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CHARACTERIZATION OF b-OPEN SOFT SETS IN SOFT TOPOLOGICAL SPACES

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Abstract – In this paper, a new class of open soft sets in a soft topological space, called b-open soft sets, is introduced and studied. Moreover, the relations this class and these different types of subsets of soft topological spaces, which introduced in [9], is studied. In particular, this class is contained in the class of β-open soft sets and contains the classes of open soft sets, pre open soft sets, semi open soft sets and α-open soft sets. Also, the authors introduce the concept of b-continuous soft functions and study some of their properties in detail. As a consequence the relations of some soft continuities are shown in a diagram.

Keywords – *Soft set, Soft topological space, Pre-open soft set, α-open soft set, Semi-open soft set, β-open soft set, b-open soft sets.*

1 Introduction

The concept of soft sets was first introduced by Molodtsov [22] in 1999 as a general mathematical tool for dealing with uncertain objects. In [22, 23], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions,game theory, operations research,Riemann integration, Perron integration, probability, theory of measurement, and so on.
After presentation of the operations of soft sets [20], the properties and applications of soft set theory have been studied increasingly [4, 17, 23, 25]. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [2, 3, 5, 18, 19, 20, 21, 23, 24, 28]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [6].
Recently, in 2011, Shabir and Naz [26] initiated the study of soft topological spaces. They defined soft topology on the collection τ of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as open and closed soft sets, soft subspace, soft closure, soft nbd of a point, soft separation axioms, soft regular spaces and soft normal spaces and established their several properties. In [9], Kandil et. al. introduced some soft operations such as semi open soft, pre open soft, α-open soft and β-open soft and investigated their properties in detail. Studies on the soft topological spaces have been accelerated [7, 8, 10, 11, 12, 13, 14, 15, 16, 27].

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The main purpose of this paper, is to introduce a new class of open soft sets in a soft topological space, called b-open soft sets, to soft topological spaces. Also, the relations this class and these different types of subsets of soft topological spaces is studied. Moreover, the authors introduced the concept of b-continuous soft functions and study some of their properties in detail.

2 Preliminary

In this section, we present the basic definitions and results of soft set theory which will be needed in the sequel.

Definition 2.1. [22] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F, A) denoted by F_A is called a soft set over X , where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parametrized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F, A) and if $e \notin A$, then $F(e) = \phi$ i.e
$F_A = \{F(e) : e \in A \subseteq E,\ F : A \to P(X)\}$. The family of all these soft sets over X denoted by $SS(X)_A$.

Definition 2.2. [20] Let $F_A,\ G_B \in SS(X)_E$. Then F_A is soft subset of G_B, denoted by $F_A \tilde{\subseteq} G_B$, if

(1) $A \subseteq B$, and

(2) $F(e) \subseteq G(e), \forall e \in A$.

In this case, F_A is said to be a soft subset of G_B and G_B is said to be a soft superset of F_A, $G_B \tilde{\supseteq} F_A$.

Definition 2.3. [20] Two soft subset F_A and G_B over a common universe set X are said to be soft equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A.

Definition 2.4. [4] The complement of a soft set (F, A), denoted by $(F, A)'$, is defined by $(F, A)' = (F', A)$, $F' : A \to P(X)$ is a mapping given by $F'(e) = X - F(e),\ \forall\, e \in A$ and F' is called the soft complement function of F.
Clearly $(F')'$ is the same as F and $((F, A)')' = (F, A)$.

Definition 2.5. [26] The difference of two soft sets (F, E) and (G, E) over the common universe X, denoted by $(F, E) - (G, E)$ is the soft set (H, E) where for all $e \in E$, $H(e) = F(e) - G(e)$.

Definition 2.6. [26] Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$ read as x belongs to the soft set (F, E) whenever $x \in F(e)$ for all $e \in E$.

Definition 2.7. [26] Let $x \in X$. Then the soft set (x, E) over X, where $x_E(e) = \{x\}\ \forall e \in E$, called the singleton soft point and denoted by x_E .

Definition 2.8. [20] A soft set (F, A) over X is said to be a NULL soft set denoted by $\tilde{\phi}$ or ϕ_A if for all $e \in A,\ F(e) = \phi$ (null set).

Definition 2.9. [20] A soft set (F, A) over X is said to be an absolute soft set denoted by $\tilde{A}$ or X_A if for all $e \in A,\ F(e) = X$. Clearly we have $X'_A = \phi_A$ and $\phi'_A = X_A$.

Definition 2.10. [20] The union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e),\ e \in A - B, \\ G(e),\ e \in B - A, \\ F(e) \cup G(e),\ e \in A \cap B \end{cases}.$$

Definition 2.11. [20] The intersection of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$. Note that, in order to efficiently discuss, we consider only soft sets (F, E) over a universe X in which all the parameter set E are same. We denote the family of these soft sets by $SS(X)_E$.

Definition 2.12. [29] Let I be an arbitrary indexed set and $L = \{(F_i, E), i \in I\}$ be a subfamily of $SS(X)_E$.

