\beta_{zy2}-\beta_{yx}\beta_{xz2}\right)S_{z_2}^2\right\}}{R_2\left(\lambda_2S_z^2+\theta S_{z_2}^2\right)}\right\}$$

By putting (12), we have

$$\alpha_d=-\left(\alpha_{sk}\right) \quad (19)$$

3.2 Proposed Estimator for Situation 2

In case of incomplete information is study variable and complete information on auxiliary variables we now propose regression –cum-ratio estimator for Situation 2:

$$t_{(2)d}=\left[\bar{y}^{*}+\hat{\beta}_{yx}\left(\bar{X}-\bar{x}\right)\right]\left(\frac{\bar{z}}{\bar{Z}}\right)^{\alpha_d^{*}}, \quad (20)$$

where α_d^{*} is a suitably chosen scalar. Using the relation $\bar{y}^{*}=\bar{Y}+\bar{e}_y^{*}$, $\bar{x}=\bar{X}+\bar{e}_x$ and $\bar{z}=\bar{Z}+\bar{e}_z$ we may write (20) as:

$$t_{(2)d}-\bar{Y}=\bar{e}_y^{*}-\hat{\beta}_{yx}\bar{e}_x+\alpha_d^{*}R_2\bar{e}_z.$$

Squaring above equation, applying expectation and using following results:

$$\left.\begin{aligned} &E\left(\bar{e}_y^{*2}\right)=\lambda_2 S_y^2+\theta S_{y_2}^2\ ;\ E\left(\bar{e}_x^2\right)=\lambda_2 S_x^2 ; E\left(\bar{e}_z^2\right)=\lambda_2 S_z^2 ; \\ &E\left(\bar{e}_y^*\bar{e}_x\right)=\lambda_2 S_{xy} ; E\left(\bar{e}_y^*\bar{e}_z\right)=\lambda_2 S_{zy} ; E\left(\bar{e}_x\bar{e}_z\right)=\lambda_2 S_{xy} \end{aligned}\right\}. \tag{21}$$

The mean square error of (20) may be written as:

$$MSE\left(t_{(2)d}\right)\approx\ \lambda_2\left(S_y^2+\beta_{yx}^2 S_x^2+\alpha_d^{*2}R_2^2 S_z^2-2\beta_{yx}S_{xy}\right.$$
$$\left.+2\alpha_d^* R_2 S_{zy}-2\alpha_d^*\beta_{yx}R_2 S_{xz}\right)+\ \theta S_{y_2}^2 \tag{22}$$

Now the optimum value of α_d^* is

$$\alpha_d^*\ =\ -(\ \alpha_{sk}^*\ ) \tag{23}$$

3.3 Proposed Estimator for New Proposed Situation 3

Now in this section we propose a new situation when the population mean of both auxiliary variables x and z , are known, incomplete information is available on study variable and incomplete information is one auxiliary variable *x* and complete information in *z*. The proposed regression –cum-ratio estimator for Situation 3:

$$t_{(3)d}\ =\left[\bar{y}^*\ +\hat{\beta}_{yx}^*\left(\bar{X}-\bar{x}^*\right)\right]\left(\frac{\bar{z}}{\bar{Z}}\right)^{\alpha_6'}, \tag{24}$$

where α_d' is a suitable scalar.

The mean square error of (24) may be written as:

$$MSE\left(t_{(3)d}\right)\approx\lambda_2\left(S_y^2+\beta_{yx}^2 S_x^2+\alpha_d'^2 R_2^2 S_z^2-2\beta_{yx}S_{xy}+2\alpha_d' R_2 S_{zy}-2\alpha_d'\beta_{yx}R_2 S_{xz}\right)$$
$$+\ \theta\left(S_{y(2)}^2+\beta_{yx}^2 S_{x(2)}^2-2\beta_{yx}S_{xy(2)}\right). \tag{25}$$

4. SPECIAL CASES OF PROPOSED ESTIMATORS

In this section we propose some special cases of simple generalized ratio-cum-regression estimators under different situations.

4.1 Special Cases of Generalized Ratio-cum-Regression Estimator under Situation 1.

If we put α_d= 0, in (16) it becomes Cochran's estimator [5] given in (3)

If we put α_d = 0, in (18), we get mean square error of Cochran's estimator 5 given in (4). If we put α_d = 1, in (16) it reduce to a new estimator

$$t_{nd(1)}\ =\left[\bar{y}^*\ +\hat{\beta}_{yx}^*\left(\bar{X}-\bar{x}^*\right)\right]\left(\frac{\bar{z}^*}{\bar{Z}}\right), \tag{26}$$

The mean square error of (26), can be easily obtained from (18), by putting α_d = 1, so the mean square error of (26) is

$$MSE\left(t_{nd(1)}\right) \approx \lambda_2\left(S_y^2 + \beta_{yx}^2 S_x^2 + S_z^2 - 2\beta_{yx}S_{xy} + 2R_2 S_{zy} - 2\beta_{yx}R_2 S_{xz}\right)$$

$$+\ \theta\left(S_{y(2)}^2 + \beta_{yx}^2 S_{x(2)}^2 + R_2^2 S_{z(2)}^2 - 2\beta_{yx}S_{xy(2)} + 2R_2 S_{zy(2)} - 2\beta_{yx}R_2 S_{xz(2)}\right). \quad (27)$$