(1) The union of L is the soft set (H, E), where $H(e) = \bigcup_{i \in I} F_i(e)$ for each $e \in E$. We write $\tilde{\bigcup}_{i \in I}(F_i, E) = (H, E)$.

(2) The intersection of L is the soft set (M, E), where $M(e) = \bigcap_{i \in I} F_i(e)$ for each $e \in E$. We write $\tilde{\bigcap}_{i \in I}(F_i, E) = (M, E)$.

Definition 2.13. [26] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if

(1) $\tilde{X}, \tilde{\phi} \in \tau$, where $\tilde{\phi}(e) = \phi$ and $\tilde{X}(e) = X$, $\forall e \in E$,

(2) the union of any number of soft sets in τ belongs to τ,

(3) the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.14. [29] The soft set $(F, E) \in SS(X)_E$ is called a soft point in X_E if there exist $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \phi$ for each $e' \in E - \{e\}$, and the soft point (F, E) is denoted by x_e.

Definition 2.15. [29] The soft point x_e is said to be belonging to the soft set (G, A), denoted by $x_e \tilde{\in} (G, A)$, if for the element $e \in A$, $F(e) \subseteq G(e)$.

Definition 2.16. [1] Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets, $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a mapping. Then;

(1) If $(F, A) \in SS(X)_A$. Then the image of (F, A) under f_{pu}, written as $f_{pu}(F, A) = (f_{pu}(F), p(A))$, is a soft set in $SS(Y)_B$ such that
$$f_{pu}(F)(b) = \begin{cases} \cup_{a \in p^{-1}(b) \cap A} \ u(F(a)), & p^{-1}(b) \cap A \neq \phi, \\ \phi, & otherwise. \end{cases}$$
for all $b \in B$.

(2) If $(G, B) \in SS(Y)_B$. Then the inverse image of (G, B) under f_{pu}, written as $f_{pu}^{-1}(G, B) = (f_{pu}^{-1}(G), p^{-1}(B))$, is a soft set in $SS(X)_A$ such that
$$f_{pu}^{-1}(G)(a) = \begin{cases} u^{-1}(G(p(a))), & p(a) \in B, \\ \phi, & otherwise. \end{cases}$$
for all $a \in A$.

The soft function f_{pu} is called surjective if p and u are surjective, also is said to be injective if p and u are injective.

Definition 2.17 ([29])**.** Let (X, τ, A) and (Y, τ^*, B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then

(1) The function f_{pu} is called continuous soft (soft-cts) if $f_{pu}^{-1}(G, B) \in \tau \ \forall \ (G, B) \in \tau^*$.

(2) The function f_{pu} is called open soft if $f_{pu}(G, A) \in \tau^* \forall \ (G, A) \in \tau$.

Definition 2.18. [9] Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Then (F, E) is said to be,

(1) Pre open soft set if $(F, E) \tilde{\subseteq} int(cl(F, E))$,

(2) Semi open soft set if $(F, E) \tilde{\subseteq} cl(int(F, E))$,

(3) α-open soft set if $(F, E) \tilde{\subseteq} int(cl(int(F, E)))$,

(4) β-open soft set if $(F, E) \tilde{\subseteq} cl(int(cl(F, E)))$.

The set of all pre open (resp. semi open, α-open, β-open) soft sets is denoted by $POS(X)$ (resp. $SOS(X)$, $\alpha OS(X)$, $\beta OS(X)$) and the set of all pre closed (resp. semi closed, α-closed, β-closed) soft sets is denoted by $PCS(X)$ (resp. $SCS(X)$, $\alpha CS(X)$, $\beta CS(X)$).

Definition 2.19. [9] Let (X, τ, E) be a soft topological space, $(F, E) \in SS(X)_E$. Then, the pre soft interior (resp. semi soft interior, α-soft interior, β-soft interior) of (F, E) is denoted by $PSint(F, E)$ (resp. $SSint(F, E)$, $\alpha Sint(F, E)$, $\beta Sint(F, E)$), which is the soft union of all pre open (resp. semi open, α-open, β-open) soft sets contained in (F, E).

Definition 2.20. [9] Let (X, τ, E) be a soft topological space, $(F, E) \in SS(X)_E$. Then, the pre soft closure (resp. semi soft closure, α-soft closure, β-soft closure) of (F, E) is denoted by $PScl(F, E)$ (resp. $SScl(F, E)$, $\alpha Scl(F, E)$, $\beta Scl(F, E)$), which is the soft intersection of all pre closed (resp. semi closed, α-closed, β-closed) soft sets containing (F, E).

Theorem 2.1. [9] Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Then

(1) $(F, E) \in SOS(X)$ if and only if $cl(F, E) = cl(int(F, E))$.

(2) If $(G, E) \in OS(X)$. Then, $(G, E)\tilde{\cap} cl(F, E)\tilde{\subseteq} cl((F, E)\tilde{\cap}(F, E))$.

(3) If $(H, E) \in CS(X)$. Then, $int[(G, E)\tilde{\cup}(H, E)]\tilde{\subseteq} int(G, E)\tilde{\cup}(H, E)$.

Definition 2.21. [9] Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be a mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then, the function f_{pu} is called,

(1) Pre-continuous soft (Pre-cts soft) if $f_{pu}^{-1}(G, B) \in POS(X)$ $\forall\ (G, B) \in \tau_2$.

(2) α-continuous soft (α-cts soft) if $f_{pu}^{-1}(G, B) \in \alpha OS(X)$ $\forall\ (G, B) \in \tau_2$.

(3) Semi-continuous soft (semi-cts soft) if $f_{pu}^{-1}(G, B) \in SOS(X)$ $\forall\ (G, B) \in \tau_2$.

(4) β-continuous soft (β-cts soft) if $f_{pu}^{-1}(G, B) \in \beta OS(X)$ $\forall\ (G, B) \in \tau_2$.

3 b-open soft sets in soft topological spaces

Definition 3.1. Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Then (F, E) is called a b-open soft set if $(F, E)\tilde{\subseteq} cl(int(F, E))\tilde{\cup}$ $int(cl(F, E))$ and its complement is said to be b-closed soft set. The set of all b-open soft sets is denoted by $BOS(X, \tau, E)$, or $BOS(X)$ and the set of all b-closed soft sets is denoted by $BCS(X, \tau, E)$, or $BCS(X)$.

Theorem 3.1. Let (X, τ, E) be a soft topological space. Then

(1) Arbitrary soft union of b-open soft sets is b-open soft.

(2) Arbitrary soft intersection of b-closed soft sets is b-closed soft.

Proof.

(1) Let $\{F_{jE} : j \in J\} \subseteq BOS(X)$. Then, $\forall\ j \in J$, $F_{jE}\tilde{\subseteq}[int(cl(F_{jE}))]\tilde{\cup}$ $[cl(int(F_{jE}))]$. It follows that, $\tilde{\bigcup}_j F_{jE}\tilde{\subseteq}\tilde{\bigcup}_j[[int(cl(F_{jE}))]\tilde{\cup}[cl(int(F_{jE}))]]$

$$= \tilde{\bigcup}_j[int(cl(F_{jE}))]\tilde{\cup}[\tilde{\bigcup}_j[cl(int(F_{jE}))]]$$
$$\tilde{\subseteq} int(\tilde{\bigcup}_j cl(F_{jE}))\tilde{\cup}[cl(int[\tilde{\bigcup}_j(F_{jE})])]$$
$$= [int(cl(\tilde{\bigcup}_j F_{jE}))]\tilde{\cup}[[cl(int[\tilde{\bigcup}_j(F_{jE})])]].$$

Hence, $\tilde{\bigcup}_j F_{jE} \in BOS(X)\ \forall\ j \in J$.

(2) By a similar way.

Remark 3.1. A finite soft intersection of b-open soft sets need not to be b-open soft, as shown in the following example. Therefore, the family of all b-open soft sets may be fail to be soft topology.1

Example 3.1. Suppose that there are three computers in the universe X given by $X = \{h_1, h_2, h_3\}$. Let $E = \{e_1, e_2\}$ be the set of decision parameters which are stands for "expensive" and "beautiful" respectively.
Let (F_1, E) be a soft set over the common universe X, which describe the composition of the computers, where
$F(e_1) = \{h_1, h_3\}, \quad F(e_2) = \{h_2, h_3\}$.
Then $\tau = \{\tilde{X}, \tilde{\phi}, (F, E)\}$ defines a soft topology on X. Hence, the sets (G, E) and (H, E) which defined as follows:
$G(e_1) = \{h_1, h_2\}, \quad G(e_2) = \{h_1\}$,
$H(e_1) = \{h_2, h_3\}, \quad H(e_2) = \{h_1\}$,
are b-open soft sets of (X, τ, E), but their soft intersection $(G, E)\tilde{\cap}(H, E) = (M, E)$ where $M(e_1) = \{h_2\}, \quad M(e_2) = \{h_1\}$ is not b-open soft set.

Remark 3.2. Note that the family of all b-open soft sets on a soft topological space (X, τ, E) forms a supra soft topology, i.e τ contains $\tilde{X}, \tilde{\phi}$ and closed under arbitrary soft union.

Definition 3.2. Let (X, τ, E) be a soft topological space, $(F, E) \in SS(X)_E$ and $x_e \in SS(X)_E$. Then

(1) x_e is called a b-interior soft point of (F, E) if $\exists\ (G, E) \in BOS(X)$ such that $x_e \in (G, E)\tilde{\subseteq}(F, E)$, the set of all b-interior soft points of (F, E) is called the b-soft interior of (F, E) and is denoted by $bSint(F, E))$ consequently, $bSint(F, E)) = \tilde{\bigcup}\{(G, E) : (G, E)\tilde{\subseteq}(F, E),\ (G, E) \in BOS(X)\}$.