4.2 Special Cases of Generalized Ratio-cum-Regression Estimator under Situation 2.

If we put α_d^* = 0, in (20) it becomes Rao's estimator 6 given in (5) and by putting α_d^* = 0, in (9) we get mean square error of Rao's estimator 6 given in (6) and

if we put α_d^* = 1, in (20) it reduces to a new estimator

$$t_{nd(2)} = \left[\bar{y}^* + \hat{\beta}_{yx}\left(\bar{X} - \bar{x}\right)\right]\left(\frac{\bar{z}}{\bar{Z}}\right), \quad (28)$$

The mean square error of (28), can be easily obtained from (22), by putting α_d^* = 1,

$$MSE\left(t_{nd(2)}\right) \approx \lambda_2\left(S_y^2 + \beta_{yx}^2 S_x^2 + R_2^2 S_z^2 - 2\beta_{yx}S_{xy} + 2R_2 S_{zy} - 2\beta_{yx}R_2 S_{xz}\right) + \theta S_{y(2)}^2. \quad (29)$$

5. THEORETICAL COMPARISON FOR THE PROPOSED ESTIMATORS

In this section we compare our proposed estimators with Singh and Kumar 1.

The comparison of proposed estimator with Singh and Kumar 1 in situation 1 is:

$$MSE\left(t_{(1)sk}\right) - MSE\left(t_{(1)d}\right) \approx \lambda_2\left(-2\alpha_{sk}\mathrm{R}_2\mathrm{S}_{\mathrm{zy}} + 2\alpha_{sk}\mathrm{R}_2\beta_{yx}\mathrm{S}_{\mathrm{xz}} - 2\alpha_d R_2 S_{zy} + 2\alpha_d\beta_{yx}R_2 S_{xz}\right)$$

$$+\ \theta\left(-2\alpha_{sk}\mathrm{R}_2\mathrm{S}_{\mathrm{zy2}} + 2\alpha_{sk}\mathrm{R}_2\beta_{yx}\mathrm{S}_{\mathrm{xz2}} - 2\alpha_d R_2 S_{zy2} + 2\alpha_d\beta_{yx}R_2 S_{xz2}\right) \quad (30)$$

We see that if we put $\alpha_d = -\left(\alpha_{sk}\right)$ in (30) then both estimators coincide and the $MSE\left(t_{(1)sk}\right) - MSE\left(t_{(1)D}\right) = 0.$ The comparison of proposed estimator with Singh and Kumar 1 in situation 2 is:

$$MSE\left(t_{(2)sk}\right) - MSE\left(t_{(2)d}\right) = \lambda_2\left(-2\alpha_{sk}^*\mathrm{R}_2\mathrm{S}_{\mathrm{zy}} + 2\alpha_{sk}^*\mathrm{R}_2\beta_{yx}\mathrm{S}_{\mathrm{xz}} - 2\alpha_d^* R_2 S_{zy} + 2\alpha_d^*\beta_{yx}R_2 S_{xz}\right). \quad (31)$$

again we see that if we put $\alpha_d^* = -(\alpha_{sk}^*)$ in (31) then $MSE\left(t_{(2)sk}\right) - MSE\left(t_{(2)d}\right) = 0$.

6. NUMERICAL COMPARISON FOR THE PROPOSED ESTIMATORS

For numerical comparison we using the data that earlier consider by Khare and Sinha [16] and Singh and Kumar 1. The description of the population data is given below:

The data on physical growth of upper socioeconomic group of 95 school children of Varanasi under an ICMR study, Department of pediatrics, B.H.U., during 1983-1984 has been taken under study. The first 25% units have been considered as non responding units. Let us consider the study and auxiliary variable as follows:

y : weight in kg of the children,

x : skull circumference in cm of the children,

z : chest circumference in cm of the children.

$\bar{Y} = 19.4968,$ $C_y = 0.15613,$ $C_{y_2} = 0.12075,$

$\bar{X} = 51.1726,$ $C_x = 0.03006,$ $C_{x_2} = 0.02478,$

$\bar{Z} = 55.8611,$ $C_z = 0.05860,$ $C_{z_2} = 0.05402,$

$\rho_{yx} = 0.328,$ $\rho_{yx_2} = 0.477,$ $\rho_{yz_2} = 0.729,$

$\rho_{yz} = 0.846,$ $\rho_{xz_2} = 0.570$ $\rho_{xz} = 0.297,$

$N = 95$, n=35, W_2=0.25,

Table 1: Mean square error of the estimators for different values of h

		$(1/h)$			
Estimators		**(1/5)**	**(1/4)**	**(1/3)**	**(1/2)**
Case 1	α_{sk}	**0.56333**	**0.39844**	**0.26198**	**0.15393**
	α_d	**-0.56333**	**-0.39844**	**-0.26198**	**-0.15393**
	$MSE\left(t_{(1)sk}\right)$	**0.25915**	**0.24463**	**0.22015**	**0.18564**
	$MSE\left(t_{(1)d}\right)$	**0.25915**	**0.24463**	**0.22015**	**0.18564**
Case 2	α_{sk}^*	**1.994478**	**1.994478**	**1.994478**	**1.994478**
	α_d^*	**-1.994478**	**-1.994478**	**-1.994478**	**-1.994478**
	$MSE\left(t_{(2)sk}\right)$	**0.286161**	**0.228486**	**0.170812**	**0.113137**
	$MSE\left(t_{(2)d}\right)$	**0.286161**	**0.228486**	**0.170812**	**0.113137**

It is observed that from table 1 that both estimators in case 1 and case 2 are equal, but one can observe that the proposed estimator is simple and easy understandable.

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