(2) x_e is called a b-closure soft point of (F, E) if $(F, E)\tilde{\cap}(H, E) \neq \tilde{\phi}\ \forall\ (H, E) \in BOS(X)$. The set of all b-closure soft points of (F, E) is called b-soft closure of (F, E) and is denoted by $bScl(F, E))$ consequently, $bScl(F, E)) = \tilde{\bigcap}\{(H, E) : (H, E) \in BCS(X),\ (F, E)\tilde{\subseteq}(H, E)\}$.

Theorem 3.2. Let (X, τ, E) be a soft topological space. Then, the following properties are satisfied for the b-soft interior operators, denoted by $(bSint)$.

(1) $bSint(\tilde{X}) = \tilde{X}$ and $bSint(\tilde{\phi}) = \tilde{\phi}$.

(2) $bSint(F, E)\tilde{\subseteq}(F, E)$.

(3) $bSint(F, E)$ is the largest b-open soft set contained in (F, E).

(4) if $(F, E)\tilde{\subseteq}(G, E)$, then $bSint(F, E)\tilde{\subseteq}bSint(G, E)$.

(5) $bSint(bSint(F, E)) = bSint(F, E)$.

(6) $bSint(F, E)\tilde{\cup}bSint(G, E)\tilde{\subseteq}bSint[(F, E)\tilde{\cup}(G, E)]$.

(7) $bSint[(F, E)\tilde{\cap}(G, E)]\tilde{\subseteq}bSint(F, E)\tilde{\cap}bSint(G, E)$.

Proof. Immediate.

Theorem 3.3. Let (X, τ, E) be a soft topological space. Then, the following properties are satisfied for the b-soft closure operators, denoted by $(bScl)$.

(1) $bScl(\tilde{X}) = \tilde{X}$ and $bScl(\tilde{\phi}) = \tilde{\phi}$.

(2) $(F, E)\tilde{\subseteq}bScl(F, E)$.

(3) $bScl(F, E)$ is the smallest b-closed soft set contains (F, E).

(4) if $(F, E)\tilde{\subseteq}(G, E)$, then $bScl(F, E)\tilde{\subseteq}bScl(G, E)$.

(5) $bScl(bScl(F, E)) = bScl(F, E)$.

(6) $bScl(F, E)\tilde{\cup}bScl(G, E)\tilde{\subseteq}bScl[(F, E)\tilde{\cup}(G, E)]$.

(7) $bScl[(F, E)\tilde{\cap}(G, E)]\tilde{\subseteq}bScl(F, E)\tilde{\cap}bScl(G, E)$.

Proof. Immediate.

Theorem 3.4. Let (X, τ, E) be a soft topological space. Then, the following properties are satisfied:

(1) $PScl(F,E) = (F,E)\tilde{\cup}cl(int(F,E))$.

(2) $PSint(F,E) = (F,E)\tilde{\cap}int(cl(F,E))$.

(3) $\alpha Scl(F,E) = (F,E)\tilde{\cup}cl(int(cl(F,E)))$.

(4) $\alpha Sint(F,E) = (F,E)\tilde{\cap}int(cl(int(F,E)))$.

(5) $SScl(F,E) = (F,E)\tilde{\cup}int(cl(F,E))$.

(6) $SSint(F,E) = (F,E)\tilde{\cap}cl(int(F,E))$.

(7) $\beta Scl(F,E) = (F,E)\tilde{\cup}int(cl(int(F,E)))$.

(8) $\beta Sint(F,E) = (F,E)\tilde{\cap}cl(int(cl(F,E)))$.

Proof. We shall prove only the first statement, the other cases are similar. Since $cl(int[(F,E)\tilde{\cup}cl(int(F,E))])\tilde{\subseteq}$ $cl[int(F,E)\tilde{\cup}cl(int(F,E))] = cl(int(F,E))\tilde{\cup}cl(int(F,E)) = cl(int(F,E))\tilde{\subseteq}(F,E)\tilde{\cup}cl(int(F,E))$ from Theorem 2.1 (3). This means that, $(F,E)\tilde{\cup}cl(int(F,E))$ is a pre closed soft set containing (F,E). So, $PScl(F,E)\tilde{\subseteq}(F,E)\tilde{\cup}cl(int(F,E))$. On the other hand, $PScl(F,E)$ is pre closed soft. So, we have $cl(int(F,E))\tilde{\subseteq}\, cl(int(PScl(F,E)))\tilde{\subseteq}PScl(F,E)$. Hence, $(F,E)\tilde{\cup}cl(int(F,E))\tilde{\subseteq}PScl(F,E)$. Therefore, $PScl(F,E) = (F,E)\tilde{\cup}cl(int(F,E))$. The rest of the proof by a similar way.

Theorem 3.5. Let (X, τ, E) be a soft topological space. Then, the following properties are satisfied:

(1) $PScl(PSint(F,E)) = PSint(F,E)\tilde{\cup}cl(int(F,E))$.

(2) $SScl(Sint(F,E)) = SSint(F,E)\tilde{\cup}cl(int(cl(F,E)))$.

Proof.

(1) Since $cl(int[PSint(F,E)\tilde{\cup}cl(int(F,E))])\tilde{\subseteq}cl[int(PSint(F,E))\tilde{\cup}cl(int(F,E))] = cl(int(PSint(F,E)))$ $\tilde{\cup}cl(int(F,E)) = cl(int(F,E))\tilde{\subseteq}PSint(F,E)\tilde{\cup}cl(int(F,E))$ from Theorem 2.1 (3). This means that, $PSint(F,E)\tilde{\cup}cl(int(F,E))$ is a pre closed soft set containing $PSint(F,E)$. So, $PScl(PSint(F,E))$ $\tilde{\subseteq}PSint(F,E)\tilde{\cup}cl(int(F,E))$. On the other hand, $PScl(PSint(F,E))$ is the largest pre closed soft set containing $PSint(F,E)$. Hence, $PSint(F,E)\tilde{\cup}cl(int(F,E))\tilde{\subseteq}PScl(PSint(F,E))$. Therefore, $PScl(PSint(F,E)) = PSint(F,E)\tilde{\cup}cl(int(F,E))$.

(2) By a similar way.

Theorem 3.6. Let (X, τ, E) be a soft topological space. Then, the following are equivalent:

(1) (F,E) is a b-open soft set.

(2) $(F,E) = PSint(F,E)\tilde{\cup}SSint(F,E)$.

(3) $(F,E)\tilde{\subseteq}PScl(PSint(F,E))$.

Proof.

(1) $\Rightarrow$ (2) Let (F,E) be a b-open soft set. Then, $(F,E)\tilde{\subseteq}cl(int(F,E))$ $\tilde{\cup}int(cl(F,E))$. By Theorem 3.4, $PSint(F,E)\tilde{\cup}SSint(F,E) = [(F,E)\tilde{\cap}int(cl(F,E))]\tilde{\cup}[(F,E)\tilde{\cap}cl(int(F,E))] =$ $(F,E)\tilde{\cap}[int(cl(F,E))\tilde{\cup}cl(int(F,E))] = (F,E)$.

(2) $\Rightarrow$ (3) $(F,E) = PSint(F,E)\tilde{\cup}SSint(F,E) = PSint(F,E)\tilde{\cup}[(F,E)\tilde{\cap}cl(int(F,E))]\tilde{\subseteq}PSint(F,E)\tilde{\cup}cl(int(F,E)) =$ $PScl(PSint(F,E))$, from Theorem 3.4 (6) and Theorem 3.5 (1).

(3) $\Rightarrow$ (1) $(F,E)\tilde{\subseteq}PScl(PSint(F,E)) = PSint(F,E)\tilde{\cup}cl(int(F,E))\tilde{\subseteq}int(cl(F,E))\tilde{\cup}cl(int(F,E))$, from Theorem 3.4 (1) and Theorem 3.5 (1).

4 Relations between b-open soft sets and some types of open soft sets of soft topological spaces

In this section, we introduce the relations between b-open soft sets and some special subsets of a soft topological space (X, τ, E) mentioned in [9].

Theorem 4.1. In a soft topological space (X, τ, E), the following statements hold,

(1) Every open (resp. closed) soft set is b-open (resp. b-closed) soft.

(2) Every pre open (resp. pre closed) soft set is b-open (resp. b-closed) soft.

(3) Every semi open (resp. semi closed) soft set is b-open (resp. b-closed) soft.

(4) Every b-open (resp. b-closed) soft set is β-open (resp. β-closed) soft.

Proof. We prove the assertion in the case of b-open soft set in (4), the other case is clear. Let $(F, E) \in BOS(X)$. Then, $(F, E)\tilde{\subseteq} int(cl(F, E))\tilde{\cup} cl(int(F, E))$

$$\begin{aligned}
&\tilde{\subseteq} cl(int(cl(F, E)))\tilde{\cup} cl(int(F, E)) \\
&= cl[int(cl(F, E))\tilde{\cup} int(F, E)] \\
&\tilde{\subseteq} cl(int[cl(F, E)\tilde{\cup} int(F, E)]) \\
&= cl(int(cl[(F, E)])).
\end{aligned}$$

Therefore, $(F, E) \in \beta OS(X)$.

Remark 4.1. It is obvious that $POS(X) \cup SOS(X) \subseteq BOS(X) \subseteq \beta OS(X)$. The following examples shall show that theses implications can not be reversed and the converse of Theorem 4.1 is not true in general.

Examples 4.1. (1) In Example 3.1, the soft set (G, E) is b-open soft set, but it is not open soft.

(2) Suppose that there are four alternatives in the universe of dresses $X = \{h_1, h_2, h_3, h_4\}$ and consider $E = \{e_1(cotton), e_2(woollen)\}$ be the set of parameters showing the material of the dresses.Let $(F_1, E), (F_2, E), (F_3, E), (F_4, E)$ be four soft sets over the common universe X which describe the goodness of the dresses, where
$F_1(e_1) = \{h_1\}, \quad F_1(e_2) = \{h_2\}$,
$F_2(e_1) = \{h_2\}, \quad F_2(e_2) = \{h_1\}$,
$F_3(e_1) = \{h_1, h_2\}, \quad F_3(e_2) = \{h_1, h_2\}$,
$F_4(e_1) = \{h_1, h_2, h_4\}, \quad F_4(e_2) = \{h_1, h_2, h_4\}$.
Then $\tau = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$ defines a soft topology on X. Hence, the soft set (G, E) which defined by;
$G(e_1) = \{h_1, h_4\}, \quad G(e_2) = \{h_2, h_4\}$ is b-open soft set, but it is not pre open soft.

(3) In Example 3.1, the soft set (H, E) is b-open soft set, but it is not semi open soft.

(4) Suppose that there are four alternatives in the universe of houses $X = \{h_1, h_2, h_3, h_4\}$ and consider $E = \{e_1, e_2\}$ be two parameter "quality of houses" and "wooden" to be the linguistic variable. Let $(F_1, E), (F_2, E), (F_3, E)$ be three soft sets over the common universe X which describe the goodness of the houses, where
$F_1(e_1) = \{h_4\}, \quad F_1(e_2) = \{h_1, h_2\}$,
$F_2(e_1) = \{h_1, h_2\}, \quad F_2(e_2) = \{h_4\}$,
$F_3(e_1) = \{h_1, h_2, h_4\}, \quad F_3(e_2) = \{h_1, h_2, h_4\}$.
Then $\tau = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$ defines a soft topology on X. Hence, the soft set (G, E) which defined by;
$G(e_1) = \{h_1\}, \quad G(e_2) = \{h_3\}$ is β-open soft set of (X, τ, E), but it is not b-open soft.

Corollary 4.1. For a soft topological space (X, τ, E) we have:

$$\begin{array}{ccccccc} OS(X) & \longrightarrow & & POS(X) & & & \\ \downarrow & & & \downarrow & & & \\ \alpha OS(X) & \longrightarrow & SOS(X) & \longrightarrow & BOS(X) & \longrightarrow & \beta OS(X) \end{array}$$

Proof. It is follows from Theorem 4.1 and [[9], Remark 4.2].

Theorem 4.2. Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Then the following hold:

(1) $bSint(F^c, E) = \tilde{X} - bScl(F, E)$.

(2) $bScl(F^c, E) = \tilde{X} - bSint(F, E))$.

Proof.

(1) $\tilde{X} - bScl(F, E) = [\tilde{\bigcap}\{(G, E) : (F, E)\tilde{\subseteq}(G, E),\ (G, E) \in BCS(X)\}]^c = \tilde{\bigcup}\{(G^c, E) : (G^c, E)\tilde{\subseteq}(F^c, E),\ (G^c, E) \in BOS(X)\} = bSint(F^c, E)$.

(2) $\tilde{X} - bSint(F, E) = [\tilde{\bigcup}\{(G, E) : (G, E)\tilde{\subseteq}(F, E),\ (G, E) \in BOS(X)\}]^c = \tilde{\bigcap}\{(G^c, E) : (F^c, E)\tilde{\subseteq}(G^c, E),\ (G^c, E) \in BCS(X)\} = bScl(F^c, E)$.

Theorem 4.3. Let (X, τ, E) be a soft topological space and $(G, E) \in BOS(X)$.

(1) If $(F, E) \in OS(X)$. Then, $F_E \tilde{\cap} G_E \in BOS(X)$.

(2) If $(F, E) \in \alpha OS(X)$. Then, $F_E \tilde{\cap} G_E \in BOS(X)$.

Proof.

(1) Let $(F, E) \in OS(X)$ and $(G, E) \in BOS(X)$. Then,
$(F, E)\tilde{\cap}(G, E)\tilde{\subseteq}int(F, E)\tilde{\cap}[cl(int(G, E))\tilde{\cup}int(cl(G, E))]$
$= [int(F, E)\tilde{\cap}cl(int(G, E))]\tilde{\cup}[int(F, E)\tilde{\cap}int(cl(G, E))]$
$\tilde{\subseteq}cl[int(F, E)\tilde{\cap}int(G, E)]\tilde{\cup}int[int(F, E)\tilde{\cap}cl(G, E)]$
$\tilde{\subseteq}cl(int[(F, E)\tilde{\cap}(G, E)])\tilde{\cup}\ int(cl[(F, E)\tilde{\cap}(G, E)])$.
from Theorem 2.1 (2). Therefore, $F_E \tilde{\cap} G_E \in BOS(X)$.

(2) Let $(F, E) \in \alpha OS(X)$ and $(G, E) \in BOS(X)$. Then,
$(F, E)\tilde{\cap}(G, E)\tilde{\subseteq}int(cl(int(F, E)))\tilde{\cap}[cl(int(G, E))\tilde{\cup}int(cl(G, E))]$
$= [int(cl(int(F, E)))\tilde{\cap}cl(int(G, E))]\tilde{\cup}[int(cl(int(F, E)))\tilde{\cap}int(cl(G, E))]$
$\tilde{\subseteq}cl[int(cl(int(F, E)))\tilde{\cap}int(G, E)]\tilde{\cup}int[cl(int(F, E))\tilde{\cap}int(cl(G, E))]$
$\tilde{\subseteq}cl(int[cl(int(F, E))\tilde{\cap}int(G, E)])\tilde{\cup}int(cl[int(F, E)\tilde{\cap}int(cl(G, E))])$
$\tilde{\subseteq}cl(int(cl[int(F, E)\tilde{\cap}int(G, E)]))\tilde{\cup}int(cl(int[int(F, E)\tilde{\cap}cl(G, E)]))$
$\tilde{\subseteq}cl(int(cl(int[(F, E)\tilde{\cap}(G, E)])))\tilde{\cup}int(cl(int(cl[(F, E)\tilde{\cap}(G, E)])))$
$\tilde{\subseteq}cl(int[(F, E)\tilde{\cap}(G, E)])\tilde{\cup}int(cl[(F, E)\tilde{\cap}(G, E)])$.
from Theorem 2.1 (2). Therefore, $F_E \tilde{\cap} G_E \in BOS(X)$.

Proposition 4.1. Let (X, τ, E) be a soft topological space and $(F, E) \in BOS(X)$.

(1) If $int(F, E) = \tilde{\phi}$, then (F, E) is a pre open soft set.

(2) If $cl(F, E) = \tilde{\phi}$, then (F, E) is a semi open soft set.

Proof. Obvious.

Proposition 4.2. Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Then $(F, E) \in BCS(X)$ if and only if $cl(int(F, E))\tilde{\cap}int(cl(F, E))\tilde{\subseteq}(F, E)$.

Proof. Obvious.

Theorem 4.4. Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Then, the following properties are satisfied:

(1) $bScl(F, E) = Scl(F, E)\tilde{\cap}PScl(F, E)$.

(2) $bSint(F, E) = Sint(F, E)\tilde{\cup}PSint(F, E)$.

Proof.

(1) Since $bScl(F, E)$ is a b-closed soft set. Then, $cl(int(bScl(F, E)))\tilde{\cap}int(cl(bScl(F, E)))\tilde{\subseteq}bScl(F, E)$. It follows that, $cl(int(F, E))\tilde{\cap}int(cl(F, E))\tilde{\subseteq}bScl(F, E)$. So, $(F, E)\tilde{\cup}[cl(int(F, E))\tilde{\cap}int(cl(F, E))]$ $\tilde{\subseteq}(F, E)\tilde{\cup}bScl(F, E) = bScl(F, E)$. Hence, $[(F, E)\tilde{\cup}cl(int(F, E))]\tilde{\cap}[(F, E)\tilde{\cup}int(cl(F, E))]$ $= Scl(F, E)\tilde{\cap}PScl(F, E)$, from Theorem 3.4. This means that, $Scl(F, E)\tilde{\cap}PScl(F, E)\tilde{\subseteq}bScl(F, E)$. The reverse inclusion is obvious from Remark 4.1.

(2) By a similar way.

5 b-soft continuity

Definition 5.1. Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then, the function f_{pu} is called a b-continuous soft (b-cts soft) if $f^{-1}_{pu}(G, B) \in BOS(X) \forall\ (G, B) \in \tau_2$.

Theorem 5.1. Let (X, τ, A) and (Y, τ^*, B) be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function.Then, the following are equivalent:

(1) f_{pu} is b-continuous soft function.

(2) $f^{-1}_{pu}(H, B) \in BCS(X)\ \forall\ (H, B) \in CS(Y)$.

(3) $f_{pu}(bScl(G, A) \subseteq cl_{\tau^*}(f_{pu}(G, A))\ \forall\ (G, A) \in SS(X)_A$.

(4) $bScl(f^{-1}_{pu}(H, B)) \subseteq f^{-1}_{pu}(cl_{\tau^*}(H, B))\ \forall\ (H, B) \in SS(Y)_B$.

(5) $f^{-1}_{pu}(int_{\tau^*}(H, B)) \subseteq bSint(f^{-1}_{pu}(H, B))\ \forall\ (H, B) \in SS(Y)_B$.

Proof.

(1) $\Rightarrow$ (2) Let (H, B) be a closed soft set over Y. Then, $(H, B)' \in OS(Y)$ and $f^{-1}_{pu}(H, B)' \in BOS(X)$ from (1). Since $f^{-1}_{pu}(H, B)' = (f^{-1}_{pu}(H, B))'$ from [[29], Theorem 3.14]. Thus, $f^{-1}_{pu}(H, B) \in BCS(X)$.

(2) $\Rightarrow$ (3) Let $(G, A) \in SS(X)_A$. Since $(G, A)\tilde{\subseteq}f^{-1}_{pu}(f_{pu}(G, A))\tilde{\subseteq}f^{-1}_{pu}(cl_{\tau^*}(f_{pu}(G, A))) \in BCS(X)$ from (2) and [[29], Theorem 3.14]. Then $(G, A)\tilde{\subseteq}bScl(G, A)\tilde{\subseteq}f^{-1}_{pu}(cl_{\tau^*}(f_{pu}(G, A)))$. Hence, $f_{pu}(bScl(G, A))\tilde{\subseteq}f_{pu}(f^{-1}_{pu}(cl_{\tau^*}(f_{pu}(G, A))))\tilde{\subseteq}cl_{\tau^*}(f_{pu}(G, A)))$ from [[29], Theorem 3.14]. Thus, $f_{pu}(bScl(G, A))\tilde{\subseteq}cl_{\tau^*}(f_{pu}(G, A))$.

(3) $\Rightarrow$ (4) Let $(H, B) \in SS(Y)_B$ and $(G, A) = f^{-1}_{pu}(H, B)$. Then $f_{pu}(bSclf^{-1}_{pu}(H, B))\tilde{\subseteq}cl_{\tau^*}(f_{pu}(f^{-1}_{pu}(H, B)))$ From (3). Hence, $bScl(f^{-1}_{pu}\ (H, B))\tilde{\subseteq}f^{-1}_{pu}(f_{pu}(bScl(f^{-1}_{pu}(H, B))))\tilde{\subseteq}f^{-1}_{pu}(cl_{\tau^*}(f_{pu}(f^{-1}_{pu}(H, B))))\tilde{\subseteq}f^{-1}_{pu}(cl_{\tau^*}(H, B))$ from [[29], Theorem 3.14]. Thus, $bScl(f^{-1}_{pu}(H, B))\tilde{\subseteq}f^{-1}_{pu}(cl_{\tau^*}(H, B))$.

(4) $\Rightarrow$ (2) Let (H, B) be a closed soft set over Y. Then $bScl(f^{-1}_{pu}(H, B))\tilde{\subseteq}f^{-1}_{pu}(cl_{\tau^*}(H, B)) = f^{-1}_{pu}(H, B)\ \forall\ (H, B) \in SS(Y)_B$ from (4). But clearly, $f^{-1}_{pu}(H, B)\tilde{\subseteq}bScl(f^{-1}_{pu}(H, B))$. This means that, $f^{-1}_{pu}(H, B) = bScl(f^{-1}_{pu}(H, B))$, and consequently $f^{-1}_{pu}(H, B) \in BCS(X)$.

(1) $\Rightarrow$ (5) Let $(H, B) \in SS(Y)_B$. Then, $f^{-1}_{pu}(int_{\tau^*}(H, B)) \in BOS(X)$ from (1). Hence, $f^{-1}_{pu}(int_{\tau^*}(H, B)) = bSint(f^{-1}_{pu}int_{\tau^*}(H, B))\tilde{\subseteq}bSint(f^{-1}_{pu}(H, B))$. Thus, $f^{-1}_{pu}(int_{\tau^*}(H, B))\tilde{\subseteq}bSint(f^{-1}_{pu}(H, B))$.

(5) $\Rightarrow$ (1) Let (H, B) be an open soft set over Y. Then $int_{\tau^*}(H, B) = (H, B)$ and $f^{-1}_{pu}(int_{\tau^*}(H, B)) = f^{-1}_{pu}((H, B))\tilde{\subseteq}bSint(f^{-1}_{pu}(H, B))$ from (5). But, we have $bSint(f^{-1}_{pu}(H, B))\tilde{\subseteq}f^{-1}_{pu}(H, B)$. This means that, $bSint(f^{-1}_{pu}(H, B)) = f^{-1}_{pu}(H, B) \in BOS(X)$. Thus, f_{pu} is continuous soft function.

Theorem 5.2. Let (X, τ, A), (Y, τ^*, B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then

(1) Every continuous soft function is b-continuous soft function.

(2) Every pre-continuous soft function is b-continuous soft function.

(3) Every semi-continuous soft function is b-continuous soft function.

(4) Every b-continuous soft function is β-continuous soft function.

Proof. Immediate from Theorem 4.1.

On accounting of Theorem 5.2 and [[9], Corollary 5.1], we have the following corollary.

Corollary 5.1. For a soft topological space (X, τ, E) we have the following implications.

$$\begin{array}{ccccccc} & cts\ \ soft & \longrightarrow & Pre - cts\ soft & & & \\ & \swarrow & & \downarrow & & & \\ \alpha - cts\ soft & \longrightarrow & semi - cts\ soft & \longrightarrow & b - cts\ soft & \longrightarrow & \beta - cts\ soft \end{array}$$

6 Conclusion

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied and improved the soft set theory, which is initiated by Molodtsov [22] and easily applied to many problems having uncertainties from social life. In this paper, a new class of open soft sets in a soft topological space, called b-open soft sets, is introduced and studied. Moreover, the relations this class and these different types of subsets of soft topological spaces ,which introduced in [9], is studied. In particular, this class is contained in the class of β-open soft sets and contains the classes of open soft sets, pre open soft sets, semi open soft sets and α-open soft sets. Also, the authors introduce the concept of b-continuous soft functions and study some of their properties in detail. As a consequence the relations of some soft continuities are shown in a diagram. In the next study, we extend the notion of b-open soft sets to supra soft topological spaces and and other topological properties. Also, we will use some topological tools in soft set application, like rough sets.

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